Design of Cross Laminated Timber (CLT)

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Summary

This paper is focused on calculation and design methods for Cross Laminated Timber (CLT) elements. The influence of the shear deformation of the cross layers and the resulting consequences to the load bearing behaviour is explained. The main focus lies in the stresses caused by concentrated loads. Normal and shear stress distribution in the area of a concentrated load is calculated and evaluated for uniaxial spanned systems according to the shear analogy and by an FEM-calculation with shell elements. The differences of the results are explained. A simplified method shows how to consider the influence of the shear deformation on the longitudinal strain of a simply supported beam stressed by a concentrated load. Furthermore theoretical considerations and experimental investigations into the twisting stiffness of CLT-elements and its influence on load bearing behaviour are presented.

1. Introduction

Advanced manufacturing technologies of modern timber construction enable the fabrication and use of large-sized plane elements. Beside timber frame construction with its high level of prefabrication the market share of CLT-constructions is constantly increasing. The cross section of CLT-elements is generally characterised by at least three glued board layers with orthogonally alternating orientation of neighboured layers. The load bearing behaviour of these plane elements is affected by the material and, due to the orthogonal orientation of the single layers, by the constructive anisotropy. Moreover the shear deformation including the ductile composite must be considered.

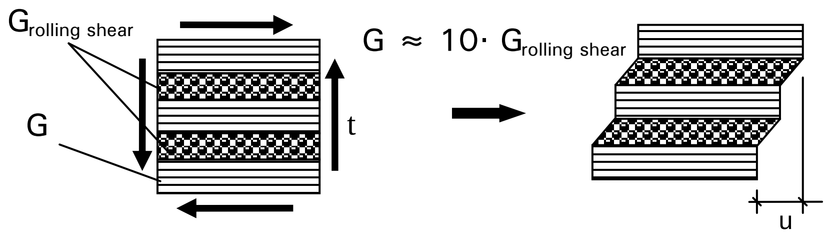

Fig. 1 Shear deformation of a CLT-element

Fig. 1 shows an element under shear stress resulting from load perpendicular to the plane. The cross layers are strained by rolling shear. Due to the low rolling shear stiffness, the deformation of the total section is caused almost exclusively by the deformation of the cross layers. The deformation figure infringes the Bernoulli hypothesis, which says, that straight lines perpendicular to the mid-surface before deformation remain straight after deformation. This means that under certain circumstances the ductile composite of the layers in each direction has to be considered.

2. Calculation Model and Design

For load perpendicular to the plane, the modelling of CLT-elements can be carried out on the basis of the composite theory. It covers the strength and the stiffness of each single layer and enables the modelling of any cross section. The stress distribution results from the stiffness of the single layers. The basic approach is described in [1] and is listed in annex D of the DIN 1052:2004-08 [2].

Each case must be considered carefully in order to decide if the shear deformation has to be taken into account. One approach to the solution of this problem is the method of shear analogy by Kreuzinger [3] which is also listed in annex D of the DIN 1052 [2]. By the use of current plane frame programmes it enables to calculate CLT-elements including the ductile composite of the layers each direction.

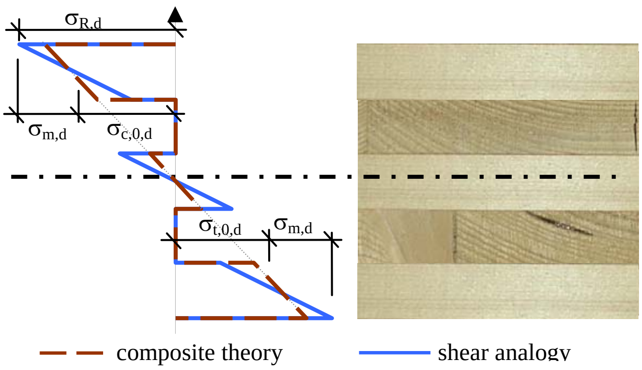

Fig. 2 Distribution of longitudinal stresses

Fig. 2 shows the normal stress distribution according to the composite theory and the shear analogy. Due to the ductile composite the stresses in the centre lines ($\sigma_{c,0,d}$ / $\sigma_{t,0,d}$) of the layers decreases while the fraction of bending stress ($\sigma_{m,d}$) increases. Consequently the values of the edge stress – relevant for design - increase.

