OPTIMAL RESOURCE ALLOCATION IN THE DOWNLINK/UPLINK OF SINGLE-USER MISO/SIMO FDD SYSTEMS WITH LIMITED FEEDBACK

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ABSTRACT

Consider a single-user FDD system with multiple antennas at the BS and a single antenna at the user. We assume that the BS performs coherent transmission and reception in the downlink and uplink, respectively. To this end, downlink resources are employed to estimate the downlink channel at the user, while uplink resources are used to estimate the uplink channel at the BS and to relay a quantized version of the downlink channel estimate back to the BS. The transmit CSI for the downlink is estimated, quantized, outdated and affected by feedback errors, while the receive CSI for the uplink is just estimated. Besides the well-known tradeoff between the training and data payload in a one-way system, in a two-way system like a downlink/uplink FDD system, there is an additional tradeoff between both links due to the feedback. We consider the resource allocation of the downlink and uplink jointly taking into account the feedback and imperfect CSI. As a figure of merit we employ the sum of the downlink and uplink capacities.

1. INTRODUCTION

The capacity of wireless communication links increases significantly by deploying multiple antennas but under the assumption of perfect CSI. However, in practice the available CSI is not perfect. For instance, in the SIMO uplink of the single-user FDD system described in the abstract, the available *receive* CSI for *maximum ratio combining* (MRC) at the BS is the estimate of the current channel which is not perfect [1]. In the MISO downlink of the same system, the *transmit* CSI for *maximum rate transmission* (MRT) becomes available at the BS in a three-step process. First, the downlink channel is *estimated* at the user. Afterwards, the channel estimate is *quantized* and finally, the quantized channel estimate is fed back to the BS. Considering an error-prone feedback link, i.e. the uplink, the feedback could be received *erroneously*. Additionally, due to the feedback delay, by the time the BS makes use of the fedback CSI for the downlink, the downlink channel could have changed and so the obtained CSI could be *outdated* by then. In such a case, the transmit CSI available for the downlink transmission is estimated, quantized, outdated and affected by feedback errors.

The tradeoff between training and data payload in one-way systems has been investigated in [1]. However, in an FDD system further resources are consumed in the uplink for the feedback of transmit CSI to the BS. The asymptotic tradeoff between training, feedback and data payload in one-way MISO systems has been discussed in [2], for a *time division duplex* (TDD) system, where the uplink is solely employed for feedback. The tradeoff between training, feedback and data payload in a multi-user FDD downlink has been treated in [3] considering transmit CSI which is imperfect as described before. Although Caire et. al. discuss the uplink channel estimation for the feedback in [3, Remark 8.1], the optimizaton of the uplink training and the feedback considering its effect on the uplink capacity is not discussed, due to the *one-way* consideration of the problem.

In a two-way system, besides the tradeoff between the training and the data payload on each link, we have an additional tradeoff between the downlink and uplink capacities due to the feedback. However, there has been only limited work that considers resource allocation in two-way systems. The tradeoff between the rates in both links of a two-way MIMO system with limited feedback and beamforming has been considered in [4, 5] with an uneven power allocation between the training, feedback and data payload. In this work, we do not allow such an uneven power allocation because of practical issues and instead, we optimize the *time* resources allocated to training and feedback. In addition, in two-way systems there is no single figure of merit as in a one-way system. As an appropriate figure of merit we consider the downlink-uplink *sum* capacity as in [6]. However, this work addresses the optimization in a different scope as [6].

In this paper, we optimize the downlink training, uplink training and feedback lengths in order to maximize a lower bound on the sum of the downlink capacity (with MRT) and uplink capacity (with MRC). We assume that the available transmit CSI at the BS for the downlink is estimated, quantized, outdated and affected by erroneous feedback, while the available receive CSI at the BS for the uplink is estimated. Although practically relevant, to our best knowledge, resource allocation in two-way systems assuming outdated and error-prone feedback has not been addressed yet. This paper is organized as follows. Section 2 discusses the system model, Section 3 explains the optimization and Section 4 present numerical results. Finally, we conclude the paper in Section 5.

