Bargaining over Fading Interference Channels

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***Abstract*—We consider the problem of bargaining over block fading interference channels, where interaction between players takes place over multiple channel realizations. Based on the assumption that the transmitters have conflicting objectives, we use axiomatic bargaining theory to derive optimal rate allocations in each block. In the setup under consideration, the Nash bargaining solution (NBS) is non-causal, i.e., cannot be implemented in a real-world system. We argue that the invariance axiom is superfluous when bargaining over a rate region. Without the invariance axiom, an equivalent solution follows from the maximization of a sum of utilities under minimum utility constraints. This alternative solution is also non-causal. We propose causal approximations to the optimal solutions. The sum utility solution allows for a more systematic approximation than the NBS. Thus, dropping the invariance axiom makes it possible to choose a solution which can be better approximated. We provide numerical results to illustrate the performance of the proposed solutions.**

I. Introduction

A simple network consisting of two transmitter-receiver pairs is considered. Transmitter k sends information to receiver k, $k = 1, 2$. Information for receiver 1 is not of interest for receiver 2, and vice versa. It is assumed, however, that both transmitters operate in the same band. From the viewpoint of information theory, such a configuration represents an interference channel (IFC) [1], [2]. With respect to practical wireless systems, the IFC is a model for interference between two base stations operating in the same band [3]. It is assumed that the transmitters are equipped with multiple transmit antennas, while the receivers have a single antenna each. Multiple antennas enable the transmitters to perform spatial signal processing – and thereby manage interference. The capacity region of such a multiple-input, single-output (MISO) IFC is not known. By treating interference as noise, however, it is straightforward to find an achievable rate region for a given channel realization [3].

The ability of each transmitter to choose from a set of transmission strategies immediately leads to the question of an optimal choice of transmission strategies. If both transmitters share a common objective, the answer is obvious: choose a pair of transmission strategies that optimizes the common objective. In this work, however, it is assumed that there is *no common good* – on the contrary, it is assumed that both transmitters have conflicting objectives. A typical scenario where the transmitters do not share a common objective is that of two base stations owned by different operators [3]. The assumption of conflicting objectives turns the problem of choosing a transmission strategy into a game theoretic problem [4].

The case that interaction between transmitters (or players) takes place for the duration of a single channel realization is investigated in [3], from both a noncooperative and cooperative game theory point of view. It is shown that in the MISO IFC, Nash Equilibria, which correspond to non-cooperative behaviour, are highly inefficient, and both players can significantly improve their outcome by cooperation. By allowing for cooperation between the players, the problem of choosing a transmission strategy can be cast as a bargaining problem. The authors in [3] propose using the Nash Bargaining Solution (NBS) [5] as outcome of the bargaining problem. The NBS has been frequently used in the context of resource allocation in communication systems [6], [7], [8], [9].

In this work, we consider the case that the players interact over multiple channel realizations. Similar to [3], we use axiomatic bargaining theory [10] to discuss the choice of transmission strategies. The interaction over multiple channel realizations, however, leads to the problem of causality: In a real-world system, the transmitters have to decide on a transmission strategy for the current channel realization without knowing about the future channel. We show that when bargaining over a block of channel realizations, the NBS leads to a non-causal solution, i.e., it is not implementable in a real-world system. As a result, the NBS has to be approximated by a causal solution.

The NBS is one possible solution to a bargaining problem. In contrast to previous work on bargaining over rate regions, we argue that when bargaining over rate regions, other solutions are equivalent from an axiomatic viewpoint, based on the fact that information rate represents a technical measure and therefore does not require scale invariance. Based on this observation, we propose sum utility maximization under minimum utility constraints (SU) as an equivalent solution to the bargaining problem. As shown in this work, the SU solution is again non-causal, and has to be approximated by a causal solution. In contrast to the NBS, approximation of SU by a causal strategy is straightforward.

Notation: Vectors and vector-valued functions are denoted by bold lowercase letters, matrices by bold uppercase letters. The transpose and the Hermitian transpose of $\boldsymbol{Q}$ are denoted by $\boldsymbol{Q}^{\mathrm{T}}$ and $\boldsymbol{Q}^{\mathrm{H}}$, respectively. The identity matrix is denoted by $\boldsymbol{I}$, and the vector of all ones is denoted by $\mathbf{1}$. The following definitions of order relations between vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^K$, with

$K > 1$, are used:

$$\begin{aligned} \boldsymbol{x} \geq \boldsymbol{y} &\Leftrightarrow \forall k : x_k \geq y_k, \\ \boldsymbol{x} > \boldsymbol{y} &\Leftrightarrow \boldsymbol{x} \geq \boldsymbol{y}, \exists k : x_k > y_k, \\ \boldsymbol{x} \gg \boldsymbol{y} &\Leftrightarrow \forall k : x_k > y_k. \end{aligned}$$

Order relations $\leq, <, \ll$ are defined in the same manner. Finally, on subsets of $\mathbb{R}^2$ we define the operator $()^{\mathrm{P}}$ as follows: $\mathcal{S}^{\mathrm{P}} = \{(x_2, x_1) : (x_1, x_2) \in \mathcal{S}\}$. Accordingly, if a set $\mathcal{S} \subseteq \mathbb{R}^2$ is symmetric, we have $\mathcal{S}^{\mathrm{P}} = \mathcal{S}$.

II. MISO Interference Channel

A. System Model

We consider the following interference channel (IFC): Two transmitters, TX_1 and TX_2, send information to two receivers, RX_1 and RX_2. The transmitters are equipped with $N > 1$ transmit antennas, while the receivers have a single receive antenna each. Such a multiple-input, single-output (MISO) IFC was previously considered in [3], [11].

The received signal at receiver k, $k \in \{1, 2\}$, is given by

$$y_k = \sum_{q=1}^{2} \boldsymbol{h}_{q,k}^{\mathrm{T}} \boldsymbol{w}_q s_q + \eta_k, \tag{1}$$

where $\boldsymbol{h}_{q,k} \in \mathbb{C}^N$ is the channel from transmitter q to receiver k, $\boldsymbol{w}_q \in \mathbb{C}^N$ and s_q are the beamforming vector (precoder) and transmitted symbol of transmitter q, respectively, and η_k is circularly symmetric AWGN with zero mean and variance σ^2.

We assume that each transmitter is subject to a peak power constraint with maximum power P, which, under the assumption of uncorrelated, unit power data symbols, translates into a constraint on the precoders $\boldsymbol{w}_k$:

$$\|\boldsymbol{w}_k\|_2^2 \leq P, k = 1, 2. \tag{2}$$

In the remainder of this section, it is assumed that the channels $\boldsymbol{h}_{1,1}, \boldsymbol{h}_{1,2}, \boldsymbol{h}_{2,1}$, and $\boldsymbol{h}_{2,2}$ are fixed and do not change over time. In Section III, the channel model is extended to a block fading model.