Comment: According to [2], the elastic modulus has to be set to zero if the single boards are not edge-glued.

The verifications for load perpendicular to the plane are to be found in chapter 10.7 of the DIN 1052 [2]. Following verifications in each layer are required:

- Longitudinal stress: $$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} \leq 1 \qquad (1)$$

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} \leq 1 \qquad (2)$$

Comment: The new technical approvals only require the verification of the edge stress.

$$\frac{\sigma_{c/t,0,d} + \sigma_{m,d}}{f_{m,d}} \leq 1 \qquad (3)$$

- Shear stress: $$\left(\frac{\tau_d}{f_{v,d}}\right)^2 + \left(\frac{\tau_{drill,d}}{f_{v,d}}\right)^2 \leq 1 \qquad (4)$$

- Combined: $$\frac{\sigma_{t,90,d}}{f_{t,90,d}} + \frac{\tau_{R,d}}{f_{R,d}} \leq 1$$ tensile stress perp. + rolling shear (5)

$$\frac{\sigma_{c,90,d}}{f_{c,90,d}} + \frac{\tau_{R,d}}{f_{R,d}} \leq 1$$ compressive stress perp. + rolling shear (6)

Several studies have shown the negligible influence of the shear deformation generated by the cross layers on the stress distribution of a simply supported beam with uniformly distributed load. The composite theory delivers sufficiently exact result for ratios of span to thickness of the CLT-elements higher than 20. So the shear deformation can be neglected within these boundary conditions.

3. Concentrated Load

3.1 Shell-model – Shear Analogy

Following a simply supported five-layer CLT-element with a width of 1.0m is stressed by a concentrated load in the middle of the span. The thickness of each layer is 30 mm and the lamellas have the material characteristics of C24. The calculation is done by using a simplified two dimensional model.

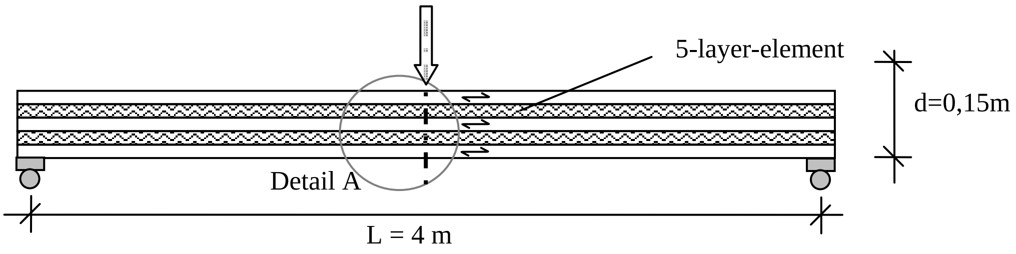

Fig. 3 System

The system was calculated with a finite element programme, using quadratic shell elements. The elements were manually cross linked. The longitudinal and shear stresses were analysed in a series of vertical sections next to the concentrated load. Simultaneously the stress distribution in the same vertical sections was calculated on the simplified model according to the shear analogy. The following figure 4 shows the results of both calculation methods. For a clear demonstration a non-scale division is used.