2. FDD DOWNLINK/UPLINK SYSTEM MODEL

Consider first the downlink (DL) MISO channel of the FDD *two-way* system with M transmit antennas at the BS. With maximum rate transmission at the BS, the effective SISO downlink channel for the D_{DL} data symbols at time slot n is

$$\mathbf{y}[n] = \sqrt{P_{\text{DL}}}\,\mathbf{w}_{\text{DL}}^{\text{H}}[n]\mathbf{h}_{\text{DL}}[n]\mathbf{r}[n] + \mathbf{v}[n], \quad (1)$$

where $\mathbf{y}[n] \in \mathbb{C}^{D_{\text{DL}}}$ are the received signals, $\mathbf{r}[n] \in \mathbb{C}^{D_{\text{DL}}}$ are the transmit symbols with unit variance, $\mathbf{w}_{\text{DL}}[n] \in \mathbb{C}^{M}$ is the beamforming vector with unit norm, $\mathbf{h}_{\text{DL}}[n] \in \mathbb{C}^{M}$ is the downlink MISO channel, $\mathbf{v}[n] \in \mathbb{C}^{D_{\text{DL}}}$ is the *additive white Gaussian noise* (AWGN) with zero mean and variance σ_v^2 and P_{DL} is the transmit power at the BS. The elements of the channel vector $\mathbf{h}_{\text{DL}}[n]$ are i.i.d. complex Gaussian random variables with zero mean and unit variance, i.e. Rayleigh fading. We assume that the downlink channel $\mathbf{h}_{\text{DL}}[n]$ is constant for T symbols, which is the duration of a time slot. The first T_{DL} symbols in time slot n are used to obtain a *minimum mean square error* (MMSE) estimate $\hat{\mathbf{h}}_{\text{DL}}[n]$ of the downlink channel and the remaining $D_{\text{DL}} = T - T_{\text{DL}}$ symbols are used to transmit data from the BS to the user. Based on the estimate $\hat{\mathbf{h}}_{\text{DL}}[n]$, the downlink channel can be written as

$$\mathbf{h}_{\text{DL}}[n] = \hat{\mathbf{h}}_{\text{DL}}[n] + \mathbf{e}_{\text{DL}}[n], \quad (2)$$

where $\mathbf{e}_{\text{DL}}[n]$ is the error vector whose elements are i.i.d. zero-mean Gaussian random variables with variance $\sigma_{e_{\text{DL}}}^2$ [1]

$$\sigma_{e_{\text{DL}}}^2 = \begin{cases} \dfrac{1+\rho_{\text{DL}}(M-T_{\text{DL}})}{1+\rho_{\text{DL}}M} & \text{for} \quad T_{\text{DL}} < M \\ \dfrac{1}{1+\rho_{\text{DL}}T_{\text{DL}}} & \text{for} \quad T_{\text{DL}} \geq M \end{cases}, \quad (3)$$

where $\rho_{\text{DL}} = \frac{P_{\text{DL}}}{M\sigma_v^2}$. Additionally, the elements of the MMSE estimate $\hat{\mathbf{h}}_{\text{DL}}$ are i.i.d. zero-mean Gaussian random variables with variance $(1-\sigma_{e_{\text{DL}}}^2)$. Contrary to the independent block fading assumption commonly found in the literature, we assume *correlated flat block* Rayleigh fading to consider the effect of the feedback delay. To this end, we assume a first order Markov model for the correlation of the downlink MISO channel between two successive time slots as in [3], i.e.

$$\mathbf{h}_{\text{DL}}[n] = \sqrt{\alpha}\,\mathbf{h}_{\text{DL}}[n-1] + \sqrt{1-\alpha}\,\mathbf{g}_{\text{DL}}[n-1], \quad (4)$$

where the elements of $\mathbf{g}_{\text{DL}}[n-1] \in \mathbb{C}^{M}$ are i.i.d. zero-mean unit-variance complex Gaussian random variables and are uncorrelated with $\mathbf{h}_{\text{DL}}[n-1]$ and $\sqrt{\alpha}$ is the correlation coefficient ($0 \leq \alpha \leq 1$), which is assumed to be unknown. Between downlink channel estimates of successive time slots, a first order Markov model also holds [7]:

$$\hat{\mathbf{h}}_{\text{DL}}[n] = \sqrt{\alpha'}\,\hat{\mathbf{h}}_{\text{DL}}[n-1] + \sqrt{1-\alpha'}\,\hat{\mathbf{g}}_{\text{DL}}[n-1], \quad (5)$$

where $\alpha' = \alpha(1-\sigma_{e_{\text{DL}}}^2)^2$ and the elements of $\hat{\mathbf{g}}_{\text{DL}} \in \mathbb{C}^{M}$ are i.i.d. zero-mean complex Gaussian random variables with variance $1-\sigma_{e_{\text{DL}}}^2$ and are uncorrelated with $\hat{\mathbf{h}}_{\text{DL}}[n-1]$.

