The Smallest Sets of Points not Determined by Their X-rays

Abstract:
Let $F$ be an $n$-point set in $K_d$ with $K \in \{\mathbb{R}, \mathbb{Z}\}$ and $d \geq 2$. A (discrete) X-ray of $F$ in direction $s$ gives the number of points of $F$ on each line parallel to $s$. We define $y_{K_d}(m)$ as the minimum number $n$ for which there exist $m$ directions $s_1, \ldots, s_m$ (pairwise linearly independent and spanning $\mathbb{R}^d$) such that two $n$-point sets in $K_d$ exist that have the same X-rays in these directions. The bound $y_{Z^2}(m) \leq 2m - 1$ has been observed many times in the literature. In this note we show $y_{K_d}(m) = O(md + 1 + \varepsilon)$ for $\varepsilon > 0$. For the cases $K_d = Z^d$ and $K_d = R^d$, $d > 2$, this represents the first upper bound on $y_{K_d}(m)$ that is polynomial in $m$. As a corollary we derive bounds on the sizes of solutions to both the classical and two-dimensional Prouhet-Tarry-Escott problem. Additionally, we establish lower bounds on $y_{K_d}$ that enable us to prove a strengthened version of Rényi’s theorem for points in $Z^2$.

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