QoS Feasibility for the MIMO Broadcast Channel: Robust Formulation and Multi-Carrier Systems

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***Abstract*—Based on recent results for the multiple-input multiple-output (MIMO) broadcast channel (BC), the feasibility of the quality of service (QoS) requirements of multi-carrier systems is investigated. Allowing for a precoding operation with arbitrary structure, the feasibility test for multi-carrier systems is analogous to that for MIMO systems with linear transceivers. However, if the precoder is restricted to be applied separately to the different carriers, additional conditions must be fulfilled for feasibility. After stating these conditions, we propose a simple test which is sufficient, but not necessary for feasibility. For a robust QoS formulation that is appropriate for erroneous channel state information, the feasibility results for the MIMO BC can be reused as long as all possible channel realizations fulfill a regularity condition.**

I. Introduction

When trying to satisfy the needs of the receivers with the least effort, the appropriate problem formulation is the QoS optimization, where the transmit power is minimized under the constraint that at least the required data rates are given to the users. Such a QoS formulation was considered in [1], [2], [3], [4], [5], [6], [7], [8] for example. Although the QoS optimization is interesting from an operator's point of view, one of its drawbacks is that it might be infeasible for linear transceivers, i.e., no solution might exist, if the requirements are too high.

If the number of degrees of freedom at the transmitter, e.g., the number of transmit antennas for single-carrier systems, is larger than or equal to the number of users and the channels fulfill a regularity condition, it is possible to invert the system by a zero-forcing precoder. The QoS requirements can then be fulfilled via power allocation. Thus, the possibility of infeasibility arises only, when the number of degrees of freedom at the transmitter in the MIMO BC is smaller than the number of users.

In [9], the feasibility region for the vector BC with linear transceivers was given. Based on this result, it was possible to identify the structure of the feasibility region together with feasibility conditions also for the MIMO BC in [10].

In this paper, we generalize the results of [10] on feasibility to multi-carrier systems and discuss feasibility for the robust QoS optimization, where the channel is known up to an uncertainty.

II. System Model and Problem Formulation

In the MIMO BC, the transmitted signal $\boldsymbol{y} \in \mathbb{C}^N$ is formed by the superposition of the precoded data signals, i.e.,

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{P}_k \boldsymbol{s}_k$$

with the linear precoder $\boldsymbol{P}_k \in \mathbb{C}^{N \times S_k}$ and the data $\boldsymbol{s}_k \in \mathbb{C}^{S_k}$ of the k-th of K users comprising S_k scalar data streams. After the transmission over the channel $\boldsymbol{G}_k \in \mathbb{C}^{M_k \times N}$ to user k, the received signal is perturbed by the noise $\boldsymbol{\eta}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{C}_{\boldsymbol{\eta}_k})$:

$$\boldsymbol{x}_k = \boldsymbol{G}_k \boldsymbol{y} + \boldsymbol{\eta}_k.$$

Assuming Gaussian codebooks, i.e., $\boldsymbol{s}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ $\forall k$, the resulting data rate for receiver k reads as

$$R_k^{\text{BC}} = \log_2 \left| \mathbf{I} + \left(\boldsymbol{C}_{\boldsymbol{\eta}_k} + \sum_{i \neq k} \boldsymbol{G}_k \boldsymbol{P}_i \boldsymbol{P}_i^{\text{H}} \boldsymbol{G}_k^{\text{H}} \right)^{-1} \boldsymbol{G}_k \boldsymbol{P}_k \boldsymbol{P}_k^{\text{H}} \boldsymbol{G}_k^{\text{H}} \right|.$$

The goal of ensuring given data rate requirements $\rho_k \geq 0$, $\forall k$, with the least possible transmit power $P_{\text{tx}}^{\text{BC}} = \sum_{k=1}^{K} \|\boldsymbol{P}_k\|_{\text{F}}^2$ leads to the QoS optimization

$$\min_{\boldsymbol{P}_1,\ldots,\boldsymbol{P}_K} P_{\text{tx}}^{\text{BC}} \quad \text{s.t.:} \quad R_k^{\text{BC}} \geq \rho_k \quad \forall k \in \{1,\ldots,K\} \tag{1}$$

where $\| \bullet \|_{\text{F}}$ denotes the Frobenius norm. Note that the rate R_k^{BC} is non-concave in the precoders of the other users $\boldsymbol{P}_i$, $i \neq k$. This non-convexity of the constraints also holds, if the transmit power is minimized w.r.t. the covariance matrices $\boldsymbol{B}_k = \boldsymbol{P}_k \boldsymbol{P}_k^{\text{H}}$, $\forall k$, instead. Therefore, (1) is difficult to solve in general. Additionally, (1) might have no solution, that is, the constraints might be infeasible. A simple example is the two user scalar BC, where the rate can be expressed as $R_k^{\text{BC}} = \log_2(1 + \text{SINR}_k)$ with

$$\text{SINR}_k = \frac{|g_k|^2 |p_k|^2}{c_{\eta_k} + |g_k|^2 |p_{3-k}|^2} \quad k \in \{1, 2\}.$$

