We contribute to the understanding of how systemic risk arises in a network of credit-interlinked agents. Motivated by empirical studies we formulate a network model which, despite its simplicity, depicts the nature of interbank markets better than a symmetric model. The components of a vector Ornstein-Uhlenbeck process living on the nodes of the network describe the financial robustnesses of the agents. For this system, we prove a LLN for growing network size leading to a propagation of chaos result. We state properties, which arise from such a structure, and examine the effect of asymmetry on several risk management issues and the possibility of contagion.