Modeling of the lossy Josephson parametric amplifier

Recent advances in superconducting nanoelectronic circuits exhibit the potential for future electronic applications. Experimental and theoretical research on superconducting quantum circuits based on the Josephson effect have recently gained more and more attention due to developments in quantum information theory and quantum metrology (Russer and Russer, 2011). The Josephson effect is observed, if two superconducting materials are weakly coupled across a tunnel barrier or a narrow conducting bridge. Electronic devices based on the Josephson effect allow the generation, detection, mixing and
parametric amplification of high-frequency signals. Moreover, Josephson devices are nanometre-sized low power devices and show high sensitivity. So far, Josephson parametric amplifiers have been treated classically as well as quantum mechanically. The existing quantum mechanical models do not include loss contributions (Russer and Russer, 2012). In this work the lossy negative-resistance Josephson parametric amplifier is analyzed. Losses are considered using the quantum Langevin method. Hereby, the resonator circuits, i.e. the signal and the idler circuit, are coupled to a heat bath, represented by a photon gas in thermal equilibrium. In this way, also temperature dependencies are taken into account. In our model, the heat bath consists of harmonic oscillators, inducing fluctuations in the resonators and causing damping of the signal energy (Jirauschek and Russer, 2012). The DC biased Josephson junction is causing a strong interaction between the idler and the signal modes. Thus, power exchange occurs between the signal and the idler mode, as well as between the DC power supply and the resonator circuits. The energy conversion in the DC-pumped Josephson parametric amplifier follows the Manley-Rowe equations, with an additional DC term (Russer, 1971). The time evolution of the signal energy and the noise contributions are derived based on the Heisenberg equations of motion. The rotating wave approximation is applied in order to simplify the Josephson coupling Hamiltonian. We assume the heat bath noise contributions to follow the statistics of a Markovian process. We have explicitly computed the time evolution of the various energy contributions for different initial settings. The signal photon number in thermal equilibrium is determined by the photon occupation number of the heat bath, given by Boltzmann statistics, and the damping constants. Our analytic results show, that this model is capable of including thermal noise into the description of the Josephson parametric amplifier. For low damping constants, the signal energy is amplified exponentially as shown in previous publications without loss considerations (Russer and Russer, 2012).