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Receive Antenna Gain of Uniform Linear Arrays of Isotrops

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***Abstract*— The receive antenna gain of an antenna array critically depends on the receiver noise covariance. Receiver noise essentially originates from two sources: the receive low-noise amplifiers (LNA), and background radiation that is received by the array. In case that the LNAs are the sole origin of noise, it is state of the art to argue that the receiver noise is spatially uncorrelated, for noise originates in components of physically separate LNAs. However, this argument ignores coupling between antennas due to their spatial proximity, and moreover the coupling introduced by the impedance matching network which is located between the antenna outputs and the LNA inputs. Because of these coupling effects, the receiver noise is usually spatially correlated even when independent LNAs are the sole source of noise in the system. The noise covariance, depends on the following four factors: 1) antenna spacing of the array, 2) properties of the impedance matching network, 3) noise-resistance of the LNAs, 4) intensity of received background noise with respect to noise generated by the LNAs. Taking these issues into account, we derive, in this paper, the receive antenna gain of a uniform linear antenna array of isotrops in closed form. It turns out that the receive antenna gain can become much larger than the number of antennas. For the case of low noise-resistance, and low relative intensity of received background noise, we show that the receive antenna gain can even grow *exponentially* with the number of antennas. These results are exciting, because they imply that the performance of communication systems which use multiple receive antennas can be much better than previously reported.**

I. Introduction

When there are multiple antennas employed only at the receiver's side of the radio communication link, one speaks of a single-input multi-output (SIMO) radio communication system. In such a SIMO system, one can use the receive antenna array for beamforming. By linearly combining the received signals, one can achieve a higher average signal to noise ratio (ASNR) than is possible with just a single isotropic receive antenna. This effect is quantified by the so-called *receive antenna gain*.

The maximum receive antenna gain achievable by optimum beamforming, critically depends on the spatial covariance of the receiver noise. In case that this noise is spatially white, optimum beamforming coincides with the technique of maximum ratio combining [1]. It is well known that in this case the resulting ASNR equals the sum of the individual ASNRs at each antenna. Hence, for an antenna array with isotropic radiators, the maximum receive antenna gain under white noise equals the number of antennas.[1]

On the other hand, if the noise is spatially correlated, optimum beamforming is no longer equivalent to maximum ratio combining. The maximum receive antenna gain can be both smaller and larger than the number of isotropic antennas in the array. It depends on how much noise power is actually contained inside the algebraic sub-space where the desired signal resides within. The key question therefore is: »How does the noise covariance matrix look like?«. In order to answer this question, we have to look more closely into the sources of noise and the mechanisms of noise coupling.

In radio communication systems, it makes sense to distinguish between *intrinsic* noise, and *extrinsic* noise. While the latter comes from background radiation which is received by the antennas, the intrinsic noise originates from components of the radio frequency front end – most importantly, from the first stage of the low noise amplifier (LNA). When intrinsic noise is the sole source of noise, it is the state of the art to conjecture that the receiver noise is spatially white – or at least uncorrelated. However, this conjecture is usually not correct! While it is true that the noise sources inside each LNA generate noise independently, the individual noise contributions superimpose because of physical coupling effects. Such coupling occurs inside the antenna array because of the proximity of closely spaced antennas (near-field coupling). Coupling also occurs because of an impedance matching network that is frequently connected between the antenna array and the input of the LNAs. In this paper, we will see that because of these effects the receiver usually experiences *correlated* noise, even when it originates solely from within independent amplifiers.

An electronic LNA generates two kinds of noise: a *voltage* noise, and a *current* noise [2]. It turns out that current noise contributes quite differently to the noise covariance than does voltage noise. Therefore, the noise covariance depends on the relative intensity of these two kinds of noise contributions. It is specified by the so-called *noise-resistance*, which is defined as the ratio of the root-mean-square (RMS) noise voltage and the RMS noise current [2]. In this paper, we will see that, the lower the noise-resistance, the higher the receive antenna gain becomes. For the case of noise of purely intrinsic origin, a noise-resistance of zero turns out to let the receive antenna gain grow exponentially with the number of antennas. Even though a noise-resistance of zero – which is achieved only by thermal noise of a resistor – is not possible to achieve with a real electronic LNA, the result nevertheless unveils remarkable capabilities of receive antenna arrays, which result from their genuine noise coupling effects. This result is exciting, since it shows that the performance of SIMO radio communication systems can be much better than previously reported. In summary, the receive antenna gain depends on the following factors:

[1]Herein, the array dimensions are assumed to be small enough such that each antenna of the array receives the desired signal with the same average power. With spatially white noise, the ASNR is the same for each antenna.

