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On chip pulse shape design for precise synchronization of DS-CDMA systems[‡]

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SUMMARY

In this work, a methodology is established to design optimum chip pulse shapes which are absolutely bandlimited, their energy mainly concentrated in the chip duration, and which minimize the variance of the time-delay estimation error of an unbiased estimator. The low-complexity optimization method is based on the prolate spheroidal wave functions. Thus, the variational problem at hand can be transformed into a tractable parametric optimization problem. It is shown that the designed optimum chip pulse shapes significantly improve the synchronization performance of direct sequence code-division multiple access systems by minimizing the Cramer–Rao lower bound (CRLB). The time-delay estimation bias under the presence of multipath is compared with the conventionally used bandlimited rectangular chip pulse shape.

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KEY WORDS: chip pulse design; synchronization; DS-CDMA

1. INTRODUCTION

Precise code synchronization is desired for direct sequence code-division multiple access (DS-CDMA) systems as the maximum achievable despreading gain and jamming margin can only be reached for perfect synchronization. Regarding any kind of application where range measurements are made, like active radar, sonar or navigation applications, accurate time-delay estimation is needed. Especially for global navigation satellite systems (GNSS) like GPS or the upcoming European Galileo system, highly accurate time of arrival estimation is essential, whereas 1 ns of timing error results in approximately 30 cm of range error and the satellite signals are deeply buried

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‡Dedicated to Professor Seán Scanlan on the occasion of his 70th birthday.

under the noise floor. Thus, for these applications, signal design shall be directed towards optimizing synchronization performance. In the following we will exemplarily consider the synchronization for GNSS.

Besides the design of the applied pseudo-random noise (PN) sequences with regard to cross-correlation and autocorrelation properties [1], the design of the chip pulse shape for these PN sequences also definitely needs to be considered in order to shape the autocorrelation function and to control the bandwidth occupancy of the signal. Not only the time-delay estimation error should be minimum, but also the signal design should enable robust code tracking and acquisition in the receiver. Furthermore, especially in the framework of navigation applications, the signal shall provide robustness against tracking errors induced by multipath signals [2]. However, as discussed in the signal design of Galileo, the ongoing Galileo evolution program and the modernization of GPS, a trade-off between meeting all these requirements and complexity of the receiver architecture needs to be accomplished [2, 3].

In this work, a methodology is established to design optimum chip pulse shapes which are absolutely bandlimited, their energy mainly concentrated in the chip duration, and that minimize the tracking error variance subject to signal and performance requirements in order to enhance signal design for DS-CDMA systems. This low-complexity optimization method is based on the prolate spheroidal wave functions (PSWFs) [4], which allow to transform the variational problem at hand into a tractable parametric optimization problem. As a cost function, we apply the Cramer–Rao lower bound (CRLB) as proposed in [5], which gives the minimum variance of the time-delay estimation error of any unbiased estimator. In the following, inter-chip interference (ICI) is also considered as the chip pulse shapes are convolved with a PN sequence. The presented approach allows a systematic treatment of the problem at hand by solving a parametric optimization problem. Related work has been previously reported in [6], however, which minimized the tracking error of a coherent delay lock loop (DLL) by considering absolutely time-limited chip pulse shapes without taking into account constraints concerning acquisition or multipath performance. In [5], pulse shapes were derived, however, not taking into account different performance requirements besides minimizing the variance of the time-delay estimation error and this approach does not allow a systematic treatment of the problem.

2. STATEMENT OF THE PROBLEM

We assume coherent downconversion of the radio frequency signal to baseband. The received DS-CDMA baseband navigation signal of one satellite is given by

$$y(t) = \sqrt{P}c(t-\tau) + n(t) \tag{1}$$

where P denotes the signal power, $c(t)$ is the PN sequence, τ is the time-delay and $n(t)$ is the white Gaussian noise with the two-sided power spectral density of $N_0/2$. For the PN sequence, we apply Gold codes as used for GPS C/A code [7] with a code period of $T = 1\,\mathrm{ms}$, with a number of chips $N_\mathrm{c} = 1023$ per code period, and each chip has a time duration of $T_\mathrm{c} = 997.52\,\mathrm{ns}$. Thus, the PN sequence is given by

$$c(t) = \sum_{k=0}^{N_\mathrm{c}} c_k \delta(t - kT_\mathrm{c}) * p(t) \tag{2}$$

where $p(t)$ denotes the chip pulse shape, and $c_k \in \{-1, 1\}$ are the code bits of the PN sequence.