B. An Achievable Rate Region

Treating the interference from other users as noise, for given precoders $(\boldsymbol{w}_1, \boldsymbol{w}_2)$ and channels $\boldsymbol{h}_{q,k}$, an achievable rate vector $\boldsymbol{R} = (R_1, R_2)$ is given by $\boldsymbol{R} \in \mathbb{R}_+^2 : R_k < r_k(\boldsymbol{w}_1, \boldsymbol{w}_2)$ [3], with

$$r_k(\boldsymbol{w}_1, \boldsymbol{w}_2) = \log_2 \left(1 + \frac{|\boldsymbol{h}_{k,k}^{\mathrm{T}} \boldsymbol{w}_k|^2}{\sigma^2 + \sum_{q \neq k} |\boldsymbol{h}_{q,k}^{\mathrm{T}} \boldsymbol{w}_q|^2} \right).$$

The set of achievable rate vectors is defined as the closure of all such vectors for given transmit power constraints,

$$\tilde{\mathcal{R}} = \left\{ \boldsymbol{r}(\boldsymbol{w}_1, \boldsymbol{w}_2) : \|\boldsymbol{w}_k\|_2^2 \leq P, k = 1, 2 \right\}. \tag{3}$$

Let $\mathcal{R}$ denote the convex hull of $\tilde{\mathcal{R}}$. By time-sharing between vectors in $\tilde{\mathcal{R}}$, any vector in $\mathcal{R}$ is also achievable. The set $\mathcal{R}$ is convex and compact [3].

C. Bargaining Over the Rate Region

While $\mathcal{R}$ provides a set of achievable rates, it is not immediately obvious how both transmitters should choose their beamforming vectors $\boldsymbol{w}_k$. In this work, it is assumed that there is *no common good*, i.e., the two transmitters do not share a common objective, but instead both aim at maximizing their individual rate.

If both transmitters (players) do not cooperate, the only reasonable outcome is an operating point which constitutes a Nash equilibrium [3]. As shown in [3], the Nash Equilibria are given by the following set of beamforming vectors:

$$\mathcal{W}_{\mathrm{NE}} = \left\{ (\boldsymbol{w}_1, \boldsymbol{w}_2) : \boldsymbol{w}_k = \frac{\alpha_k \boldsymbol{h}_{k,k}^*}{\|\boldsymbol{h}_{k,k}\|_2}, |\alpha_k|^2 = P \right\}. \tag{4}$$

Let $\boldsymbol{r}_{\mathrm{NE}}$ denote the corresponding rate vector. given by

$$\boldsymbol{r}_{\mathrm{NE}} = \boldsymbol{r} \left(\frac{\sqrt{P} \boldsymbol{h}_{1,1}^*}{\|\boldsymbol{h}_{1,1}\|_2}, \frac{\sqrt{P} \boldsymbol{h}_{2,2}^*}{\|\boldsymbol{h}_{2,2}\|_2} \right). \tag{5}$$

The rate vector $\boldsymbol{r}_{\mathrm{NE}}$ is in general not Pareto optimal, i.e., it does not lie on the Pareto boundary of $\mathcal{R}$. Based on this result, both players can jointly improve their outcome by cooperation [3].

By allowing for cooperation between the players, the problem of choosing a point from $\mathcal{R}$ turns into a bargaining problem. The authors in [3] propose using the Nash Bargaining Solution (NBS) [5] as outcome of the bargaining problem (cf. Section IV for details on the NBS).

D. Weighted Sum Rate Maximization

An important problem in the MISO IFC is the maximization of a weighted sum of rates over the set of achievable rates:

$$\max_{\boldsymbol{r} \in \mathcal{R}} \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{r}, \tag{6}$$

for a given weight vector $\boldsymbol{\lambda} \geq \boldsymbol{0}$. As time-sharing is not required to maximize a weighted sum of rates, the problem is equivalent to maximizing over $\tilde{\mathcal{R}}$. Moreover, as the objective function is increasing in $\boldsymbol{r}$, the maximum is achieved on the Pareto boundary of $\tilde{\mathcal{R}}$ [12]. A parameterization of the Pareto boundary is provided in [11]. Based on the parameterization from [11], we can write the weighted sum rate maximization problem as a monotonic optimization problem [12]. As a result, using methods from monotonic optimization, we can solve the weighted sum rate maximization on the MISO IFC to global optimality.

III. Block-Fading IFC

Section II summarized the case of two players bargaining over a single channel realization of a MISO IFC. In a wireless communication system, players will usually not meet for the duration of only a single channel realization, but their interaction will last for the duration of several channel realizations. We adopt a block-fading channel model, where the quadrupel of channel vectors $(\boldsymbol{h}_{1,1}, \boldsymbol{h}_{1,2}, \boldsymbol{h}_{2,1}, \boldsymbol{h}_{2,2})$ at block ℓ is the realization of a random variable H^{ℓ}. Random variables H^m and H^n, with $m \neq n$, are assumed to be statistically independent and identically distributed. The probability density function of H^ℓ is denoted by p_H. The problem of bargaining over a sequence of L blocks is considered.

Let $\mathcal{R}^\ell$ denote the achievable rate region at the l-th block. Depending on the transmit strategies of both players, player k gets a rate r_k^ℓ in block l. The players' utility is defined as the average rate over the L blocks:

$$u_k = \frac{1}{L}\sum_{l=1}^{L} r_k^\ell. \tag{7}$$

Accordingly, the achievable utility region is given by

$$\mathcal{U} = \left\{ \boldsymbol{u} \in \mathbb{R}_+^2 : \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell, \boldsymbol{r}^\ell \in \mathcal{R}^\ell \right\}. \tag{8}$$

The achievable utility region is a convex set: Let $\boldsymbol{u}, \tilde{\boldsymbol{u}} \in \mathcal{U}$. Define

$$\begin{aligned} \boldsymbol{u}(\alpha) &= \alpha \boldsymbol{u} + (1-\alpha)\tilde{\boldsymbol{u}} \\ &= \frac{1}{L}\sum_{l=1}^{L} \alpha \boldsymbol{r}^\ell + (1-\alpha)\tilde{\boldsymbol{r}}^\ell = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell(\alpha), \end{aligned}$$

with $\boldsymbol{r}^\ell, \tilde{\boldsymbol{r}}^\ell \in \mathcal{R}^\ell$. Due to the convexity of $\mathcal{R}^\ell$, $\boldsymbol{r}^\ell(\alpha) \in \mathcal{R}^\ell$, thus $\boldsymbol{u}(\alpha) \in \mathcal{U}$ for $\alpha \in [0,1]$. Moreover, $\mathcal{U}$ is compact, due to the compactness of $\mathcal{R}^\ell$.