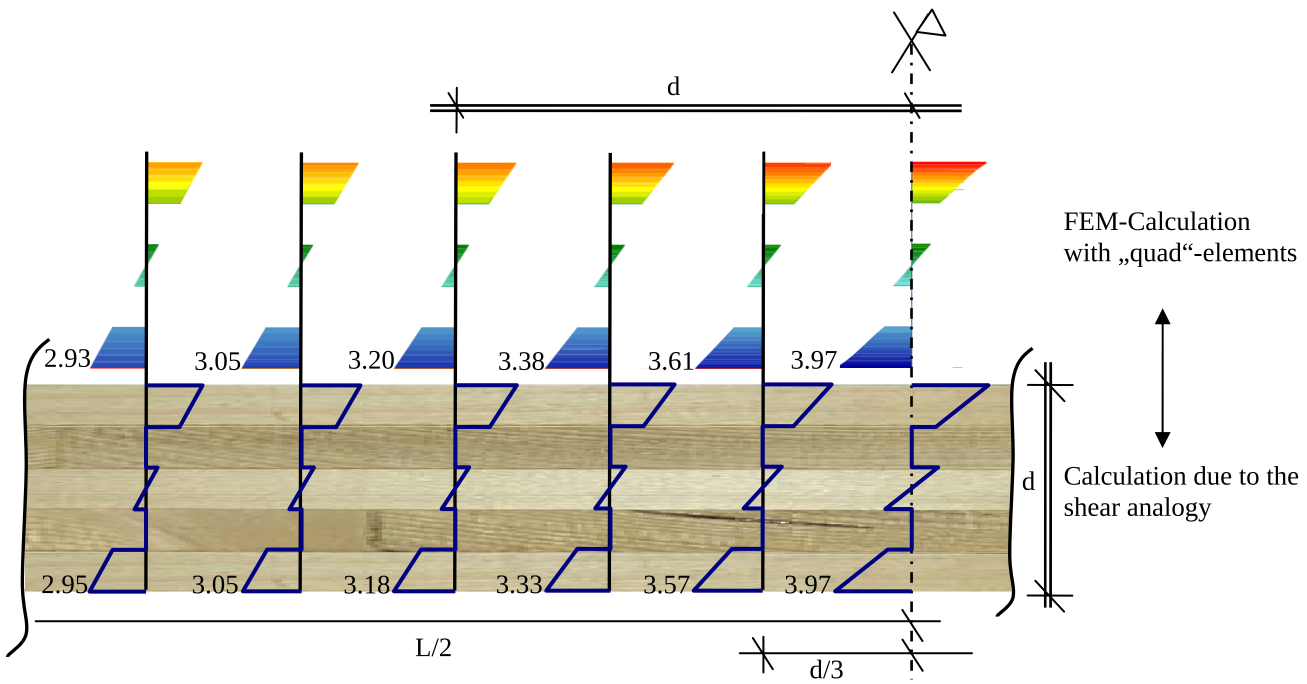

Fig. 4 Detail A: distribution of longitudinal stresses in several sections [N/mm^2]

The deviation of the results lies at about two percent. Immediately next to the concentrated load the influence of the shear deformation of the cross layers is obvious. So the stress distribution differs from a straight through the centre line. The Bernoulli Hypothesis of a plane section is not fulfilled - consequently the edge stress reaches stress peaks in that area, but their influence decreases rapidly. In the outmost section (about 5/3·d distance to the centre of the system) the stress distribution already shows an almost plane gradient, so that the composite theory delivers sufficiently precise results.

