HOLMES: convergent meshfree approximation schemes of arbitrary order and smoothness

Abstract:
Local Maximum-Entropy (LME) approximation schemes are meshfree approximation schemes that satisfy consistency conditions of order one, i.e., they approximate affine functions exactly. In addition, LME approximation schemes converge in the Sobolev space $W^{1,p}$, i.e., they are $C^0$-continuous in the conventional terminology of finite-element interpolation.

Here we present a generalization of the Local Max-Ent approximation schemes that are consistent to arbitrary order, i.e., interpolate polynomials of arbitrary degree exactly, and which converge in $W^{k,p}$, i.e., they are $C^k$-continuous to arbitrary order $k$. We refer to these approximation schemes as High Order Local Maximum-Entropy Approximation Schemes (HOLMES). We prove uniform error bounds for the HOLMES approximates and their derivatives up to order $k$.

Moreover, we show that the HOLMES of order $k$ is dense in the Sobolev Space $W^{k,p}$, for any $1 \leq p < \infty$. The good performance of HOLMES relative to other meshfree schemes in selected test cases is also critically appraised.