IV. Bargaining over the Utility Region

In this work, we consider the case of cooperative bargaining, i.e., both players cooperate in order to achieve an outcome $\boldsymbol{u} \in \mathcal{U}$ that is better than an outcome which would result from non-cooperative behaviour. The non-cooperative outcome is given by the utility point that results from both users choosing their transmission strategy to optimize their rate without considering the interference caused to the other player:

$$\boldsymbol{d} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{d}^\ell = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}_{\mathrm{NE}}^\ell, \tag{9}$$

where $\boldsymbol{r}_{\mathrm{NE}}^\ell$ is defined in (5). In the following, we will use the theory of axiomatic bargaining to discuss possible methods to choose an outcome $\boldsymbol{u} \in \mathcal{U}$. In the language of axiomatic bargaining, a pair $(\mathcal{U}, \boldsymbol{d})$, with $\mathcal{U}$ compact and convex, and $\boldsymbol{d} \in \operatorname{int} \mathcal{U}$, represents a bargaining problem [10]. Let $\mathcal{B}$ denote the set of all bargaining problems. Then a function $f : \mathcal{B} \to \mathbb{R}^2$ that assign to each problem $(\mathcal{U}, \boldsymbol{d})$ a unique element of $\mathcal{U}$ is a bargaining solution.

A. Nash Bargaining Solution

One popular solution to a family of bargaining problems $\mathcal{B}$ is the *Nash Bargaining Solution* (NBS),

$$\boldsymbol{f}^{\mathrm{NBS}}(\mathcal{U}, \boldsymbol{d}) = \underset{\boldsymbol{u} \in \mathcal{U}, \boldsymbol{d} \leq \boldsymbol{u}}{\operatorname{argmax}} (u_1 - d_1)(u_2 - d_2). \tag{10}$$

The NBS can be found by computing the optimal rate allocation in each block:

$$\max_{\boldsymbol{r}^1,\ldots,\boldsymbol{r}^L} \frac{1}{L^2}\prod_{k=1}^{2}\left(\sum_{l=1}^{L}(r_k^\ell - d_k^\ell)\right) \quad \text{s.t.} \quad \frac{1}{L}\sum_{l=1}^{L}\boldsymbol{r}^\ell \geq \boldsymbol{d}, \boldsymbol{r}^\ell \in \mathcal{R}^\ell. \tag{11}$$

Let $\hat{\boldsymbol{r}} = (\hat{\boldsymbol{r}}^1, \ldots, \hat{\boldsymbol{r}}^L)$ denote a maximizer of (11). Then

$$\boldsymbol{f}^{\mathrm{NBS}}(\mathcal{U}, \boldsymbol{d}) = \frac{1}{L}\sum_{l=1}^{L} \hat{\boldsymbol{r}}^\ell. \tag{12}$$

Equation (11) shows that the NBS is a non-causal solution: In order to determine the optimal rate allocation in the first block, knowledge of the rate region and Nash Equilibrium of the blocks $2, \ldots, L$ is required. This is due to the fact that the optimization problem in (11) does not decouple, thus the optimal rates $\hat{\boldsymbol{r}}^1, \ldots, \hat{\boldsymbol{r}}^L$ have to be computed jointly.

B. The Invariance Axiom is Superfluous

The NBS is the only bargaining solution that fulfills

$$\boldsymbol{f}(\mathcal{U}, \boldsymbol{d}) \geq \boldsymbol{d} \tag{13}$$

and the following four axioms:

1) Weak Pareto Optimality (WPO). Let $\boldsymbol{u}, \boldsymbol{u}' \in \mathcal{U}$, with $u_k' > u_k, \forall k$. Then $\boldsymbol{u} \neq \boldsymbol{f}(\mathcal{U}, \boldsymbol{d})$.
2) Symmetry (SYM). If $\mathcal{U} = \mathcal{U}^{\mathrm{P}}$ and $d_1 = d_2$, then $f_1(\mathcal{U}, \boldsymbol{d}) = f_2(\mathcal{U}, \boldsymbol{d})$.
3) Independence of Irrelevant Alternatives (IIA). Let $(\mathcal{U}, \boldsymbol{d})$ and $(\mathcal{U}', \boldsymbol{d})$ be bargaining problems, with $\mathcal{U}' \subset \mathcal{U}$ and $\boldsymbol{f}(\mathcal{U}, \boldsymbol{d}) \in \mathcal{U}'$. Then $\boldsymbol{f}(\mathcal{U}, \boldsymbol{d}) = \boldsymbol{f}(\mathcal{U}', \boldsymbol{d})$.
4) Scale Invariance (INV). Define an affine transformation $\boldsymbol{g}(\boldsymbol{u}) = \boldsymbol{T}\boldsymbol{u} + \boldsymbol{b}$, where $\boldsymbol{T} = \operatorname{diag}(t_1, t_2)$ is a positive definite diagonal matrix, and let $\mathcal{U}' = \{\boldsymbol{u}' = \boldsymbol{g}(\boldsymbol{u}), \boldsymbol{u} \in \mathcal{U}\}, \boldsymbol{d}' = \boldsymbol{g}(\boldsymbol{d})$. Then $\boldsymbol{f}(\mathcal{U}', \boldsymbol{d}') = \boldsymbol{g}(\boldsymbol{f}(\mathcal{U}, \boldsymbol{d}))$.

Judging whether NBS is a good solution strategy for the problem at hand has to be based on a discussion of the four axioms – if all four axioms are desired, there is no other option. For a detailed discussion of the axioms WPO, SYM, and IIA, see, e.g., [5], [10]. For the bargaining problem considered in this paper, they all imply desirable properties.

The desirability of the axiom INV is less obvious. Assume that INV holds. Then

$$f_1^{\mathrm{NBS}}(\mathcal{U}', \boldsymbol{d}') = t_1 f_1^{\mathrm{NBS}}(\mathcal{U}, \boldsymbol{d}) + b_1,$$

i.e., the utility of user 1 is independent of the scale transformations t_2 and b_2 of user 2. In other words, user 2 can choose any values $t_2 > 0$ and b_2 without affecting user 1. Why would such a property be desirable? The answer lies in the utility model that Nash assumed in his work: Nash assumed that the users' utility functions are *von Neumann-Morgenstern utilities*, and such utility functions are unique only up to a positive affine transformation [4], [10]. Obviously, if the utility of user 1 shows such an ambiguity, it is desirable to not have this ambiguity impact the outcome for user 2, and vice-versa.