Here, g_k, p_k, and c_{η_k} are the scalar channel, the scalar precoder, and the noise variance for user k, respectively. For the product of the SINRs, we get

$$\text{SINR}_1 \text{SINR}_2 = 1 - \frac{c_{\eta_1} c_{\eta_2} + c_{\eta_1} |g_2|^2 |p_1|^2 + c_{\eta_2} |g_1|^2 |p_2|^2}{(c_{\eta_1} + |g_1|^2 |p_2|^2)(c_{\eta_2} + |g_2|^2 |p_1|^2)}.$$

Denoting the SINR requirements as γ_k with $\gamma_k = 2^{\rho_k} - 1$, the result for $\text{SINR}_1\text{SINR}_2$ shows that only requirements with $\gamma_1 \gamma_2 < 1$ are feasible or, equivalently, $(2^{\rho_1} - 1)(2^{\rho_2} - 1) < 1$.

Based on the duality between the MIMO BC and the MIMO MAC with linear transceivers [11], the QoS optimization (1) can be reformulated and solved in the dual MIMO MAC, and the resulting MIMO MAC filters can be transformed to MIMO BC filters achieving the same minimal transmit power. Therefore, we concentrate on the MIMO MAC QoS formulation

$$\min_{\boldsymbol{T}_1,\ldots,\boldsymbol{T}_K} P_{\text{tx}} \qquad \text{s.t.:} \quad R_k \geq \rho_k \quad \forall k \in \{1,\ldots,K\} \tag{2}$$

with the MIMO MAC data rate of user k

$$R_k = \log_2 \left| \mathbf{I} + \left(\mathbf{I} + \sum_{i \neq k} \boldsymbol{H}_i \boldsymbol{T}_i \boldsymbol{T}_i^{\text{H}} \boldsymbol{H}_i^{\text{H}} \right)^{-1} \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{T}_k^{\text{H}} \boldsymbol{H}_k^{\text{H}} \right|.$$

Here, we introduced precoders $\boldsymbol{T}_k \in \mathbb{C}^{M_k \times S_k}$, the sum transmit power $P_{\text{tx}} = \sum_{k=1}^{K} \|\boldsymbol{T}_k\|_{\text{F}}^2$, and the channels $\boldsymbol{H}_k = \boldsymbol{G}_k^{\text{H}} \boldsymbol{C}_{\boldsymbol{\eta}_k}^{\text{H},-1/2}$, $k = 1,\ldots,K$, in the dual MIMO MAC. The noise in the MIMO MAC is $\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0},\mathbf{I})$.

III. Previous Results

First, we define the required effective degrees of freedom (EDoF) of user k

$$d_k = 1 - 2^{-\rho_k} \tag{3}$$

which fulfill $d_k \in [0,1)$ for $\rho_k \geq 0$. In [12], the EDoF are called the effective bandwidth. With above definition, the feasibility results of [10] can be formulated as the following theorem.

Theorem 1. *The QoS optimization* (2) *has a solution for systems with linear transceivers, if and only if the corresponding EDoF requirements* $d_1,\ldots,d_K$ *fulfill*

$$\sum_{k=1}^{K} d_k < \operatorname{rank}(\boldsymbol{H})$$

and

$$\sum_{i \in \mathcal{I}} d_i < \operatorname{rank}(\boldsymbol{H}_{\mathcal{I}})$$
$$\forall \mathcal{I} \subset \{1,\ldots,K\}: \quad \operatorname{rank}(\boldsymbol{H}_{\mathcal{I}}) < \min(|\mathcal{I}|, N)$$

where $\boldsymbol{H}_{\mathcal{I}} \in \mathbb{C}^{N \times \sum_{i\in\mathcal{I}} M_i}$ *comprises all channels* $\boldsymbol{H}_i$ *with* $i \in \mathcal{I}$ *in a block row.*

This result motivates the name required effective degrees of freedom for d_k. The system has $\operatorname{rank}(\boldsymbol{H})$ degrees of freedom ($\operatorname{rank}(\boldsymbol{H}_{\mathcal{I}})$ degrees of freedom for the user subset $\mathcal{I}$) and the requirement for user k reduces the degrees of freedom available to serve the other users by d_k.