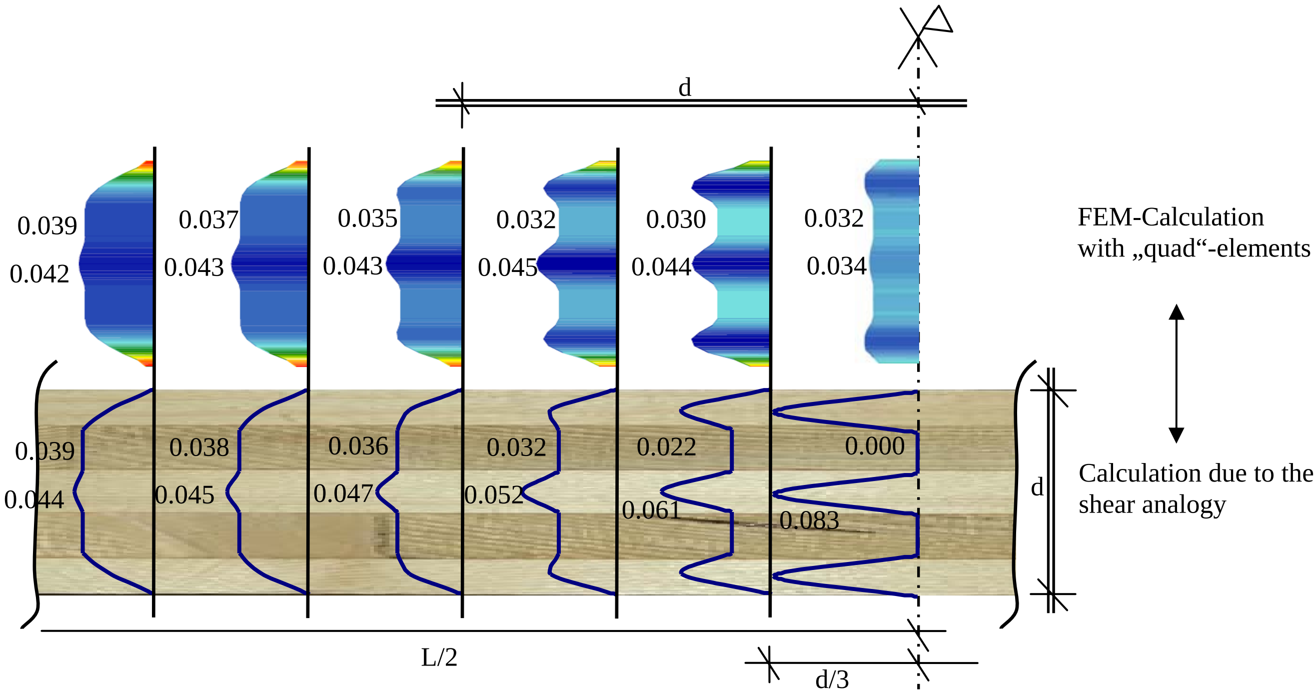

Fig. 5 Detail A: distribution of shear stresses in several sections [N/mm^2]

A comparison of the shear stress distributions according to the shear analogy and the FE-shell model shows, that the progresses of stress offer qualitative similarity – except for the progress in the vertical section directly next to the concentrated load. This obvious difference results from the modelling of the concentrated load in the FE-shell model. To avoid stress peaks in longitudinal stress distribution the load was applied by a series of single loads in the knots of the vertical centreline of the system, as shown in Fig. 6. This continuous loading causes an almost constant progress of shear stress in that area.

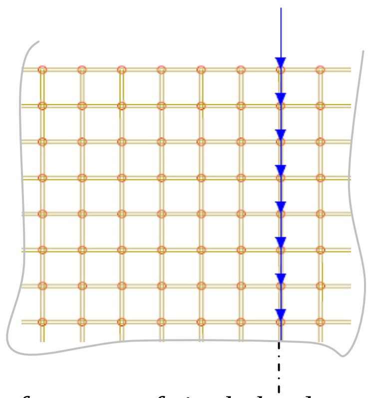

Fig. 6 Point of contact of single loads

Contrary to this effect the shear stress according to the shear analogy only appears in the layers with boards parallel to the span. This has to be the case, because corresponding to the shear analogy beam A is shear resistant while only beam B represents the shear deformation of the total cross section. So this method is not able to demonstrate the actual shear stress distribution immediately next to a concentrated load. In the vertical section, however, already at the distance of the element thickness d to the concentrated load the difference for measuring the rolling shear stress is less than three percent. This means that one of the most relevant parameter for design - the rolling shear stress – is presented almost accurately.

3.2 Simplified Method - Consideration of Shear Deformation

In order to gain knowledge about the influence of the shear deformation on the increase of the longitudinal stresses in the area of a concentrated load, comparative analyses at uniaxial CLT-elements have been done. The static systems consisted of simply supported beams loaded in the middle of the span. Their cross sections were always symmetric with a constant thickness of the single layers. The varying parameters were the ratio of span to thickness and the number of layers of the panel. This analysis focuses on the comparison of the longitudinal stresses according to the shear analogy (SA) and respectively to the composite theory (CT).