In order to perform precise synchronization in a navigation receiver, the delay τ needs to be estimated with high accuracy. The variance of the delay estimation error σ_τ^2 of any unbiased estimator is lower bounded by the CRLB. The CRLB can be given [8]

$$\sigma_\tau^2 \geqslant \frac{B_L}{C/N_0\, 8\pi^2} \frac{\int_{-\infty}^{\infty} |P(f)|^2 \, \mathrm{d}f}{\int_{-\infty}^{\infty} f^2 |P(f)|^2 \, \mathrm{d}f} \tag{3}$$

where B_L denotes the equivalent noise bandwidth of the generic estimator, $P(f)$ is the Fourier transform of the chip pulse shape $p(t)$, and C/N_0 denotes the carrier-to-noise density ratio of the received signal. For long PN sequences, the autocorrelation function $R(\varepsilon)$ can be approximated as

$$R(\varepsilon) \approx \int_{-\infty}^{\infty} p(t) p(t-\varepsilon) \, \mathrm{d}t = \int_{-\infty}^{\infty} |P(f)|^2 \mathrm{e}^{\mathrm{j}2\pi f \varepsilon} \, \mathrm{d}f \tag{4}$$

3. OPTIMIZATION

The objective is to find the optimum chip pulse shape $p(t)$ which is absolutely bandlimited to $[-B, B]$, its energy mainly concentrated within $[-T_\mathrm{c}/2, T_\mathrm{c}/2]$ and which minimizes the tracking error variance σ_τ^2 as given in Equation (3). The interdependency of $B>0$ and $T_\mathrm{c}>0$ is given by the normalized time-bandwidth product $2\varrho = 2\pi T_\mathrm{c} B$ [4, 9]. The minimization is subject to the constraint

$$\int_{-\infty}^{\infty} |P(f)|^2 \, \mathrm{d}f = 1 \tag{5}$$

We introduce an additional constraint in order to influence the shape of the autocorrelation function of the PN sequence $R(\varepsilon)$ such that

$$\underset{i \in \mathbb{N}}{\forall} |v_i| \leqslant \kappa \tag{6}$$

where v_i denotes the value of $R(\varepsilon)$ at the local extrema besides the global maximum of $R(\varepsilon)$ for $\varepsilon = 0$. Thus, we limit the absolute value of the local extrema of $R(\varepsilon)$ to $\kappa \in \mathbb{R}^+$. The shaping threshold κ typifies the trade-off between the different performance requirements as the shape of the autocorrelation function directly affects tracking accuracy, tracking robustness, acquisition performance, and multipath performance.

3.1. Methodology

Although the resulting optimization problem is not tractable, it is converted to an equivalent discrete formulation with reduced dimensions. This will be achieved by expanding the chip pulse shape $p(t)$ of a PN sequence using an adequate set of orthogonal basis functions. This approach transforms the apparent variational problem into a parameter optimization problem solving for the expansion coefficients that minimize the cost function. Special functions known as the PSWFs are

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particularly well suited to form such a set of basis functions [4]. Thus, for any $B>0$ and $T_c>0$, the PSWFs form an infinite set of real functions $\psi_0(\varrho, t), \psi_1(\varrho, t), \psi_2(\varrho, t), \ldots$ with associated real positive eigenvalues $\lambda_0(\varrho)>\lambda_1(\varrho)>\lambda_2(\varrho), \ldots$. The ψ's and λ's are functions of the normalized time-bandwidth product $2\varrho = 2\pi T_c B$. The $\psi_i(\varrho, t)$ are bandlimited to $[-B, B]$ and form a complete and orthonormal set of functions [4]:

$$\int_{-\infty}^{\infty} \psi_i(\varrho, t)\psi_j(\varrho, t)\,\mathrm{d}t = \begin{cases} 1, & i=j \\ 0, & i\neq j \end{cases} \tag{7}$$