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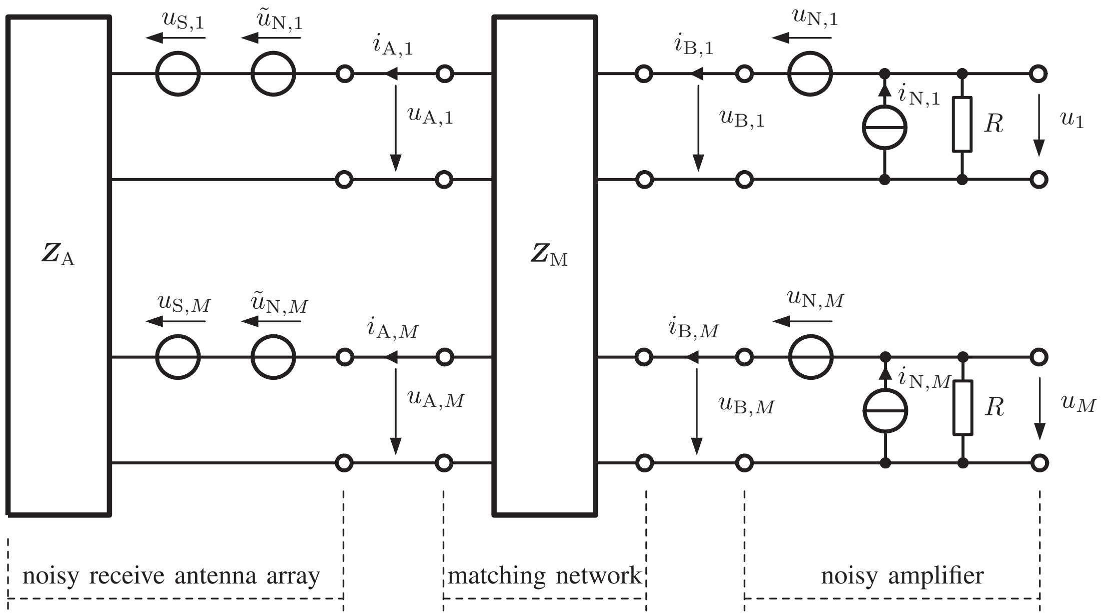


Figure 1: Circuit-theoretic model of a receive antenna array cascaded with an impedance matching network and input of a low-noise amplifier.

① near-field antenna coupling,
② impedance matching network,
③ noise-resistance of the LNA,
④ relative strength (and correlation) of extrinsic noise,
⑤ the direction of beamforming.

In this paper, we derive the noise covariance and the resulting receive antenna gain of a ULA of isotrops in closed form, making the following two assumptions:

① The signal bandwidth is very small compared to the carrier frequency and to the inverse of the time it takes for an electro-magnetic wave to propagate across the longest part of the antenna aperture (so-called narrow-band assumption). This assumption is met in existing mobile communication systems, like GSM, or UMTS.

② The impedance matching network is designed such as to compensate the imaginary part of the impedance matrix of the antenna array (so-called power-matching). While this may not be the best way to design a matching network, this approach has the advantage that we only need to know the real-part of the impedance matrix of the antenna array in order to compute the receive antenna gain. This is advantageous because for computing the real-part of the impedance matrix we do not have to go into complicated near-field calculations, but instead can find the solution from a far-field point of view.

II. Circuit-Theoretic System Model

We build our calculation of noise covariance on the circuit theoretic system model from Figure 1. It consists of 3 blocks: the receive array, the impedance matching network, and the input of the LNA. The M–antenna receive array is described as a passive, strictly-linear M–port, while the received signals are brought into the play by two voltage sources per port. The voltage $u_{S,i}$, with $i \in \{1, 2, \ldots, M\}$, models the desired received signal, while $\tilde{u}_{N,i}$ models the received background noise (extrinsic noise). Mutual coupling between the antennas is described by the impedance matrix $\boldsymbol{Z}_A \in \mathbb{C}^{M \times M} \times \mathrm{V/A}$, such that the vector $\boldsymbol{u}_A = (u_{A,1} \;\; u_{A,2} \;\; \cdots \;\; u_{A,M})^{\mathrm{T}}$, of the receive voltage envelopes can be written as:

$$\boldsymbol{u}_A = \boldsymbol{Z}_A \boldsymbol{i}_A + \boldsymbol{u}_S + \tilde{\boldsymbol{u}}_N, \tag{1}$$

where $\boldsymbol{i}_A = (i_{A,1} \;\; i_{A,2} \;\; \cdots \;\; i_{A,M})^{\mathrm{T}}$, denotes the vector of the envelopes of the electric currents that flow into the antennas, and the vectors $\boldsymbol{u}_S \in \mathbb{C}^{M \times 1} \times \mathrm{V}$, and $\tilde{\boldsymbol{u}}_N \in \mathbb{C}^{M \times 1} \times \mathrm{V}$, contain as its elements the voltage envelopes of the desired signal and the background noise, respectively. The received voltages $\boldsymbol{u}_A$ are then fed to the input ports of an impedance matching network, which operation is described by:

$$\begin{pmatrix} \boldsymbol{u}_A \\ \boldsymbol{u}_B \end{pmatrix} = \begin{pmatrix} -\mathrm{j}\,\mathrm{Im}\{\boldsymbol{Z}_A\} & \mathrm{j}\,\mathrm{Re}\{\boldsymbol{Z}_A\} \\ \mathrm{j}\,\mathrm{Re}\{\boldsymbol{Z}_A\} & \mathbf{O} \end{pmatrix} \begin{pmatrix} -\boldsymbol{i}_A \\ \boldsymbol{i}_B \end{pmatrix}. \tag{2}$$