Restricting to regular channels, i.e.,

$$\forall \mathcal{I} \subseteq \{1,\ldots,K\}: \quad \operatorname{rank}(\boldsymbol{H}_{\mathcal{I}}) \geq \min(|\mathcal{I}|, N) \tag{4}$$

leads to the following corollary.

Corollary 1. *For regular channels fulfilling* (4)*, the QoS optimization* (2) *has a solution for linear transceivers, if and only if*

$$\sum_{k=1}^{K} d_k < N.$$

Note that the results in Theorem 1 and Corollary 1 are independent of the number of antennas at the users, since only the existence of a solution to the optimization (2) is discussed, as was demonstrated in [10]. In contrast, the optimum of (2) depends on the number of antennas deployed at the user terminals.

IV. Time Sharing

For Theorem 1 and Corollary 1, it is assumed that no time sharing is applied. In the following, we will show that any requirements are feasible, if time sharing is allowed and the number of time slots is arbitrary. However, if the number of time slots has an upper bound, feasibility is not always possible (see Section VI).

If the number of time slots has no limit, a time slot can be allocated to every user such that only user k is served during the k-th of the K time slots. Assuming that the time slots have the same duration for all users, user k must have a data rate of $K\rho_k$ in the k-th time slot to fulfill the requirement that the average data rate is ρ_k. Since only user k uses the channel during time slot k, this data rate can be achieved with finite transmit power. Thus, any requirements are feasible for an unlimited number of time slots. Note, however, that aforementioned strategy to allocate a separate time slot to every user is sufficient to prove feasibility of arbitrary requirements, but is in general suboptimal w.r.t. the needed transmit power.

The considerations of this section also hold for the case of frequency division multiplexing, if the number of subcarriers is not upper bounded.

V. Robust QoS Optimization

In [13], [14], the case of erroneous channel state information is considered. Based on the model that for the channel of user k, a region $\mathcal{U}_k$ is known in which the channel lies, the requirements can be met for any channel state by the following reformulation of (1) (see [14])

$$\min_{\boldsymbol{P}_1,\ldots,\boldsymbol{P}_K} P_{\text{tx}}^{\text{BC}} \quad \text{s.t.: } R_k^{\text{BC}} \geq \rho_k \ \forall \boldsymbol{G}_k \in \mathcal{U}_k, \ \forall k \in \{1,\ldots,K\}. \tag{5}$$

Equivalently, the requirements are met for all channels in the region $\mathcal{U}_k$, if they are achieved in the worst case (see [13])

$$\min_{\boldsymbol{P}_1,\ldots,\boldsymbol{P}_K} P_{\text{tx}}^{\text{BC}} \quad \text{s.t.: } \min_{\boldsymbol{G}_k \in \mathcal{U}_k} R_k^{\text{BC}} \geq \rho_k \ \forall k \in \{1,\ldots,K\}. \tag{6}$$

Note that we can conclude from (6) that the solution to (5) and (6) could be obtained, if the worst case channels for the optimal precoder were known, just by substituting these channels into (2).

The test of feasibility for (5) and (6) is difficult in general, since for any combination of $\boldsymbol{G}_k \in \mathcal{U}_k \ \forall k$ feasibility must be ensured. However, it is easy to see that (5) is infeasible [and thus, (6)], if there exists at least one combination of $\boldsymbol{G}_k \in \mathcal{U}_k \ \forall k$ that are infeasible. Similarly, (6) is feasible [and therefore, (5)], if the requirements are feasible for the worst case channel and, hence, for all channels.

These observations lead to following conclusion. Assume that the regions $\mathcal{U}_1,\ldots,\mathcal{U}_K$ have the structure such that any combination of the channels $\boldsymbol{G}_k \in \mathcal{U}_k\ \forall k$ fulfill (4).[1] Therefore, the requirements are feasible for any of the channel combinations, if and only if the condition in Corollary 1 is fulfilled. Thus, we can conclude that (6) [and (5)] is feasible, when the QoS requirements meet the conditions in Corollary 1. And we can infer that (5) [and (6)] is infeasible, if the requirements violate the conditions in Corollary 1.