They also form a complete and orthogonal set in the interval $[-T_c/2, T_c/2]$ [4]:

$$\int_{-T_c/2}^{T_c/2} \psi_i(\varrho, t)\psi_j(\varrho, t)\,\mathrm{d}t = \begin{cases} \lambda_i(\varrho), & i=j \\ 0, & i\neq j \end{cases} \tag{8}$$

The PSWFs are solutions of the integral equation [4]:

$$\lambda_i \psi_i(\varrho, z) = \int_{-T_c/2}^{T_c/2} \frac{\sin(2\pi B(t-s))}{\pi(t-s)} \psi_i(\varrho, s)\,\mathrm{d}s \tag{9}$$

Finally, we propose the expansion

$$p(t) = \sum_{m=0}^{\infty} \alpha_m \psi_m(\varrho, t) \tag{10}$$

where $\{\alpha_m\}_{m=0}^{\infty}$ are the expansion coefficients. We can now transform the primal variational problem into the dual parametric optimization problem by setting $P(f) = \sum_{m=0}^{\infty} \alpha_m \Psi_m(\varrho, f)$, where $\{\Psi_m(\varrho, f)\}_{m=0}^{\infty}$ denotes the Fourier transforms of the PSWFs.

3.2. Problem reduction

For the current GPS C/A code, a rectangular chip shape is used and most of its signal power is contained in the one-sided bandwidth of $B = 1.023\,\text{MHz}$. In order to exemplarily demonstrate the proposed optimization methodology and the achievable performance using the designed chip pulse shapes, we will follow the approach of designing an optimized chip pulse shape for a GNSS with $B = 1/T_c = 1.023\,\text{MHz}$, $\varrho = \pi$, and with most of its energy concentrated within the interval $[-T_c/2, T_c/2]$. Thus, in the following, we simply write $\psi_i(t)$, $\Psi_i(f)$ and λ_i without accounting for the dependency concerning ϱ. For simplicity, we also restrict our attention to chip pulse shapes $p(t)$ of even symmetry as done in [6] which allows us to solve only for $\{\alpha_m\}_{m=0}^{\infty}$ if m is even, since $\{\psi_m(t)\}_{m=0}^{\infty}$ are even if m is even and odd otherwise [4]. Because of Equations (7) and (8), a small value of λ_m implies that $\psi_m(t)$ will have most of its energy outside $[-T_c/2, T_c/2]$, whereas a value of λ_m near 1 implies that $\psi_m(t)$ contains most of its energy within $[-T_c/2, T_c/2]$. For a fixed value of ϱ, the λ_m decreases to zero rapidly with increasing m once m has exceeded $(2/\pi)\varrho$. As in our case,

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we have $\varrho=\pi$ only the $\{\psi_m(t)\}_{m=0}^{\infty}$ with $m\leqslant 2$ have considerable large energy concentration within $[-T_c/2, T_c/2]$.

Thus, with negligible impact on accuracy, we propose the truncated expansion:

$$p(t)=\alpha_0\psi_0(t)+\alpha_2\psi_2(t) \tag{11}$$

where $\alpha_0\in\mathbb{R}$ and $\alpha_2\in\mathbb{R}$ and with Equations (11), (3), (5), and (6) we obtain the parametric optimization problem:

$$(\alpha_0^*, \alpha_2^*)=\arg\min_{(\alpha_0,\alpha_2)}\left\{\frac{\int_{-\infty}^{\infty}(\alpha_0\Psi_0(f)+\alpha_2\Psi_2(f))^2\,\mathrm{d}f}{\int_{-\infty}^{\infty}f^2(\alpha_0\Psi_0(f)+\alpha_2\Psi_2(f))^2\,\mathrm{d}f}\right\} \tag{12}$$

s.t.

