We study some descriptional complexity aspects of regular expressions. In particular, we give lower bounds on the minimum required size for the conversion of deterministic finite automata into regular expressions and on the required size of regular expressions resulting from applying some basic language operations on them, namely intersection, shuffle, and complement. Some of the lower bounds obtained are asymptotically tight, and, notably, the examples we present are over an alphabet of size two. To this end, we develop a new lower bound technique that is based on the star height of regular languages. It is known that for a restricted class of regular languages, the star height can be determined from the digraph underlying the transition structure of the minimal finite automaton accepting that language. In this way, star height is tied to cycle rank, a structural complexity measure for digraphs proposed by Eggan and Buchi, which measures the degree of connectivity of directed graphs. This measure, although a robust graph theoretic notion, appears to have remained widely unnoticed outside formal language theory. We establish some basic theory for cycle rank and put it into context with other digraph connectivity measures proposed recently.