We argue that the utility defined in (7) is unique, i.e., it is not invariant to positive affine transformations: The utility u_k as defined in (7) is the average of information rates, and there is a clear understanding that $r = 0$ means no information (thus we cannot choose b_k arbitrarily), and $r' = 100r$ means 100-fold more information (thus we cannot choose t_k arbitrarily). *Consequently, the axiom INV is not necessary for bargaining over rate regions.*

C. Maximum Sum Utility

From an axiomatic viewpoint, if the axiom INV is not needed, any solution $\boldsymbol{f}$ that fulfills (13) and the axioms WPO, SYM, and IIA is as good as the NBS. Consider a solution $\boldsymbol{f}^{\text{SU}}$ defined as follows:

$$\boldsymbol{f}^{\text{SU}}(\mathcal{U}, \boldsymbol{d}) = \boldsymbol{s}(\mathcal{M}(\mathcal{U}, \boldsymbol{d}), \boldsymbol{d}) \tag{14}$$

where

$$\mathcal{M}(\mathcal{U}, \boldsymbol{d}) = \operatorname{argmax} \mathbf{1}^{\text{T}}\boldsymbol{u} \quad \text{s.t.} \quad \boldsymbol{u} \in \mathcal{U}, \boldsymbol{u} \geq \boldsymbol{d}, \tag{15}$$

and $\boldsymbol{s}$ is a function satisfying

$$\boldsymbol{s}(\mathcal{M}, \boldsymbol{d}) \in \mathcal{M}, \tag{16}$$

$$\mathcal{M} = \mathcal{M}^{\text{P}}, d_1 = d_2 \Rightarrow s_1(\mathcal{M}, \boldsymbol{d}) = s_2(\mathcal{M}, \boldsymbol{d}). \tag{17}$$

The solution $\boldsymbol{f}^{\text{SU}}$ fulfills (13) and the axioms WPO, SYM, and IIA [10]. The function $\boldsymbol{s}$ is needed to select one maximizer if the set of maximizers has multiple elements. Property (17) ensures that the solution fulfills the SYM axiom.

Consider the problem

$$\max_{\boldsymbol{r}^1,\ldots,\boldsymbol{r}^L} \frac{1}{L}\sum_{l=1}^{L} \mathbf{1}^{\text{T}}\boldsymbol{r}^\ell \quad \text{s.t.} \quad \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell \geq \boldsymbol{d}, \boldsymbol{r}^\ell \in \mathcal{R}^\ell. \tag{18}$$

Let $\hat{\boldsymbol{r}} = (\hat{\boldsymbol{r}}^1, \ldots, \hat{\boldsymbol{r}}^L)$ denote a maximizer of (18). Define

$$\boldsymbol{u}^{\text{SU}} = \frac{1}{L}\sum_{l=1}^{L} \hat{\boldsymbol{r}}^\ell.$$

Clearly, $\boldsymbol{u}^{\text{SU}} \in \mathcal{M}(\mathcal{U}, \boldsymbol{d})$. The solution $\boldsymbol{f}^{\text{SU}}$ requires that $\boldsymbol{f}^{\text{SU}}(\mathcal{U}, \boldsymbol{d})$ is chosen from $\mathcal{M}$ by a function satisfying (17). Under the assumption that $\mathcal{M} \neq \mathcal{M}^{\text{P}}$ or $d_1 \neq d_2$, we can set

$$\boldsymbol{f}^{\text{SU}}(\mathcal{U}, \boldsymbol{d}) = \boldsymbol{u}^{\text{SU}}. \tag{19}$$

On the other hand, if $\mathcal{M} = \mathcal{M}^{\text{P}}$ and $d_1 = d_2$, we have to find the point in $\mathcal{M}$ with $u_1 = u_2$. This corresponds to computing the intersection between the line $\{x\mathbf{1}, x \in \mathbb{R}\}$ and $\mathcal{M}$. As all points in $\mathcal{M}$ are Pareto optimal, this corresponds to finding the maximum x such that $x\mathbf{1} \in \mathcal{U}$. The optimization problem to find the intersection point is thus formulated as

$$\max_{x,\boldsymbol{r}^1,\ldots,\boldsymbol{r}^L} x \quad \text{s.t.} \quad x\mathbf{1} \leq \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell, \boldsymbol{r}^\ell \in \mathcal{R}^\ell. \tag{20}$$

Let $\check{x}$ denote the optimum solution of (20). While it is simple to test whether $d_1 = d_2$, it is not obvious whether $\mathcal{M} = \mathcal{M}^{\text{P}}$. Still, if $\check{x}\mathbf{1} \in \mathcal{M}$, then $\check{x}\mathbf{1}$ achieves the same sum utility as $\boldsymbol{u}^{\text{SU}}$. Accordingly, one possible algorithm for computing a solution $\boldsymbol{f}^{\text{SU}}(\mathcal{U}, \boldsymbol{d})$ is specified by

$$\boldsymbol{f}^{\text{SU}}(\mathcal{U}, \boldsymbol{d}) = \begin{cases} \check{x}\mathbf{1}, \check{x}\mathbf{1}^{\text{T}}\mathbf{1} = \mathbf{1}^{\text{T}}\boldsymbol{u}^{\text{SU}}, \\ \boldsymbol{u}^{\text{SU}}, \text{otherwise.} \end{cases} \tag{21}$$

Similar to the NBS solution, problem (18) is non-causal, due to the sum over all blocks in the constraint. The objective function is decomposable: it is a sum of terms that are independent for each block. This property allows for a systematic causal approximation and represents the main motivation for considering the SU solution. Another solution that satisfies the axioms WPO, SYM, and IIA is the so-called egalitarian solution [13]. While the egalitarian solution has an interesting robustness property [13], it does not provide a decoupling in the objective.

D. A Special Case: Infinite Horizon and Symmetric Physical Layer

In general, the solutions $\boldsymbol{f}^{\text{NBS}}$ and $\boldsymbol{f}^{\text{SU}}$ will yield different outcomes to a given bargaining problem. There is an important special case, however, in which both solutions yield an identical outcome: The interaction between both players lasts for infinitely many blocks and the interference channel is statistically symmetric. We say that the channel is statistically symmetric if the probability density function p_H fulfills

$$p_H(\boldsymbol{h}_{1,1}, \boldsymbol{h}_{1,2}, \boldsymbol{h}_{2,1}, \boldsymbol{h}_{2,2}) = p_H(\boldsymbol{h}_{2,2}, \boldsymbol{h}_{2,1}, \boldsymbol{h}_{1,2}, \boldsymbol{h}_{1,1}).$$

Define the ergodic utility region $\bar{\mathcal{U}}$ and disagreement point $\bar{\boldsymbol{d}}$ as follows:

$$\bar{\mathcal{U}} = \lim_{L\to\infty} \mathcal{U}, \quad \bar{\boldsymbol{d}} = \lim_{L\to\infty} \boldsymbol{d}.$$

From the symmetry of the physical layer immediately follows that in the case of a statistically symmetric channel,

$$\bar{\mathcal{U}} = \bar{\mathcal{U}}^{\text{P}}, \quad \bar{d}_1 = \bar{d}_2.$$

In other words, the bargaining problem $(\bar{\mathcal{U}}, \bar{\boldsymbol{d}})$ is symmetric if the channel is statistically symmetric. As both $\boldsymbol{f}^{\text{NBS}}$ and $\boldsymbol{f}^{\text{SU}}$ fulfill the SYM axiom, we have

$$\boldsymbol{f}^{\text{NBS}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}}) = \boldsymbol{f}^{\text{SU}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}}).$$

As a result, if the channel is statistically symmetric and the interaction between the two players is long enough, it does not matter whether the players cooperate based on a Nash or sum utility strategy, as both yield the same result.