We can rewrite (1) using (2) as

$$\mathbf{y}[n] = \sqrt{P_{\text{DL}}}\,\mathbf{w}_{\text{DL}}^{\text{H}}[n]\hat{\mathbf{h}}_{\text{DL}}[n]\mathbf{r}[n] + \mathbf{z}[n], \quad (6)$$

where $\mathbf{z}[n] = \sqrt{P_{\text{DL}}}\,\mathbf{w}_{\text{DL}}^{\text{H}}[n]\mathbf{e}_{\text{DL}}[n]\mathbf{r}[n] + \mathbf{v}[n]$ is the effective noise, whose elements have variance $\sigma_z^2 = \sigma_{e_{\text{DL}}}^2 P_{\text{DL}} + \sigma_v^2$.

The uplink (UL) of the FDD *two-way* system is a SIMO channel where the M receive antennas at the BS are used to perform maximum ratio combining and hence, the equivalent SISO channel of the D_{UL} data symbols at time slot n is

$$\mathbf{x}[n] = \sqrt{P_{\text{UL}}}\mathbf{w}_{\text{UL}}^{\text{H}}[n]\mathbf{h}_{\text{UL}}[n]\mathbf{s}[n] + \mathbf{u}[n], \quad (7)$$

where $\mathbf{x}[n] \in \mathbb{C}^{D_{\text{UL}}}$ are the received signals at the BS after the beamforming, $\mathbf{s}[n] \in \mathbb{C}^{D_{\text{UL}}}$ are the unit-variance transmit signals, $\mathbf{w}_{\text{UL}}[n] \in \mathbb{C}^{M}$ is the received beamforming vector with unit norm, $\mathbf{h}_{\text{UL}}[n] \in \mathbb{C}^{M}$ is the uplink SIMO channel, $\mathbf{u}[n] \in \mathbb{C}^{D_{\text{UL}}}$ is the AWGN and P_{UL} is the transmit power at the user. The elements of the uplink channel $\mathbf{h}_{\text{UL}}[n]$ are i.i.d. complex Gaussian random variables with zero mean and unit variance and like in the DL, the uplink channel $\mathbf{h}_{\text{UL}}[n]$ is constant for the duration of a time slot, i.e., T symbols. The first T_{UL} symbols in the time slot are used to obtain an MMSE estimate $\hat{\mathbf{h}}_{\text{UL}}[n]$ of the uplink channel, F symbols are used to feedback the quantized version of the downlink channel estimate $\hat{\mathbf{h}}_{\text{DL}}[n]$ to the BS, and the remaining $D_{\text{UL}} = T - T_{\text{UL}} - F$ symbols are used to transmit data from the user to the BS as depicted in Fig. 1. Hence, the available receive CSI in the UL at the BS is $\hat{\mathbf{h}}_{\text{UL}}[n]$. Similar to (2), the uplink channel $\mathbf{h}_{\text{UL}}[n]$ can be written as $\mathbf{h}_{\text{UL}}[n] = \hat{\mathbf{h}}_{\text{UL}}[n] + \mathbf{e}_{\text{UL}}[n]$, where $\mathbf{e}_{\text{UL}}[n]$ is the error vector whose elements are i.i.d. zero-mean complex Gaussian random variables with variance $\sigma_{e_{\text{UL}}}^2$, where $\sigma_{e_{\text{UL}}}^2 = \frac{1}{1+P_{\text{UL}}T_{\text{UL}}/\sigma_u^2}$. The elements of $\hat{\mathbf{h}}_{\text{UL}}[n]$ are i.i.d. zero-mean complex Gaussian random variables with variance $1-\sigma_{e_{\text{UL}}}^2$. With the available receive CSI, the beamforming vector for MRC is $\mathbf{w}_{\text{UL}}[n] = \frac{\hat{\mathbf{h}}_{\text{UL}}[n]}{\|\hat{\mathbf{h}}_{\text{UL}}[n]\|_2}$, so we rewrite (7) as

$$\mathbf{x}[n] = \sqrt{P_{\text{UL}}}\,\|\hat{\mathbf{h}}_{\text{UL}}[n]\|_2\mathbf{s}[n] + \mathbf{z}'[n], \quad (8)$$

where $\mathbf{z}'[n] = \sqrt{P_{\text{UL}}}\,\mathbf{w}_{\text{UL}}^{\text{H}}[n]\mathbf{e}_{\text{UL}}[n]\mathbf{s}[n] + \mathbf{u}[n]$ is the effective noise with variance $\sigma_{z'}^2 = \sigma_{e_{\text{UL}}}^2 P_{\text{UL}} + \sigma_u^2$.