$$\int_{-\infty}^{\infty}(\alpha_0\Psi_0(f)+\alpha_2\Psi_2(f))^2\,\mathrm{d}f=1 \tag{13}$$

and

$$\forall_{i\in\mathbb{N}}\ |v_i|\leqslant\kappa \tag{14}$$

Please note that in Equations (12) and (13) we can write parentheses instead of the absolute value as done in Equations (5) and (3), because $\psi_0(t)$ and $\psi_2(t)$ are real and even and therefore, $\Psi_0(f)$ and $\Psi_2(f)$ are real. Applying Parseval's theorem to Equation (7) the constraint given in Equation (13) reduces to

$$\alpha_0^2+\alpha_2^2-1=0 \tag{15}$$

Hence, the parametric optimization problem given in Equations (12) and (13) can be reduced further as the additional constraint given in Equation (15) is a cycle for α_0 and α_2. Therefore, $\alpha_0\in[-1, 1]$, $\alpha_2\in[-1, 1]$ and the whole problem is symmetric, because (α_0^*, α_2^*) and $(-\alpha_0^*, -\alpha_2^*)$ are solutions of the problem.

4. RESULTS

In order to obtain the optimum chip pulse shape $p(t)$ which is absolutely bandlimited to $[-B, B]$, mainly concentrated within $[-T_c/2, T_c/2]$ and which minimizes the tracking error variance σ_τ^2 while taking into account the shaping threshold κ, we solved the parametric optimization problem as given in Equation (12) subject to the constraints (15) and (14) for $B=1.023$ MHz and $T_c=997.52$ ns by a simple line search. For the generation of the PSWFs, we followed [4, 10, 11].

Figures 1 and 2 depict the optimum chip pulse shape (OPT) for $\kappa=1$, 0.5, and 0.25 in time and frequency domain. In Figure 3 the autocorrelation function $R(\varepsilon)$ of the optimum chip pulse shape (OPT) for $\kappa=1$, 0.5, and 0.25 is illustrated.

In order to compare the performance of the optimum chip pulse shape (OPT) with the bandlimited rectangular pulse (RECT) which is used for the GPS C/A signal at the moment, we generated GPS C/A PN sequences as described in Section 2 with the different chip pulse shapes. The bandwidth of the signal using a rectangular chip pulse is also $B=1.023$ MHz. As we convolved, the different

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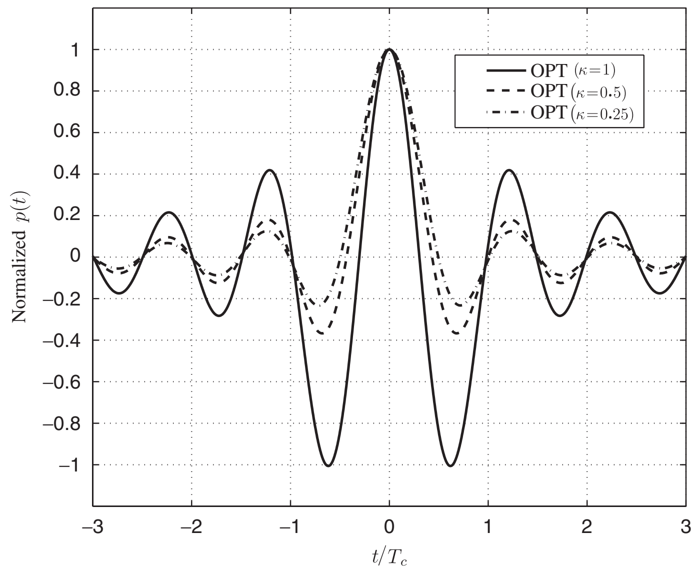

Figure 1. Optimum chip pulse shape in time domain.

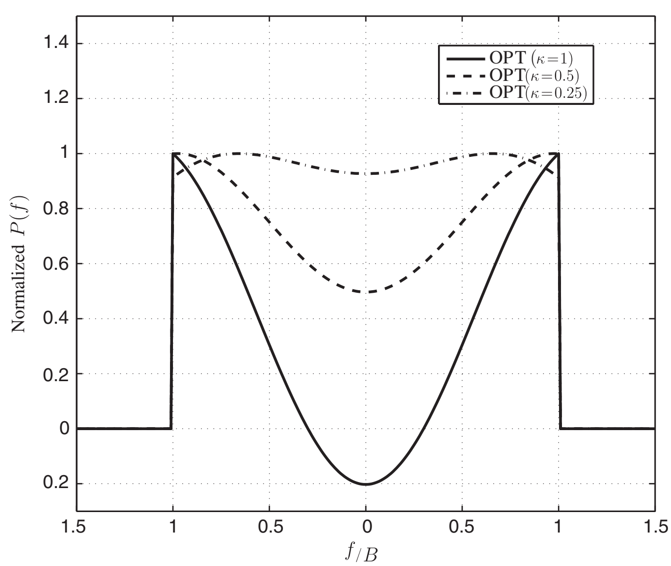

Figure 2. Optimum chip pulse shape in frequency domain.