Similarly, the case can be handled, where at least one channel combination exists that does not fulfill (4). If any combination of $\boldsymbol{G}_k \in \mathcal{U}_k\ \forall k$ exists such that the conditions for the respective dual MIMO MAC channels $\boldsymbol{H}_k\ \forall k$ given in Theorem 1 are violated, (5) [and (6)] has no solution. And (6) [and (5)] is feasible, if the conditions in Theorem 1 are fulfilled by any combination of channels.

VI. Feasibility in Multicarrier Settings

In this section, we consider a multi-carrier downlink system where the C carriers are assumed to be orthogonal to each other. It is further assumed that the division of frequency is fixed and cannot be made finer (it is impossible to increase the number of subcarriers) such that a scheme as discussed in Section IV is impossible. In this setting a feasibility check is necessary if the number of users K exceeds the product of the number of base station antennas and the number of carriers NC.

The considered system can be seen as a set of parallel broadcast channels, each characterized by a set of dual MIMO MAC channel matrices $\{\boldsymbol{H}_k^{(c)}\}_{k\in\{1,\ldots,K\}}$ where $c \in \{1,\ldots,C\}$ is the carrier index. If we write these matrices in blockdiagonal per-user channel matrices

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{H}_k^{(1)} & & \\ & \ddots & \\ & & \boldsymbol{H}_k^{(C)} \end{bmatrix} \in \mathbb{C}^{NC\times M_k C} \tag{7}$$

and assume that arbitrary precoders $\boldsymbol{T}_k$ are allowed, this setting complies with the system model from Section II. Consequently, the feasibility conditions from Section III are directly applicable for precoders $\boldsymbol{T}_k$ with no special structure. In the following, we will call a rate requirement *potentially feasible*, if these feasibility conditions for precoders with arbitrary structure are fulfilled.

A. Carrier-noncooperative Transmission

When transmit strategies for multi-carrier systems are designed, it is often assumed that no data stream of any user is split across several carriers, i.e., the precoding matrices $\boldsymbol{T}_k$ are chosen such that their Gram product $\boldsymbol{T}_k\boldsymbol{T}_k^{\mathrm{H}}$ is blockdiagonal matching the blockdiagonal structure of the channel. Examples for this can be found for a variety of different transmit strategies, (e.g., in [7], [6], [15], and [8]). Nevertheless, when the transmit strategy is limited to linear beamforming without time sharing, this assumption is not necessarily optimal as will be seen in the following discussion on feasibility. In a related discussion for a single-user MIMO system with per-stream QoS constraints presented in [16], it turned out that carrier-cooperative transmission (equivalent to precoders with arbitrary structure) performs better than carrier-noncooperative transmission. However, the results of [16] are not applicable to our setting due to the fundamental differences between per-stream and per-user constraints.

Assuming separate precoding on each carrier, each user can be considered as C virtual users that are only coupled by the user's QoS constraint. Each of these virtual users is constrained to use a precoder that can be written as

$$\tilde{\boldsymbol{T}}_k^{(c)} = (\boldsymbol{e}_c \otimes \mathbf{I}_{M_k})\boldsymbol{T}_k^{(c)} \ \in \mathbb{C}^{M_k C\times S_k^{(c)}} \tag{8}$$

where $\mathbf{I}_{M_k}$ is the identity matrix of size M_k, $\otimes$ denotes the Kronecker product, $\boldsymbol{e}_c \in \mathbb{C}^C$ has a one in the cth entry and zeros elsewhere, $S_k^{(c)}$ is the number of streams transmitted by the virtual user (k,c), and $\boldsymbol{T}_k^{(c)} \in \mathbb{C}^{M_k\times S_k^{(c)}}$ is an arbitrary precoding matrix. We define the effective channel

$$\tilde{\boldsymbol{H}}_k^{(c)} = \boldsymbol{H}_k(\boldsymbol{e}_c \otimes \mathbf{I}_{M_k}) \ \in \mathbb{C}^{NC\times M_k} \tag{9}$$

of the virtual user (k,c) so that the product of channel and beamformer can be written as $\tilde{\boldsymbol{H}}_k^{(c)}\boldsymbol{T}_k^{(c)}$. Due to the special structure of these effective channels, the resulting effective system is a singular channel scenario as soon as the number of real users K is larger than the number of base station antennas N, even if the original system fulfills the regularity condition (4). As this was the optimal strategy in the original system [10], we will for the moment assume that each user transmits only one data stream, i.e., for each user k one virtual user (k,c_k) is active with $S_k^{(c_k)} = 1$ stream and the EDoF requirement $d_k^{(c_k)} = d_k$, while $d_k^{(c)} = 0$ for $c \neq c_k$. Due to Theorem 1, the condition on the sum of all EDoF is the same as in the original problem, but additional conditions have been introduced due to the new rank deficiencies. Consequently, this choice for the precoding matrices might render a potentially feasible QoS requirement infeasible.