In the DL, the user obtains, at time slot $n-1$, the downlink channel estimate $\hat{\mathbf{h}}_{\text{DL}}[n-1]$, which needs to be quantized to relay it back to the BS with the limited feedback of B bits. For this, we employ *random vector quantization* (RVQ) [2], where we have at the transmitter (BS) and receiver (user) a codebook with 2^B random beamforming vectors $\mathbf{t}_j$, $j = 1, \ldots, 2^B$, i.i.d isotropically distributed over the M-dimensional unit sphere. $\hat{\mathbf{h}}_{\text{DL}}[n-1]$ is quantized by selecting the beamforming vector $\mathbf{w}'_{\text{DL}}[n]$ (to be used at time slot n) that best matches the current channel estimate $\hat{\mathbf{h}}_{\text{DL}}[n-1]$, i.e.

$$\mathbf{w}'_{\text{DL}}[n] = \underset{\mathbf{t}_j}{\text{argmax}}\;\; |\mathbf{t}_j^{\text{H}}\hat{\mathbf{h}}_{\text{DL}}[n-1]|^2. \quad (9)$$

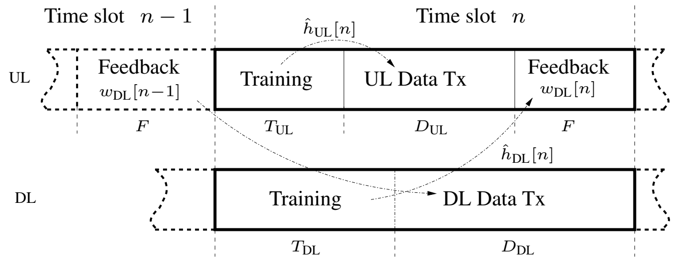

Fig. 1. Resource Allocation in an FDD System.

Due to feedback delay, the transmit CSI obtained with the feedback at time slot $n-1$ becomes available at the BS for the downlink until time slot n and hence, the beamforming vector $\mathbf{w}'_{\mathrm{DL}}[n]$ is outdated by one time slot. The degree of outdating of the beamforming vector $\mathbf{w}'_{\mathrm{DL}}[n]$ depends on α. A general overview of the considered two-way FDD system with feedback is depicted in Fig. 1.

In addition, due to the fading and noise in the uplink, the feedback can be received with errors. At time slot $n-1$, the user feeds back the B uncoded bits representing the index of $\mathbf{w}'_{\mathrm{DL}}[n]$ using $F = \frac{B}{2}$ QPSK symbols. We employ QPSK for the feedback due to robustness, since higher modulation schemes suffer from higher symbol error probabilites. The feedback bits are sent uncoded in the uplink without any error detection, such that if at least one feedback symbol is in error, there is a *total* loss of the feedback, since no optimized labeling scheme of the feedback bits is employed as in [3]. After an error, the index received by the BS corresponds to a different beamforming vector than the one intended by the user. Hence, the beamforming vector applied by the BS, i.e., $\mathbf{w}_{\mathrm{DL}}[n]$ (c.f. (6)), would be different than the beamforming vector $\mathbf{w}'_{\mathrm{DL}}[n]$, which was fedback by the user (c.f. (9)). In this case, $\mathbf{w}_{\mathrm{DL}}[n]$ would be completely *uncorrelated* with the actual downlink channel, as if there were no feedback.