The vectors $\boldsymbol{u}_B \in \mathbb{C}^{M \times 1} \times \mathrm{V}$, and $\boldsymbol{i}_B \in \mathbb{C}^{M \times 1} \times \mathrm{A}$, contain the voltage and current envelopes at the output ports of the matching network. From (1) and (2) follows:

$$\boldsymbol{u}_B = \mathrm{Re}\{\boldsymbol{Z}_A\}\, \boldsymbol{i}_B + \mathrm{j}\,(\boldsymbol{u}_S + \tilde{\boldsymbol{u}}_N), \tag{3}$$

from which the rationale behind our specific choice of the matching network becomes clear: the antenna coupling now depends only on the real-part of the impedance matrix of the antenna array. Moreover, the received voltages $\boldsymbol{u}_S$, and $\tilde{\boldsymbol{u}}_N$ appear essentially unchanged – merely shifted by 90° in phase – at the output of the matching network. The voltage envelopes in $\boldsymbol{u}_B$ are fed to the inputs of the LNAs. The latter are modeled by a noiseless resistor R, and two sources which introduce the amplifier's current and voltage noise envelopes, $u_{\mathrm{N},i}$, and $i_{\mathrm{N},i}$, respectively. When we collect these envelopes into the vectors $\boldsymbol{u}_\mathrm{N} \in \mathbb{C}^{M\times 1} \times \mathrm{V}$, and $\boldsymbol{i}_\mathrm{N} \in \mathbb{C}^{M\times 1} \times \mathrm{A}$, we can write $\boldsymbol{u}_\mathrm{B} = R\boldsymbol{i}_\mathrm{N} - \boldsymbol{u}_\mathrm{N} - R\boldsymbol{i}_\mathrm{B}$. Substituting this expression into (3), and solving for $\boldsymbol{i}_\mathrm{B}$, we find

$$\boldsymbol{i}_\mathrm{B} \;=\; R^{-1}\boldsymbol{Q}\left(R\boldsymbol{i}_\mathrm{N} - \boldsymbol{u}_\mathrm{N} - \mathrm{j}\tilde{\boldsymbol{u}}_\mathrm{N} - \mathrm{j}\boldsymbol{u}_\mathrm{S}\right), \qquad (4)$$

where the matrix $\boldsymbol{Q} \in \mathbb{C}^{M\times M}$ is defined as:

$$\boldsymbol{Q} \;=\; R\left(R\mathbf{I}_M + \mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}\right)^{-1}, \qquad (5)$$

and $\mathbf{I}_M$ is the $M \times M$ identity matrix. Note that $\boldsymbol{Q} = \boldsymbol{Q}^\mathrm{H}$, because $\boldsymbol{Z}_\mathrm{A} = \boldsymbol{Z}_\mathrm{A}^\mathrm{T}$ holds due to reciprocity of antennas. The vector of the output voltage envelopes $\boldsymbol{u} = (u_1 \;\; u_2 \;\; \cdots \;\; u_M)^\mathrm{T}$, can be written as $\boldsymbol{u} = R\boldsymbol{i}_\mathrm{N} - R\boldsymbol{i}_\mathrm{B}$. Together with (4) it therefore follows with $\mathbf{I}_M - \boldsymbol{Q} = R^{-1}\boldsymbol{Q}\,\mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}$, that:

$$\boldsymbol{u} \;=\; \boldsymbol{Q}\left(\mathrm{j}\boldsymbol{u}_\mathrm{S} + \mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}\,\boldsymbol{i}_\mathrm{N} + \boldsymbol{u}_\mathrm{N} + \mathrm{j}\tilde{\boldsymbol{u}}_\mathrm{N}\right). \qquad (6)$$

Clearly, the amplifier noise current $\boldsymbol{i}_\mathrm{N}$ contributes to the noise portion of $\boldsymbol{u}$ quite differently than the amplifier noise voltage $\boldsymbol{u}_\mathrm{N}$, or the background noise $\tilde{\boldsymbol{u}}_\mathrm{N}$. As we will see later, it makes a big difference in performance, whether current noise or voltage noise dominates the receiver noise.

III. Signal Model

In the following, let us consider only the case where the antenna array receives a single planar wave, as is the case in a line of sight (LOS) environment. The receive signal voltage envelope vector $\boldsymbol{u}_\mathrm{S}$ can then be written (see e.g. [3], page 135):

$$\boldsymbol{u}_\mathrm{S} \;=\; u_\mathrm{S}\cdot\underbrace{\left(1 \;\; \mathrm{e}^{-\mathrm{j}\mu} \;\; \mathrm{e}^{-2\mathrm{j}\mu} \;\; \cdots \;\; \mathrm{e}^{-(M-1)\mathrm{j}\mu}\right)^\mathrm{T}}_{\boldsymbol{a}(\theta)}, \qquad (7)$$

where $u_\mathrm{S} \in \mathbb{C} \times \mathrm{V}$, is the information bearing signal voltage envelope carried by the electro-magnetic wave, and

$$\mu = 2\pi\frac{\Delta}{\lambda}\cos\theta, \qquad (8)$$

where Δ is the distance between the antennas in the ULA, and λ is the wavelength of the carrier. The angle θ denotes the angle of arrival of the planar wave with respect of the array axis. By defining the channel vector:

$$\boldsymbol{h} \;=\; \mathrm{j}\boldsymbol{Q}\boldsymbol{a}(\theta) \;\in \mathbb{C}^{M\times 1}, \qquad (9)$$

with the array steering vector $\boldsymbol{a}(\theta)$ defined in (7), and further defining a receiver noise voltage vector

$$\boldsymbol{\eta} \;=\; \boldsymbol{Q}\left(\mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}\,\boldsymbol{i}_\mathrm{N} + \boldsymbol{u}_\mathrm{N} + \mathrm{j}\tilde{\boldsymbol{u}}_\mathrm{N}\right) \;\in \mathbb{C}^{M\times 1} \times \mathrm{V}, \quad (10)$$

we can rewrite the system equation (6) as:

$$\boldsymbol{u} \;=\; \boldsymbol{h}\cdot u_\mathrm{S} + \boldsymbol{\eta}. \qquad (11)$$

The receiver noise covariance matrix is then defined as:

$$\boldsymbol{R}_{\boldsymbol{\eta}} \;=\; \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^\mathrm{H}\right] \;\in \mathbb{C}^{M\times M} \times \mathrm{V}^2. \qquad (12)$$

In the following, we assume that

$$\mathrm{E}\left[\boldsymbol{i}_\mathrm{N}\boldsymbol{i}_\mathrm{N}^\mathrm{H}\right] \;=\; \beta\mathbf{I}_M, \qquad (13)$$

where $\beta \in \mathbb{R}_+ \times \mathrm{A}^2$, is a constant, specific to the LNA, and proportional to the noise bandwidth. All LNAs have the same current noise intensity, but the current noises of different LNAs are uncorrelated. We further assume

$$\mathrm{E}\left[\boldsymbol{u}_\mathrm{N}\boldsymbol{u}_\mathrm{N}^\mathrm{H}\right] \;=\; \beta' R^2\mathbf{I}_M, \qquad (14)$$

where $\beta' \in \mathbb{R}_+ \times \mathrm{A}^2$, is a constant, specific to the LNA, and proportional to the noise bandwidth. Furthermore, let

$$\mathrm{E}\left[\tilde{\boldsymbol{u}}_\mathrm{N}\tilde{\boldsymbol{u}}_\mathrm{N}^\mathrm{H}\right] \;=\; \tilde{\beta} R^2\boldsymbol{\Phi}, \qquad (15)$$

where $\tilde{\beta} \in \mathbb{R}_+ \times \mathrm{A}^2$, is a constant, that is proportional to the intensity of the received background noise, and $\boldsymbol{\Phi} \in \mathbb{C}^{M\times M}$ describes the spatial correlations. In general, $(\boldsymbol{\Phi})_{i,i} = 1$, for $i \in \{1, 2, \ldots, M\}$, because all antennas receive background noise with the same power. When we assume that all of the three noise sources above are uncorrelated, we find for the resulting receiver noise covariance:

$$\boldsymbol{R}_{\boldsymbol{\eta}} = \beta R^2\boldsymbol{Q}\underbrace{\left(\frac{1}{R^2}\left(\mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}\right)^2 + \frac{R_\mathrm{N}^2}{R^2}\mathbf{I}_M + \frac{\tilde{\beta}}{\beta}\boldsymbol{\Phi}\right)}_{\boldsymbol{\Upsilon}}\boldsymbol{Q}, \qquad (16)$$

where the *noise-resistance* is defined as:

$$R_\mathrm{N} = \sqrt{\frac{\mathrm{E}\left[|u_{\mathrm{N},i}|^2\right]}{\mathrm{E}\left[|i_{\mathrm{N},i}|^2\right]}} = R\sqrt{\frac{\beta'}{\beta}}. \qquad (17)$$

IV. Optimum Receive Beamforming

The receiver performs linear beamforming, according to:

$$s \;=\; \boldsymbol{w}^\mathrm{H}\boldsymbol{u} \;=\; \boldsymbol{w}^\mathrm{H}\boldsymbol{h}\cdot u_\mathrm{S} + \boldsymbol{w}^\mathrm{H}\boldsymbol{\eta}, \qquad (18)$$

where $\boldsymbol{w} \in \mathbb{C}^{M\times 1}$ is the beamforming vector, and $s \in \mathbb{C} \times \mathrm{V}$ is the resulting scalar voltage envelope. The signal to noise ratio after beamforming is given by

$$\mathrm{SNR} \;=\; \mathrm{E}\left[|u_\mathrm{S}|^2\right]\frac{\boldsymbol{w}^\mathrm{H}\boldsymbol{h}\boldsymbol{h}^\mathrm{H}\boldsymbol{w}}{\boldsymbol{w}^\mathrm{H}\boldsymbol{R}_{\boldsymbol{\eta}}\boldsymbol{w}}. \qquad (19)$$

Optimum beamforming results in maximization of this SNR, which leads to the well known solution $\boldsymbol{w}_\mathrm{opt} = \boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{h}$ (see e.g. [3]). Hence, the maximum achievable SNR becomes:

$$\mathrm{SNR}_\mathrm{max} \;=\; \mathrm{E}\left[|u_\mathrm{S}|^2\right]\,\boldsymbol{h}^\mathrm{H}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{h}. \qquad (20)$$

When we substitute (9) and (16) into (20) we arrive at:

$$\mathrm{SNR}_\mathrm{max} \;=\; \mathrm{E}\left[|u_\mathrm{S}|^2\right]\,\frac{\boldsymbol{a}^\mathrm{H}(\theta)\boldsymbol{\Upsilon}^{-1}\boldsymbol{a}(\theta)}{\beta R^2}, \qquad (21)$$

where the matrix $\boldsymbol{\Upsilon} \in \mathbb{C}^{M\times M}$ is defined in (16).