In addition, from the SYM axiom and (13) it follows that

$$\boldsymbol{f}^{\text{SU}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}}) = \gamma\bar{\boldsymbol{d}},$$

with $\gamma \geq 1$. Accordingly,

$$\boldsymbol{f}^{\text{SU}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}}) \in \operatorname*{argmax}_{\boldsymbol{u}\in\bar{\mathcal{U}}} \mathbf{1}^{\text{T}}\boldsymbol{u}.$$

In other words, the sum utility solution can be obtained without considering the constraint $\boldsymbol{u} \geq \bar{\boldsymbol{d}}$. Now let

$$\hat{\boldsymbol{r}}^\ell \in \operatorname*{argmax}_{\boldsymbol{r}^\ell\in\mathcal{R}^\ell} \mathbf{1}^{\text{T}}\boldsymbol{r}^\ell. \tag{22}$$

Define

$$\hat{\boldsymbol{u}} = \lim_{L\to\infty} \frac{1}{L}\sum_{\ell=1}^{L} \hat{\boldsymbol{r}}^\ell.$$

It follows that

$$\hat{\boldsymbol{u}} \in \operatorname*{argmax}_{\boldsymbol{u}\in\bar{\mathcal{U}}} \mathbf{1}^{\text{T}}\boldsymbol{u}.$$

Moreover, if the algorithm to compute $\hat{\boldsymbol{r}}^\ell$ is not biased towards one user, symmetry implies that $\hat{u}_1 = \hat{u}_2$. As a result,

$$\boldsymbol{f}^{\text{SU}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}}) = \hat{\boldsymbol{u}}.$$

Accordingly, in the symmetric case, both $\boldsymbol{f}^{\text{NBS}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}})$ and $\boldsymbol{f}^{\text{SU}}(\bar{\mathcal{U}}, \bar{\boldsymbol{d}})$ can be obtained in a causal manner by solving (22) in each block.

V. Causal Solutions

A causal solution denotes a solution which provides a rule to choose the rate $\boldsymbol{r}^\ell$ at time instant ℓ based on knowledge from the current and past time instants. Any such solution has to ensure that cooperation does not make any player worse-off than noncooperation, i.e., the solution has to guarantee that

$$\frac{1}{L}\sum_{l=1}^{L}\boldsymbol{r}^\ell \geq \boldsymbol{d}, \tag{23}$$

otherwise it would clearly be irrational of the players to cooperate.

A. Causal NBS

A straightforward causal solution is simply to ignore that the bargaining takes place over multiple blocks and compute the NBS for each block separately. The corresponding rates are maximizers of

$$\max_{\boldsymbol{r}^\ell\in\mathcal{R}^\ell}(r_1^\ell - d_1^\ell)(r_2^\ell - d_2^\ell) \quad \text{s.t.} \quad \boldsymbol{r}^\ell \geq \boldsymbol{d}^\ell. \tag{24}$$

Let $\hat{\boldsymbol{r}}^\ell$ denote a maximizer of (24). Clearly, (23) is satisfied, as

$$\boldsymbol{u}^{\text{CNBS}} = \frac{1}{L}\sum_{l=1}^{L}\hat{\boldsymbol{r}}^\ell \geq \frac{1}{L}\sum_{l=1}^{L}\boldsymbol{d}^\ell = \boldsymbol{d}.$$

Still, by considering each block separately, this solution does not take into account the rate allocation at the previous blocks. Consider the following set of inequalities:

$$\sum_{q=1}^{\ell}\boldsymbol{r}^q \geq \sum_{q=1}^{\ell}\boldsymbol{d}^q, \ell = 1,\dots,L. \tag{25}$$

Clearly, (25) implies (23). Re-arranging (25) yields

$$\boldsymbol{r}^\ell \geq \tilde{\boldsymbol{d}}^\ell, \ell = 1,\dots,L. \tag{26}$$

with

$$\tilde{\boldsymbol{d}}^\ell = \sum_{q=1}^{\ell}\boldsymbol{d}^q - \sum_{q=1}^{\ell-1}\boldsymbol{r}^q. \tag{27}$$

Note that with (26) and (27),

$$\tilde{\boldsymbol{d}}^\ell = \boldsymbol{d}^\ell - \left(\boldsymbol{r}^{\ell-1} - \tilde{\boldsymbol{d}}^{\ell-1}\right) \leq \boldsymbol{d}^\ell. \tag{28}$$

Equation (28) shows that the constraint in problem (24) is unnecessarily tight. Accordingly, we can modify problem (24) by replacing $\boldsymbol{d}^\ell$ by $\tilde{\boldsymbol{d}}^\ell$:

$$\max_{\boldsymbol{r}^\ell\in\mathcal{R}^\ell}(r_1^\ell - \tilde{d}_1^\ell)(r_2^\ell - \tilde{d}_2^\ell) \quad \text{s.t.} \quad \boldsymbol{r}^\ell \geq \tilde{\boldsymbol{d}}^\ell. \tag{29}$$

While the original version does not consider the rate allocations at the previous blocks, the modified version subtracts any surplus $\boldsymbol{r}^{\ell-1} - \tilde{\boldsymbol{d}}^{\ell-1}$ from the previous block from the disagreement point $\boldsymbol{d}^\ell$ of the current block.

Assume that (29) was solved for $\ell = 1,\dots,L$, thus the corresponding $\tilde{\boldsymbol{d}}^\ell$ are known. Then solving problem (29) for $\ell = 1,\dots,L$ is equivalent to solving the problem

$$\begin{aligned}&\max_{\boldsymbol{r}^1,\dots,\boldsymbol{r}^L}\frac{1}{L^2}\sum_{l=1}^{L}(r_1^\ell - \tilde{d}_1^\ell)(r_2^\ell - \tilde{d}_2^\ell)\\ &\text{s.t.} \quad \boldsymbol{r}^\ell \geq \tilde{\boldsymbol{d}}^\ell, \boldsymbol{r}^\ell \in \mathcal{R}^\ell, \ell = 1,\dots,L\end{aligned} \tag{30}$$

with $\tilde{\boldsymbol{d}}^\ell$ set to the previously obtained values. Comparing (30) with the non-causal NBS problem (11) shows that to obtain the modified causal NBS as an approximation of the noncausal NBS, the constraint on $\boldsymbol{r}^\ell$ is tightened, while in the objective function $\boldsymbol{d}^\ell$ is replaced by $\tilde{\boldsymbol{d}}^\ell$ and the order of multiplication and summation is reversed. Due to the significant modifications of the cost function, an assessment of the quality of this approximation other than numerical seems infeasible.

