Estimation for non-negative Lévy driven Ornstein-Uhlenbeck processes

Continuous time autoregressive moving average (CARMA) processes with a non-negative kernel and driven by a non-decreasing Lévy process constitute a very general class of stationary non-negative continuous-time processes. In financial econometrics a stationary Ornstein-Uhlenbeck (or CAR(1)) process, driven by a non-decreasing Lévy process, was introduced by Barndorff-Nielsen and Shephard (2001) as a model for stochastic volatility to allow for a wide variety of possible marginal distributions and the possibility of jumps. For such processes we take advantage of the non-negativity of the increments of the driving Lévy process to study the properties of a highly efficient estimation procedure for the parameters when observations are available at uniformly spaced times 0, h, ..., Nh. We also show how to reconstruct the background driving Lévy process from a continuously observed realization of the process and use this result to estimate the increments of the Lévy process itself when h is small. Asymptotic properties of the coefficient estimator are derived and the results illustrated using a simulated gamma-driven Ornstein-Uhlenbeck process.