V. The Real-Part of the Impedance Matrix

In order to compute the optimum SNR we still need to know the real-part of $\boldsymbol{Z}_\mathrm{A}$. This can be computed in the following way. Suppose that the array does not receive any signal, such that both $u_{\mathrm{S},i} \equiv 0$, and $\tilde{u}_{\mathrm{N},i} \equiv 0$, for all antennas. When in this case a current $\boldsymbol{i}_\mathrm{A}$ flows through the antennas, the array actually becomes a transmitter rather than a receiver. As a result, an electric field is excited by the array. In a distance r, well in the far-field, and a direction θ, the magnitude of the electric field becomes (see e.g., [4] on pp 250 and 258):

$$E = \alpha \frac{\mathrm{e}^{-\mathrm{j}2\pi r/\lambda}}{r} \boldsymbol{a}^\mathrm{T}(\theta)\boldsymbol{i}_\mathrm{A}, \tag{22}$$

where α is a constant. The power density is then given by (see e.g., [5], eq. (8) on page 571):

$$\begin{aligned} S &= \alpha' \,\mathrm{E}\left[|E|^2\right] \\ &= \frac{\alpha' |\alpha|^2}{r^2} \mathrm{E}\left[\boldsymbol{i}_\mathrm{A}^\mathrm{H} \boldsymbol{a}^*(\theta)\boldsymbol{a}^\mathrm{T}(\theta)\boldsymbol{i}_\mathrm{A}\right], \end{aligned} \tag{23}$$

where $\alpha' > 0$ is another constant. The radiated power is then obtained by integrating the power density over a sphere in the far-field around the array:

$$\begin{aligned} P_\mathrm{rad} &= \iint_\text{sphere} S \,\mathrm{d}A \\ &= \underbrace{4\pi\alpha' |\alpha|^2}_{R_0} \mathrm{E}\left[\boldsymbol{i}_\mathrm{A}^\mathrm{H} \underbrace{\left(\frac{1}{2}\int_0^\pi \boldsymbol{a}^*(\theta)\boldsymbol{a}^\mathrm{T}(\theta)\sin(\theta)\,\mathrm{d}\theta\right)}_{\boldsymbol{C}} \boldsymbol{i}_\mathrm{A}\right]. \end{aligned} \tag{24}$$

On the other hand, the electric power put into the array is given by:

$$P_\mathrm{in} = \mathrm{E}\left[\mathrm{Re}\{\boldsymbol{i}_\mathrm{A}^\mathrm{H}\boldsymbol{u}_\mathrm{A}\}\right]. \tag{25}$$

When we substitute (1) into (25), set $\boldsymbol{u}_\mathrm{S} = \tilde{\boldsymbol{u}}_\mathrm{N} = \boldsymbol{0}$, and recall that, due to reciprocity of antennas, $\boldsymbol{Z}_\mathrm{A} = \boldsymbol{Z}_\mathrm{A}^\mathrm{T}$, we find

$$P_\mathrm{in} = \mathrm{E}\left[\boldsymbol{i}_\mathrm{A}^\mathrm{H}\,\mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\}\,\boldsymbol{i}_\mathrm{A}\right]. \tag{26}$$

It is reasonable to assume that the antennas in the array are *lossless*. Then the principle of conservation of energy demands that $P_\mathrm{in} = P_\mathrm{rad}$. Comparison of (26) with (24) shows that

$$\mathrm{Re}\{\boldsymbol{Z}_\mathrm{A}\} = R_0 \boldsymbol{C}, \tag{27}$$

where both the coupling matrix $\boldsymbol{C} \in \mathbb{R}^{M\times M}$, and the radiation resistance $R_0 \in \mathbb{R}_+ \times \mathrm{V/A}$, are defined in (24). With the array steering vector defined in (7), we obtain from (24) by standard integration:

$$(\boldsymbol{C})_{m,n} = \mathrm{sinc}\left(2\pi\frac{\Delta}{\lambda}(m-n)\right), \tag{28}$$

where

$$\mathrm{sinc}(x) = \begin{cases} \dfrac{\sin x}{x} & \text{for } x \neq 0, \\ 1 & \text{for } x = 0. \end{cases} \tag{29}$$