Let us now present the feedback error probability. The feedback, like the data, can be detected with MRC and hence from (8) the uplink SNR at time slot n would be given by

$$\gamma_{\mathrm{UL}}[n] \quad = \quad \frac{P_{\mathrm{UL}}\|\hat{\mathbf{h}}_{\mathrm{UL}}[n]\|_2^2}{\sigma_u^2 + P_{\mathrm{UL}}\sigma_{e_{\mathrm{UL}}}^2}. \tag{10}$$

One symbol error leads to a total feedback loss, so the average feedback error probability p_ϵ is given approximately as [9]

$$p_\epsilon \quad \approx \quad 1 - \left(1 - \mathrm{E}[p_{\mathrm{s}}(\gamma_{\mathrm{UL}})]\right)^{\frac{B}{2}}, \tag{11}$$

where p_{s} is the symbol error probabiliy of uncoded QPSK and

$$\mathrm{E}[p_{\mathrm{s}}(\gamma_{\mathrm{UL}})] = 2\left(\frac{1-\kappa}{2}\right)^{M}\sum_{m=0}^{M-1}\binom{M-1+m}{m}\left(\frac{1+\kappa}{2}\right)^{m},$$

where

$$\kappa \quad = \quad \sqrt{\frac{1-\sigma_{e_{\mathrm{UL}}}^2}{1+2\frac{\sigma_u^2}{P_{\mathrm{UL}}}+\sigma_{e_{\mathrm{UL}}}^2}}.$$

With a probability $1-p_\epsilon$ (correct feedback) we have that $\mathbf{w}_{\mathrm{DL}}[n] = \mathbf{w}'_{\mathrm{DL}}[n]$ and with a probability p_ϵ (erroneous feedback) we have that $\mathbf{w}_{\mathrm{DL}}[n] \neq \mathbf{w}'_{\mathrm{DL}}[n]$, i.e. the user's assumption about the beamforming vector is wrong! The BS, nevertheless, is unaware if a feedback error has occurred or not. But since the feedback error probability p_ϵ depends only on $\frac{\sigma_u^2}{P_{\mathrm{UL}}}$, which is a long term parameter, and on the uplink training length T_{UL} and the number of quantization bits B, which are fixed, we have that p_ϵ remains constant over many transmission blocks. So it can be assumed that p_ϵ is known even if the BS is unaware of the feedback errors.

3. OPTIMAL RESOURCE ALLOCATION

Since the capacity with imperfect CSI is unknown, we focus on the optimization of lower bounds on the downlink and uplink capacities. A lower bound on the downlink capacity with beamforming with the described imperfect transmit CSI is given by [7, Theorem 3.3]

$$\begin{aligned} C_{\mathrm{DL,lb}} \; = \; & \frac{T-T_{\mathrm{DL}}}{T}\, p_\epsilon \log_2(\mathrm{e})\, \mathrm{e}^{\frac{\sigma_v^2}{P_{\mathrm{DL}}}} \mathrm{E}_1\left(\frac{\sigma_v^2}{P_{\mathrm{DL}}}\right) \\ & + \frac{T-T_{\mathrm{DL}}}{T}(1-p_\epsilon)\left(1-\frac{1}{\sqrt{2\pi}}\frac{\sigma_{\eta_{\mathrm{DL}}}}{\mu_{\eta_{\mathrm{DL}}}}\right) \times \\ & \log_2\left(1+\frac{(\alpha'(M\mathrm{E}\left[\nu\right]-1)+1)(1-\sigma_{e_{\mathrm{DL}}}^2)}{\sigma_{e_{\mathrm{DL}}}^2+\frac{\sigma_v^2}{P_{\mathrm{DL}}}}\right) \\ & - \frac{1}{T}\left(p_\epsilon \log_2(\mathrm{e})\, \mathrm{e}^{\frac{\sigma_v^2}{P_{\mathrm{DL}}}} \sum_{k=1}^{D_{\mathrm{DL}}} \mathrm{E}_k\left(\frac{\sigma_v^2}{P_{\mathrm{DL}}}\right) + h_b(p_\epsilon)\right) \end{aligned} \tag{12}$$

where $h(p_\epsilon)$ is the binary entropy function, while $\frac{\sigma_{\eta_{\mathrm{DL}}}}{\mu_{\eta_{\mathrm{DL}}}}$ and $\mathrm{E}\left[\nu\right]$ are given by (24) and (19) in [7]. E_k denotes the generalized exponential integral $\mathrm{E}_k(z) = \int_1^\infty \frac{\mathrm{e}^{-zt}}{t^k}\mathrm{d}t$.