B. Causal Sum Utility

A causal approximation of the sum utility solution is found in two steps: First, the constraint (23) in (18) is replaced by the tighter constraint (26), yielding the problem:

$$\max_{\boldsymbol{r}^1,\dots,\boldsymbol{r}^L}\frac{1}{L}\sum_{l=1}^{L}\boldsymbol{1}^{\text{T}}\boldsymbol{r}^\ell \quad \text{s.t.} \quad \boldsymbol{r}^\ell \geq \tilde{\boldsymbol{d}}^\ell, \boldsymbol{r}^\ell \in \mathcal{R}^\ell, \ell = 1,\dots,L. \tag{31}$$

Second, by neglecting the dependency of $\tilde{\boldsymbol{d}}^\ell$ on $\boldsymbol{r}^1,\dots,\boldsymbol{r}^{\ell-1}$, problem (31) decouples into L independent problems, with the problem at the l-th time instant given by

$$\max_{\boldsymbol{r}^\ell\in\mathcal{R}^\ell}\boldsymbol{1}^{\text{T}}\boldsymbol{r}^\ell \quad \text{s.t.} \quad \boldsymbol{r}^\ell \geq \tilde{\boldsymbol{d}}^\ell. \tag{32}$$

Let $\hat{\boldsymbol{r}}^\ell$ denote a maximizer of (32) in the ℓ-th block. The resulting utility point $\boldsymbol{u}^{\text{CSU}} \in \mathcal{U}$ is given by

$$\boldsymbol{u}^{\text{CSU}} = \frac{1}{L}\sum_{l=1}^{L}\hat{\boldsymbol{r}}^\ell. \tag{33}$$

Let the set $\mathcal{L}$ contain the indices of the blocks in which the inequality constraints in (32) are inactive at the optimal rate $\hat{\boldsymbol{r}}^\ell$. In contrast to the causal approximations of the NBS, the causal sum utility solution (32) uses the same cost function as the non-causal one (18). As a result, if for all $\ell = 1,\dots,L$ the inequality constraints in (32) are inactive in the optimum, problems (18) and (32) are equivalent in the sense that the L maximizers of (32) also represent a maximizer of (18). As an immediate result, if $\mathcal{L} = \{1,\dots,L\}$, the point $\boldsymbol{u}^{\text{CSU}}$ is Pareto optimal. Note that $\tilde{\boldsymbol{d}}^\ell$ is smaller the larger the surplus from previous blocks. Thus, if channel realizations are such that a surplus can be accumulated over time, a good approximation can be expected.

VI. Computing the Optimal Rate Allocations

In this section, we briefly discuss how to compute the optimum rate allocations for the bargaining solutions presented in Sections IV and V by solving the corresponding optimization problems.

A. Nash Bargaining Solution

Let $\boldsymbol{u}^* = \boldsymbol{f}^{\text{NBS}}(\mathcal{U},\boldsymbol{d})$. From the definition of $\boldsymbol{f}^{\text{NBS}}$ and the assumption that there exists $\boldsymbol{u} \in \mathcal{U}$ with $\boldsymbol{u} \gg \boldsymbol{d}$, it follows that $\boldsymbol{u}^* \gg \boldsymbol{d}$.

We compute the optimum rate allocations $\hat{\boldsymbol{r}}^\ell, \ell = 1, \ldots, L$, of (11) by solving the following optimization problem:

$$\max_{\boldsymbol{s},\boldsymbol{r}^1,\ldots,\boldsymbol{r}^L} \sum_{k=1}^{K} \ln(s_k) \quad \text{s.t.} \quad \boldsymbol{s} \leq \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell - \boldsymbol{d}, \boldsymbol{r}^\ell \in \mathcal{R}^\ell, \tag{34}$$

with an extended definition of ln: $\ln(x) = -\infty, x \leq 0$ and slack variables $\boldsymbol{s} \in \mathbb{R}^2$. Problem (34) is solved by Lagrange duality. Define the Lagrangian as

$$L(\boldsymbol{s}, \boldsymbol{r}^1, \ldots, \boldsymbol{r}^L, \boldsymbol{\lambda}) = \sum_{k=1}^{2} \ln(s_k) + \boldsymbol{\lambda}^{\mathrm{T}}(\frac{1}{L}\sum_{l=1}^{L} \boldsymbol{r}^\ell - \boldsymbol{d} - \boldsymbol{s}), \tag{35}$$

with $\boldsymbol{\lambda} \in \mathbb{R}^2_+$. The dual function follows as

$$q(\boldsymbol{\lambda}) = -\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{d} + \sup_{\boldsymbol{s}} \sum_{k=1}^{2} \ln(s_k) - \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{s} + \frac{1}{L}\sum_{l=1}^{L} \max_{\boldsymbol{r}^\ell \in \mathcal{R}^\ell} \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{r}^\ell. \tag{36}$$

Accordingly, evaluating the dual function at $\boldsymbol{\lambda} \gg \boldsymbol{0}$ basically corresponds to solving L weighted sum rate maximization problems on the underlying MISO IFCs. This actually is the motivation for introducing the variables $\boldsymbol{s}$ in (34): All problems that directly involve $\mathcal{R}^\ell$ are weighted sum rate problems, and globally optimal solution methods for this problem are available, cf. Section II.

The dual solution is obtained by minimizing the dual function with respect to $\boldsymbol{\lambda} \geq \boldsymbol{0}$. We use an outer approximation method to obtain the dual optimizer $\boldsymbol{\lambda}^*$ [14]. A primal optimum rate allocation $\hat{\boldsymbol{r}} = (\hat{\boldsymbol{r}}^1, \ldots, \hat{\boldsymbol{r}}^L)$ is obtained by primal recovery, using the method described in [14]. Note that $\hat{\boldsymbol{r}}$ may only be achievable by time-sharing. Strong duality holds, thus the primal recovery yields the primal optimal solution – and implicitly performs a time-sharing in the case of time-sharing optimality.