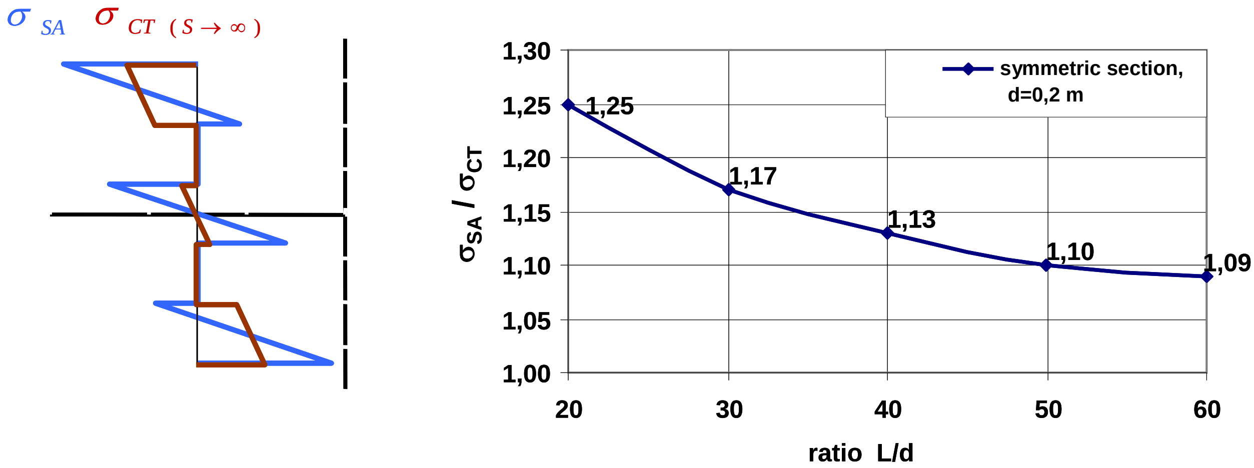

Fig. 7 Ratio of σ_{SA} to σ_{CT}

The ratio SA/CT of the stresses in the diagram was calculated for an element with a thickness of 200 mm. Comparative analyses show that generally the ratios are getting insignificantly minor with decreasing thickness. The ratio of span to thickness proved to be the crucial factor. For symmetric cross sections the number of layers has an irrelevant influence.

The diagram can also be used for the design of a double span beam with a uniformly distributed load. But it must be considered, that the influence of the shear deformation on the longitudinal stress progress in the area of the centre support is significantly higher. This is the consequence of the superposition of the action due to the uniformly distributed load and the supporting force. So the stresses must be calculated separately for the uniformly distributed load and a contrarily acting concentrated load using the total length of the system, as it is shown in Fig. 8. Finally the calculated stresses must be superposed.

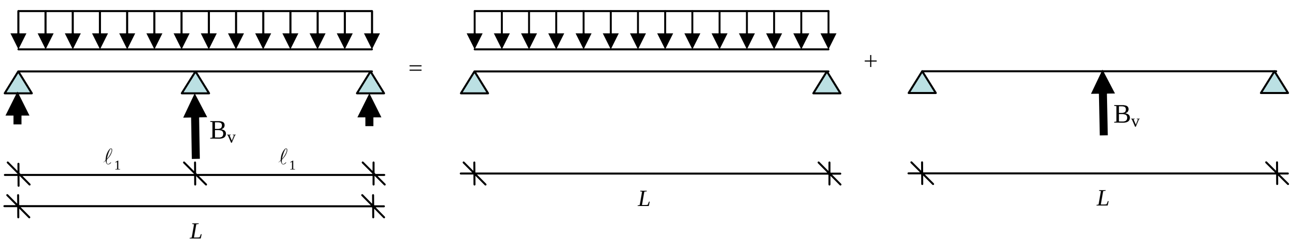

Fig. 8 Interaction of loading case

This simplified method can only be used for systems and loading conditions that can be reduced to two dimensional calculation models. Biaxial load bearing can not be modelled by this simplified method.

