This paper presents an improved mixed-integer model for the Thermal Unit Commitment Problem. By introducing new variables for the temperature of each thermal unit, the off-time-dependent start-up costs are modeled accurately while using fewer inequalities than state-of-the-art formulations. This new approach significantly improves computational efficiency, even compared to existing formulations which only roughly approximate start-up costs. Our findings were validated on real-world test cases using CPLEX and Xpress.