B. Sum Utility Maximization

To find an optimum rate allocation of (18), we proceed similar to the NBS solution. Again, the solution is found via Lagrange duality. Define the Lagrangian of (18) as

$$L(\boldsymbol{r}^1, \ldots, \boldsymbol{r}^L, \boldsymbol{\lambda}) = \sum_{l=1}^{L} \boldsymbol{1}^{\mathrm{T}}\boldsymbol{r}^\ell + \sum_{l=1}^{L} \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{r}^\ell - \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{d} \tag{37}$$

for $\boldsymbol{\lambda} \geq \boldsymbol{0}$. Then the dual function follows as

$$q(\boldsymbol{\lambda}) = -\boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{d} + \sum_{l=1}^{L} \max_{\boldsymbol{r}^\ell \in \mathcal{R}^\ell} (\boldsymbol{1} + \boldsymbol{\lambda})^{\mathrm{T}}\boldsymbol{r}^\ell. \tag{38}$$

Evaluating the dual function at $\boldsymbol{\lambda} \geq \boldsymbol{0}$ corresponds to solving L weighted sum rate maximization problems on the underlying MISO IFCs, with weight $\boldsymbol{\mu} = \boldsymbol{1} + \boldsymbol{\lambda}$. The dual solution is again found by an outer linearization method, and a primal optimal rate allocation $\check{\boldsymbol{r}}$ is obtained by primal recovery. Note that [15] solves a similar problem in the context of MIMO broadcast channels.

Problem (20) can also be solved via Lagrange duality, for details, see, e.g., [16].

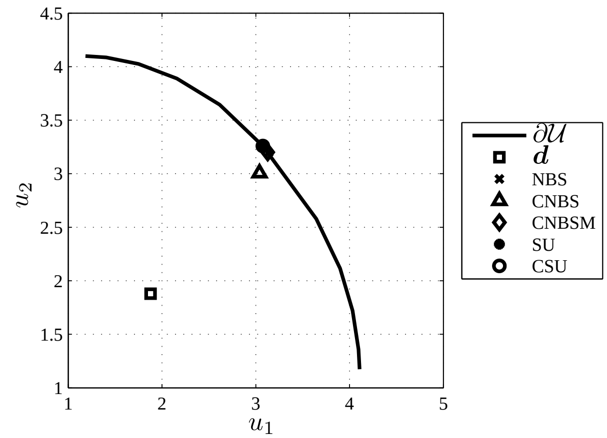

Figure 1. Bargaining solutions, $L = 500$, identical channel distributions

C. Causal NBS

The optimum rate allocation in the ℓ-th block under the causal NBS strategy is obtained analogous to the non-causal NBS solution, with $\boldsymbol{r}_1, \ldots, \boldsymbol{r}^L$ replaced by $\boldsymbol{r}^\ell$ and $\boldsymbol{d}$ replaced by $\tilde{\boldsymbol{d}}^\ell$, resulting in the Lagrangian

$$L(\boldsymbol{s}, \boldsymbol{r}^\ell, \boldsymbol{\lambda}) = \sum_{k=1}^{2} \ln(s_k) + \boldsymbol{\lambda}^{\mathrm{T}}(\boldsymbol{r}^\ell - \tilde{\boldsymbol{d}}^\ell - \boldsymbol{s}). \tag{39}$$

D. Causal Sum Utility

The optimum rate allocation in the ℓ-th block under the causal maximum sum utility strategy is obtained analogous to the non-causal solution, with $\boldsymbol{r}_1, \ldots, \boldsymbol{r}^L$ replaced by $\boldsymbol{r}^\ell$ and $\boldsymbol{d}$ replaced by $\tilde{\boldsymbol{d}}^\ell$, resulting in the Lagrangian

$$L(\boldsymbol{r}^\ell, \boldsymbol{\lambda}) = \boldsymbol{1}^{\mathrm{T}}\boldsymbol{r}^\ell + \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{r}^\ell - \boldsymbol{\lambda}^{\mathrm{T}}\tilde{\boldsymbol{d}}^\ell. \tag{40}$$

VII. NUMERICAL RESULTS

In order to investigate the performance of the causal solutions proposed in Section V, we compute the bargaining solutions for different number of blocks and different distributions of the channels $\boldsymbol{h}_{q,k}$. In all simulations, $P/\sigma^2 = 10$ and $N = 2$. Figure 1 shows the utility set $\mathcal{U}$, the disagreement point $\boldsymbol{d}$, and the corresponding bargaining solutions for $L = 500$ blocks. For each block, the channels $\boldsymbol{h}_{q,k}$ are drawn from a circularly symmetric Gaussian distribution with zero mean and covariance matrix $\boldsymbol{C}_{\boldsymbol{h}_{q,k}} = \boldsymbol{I}$. The solid line in Figure 1, labelled by $\partial\mathcal{U}$, corresponds to the boundary of $\mathcal{U}$. Shown are the following solutions: non-causal NBS (NBS), causal NBS (CNBS), modified causal NBS (CNBSM), non-causal sum utility maximization (SU), and causal sum utility maximization (CSU).

As expected, for large L and identical channel distributions, the utility set $\mathcal{U}$ is almost symmetric and $d_1 = d_2$. Moreover, NBS and SU almost coincide. The causal NBS solution is clearly not Pareto optimal. On the other hand, the modified causal NBS and the causal SU solution perform very close to the optimum non-causal solutions.

Figure 2 shows the boundary of $\mathcal{U}$, the disagreement point $\boldsymbol{d}$, and the corresponding bargaining solutions for $L = 10$ blocks.

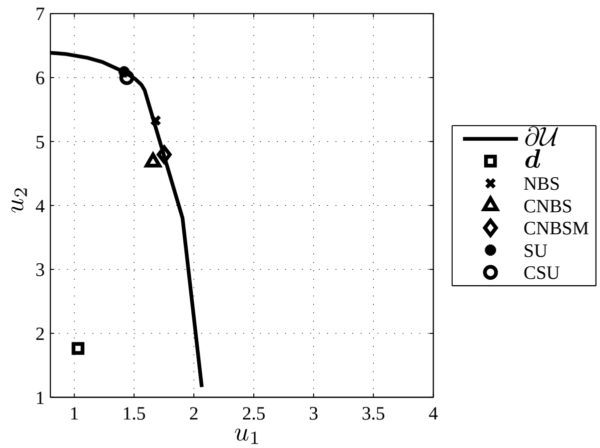

Figure 2. Bargaining solutions, $L = 10$, asymmetric scenario

For each block, the channel realizations are drawn from zero-mean Gaussian distributions with the following covariance matrices: $\boldsymbol{C}_{\boldsymbol{h}_{1,1}} = \boldsymbol{C}_{\boldsymbol{h}_{2,1}} = 0.5\boldsymbol{I}$, and $\boldsymbol{C}_{\boldsymbol{h}_{1,2}} = \boldsymbol{C}_{\boldsymbol{h}_{2,2}} = 5\boldsymbol{I}$.