A lower bound on the uplink capacity can be computed by replacing $\mathbf{z}'[n]$ in (8) with a zero-mean complex Gaussian random variable with variance $\sigma_{z'}^2$ which is independent of the signal $\mathbf{s}[n]$ [1]. Hence,

$$\begin{aligned} C_{\mathrm{UL,lb}} = \; & \frac{T-T_{\mathrm{UL}}-\frac{B}{2}}{T}\mathrm{E}\left[\log\left(1+\frac{P_{\mathrm{UL}}\|\hat{\mathbf{h}}_{\mathrm{UL}}\|_2^2}{\sigma_{e_{\mathrm{UL}}}^2 P_{\mathrm{UL}}+\sigma_u^2}\right)\right] \\ = & \frac{D_{\mathrm{UL}}}{T}\log_2(\mathrm{e})\mathrm{e}^{\frac{\sigma_u^2+P_{\mathrm{UL}}\sigma_{e_{\mathrm{UL}}}^2}{P_{\mathrm{UL}}(1-\sigma_{e_{\mathrm{UL}}}^2)}}\sum_{k=1}^{M}\mathrm{E}_k\left(\frac{\sigma_u^2+P_{\mathrm{UL}}\sigma_{e_{\mathrm{UL}}}^2}{P_{\mathrm{UL}}(1-\sigma_{e_{\mathrm{UL}}}^2)}\right), \end{aligned} \tag{13}$$

where the second steps follows from [8].

Although there is no single figure of merit in two-way systems as in a one-way system, we optimize the FDD system by maximizing the lower bound of the sum of the downlink and uplink capacities. The optimization problem we tackle is

$$\max_{B,T_{\mathrm{DL}},T_{\mathrm{UL}}} \quad C_{\mathrm{DL,lb}} + C_{\mathrm{UL,lb}} \tag{14}$$

$$\text{subj. to} \quad 0 \leq B < 2T,\; 0 \leq T_{\mathrm{DL}} \leq T,\;\; 0 \leq T_{\mathrm{UL}} \leq T - B/2,$$

which is an *integer* optimization problem. We solve (14) in two steps: first we find the optimal T_{DL} and T_{UL} for a given

B and then we perform a search to find the optimum B. In Fig. 2, we depict an example of the first step of the optimization for $B = 16$, i.e. $C_{\text{DL,lb}}$ and $C_{\text{UL,lb}}$ as a function of T_{DL} and T_{UL}, respectively. The other system parameters are $M = 8$, $T = 100$, $\frac{P_{\text{DL}}}{\sigma_v^2} = 9$ dB, $\frac{P_{\text{UL}}}{\sigma_u^2} = 0$ dB and $\alpha = 0.85$. For a fixed B, $C_{\text{UL,lb}}$ only depends on T_{UL}. The tradeoff between training and data payload in each link can be observed. As $T_{\text{UL}} \to T - \frac{B}{2}$ the estimation error $\sigma_{e\text{UL}} \to 0$ but $C_{\text{UL,lb}} \to 0$ since there are less data symbols available in the time slot.

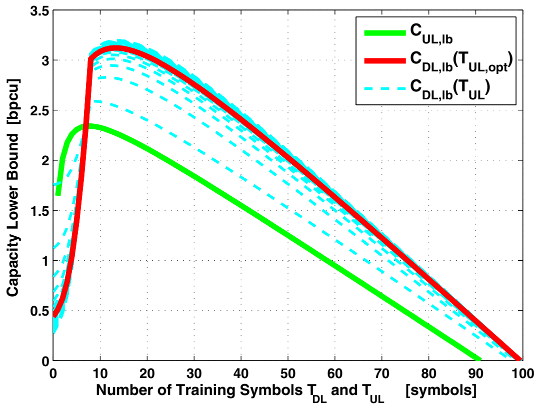

Fig. 2. Sum Capacity Optimization: First Step.

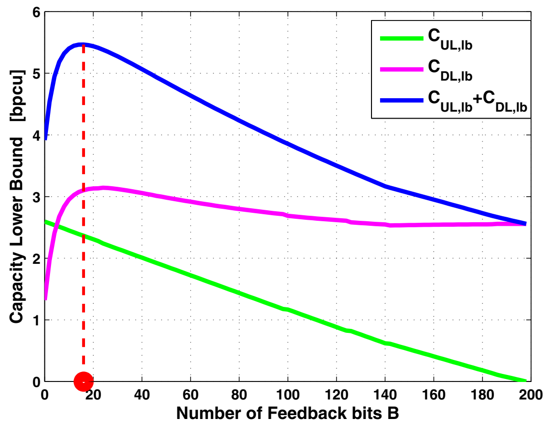

Fig. 3. Sum Capacity Optimization: Second Step.

