Technische Berichte

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Titel des Berichts: Inequalities for the Number of Walks in Trees and General Graphs and a Generalization of a Theorem of Erdös and Simonovits
Abstract: We investigate the growth of the number \( w_k \) of walks of length \( k \) in undirected graphs as well as related inequalities. We provide an insight into the relations between certain combinatorial properties of graphs and (spectral) algebraical characterizations in terms of eigenvalues and eigenvectors of the adjacency matrix. We show that the inequality \( w_2aw_2(a+b+c) \geq w_2a+cw_2(a+b)+c \) is valid for all graphs, which is a generalization of the inequality \( w_2aw_2b \geq wa+b^2 \) published by Dress and Gutman. We also show a similar sandwich theorem for the number of closed walks starting at a given vertex. Further, we prove that \( w_2l+pkw_2lk-1 \geq w_2l+pk \) for all \( k,l,p \in \mathbb{N} \), which is another generalization of the inequality by Dress and Gutman and at the same time also a generalization of an inequality published by Erdös and Simonovits. Both results can be translated directly into the corresponding forms using the higher order densities instead of the number of walks. Furthermore, we provide two families of lower bounds for the spectral radius of the adjacency matrix (in terms of the number of closed walks starting at a specified vertex) that generalize a bound published by Nosal. We also show monotonicity, i.e., the method yields better bounds with increasing walk lengths. In the last part of the paper, we investigate several special cases w.r.t. the graph class as well as regarding the inequality. We show, that there are connected bipartite graphs as well as unconnected cycle-free graphs that...
violate the inequality $w_3 \geq dw_2$. In contrast, we show that surprisingly this inequality is always satisfied for trees and we show how to construct worst-case instances (w.r.t. the difference of both sides of the inequality) for a given degree sequence. We also provide a proof for the inequality $w_5 \geq dw_4$ for trees and conclude with a corresponding conjecture for longer walks.

**Stichworte:**

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