In such a non-symmetric scenario, the solutions NBS and SU yield clearly different outcomes. Still, from an axiomatic viewpoint, they are equivalent – they are different, but both provide a gain compared to the disagreement point $\boldsymbol{d}$ and fulfill the axioms WPO, IIA, and SYM. The causal SU solution closely approximates the SU solution, although it is not strictly PO. The causal NBS is clearly not Pareto optimal. The modified causal NBS fares much better than the unmodified version. While the CNBSM point seems to lie on the Pareto boundary, it is not very close to the NBS solution – in other words, while being PO, it does not provide a good approximation of NBS.

To further investigate the quality of approximation provided by the causal strategies, Figure 3 shows the average relative distance between the solutions for different numbers of blocks L. The relative distances are computed as follows:

$$\frac{\left\|\boldsymbol{u}^{\mathrm{SU}} - \boldsymbol{u}^{\mathrm{CSU}}\right\|_2}{\left\|\boldsymbol{u}^{\mathrm{SU}}\right\|_2} \quad \text{(SU)}, \qquad \frac{\left\|\boldsymbol{u}^{\mathrm{NBS}} - \boldsymbol{u}^{\mathrm{CNBS}}\right\|_2}{\left\|\boldsymbol{u}^{\mathrm{NBS}}\right\|_2} \quad \text{(NBS)},$$
$$\frac{\left\|\boldsymbol{u}^{\mathrm{NBS}} - \boldsymbol{u}^{\mathrm{CNBSM}}\right\|_2}{\left\|\boldsymbol{u}^{\mathrm{NBS}}\right\|_2} \quad \text{(NBSM)}.$$

For each L, results are averaged over 900 realizations of $\mathcal{U}$. For each block, the channel realizations are drawn from a circularly symmetric Gaussian distribution with zero mean and covariance matrix $\boldsymbol{C}_{\boldsymbol{h}_{q,k}} = \boldsymbol{I}$. The causal solution obtained by simply computing the NBS at each block provides the worst approximation. Taking into account the surplus from previous blocks significantly improves the performance of the causal NBS. The causal sum utility strategy provides the best approximation, and its outcome is close to the optimum solution if the number of blocks is sufficiently large.

For the results in Figure 4, the channel realizations are drawn from zero-mean Gaussian distributions with the following covariance matrices: $\boldsymbol{C}_{\boldsymbol{h}_{1,1}} = \boldsymbol{C}_{\boldsymbol{h}_{2,1}} = 0.5\boldsymbol{I}$, and $\boldsymbol{C}_{\boldsymbol{h}_{1,2}} = \boldsymbol{C}_{\boldsymbol{h}_{2,2}} = 5\boldsymbol{I}$. Again, the block-wise NBS solution provides worst performance. The performance of the causal

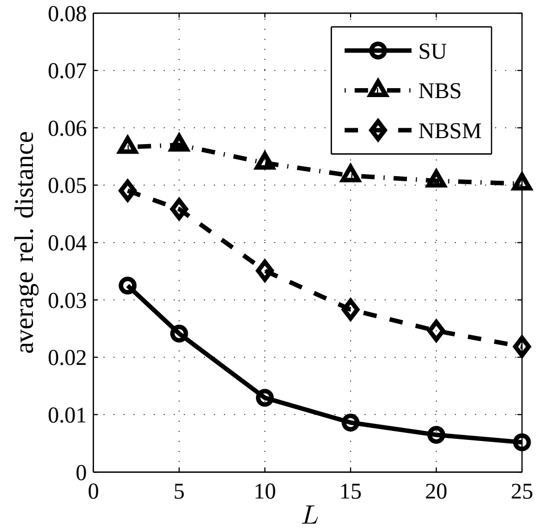

Figure 3. Average relative distances, identical channel distributions

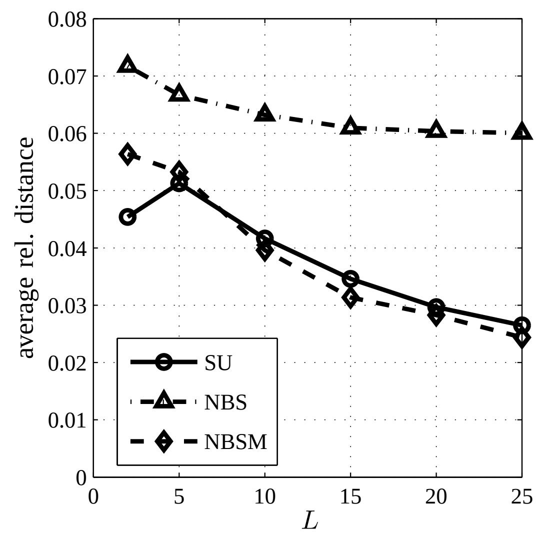

Figure 4. Average relative distances, asymmetric scenario

sum utility solution is considerably worse than in the symmetric scenario. As a result, the modified block-wise NBS and the causal sum utility provide a similar performance. The reduced performance of the causal sum utility solution can be explained as follows: Due to the strong asymmetry of the scenario, with much better channels to user 2, in many channel realizations sum rate is maximized by allocating a large rate to user 2 and only the minimal required rate $\tilde{d}_1^\ell$ to user 1. As a result, user 1 cannot accumulate a surplus over time, resulting in constraints $r_1^\ell \geq \tilde{d}_1^\ell$ that are much tighter than necessary.

VIII. Conclusions

We investigated the problem of bargaining over fading interference channels, where interaction between players takes place over multiple blocks, with each block corresponding to a channel realization. We showed that the Nash bargaining solution is non-causal. Based on the argument that an invariance axiom is not needed when bargaining over rate region, an equivalent solution to the bargaining problem is obtained by maximizing a sum of utilities under minimum utility constraints. This alternative solution is also non-causal. We showed that a causal solution can be obtained in the special case of infinite horizon and symmetric physical layer. To deal with the general case of a finite number of blocks and a nonsymmetric setup, we proposed causal approximations to the optimal strategies. For the NBS, the approximation is rather ad-hoc and basically corresponds to computing an NBS in each block. For the sum utility solution, an approximation is obtained by tightening the constraints on the rate allocation in each block. Thereby, the sum utility solution allows for a more systematic approximation than the NBS.

Numerical results show the quality of the proposed approximations. It turns out that simply computing the NBS separately in each block often leads to inefficient results. A modified version of the block-wise NBS that takes into account the surplus from previous blocks performs significantly better. In the statistically symmetric scenario, the causal approximation of the sum utility strategy outperforms the other causal strategies and yields an outcome close to the non-causal solution if the number of blocks is large. In the asymmetric scenario, the causal sum utility solution and the modified block-wise NBS solution provide similar performance.

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