However, we have an additional tradeoff in two-way systems between the uplink and the downlink due to the feedback. The downlink capacity not only depends on T_{DL} and B, but also on the uplink channel estimation, i.e. T_{UL}! A poor channel estimation leads to a larger feedback error probability p_ϵ (c.f. (11)), which in turn decreases the downlink capacity The downlink capacity is then a function of T_{UL}. With $B = 16$ and $0 \leq T_{\text{UL}} \leq T - \frac{B}{2}$, there are 93 possible values for T_{UL} and also for the downlink capacity denoted by $C_{\text{DL,lb}}(T_{\text{UL}})$, which are given by the dashed curves in Fig. 2. One can observe how the downlink depends on the uplink even for a fixed B.

The first step in the optimization includes finding for each T_{UL}, the downlink training length $T_{\text{DL}}(T_{\text{UL}})$, as a function of T_{UL}, which maximizes $C_{\text{DL,lb}}(T_{\text{UL}})$. Then we find the uplink training length T_{UL} which maximizes $C_{\text{UL,lb}}(T_{\text{UL}}) + C_{\text{DL,lb}}(T_{\text{UL}})$, i.e. the sum downlink-uplink capacity for the given B. For the given example, the optimum values are $T_{\text{UL,opt}} = 9$ and $T_{\text{DL,opt}} = 13$. In Fig. 2, we have also depicted $C_{\text{DL,lb}}(T_{\text{UL,opt}} = 9)$. Note that $C_{\text{UL,lb}}$ is not maximized with $T_{\text{UL}} = 9$, but rather with $T_{\text{UL}} = 7$, i.e. besides the $\frac{B}{2} = 8$ symbols reserved for the feedback, the uplink *sacrifices* additionally two data symbol to help the downlink achieve a higher capacity and in turn to achieve a higher downlink-uplink sum capacity.

The second step in the optimization requires finding the optimum B. So the first step is performed for $0 \leq B < 2T$ as depicted in Fig. 3 for the same setting as before. The optimum B is the one which maximizes the sum downlink-uplink capacity, while the optimal training lengths are the ones found in the first step. For this example, the optimal resource allocation would be $T_{\text{DL,opt}} = 13, T_{\text{UL,opt}} = 9$ and $B_{\text{opt}} = 16$. Observe in Fig. 3 how $C_{\text{DL,lb}}$ increases at first with B, since the quantization error is reduced with a negligible p_ϵ. However, after $B = 24$, $C_{\text{DL,lb}}$ decreases since the reduction in the quantization error cannot compensate the increased feedback error probability. Notice also that if the figure of merit were solely the uplink capacity, the optimum would be of course $B = 0$. If the figure of merit were the downlink capacity, the optimum would be around $B = 24$ with the rest of the uplink symbols employed for training, i.e. C_{UL} would be $C_{\text{UL}} = 0$!

4. NUMERICAL RESULTS

Now we present some numerical results of the optimization problem (14). Fig. 4 plots the optimum T_{UL}, T_{DL}, B vs. $\frac{P_{\text{DL}}}{\sigma_{\text{DL}}^2}$ in dB. We assume $T = 200$, $M = 8$, $\alpha = 1$, $\sigma_v^2 = \sigma_u^2$ and $P_{\text{UL}} = \frac{P_{\text{DL}}}{M}$ in order to have the same power per transmit antenna in the UL and DL. As expected, less training symbols are required as the SNR increases. $T_{\text{DL,opt}}$ decreases slower than $T_{\text{UL,opt}}$ due to the difference between the channel estimation in the downlink and the uplink. B_{opt} increases with SNR since the feedback becomes more reliable.

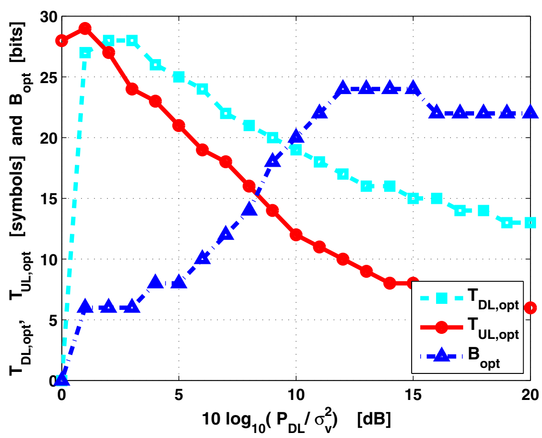

Fig. 4. Optimal values of T_{DL}, T_{UL} and B vs. $\frac{P_{\text{DL}}}{\sigma_{\text{DL}}^2}$