On the other hand, the optimality of single-stream transmission, which we assumed in the last paragraph, is no longer true for our case. In the original system, the only matter of interest was the sum of EDoF required by all users or by a certain subset of users, and the optimality of single-stream transmission was a consequence of minimizing the EDoF per user. However, in the effective system for the multi-carrier case, we no longer have per-user rate requirements, but sum rate requirements for groups of virtual users. Therefore, as long as the condition on the sum of all EDoF is not the limiting one we can activate additional virtual users and redistribute the per-virtual-user EDoF requirements $d_k^{(c)}$. Due to the non-linear relation between rate and EDoF requirements, this will increase the overall number of necessary EDoF, but it might lead to a distribution which does not violate any of the conditions that were introduced due to the artificial rank

[1]For the test in (4), the MIMO BC channels must be transformed to the respective dual MIMO MAC channels, i.e., $\boldsymbol{H}_i = \boldsymbol{G}_i^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{\eta}_i}^{\mathrm{H},-1/2}$.

deficiencies. This possibility will be discussed in detail later in this section.

B. Per-carrier Feasibility

Before we continue our analysis, we will introduce a second perspective on the carrier-noncooperative feasibility problem, which is more handy than the one presented above. Due to the separate precoding, we can consider the carriers as independent channels which are only coupled by the users' rate requirements. In each of these channels we can apply the feasibility condition of Corollary 1 and we get the per-carrier feasibility condition

$$\sum_{k=1}^{K} d_k^{(c)} < N \quad \forall c \in \{1, \ldots, C\} \tag{10}$$
$$\text{with} \quad d_k^{(c)} \in [0, 1) \quad \forall k, c$$
$$\text{and} \quad \sum_{c=1}^{C} -\log_2\left(1 - d_k^{(c)}\right) = \rho_k \quad \forall k.$$

In order to show feasibility, it is sufficient to find a set of per-carrier EDoF requirements $d_k^{(c)}$ such that (10) is fulfilled. However, to show infeasibility of potentially feasible rate requirements it is necessary to prove that no such set of $d_k^{(c)}$ exists.

When (10) is considered as a feasibility problem in the real variables $d_k^{(c)}$, the inequality constraints do not pose problems as they define halfspaces.[2] However, the equality constraints which are imposed by convex functions cannot be relaxed to convex inequality constraints: the rate requirements ρ_k have to be considered as minimum rate requirements and the superlevel set of a convex function is in general non-convex. Thus, the feasibility problem is non-convex, which makes it difficult to handle. We will discuss an approximate solution to the problem in the remainder of this section.

C. Single Data Streams

From [10], we know that in order to minimize the number of occupied EDoF we have to transmit a single data stream per user. If we restrict ourselves to this strategy we end up with a problem that is equivalent to a decision version of the famous bin packing problem (cf. e.g. [17], [18]): given a list $\mathcal{L}$ of real numbers between 0 and 1 (in our case $d_k = 1 - 2^{-\rho_k}$) that are denoted as items, decide if it is possible to place the elements of $\mathcal{L}$ in a given number (in our case C) of bins such that no bin contains items whose sum exceeds a certain capacity (in our case $N - \epsilon$).[3]

As the bin packing problem is known to be NP-hard [17], we note that no polynomial time solution to it is known. One of the most famous heuristics for packing the elements of $\mathcal{L}$ in a close-to-optimum number of bins is the *Best-fit decreasing* (BFD) strategy [19]. The elements of $\mathcal{L}$ are arranged in decreasing order and successively each element is placed into the bin where it fits best, i.e., among all bins that still have enough free space to carry the element, the one with the least amount of free space is chosen. To apply this algorithm to the decision version we simply compare the obtained number of bins with the given limit.

However, there is no guarantee that existing feasible solutions are found using this strategy so that it cannot be used to prove the absence of a feasible single-stream solution. To this end, we could instead apply an exponential time algorithm which in the worst case tests all possible packings. Moreover, even if no single-stream solution exists, the requirements could be feasible using multiple streams per user as will be discussed in the next section.