VI. The Receive Antenna Gain

If only a single isotropic radiator is present, we can see from (21) applied for $M = 1$, that the SNR becomes

$$\mathrm{SNR}_\mathrm{single} = \frac{\mathrm{E}\left[|u_\mathrm{S}|^2\right]}{\beta\left(R_0^2 + R_\mathrm{N}^2\right) + R^2\tilde{\beta}}, \tag{30}$$

as (16) returns $\Upsilon = (R_0^2 + R_\mathrm{N}^2)/R^2 + \tilde{\beta}/\beta$, for $M = 1$, as $(\boldsymbol{\Phi})_{i,i} = 1$, per definition. The receive antenna gain A_Rx, then quantifies how much more SNR we can obtain by using all antennas of the array, compared to a single antenna of the same type:

$$\begin{aligned} A_\mathrm{Rx} &= \frac{\mathrm{SNR}_\mathrm{max}}{\mathrm{SNR}_\mathrm{single}} && (31) \\ &= \boldsymbol{a}^\mathrm{H}(\theta)\boldsymbol{\Upsilon}^{-1}\boldsymbol{a}(\theta)\left(\frac{R_0^2 + R_\mathrm{N}^2}{R^2} + \frac{\tilde{\beta}}{\beta}\right). && (31\mathrm{a}) \end{aligned}$$

In the following, we set the input resistance R of the LNAs to be equal to the radiation resistance R_0 of the isotropic antenna. As $\boldsymbol{a}^\mathrm{H}(\theta)\boldsymbol{a}(\theta) = M$, we find for the receive antenna gain:

$$A_\mathrm{Rx} = M\frac{\boldsymbol{a}^\mathrm{H}(\theta)\boldsymbol{\Upsilon}^{-1}\boldsymbol{a}(\theta)}{\boldsymbol{a}^\mathrm{H}(\theta)\boldsymbol{a}(\theta)}\left(1 + \frac{R_\mathrm{N}^2}{R_0^2} + \frac{\tilde{\beta}}{\beta}\right). \tag{32}$$

With the result (27), we have from (16) with $R = R_0$:

$$\boldsymbol{\Upsilon} = \boldsymbol{C}^2 + \frac{R_\mathrm{N}^2}{R_0^2}\mathbf{I}_M + \frac{\tilde{\beta}}{\beta}\boldsymbol{\Phi}. \tag{33}$$

Note that $\boldsymbol{\Upsilon}$ depends on the antenna spacing Δ/λ, by virtue of the matrix $\boldsymbol{C}$. Hence, the receive antenna gain depends on:

① number of antennas: M,
② antenna spacing: Δ/λ,
③ relative noise-resistance: R_N/R_0,
④ relative intensity of background noise: $\tilde{\beta}/\beta$,
⑤ correlation of background noise: $\boldsymbol{\Phi}$,
⑥ direction of beamforming: θ.

VII. Numerical Evaluation and Discussion

Let us now have a look how much receive antenna gain can be obtained by a ULA of isotrops. To keep it simple, we assume that the background radiation arrives at the array from all angles with equal intensity. The correlation matrix $\boldsymbol{\Phi}$ is therefore given by [6]:

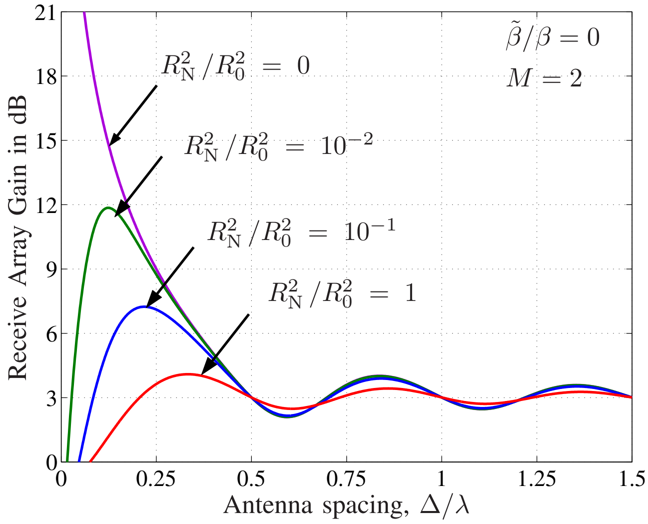
$$\boldsymbol{\Phi} = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{\pi} \boldsymbol{a}^*(\theta)\boldsymbol{a}^\mathrm{T}(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{34}$$

With the definition of the steering vector from (7), it follows from (34) by standard integration that

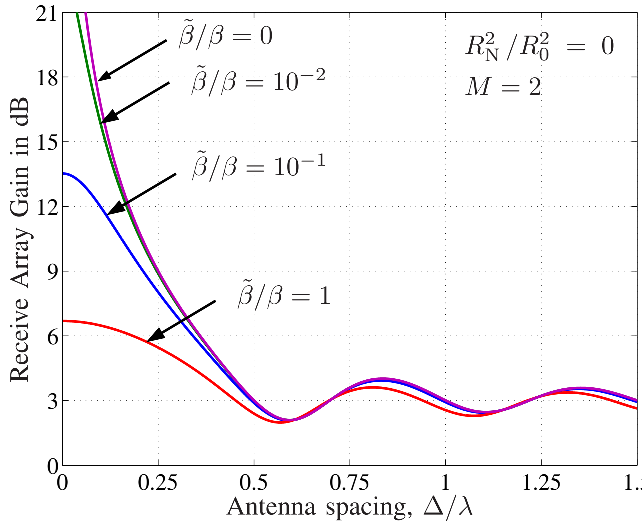
$$(\boldsymbol{\Phi})_{m,n} = \mathrm{J}_0\left(2\pi\frac{\Delta}{\lambda}(m-n)\right), \tag{35}$$

where $\mathrm{J}_0(\cdot)$ is the Bessel function of the first kind, and zero-th order. Note that $(\boldsymbol{\Phi})_{i,i} = 1$, as desired.

