This paper presents a fully automated high-order hexahedral mesh generation algorithm for shell-like structures based on enhanced sweeping methods. Traditional sweeping techniques create all-hexahedral element meshes for solid structures by projecting an initial single surface mesh along a specified trajectory to a specified target surface. The work reported here enhances the traditional method for thin solids by creating conforming all-hexahedral finite element meshes on an enhanced surface model with surfaces intersecting in parallel, perpendicular and skew-angled directions. The new algorithm is based on cheap projection rules separating the original surface model into a set of disjoint single surfaces and a so-called interface skeleton. The core of this process is reshaping the boundary representations of the initial surfaces, generating new sweeping templates along the intersection curves and joining the single swept hex meshes in an independently generated interface mesh.