A classification of Lévy processes via their symbols and its application to Finance

Abstract:
We classify Lévy processes according to the solution spaces of the associated parabolic PIDEs. This classification reveals structural characteristics of the processes and is relevant for applications such as for solving PIDEs numerically for pricing options in Lévy models. The classification is done via the Fourier transform i.e. via the symbol of the process. We define the Sobolev index of a Lévy process by a certain growth condition on the symbol. It follows that for Lévy processes with Sobolev index alpha the corresponding evolution problem has a unique weak solution in the Sobolev-Slobodeckii space $H^{\alpha/2}$. We show that this classification applies to a wide range of processes. Examples are the Brownian motion with or without drift, generalised hyperbolic (GH), CGMY and (semi) stable Lévy processes. A comparison of the Sobolev index with the Blumenthal-Getoor index sheds light on the structural implication of the classification. More precisely, we discuss the Sobolev index as an indicator of the smoothness of the distribution and of the variation of the paths of the process. This highlights the relation between the p-variation of the paths and the degree of smoothing effect that stems from the distribution.

Stichworte:
Lévy processes, PIDEs, weak solutions, parabolic evolution equation, Sobolev-Slobodeckii-spaces, pseudo differential operator, option pricing.

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