In the following, we fix the angle θ of the beamforming to the value of $\theta = 0$, which corresponds to the direction of the ULA line-up – the so-called »end-fire« direction. Let us start the numerical evaluation by looking at a (theoretical) scenario of zero background noise. That is, all noise origins



(a) Dependence on noise-resistance



(b) Dependence on isotropic background noise

Figure 2: Receive antenna gain in dB for two antennas, and *zero* background noise, as function of antenna spacing. The remaining parameters are $\theta = 0$, $R = R_0$.

from the LNAs. For a small array of $M = 2$ isotrops, we see in Figure 2a, the receive antenna gain in dB, as function of antenna separation. As was pointed out earlier, current noise contributes differently to the overall receiver noise than does voltage noise. Figure 2a therefore contains four different curves, which correspond to different values of the noise resistance. A noise resistance of zero, means that the noise originates solely from current noise. This is the case for thermal noise of a resistor. One can therefore argue that zero noise resistance corresponds to the case of having an *ideal* amplifier, which does not introduce any additional noise besides the inescapable thermal noise of the real-part of its input resistance. Hence, as can be seen from the top most curve in Figure 2a, with ideal amplifiers and no reception of background radiation, the receive antenna gain grows unboundedly, as the distance of the antennas is reduced towards zero. However, as soon as the noise resistance is even slightly greater than zero, the receive antenna gain first grows towards a maximum value as the antenna separation is reduced, and then begins to drop. We can see from Figure 2a, that the peak receive antenna gain increases with decreasing noise resistance. For high antenna gain it is therefore important, that the LNAs are designed such that current noise dominates over voltage noise. A low noise resistance is therefore more important than a low noise figure! Note that for an antenna separation of half of the wavelength, or integer multiples thereof, the receive antenna gain always equals the number of antennas. This effect comes about, because the antennas cease to be coupled for that specific separations.

Let us now have a look at what happens when we allow some amount of isotropic background radiation to be received by the array. For the case of zero noise resistance – that is, for ideal amplifiers which contribute only thermal noise in the real-part of their input resistance – Figure 2b shows the resulting receive antenna gain in dB for a small array of $M = 2$ isotrops. Interestingly, the supreme receive antenna gain now occurs for infinitesimal antenna separation, regardless of the relative intensity of the background radiation. This intensity only affects the value of the supreme antenna gain. As can be seen from Figure 2b, the supreme antenna gain reduces as the background noise gains importance.

It is interesting to study the behavior of receive antenna gain when the number of antennas is increased. Figure 3 shows the results for different noise scenarios and a fixed antenna spacing $\Delta = 0.35\lambda$. Let us start with the top most curve, which is obtained for the case of ideal amplifiers, and no reception of background radiation. Apparently, the receive antenna gain grows *exponentially* with the number of antennas! The second curve from the top shows what happens when we allow reception of some background radiation, while still keeping our amplifiers ideal. As we can observe, the receive antenna gain starts out growing exponentially with the number of antennas, but at a certain number (in this case around 4 or 5 antennas), begins to flatten out towards a linear growth. For the third curve from the top, we leave the ideal amplifiers, and allow some positive noise resistance. Clearly, the effect of flattening out of the exponential growth is now more pronounced. The more dominant background radiation becomes, and/or the more dominant amplifier voltage noise becomes with respect to the amplifier current noise, the less receive antenna gain one obtains, and the less super-linear it grows with the number of antennas. What we can learn from Figure 3, is that even though exponential growth of receive antenna gain with the number of antennas is possible, such growth is highly sensitive to the presence of background noise and noise properties of the LNAs, such that it may not be usable in practice. Nevertheless, a moderate increase of the receive antenna gain over the number of antennas should be practical. In the case $\tilde{\beta}/\beta = R_N^2/R_0^2 = 0.1$, we can still obtain 10.5dB receive an-

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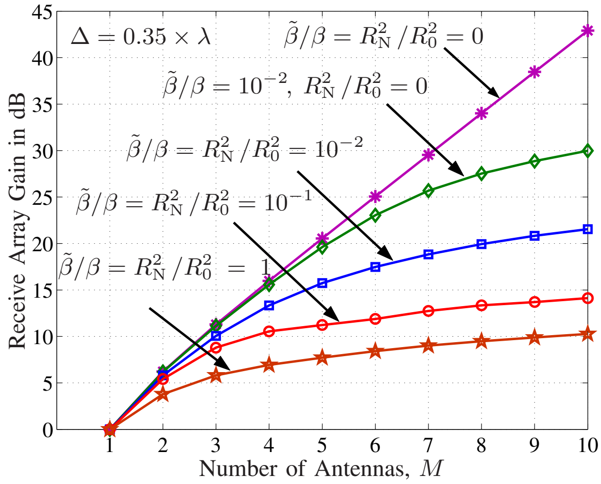

Figure 3: The receive antenna gain in dB, as function of the number of antennas for several amounts of background noise and different noise resistance. Here: $\theta = 0$, and $R = R_0$.