4. Twisting Stiffness

It is not possible to reduce all side or punctually supported large-sized elements to plane calculation models. The load bearing behaviour of these systems is influenced by the biaxial load distribution and the twisting stiffness of the elements. The German design standard recommends for elements with edge-glued boards the following twisting stiffness:

$$B_{xy} = B_{xyS} + B_{xyE} = \sum B_{xyS} + \sum B_{xyE} = \sum 2 \cdot G_{xy,i} \cdot d_i \cdot z_i^{\,2} + \sum G_{xy,i} \cdot \frac{d_i^{\,2}}{6} \tag{7}$$

For CLT-elements without edge-glued boards in the layers, a lower value of the twisting stiffness is estimated – it may be set to approximately zero. This is a simplification and does not reflect the real load bearing performance. Therefore some experimental investigations with quadratic CLT-elements have been done and the measured deformations have been compared to theoretical models. The following theoretical considerations form the basics for the experimental tests.

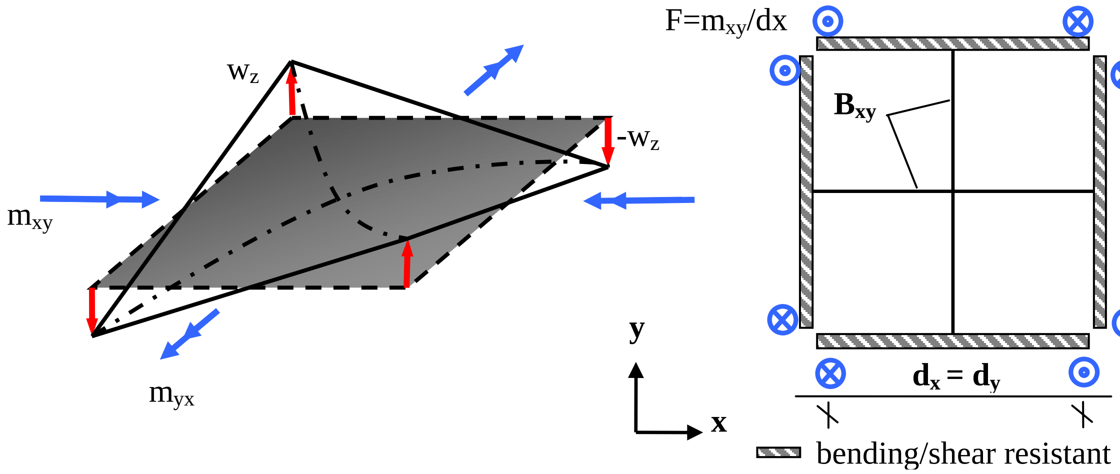

Fig. 9 Theoretical twisting-model

A quadratic element loaded by an ideal twisting stress shows a deformation figure like the one demonstrated in Fig. 9. There is a lift-off respectively a lowering in the diagonally opposite corners. Plate stripes parallel to the edges stay plane and do not show any bending deformation. Since the deformation figure is exclusively caused by the twisting of the elements due to the twisting moment, this model is also valid for orthotropic plates. Without transverse shear deformation, the correlation of deformation and twisting moments can be described, according to the geometrical and material equilibrium of the Kirchhoff Theory, by the following equations:

$$\kappa_{xy} = \frac{m_{xy}}{K} \tag{8}$$

with: $$B_{xy} = 2 \cdot K = G_{xy} \cdot t^3 / 6 \tag{9}$$

$$\kappa_{xy} = -w'' = -\frac{\partial^2 w}{\partial x \partial y} \tag{10}$$

A comparable figure can be represented by a girder grid – in the easiest case modelled by two crossed girders. The twisting moments can be replaced by a pair of forces at the ends of bending and shear resistant cantilevers. Each main girder holds the plate-stiffness in x- respectively in y-direction.