D. Multiple Data Streams

If no feasible allocation with single data streams per user can be found although the problem is potentially feasible, this is a result of an unsuitable combination of the individual requirement sizes. For example in a system with $N = 1$ transmit antenna and $C = 2$ carriers the EDoF requirements $d_1 = d_2 = d_3 = 0.6$ cannot be fulfilled with single streams altough they sum up to $d_{\text{total}} = 1.8 < CN$. In such a case, we can make use of the gap between d_{total} and CN and cut requirements into smaller chunks by the cost of increasing the sum of required EDoF.

As the multistream feasibility test consists of finding the correct cuts and packing the resulting streams into the carriers, it is more difficult than the packing problem discussed for single data streams. Consequently, this problem is NP-hard, too. In order to find a heuristic solution we will first study the behaviour of the sum of EDoF for different possible cuts.

Observation 1. *The function*

$$x \mapsto (1 - 2^{-x}) + (1 - 2^{-(\rho_k - x)}) \quad x \in [0; \rho_k] \tag{11}$$

is minimized for $x = 0$ and $x = \rho_k$. Thus, in order to keep the sum of necessary EDoF low, it is favorable to cut as few requirements as possible.

Observation 2. *The function (11) is monotonically increasing for $x < \rho_k/2$ and monotonically decreasing for $x > \rho_k/2$. Thus, when cutting a requirement, it is favorable to cut it in pieces whose sizes are as unequal as possible.*

Observation 3. *The expression*

$$\Big((1 - 2^{-x}) + (1 - 2^{-(\rho_k - x)})\Big) - (1 - 2^{-\rho_k}) \quad x \in]0; \rho_k[\tag{12}$$

is increasing in ρ_k. Since the first term represents the EDoF necessary for a rate requirement cut into two pieces and the second term represents the EDoF of the uncut requirement, we can conclude that cutting a larger rate requirement is worse than cutting a smaller one.

Based on these three observations and motivated by the BFD algorithm, we propose the following heuristic approach to find a feasible allocation with multiple data streams per user:

[2] The strict inequality $d_k < 1$ can be relaxed to $d_k \leq 1$ without any problems and in the first constraint the strict inequality can be relaxed by changing the right side to $N - \epsilon$ where $\epsilon > 0$ is a small constant.

[3] The subtraction of a small $\epsilon > 0$ is to ensure strict inequality in the feasibility condition.

1) Sort the EDoF requirements d_k in decreasing order.
2) For the highest d_k:
   a) If there are carriers with more than $d_k + \epsilon$ free EDoF place the element according to the best-fit rule and set $d_k = 0$.
   b) Otherwise choose the carrier with the maximum amount of free EDoF and fill it up to $N-\epsilon$. Reduce d_k to the number of EDoF necessary to fulfill the remaining rate requirement.
3) Break if all d_k are zero or all carriers are full. Otherwise go back to step 1).

A feasible solution has been found if $d_k = 0\ \forall k$ after the execution of the algorithm. The sorting step 1) is not only motivated by BFD, but also in compliance with Observation 3: the earlier the large requirements are placed, the lower the risk that they have to be split. Sorting has to be repeated in each iteration to account for d_k values changed in step 2). In step 2b) we place the largest possible fraction of the requirement. If this portion is more than $d_k/2$, this is clearly in compliance with Observation 2. Otherwise this rule seems to be reasonable, too, because in subsequent steps there will not be a bin which can carry a larger fraction and thus, placing less than possible might increase the number of total cuts which would counteract Observation 1.

Note that a positive outcome of the proposed feasibility test is sufficient, but not necessary for feasibility, i.e. if the algorithm does not find a feasible packing this does not necessarily imply infeasibility: Firstly, the algorithm is a polynomial time approximation for an NP-hard problem and cannot guarantee the absence of feasible solutions. Secondly, the parameter ϵ might have been chosen too large. On the other hand, a feasible allocation that has been found for a very small ϵ might require a very high (though finite) transmit power. Such an allocation might cause numerical problems when used as the initialization of a subsequent optimization procedure. Thus, the value of ϵ has to be chosen carefully in a practical implementation.

E. Discussion

In this section we have seen that in a multi-carrier MIMO system there is a class of QoS requirements which are feasible only if the carriers are treated jointly, i.e., if the precoders do not emulate the blockdiagonal structure of the channel matrix. This result is somehow surprising as separate treatment of the carriers is a common assumption. In some contexts this strategy is even the optimal one, e.g. when the whole capacity region (DPC region) of the broadcast channel is exploited [20]. However, in the case of linear precoding without time sharing, separate treatment might render potentially feasible requirements infeasible and is therefore clearly suboptimal in general.

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