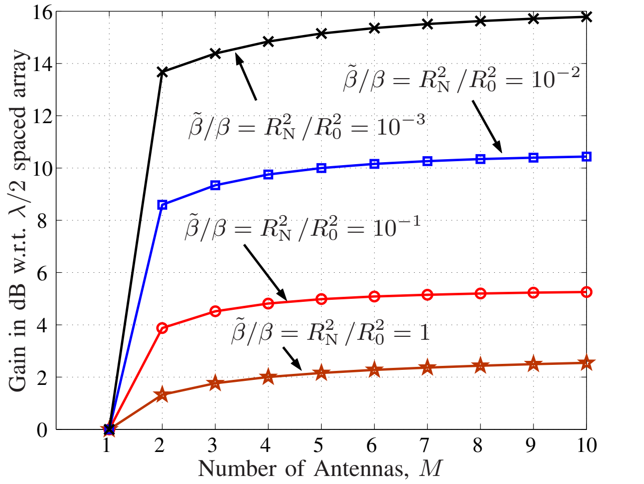

Figure 5: Increase of receive antenna gain in dB, of optimally spaced antennas, compared to half wavelength spacing. Here: $\theta = 0$, and $R = R_0$.

tenna gain from just 4 antennas. This is 4.5dB more than the usually expected 6dB gain!

We have already seen that the amount of achievable receive antenna gain depends on the noise scenario, and on antenna spacing. It therefore makes sense to optimize the antenna spacing for maximum receive antenna gain. Figure 4 shows the results. We can observe that the optimum spacing is always below half of the wavelength. However, the larger the number of antennas in the array, the more close the spacing should be to half the wavelength. On the other hand, the less background noise is received, and/or the more amplifier current noise dominates over amplifier voltage noise (low LNA noise-resistance), the more closely should the antennas be spaced.

Suppose now the antenna separation is chosen optimally for receive antenna gain, how much more antenna gain can be achieved compared to a standard half-wavelength antenna spacing? The result is shown in Figure 5. While the increase in receive antenna gain grows with more antennas, most of the possible increase in gain is achieved by $M = 2$ antennas. This property makes a two element receive antenna array a promising candidate for radio communication SIMO systems, for it combines high relative gain in performance, with low number of radio frequency frontends and low required space. All presented results qualitatively hold true also for arrays of Hertzian dipoles [7].

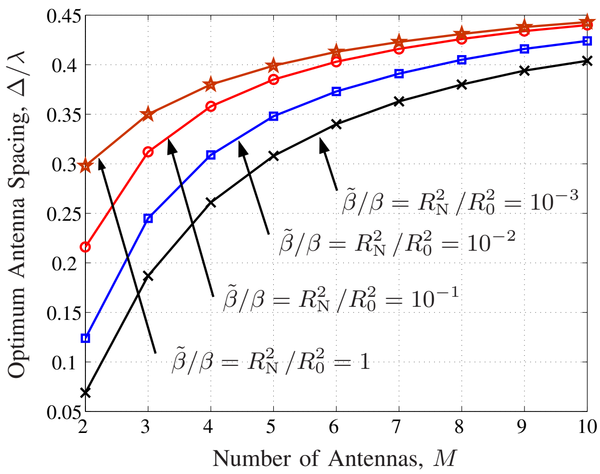

Figure 4: Optimum antenna spacing as function of number of antennas in different noise scenarios.

VIII. Conclusion

The receive antenna gain depends on the following four key factors: 1) number of antennas, their radiation characteristics and spacing, 2) relative intensity and correlation of received background radiation, 3) noise-resistance of the receive amplifiers, and 4) direction of beamforming. In order to obtain large antenna gain, the amplifier current noise should dominate over its voltage noise and the received background radiation. Large receive antenna gains are possible, even increasing exponentially with the number of antennas. Therefore, it is theoretically possible that channel capacity of wireless SIMO systems grows linearly with the number of antennas – a property which has up to now been ascribed only to multi-input multi-output (MIMO) systems. However, such exponential growth is sensitive to noise-resistance and background noise. Moderate super-linear growth might be implementable, though.

References

[1] W.C. Jakes Jr., *Mobile Microwave Communication*, New York: Wiley, 1974.

[2] M. J. Buckingham, *Noise in Electronic Devices and Systems*, New York: John Wiley & Sons Ltd., 1983.

[3] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing*, Prentice Hall, 1993.

[4] A. Balanis, *Antenna Theory*, Second Edition, John Wiley & Sons, 1997.

[5] D. Kraus, *Electromagnetics*, Fourth Edition, McGraw-Hill, 1992.

[6] M.T. Ivrlač, *Wireless MIMO Systems – Models, Performance, Optimization*, Shaker Verlag, Aachen, 2005.

[7] H. Yordanov, M. T. Ivrlač, P. Russer, and J. A. Nossek, "Arrays of Isotropic Radiators – A Field-theoretic Justification," in *Proc. ITG/IEEE Workshop on Smart Antennas*, Berlin, Germany, Feb. 2009.