Fig. 5 depicts the optimum T_{UL}, T_{DL}, B as a function of the coherence interval T for the same scenario as before but with $\frac{P_{\text{DL}}}{\sigma_v^2} = 9$ dB and for $M = 4$ and $M = 8$. The training lengths grow sublinearly in T, while the feedback length converges to a finite value since further increase of B does not improve the downlink capacity due to an eventual increase of the feedback error probability. The $T_{\text{UL,opt}}$ for $M = 4$ and $M = 8$ behave similarly because the uplink channel estimation is independent of M, since the M channels from the user to each receive antenna at the BS are estimated orthogonally in space.

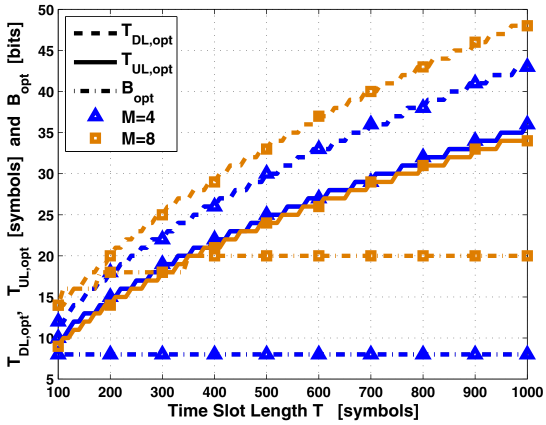

Fig. 5. Optimal values of T_{DL}, T_{UL} and B vrs T.

The achievable downlink rate (12), uplink rate (13) and downlink-uplink sum rate under imperfect CSI with the optimum resource allocation as a function of T are shown in Fig. 6 for the same setting as before. We include also a case with outdated CSI, i.e. $\alpha = 0.75$. Recall that we have outdated CSI only in the downlink. As a reference, we depict additionally the capacity with perfect CSI. From Fig. 5, we can observe that the overhead due to the training and feedback becomes negligible as $T \to \infty$, due to the sublinear increase of the training and feedback with increasing T. Nevertheless, as $T \to \infty$, the sum capacity with imperfect CSI will not converge to the sum capacity with perfect CSI due to the errors and delay in the feedback link and additionally for $\alpha = 0.75$ due to the outdating. The gap between the imperfect CSI case and the perfect CSI case is smaller for the uplink than for the downlink since the uplink CSI is only affected by estimation errors.

5. CONCLUSIONS

We have developed a framework which appropriately considers the inherent tradeoff between training, feedback and data payload for a single-user MISO-downlink/SIMO-uplink FDD system. In such a two-way system, the receive CSI at the BS is estimated while the transmit CSI is estimated, quantized, outdated and affected by feedback errors. It turns out that the optimal number of symbols dedicated to training varies sublinearly in the coherence time T. However, due to errors in

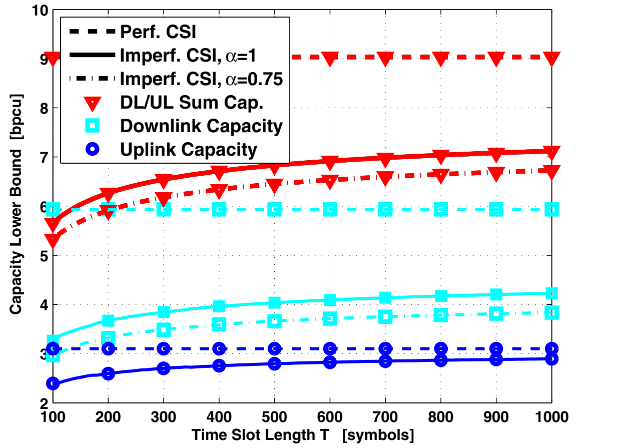

Fig. 6. Achievable sum and individual rates vs. T for $M = 8$.

the feedback link, the resources dedicated for feedback converges to a fixed finite value as $T \to \infty$. The resource allocation optimization can be also performed based on capacity upper bounds derived in [7]. Interestingly, the resulting solution turns to be nearly the same, which shows the usefulness of the proposed approach. Future work includes extending the presented formulation to the multiuser case.

6. REFERENCES

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