Comment: When modelling a plate by using a girder grid it must be considered, that the twisting of the grid activates the girders in x- as well as in y-direction. So the twisting stiffness of a girder that represents a plate is half the twisting stiffness of a single beam with an equal width.

The described model relates to a quadratic plate that is supported in three corners and stressed by a single load perpendicular to the plane in the free corner. Based on this, an experimental rig was developed and tests were carried out. The quadratic test pieces had an edge length of 1.20 m. In each case two pieces of a three- and a five-layer element were tested. The three-layer element had a total thickness of 81 mm and the five-layer of 85 mm. Fabrication was done by vacuum method. The single boards with material characteristics of C24 are not edge-glued. In the free corner deformation depending of the load was measured.

In addition the experimental tests were modelled with a plane frame programme. The plane test pieces were modelled using a girder grid. To achieve results as accurate as possible, the grid space chosen measured 20 mm. The influence of the shear deformation was to be investigated in this context, too. So the calculation was done by the shear analogy as well as by the before described simplified shear resistant model. In Fig. 10 the results of a five-layer test piece are demonstrated.

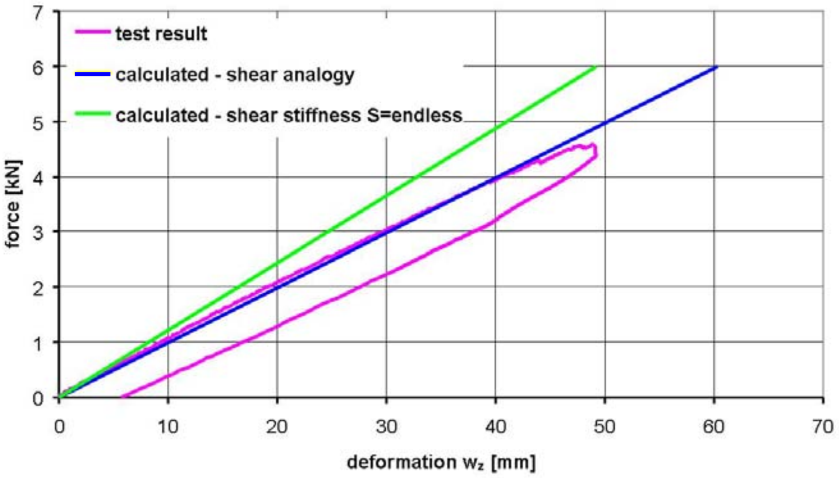

Fig. 10 Deformation diagram

The deformation according to the shear analogy (blue line) shows a good correlation with the measured deformation of the experimental tests. The calculated deformation based on the differential equation of the Kirchhoff Theory (green line) undervalues the total deformation of the plate element because it does not include the shear deformation and the ductile composite of the layers.

5. Conclusion

Comparative analysis on uniaxial spanned CLT-elements using the shear analogy and an FE-shell model have shown, that in the immediate area of a concentrated load the longitudinal stress does not stay plane. This causes stress peaks in the single layers, which can be taken into account by the method of the shear analogy. The shear stress can also be calculated by this approach. However theoretically caused stress peaks that do not reflect the actual shear stress distribution, have to be expected to appear in the immediate area of a concentrated load. Yet at a distance of plate thickness the rolling shear strain, which is normally the relevant parameter for design, already shows sufficient correlation. Analyses were done of symmetric cross sections with constant thickness of the single layers with simply supported beams with concentrated loads in the centre of the systems. They demonstrate that the increase of the longitudinal stresses due to the influence of the shear deformation only depends on the ratio of span to thickness. Finally theoretical and experimental investigations have shown, that the valuation of the twisting stiffness according to the DIN 1052 [2] also can be used for CLT-elements with non edge-glued boards in the single layers.

6. Acknowledgement

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7. References

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