chip pulse shapes with a PN sequence ICI is accounted for. In Figure 4, the standard deviation of the time-delay estimation error σ_τ is evaluated through the CRLB (3) for C/N_0 within the range from 30 to 50 dB Hz. We assume $B_{\mathrm{L}} = 1$ Hz. If the generic estimator which is used to estimate the time-delay is a DLL, then B_{L} is also called the loop bandwidth. In Figure 5 the absolute maximum

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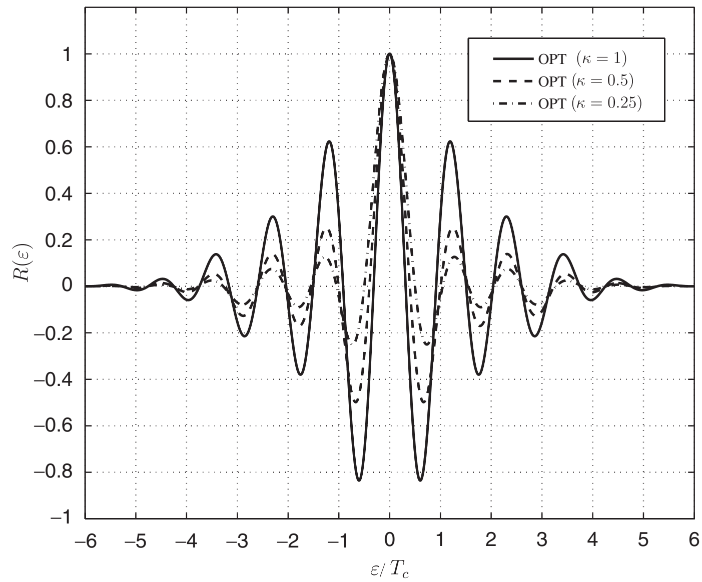

Figure 3. Autocorrelation function of optimum chip pulse shape.

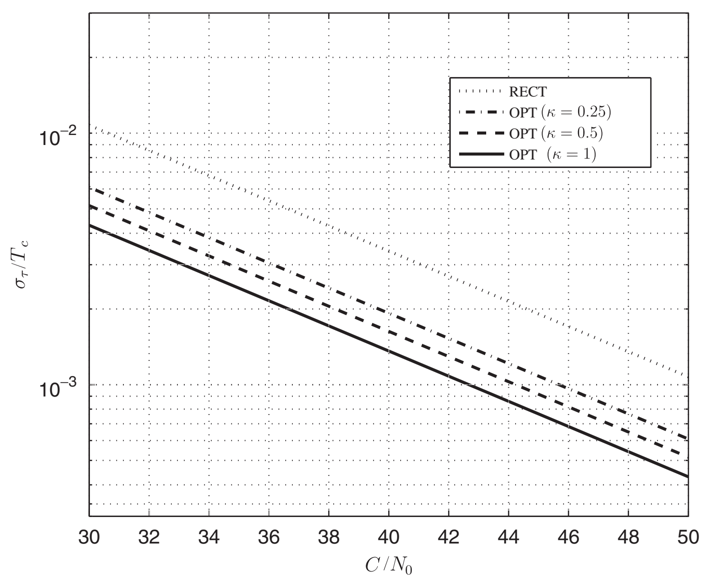

Figure 4. Standard deviation of the time-delay estimation error *versus* C/N_0.

biases of the time-delay estimates produced by a single reflection *versus* the relative delay between the line-of-sight signal (LOSS) and the multipath signal $\Delta\tau/T_c$ are plotted for the different chip pulse shapes. The attenuation of the multipath signal is considered to be -3 dB. We assume a

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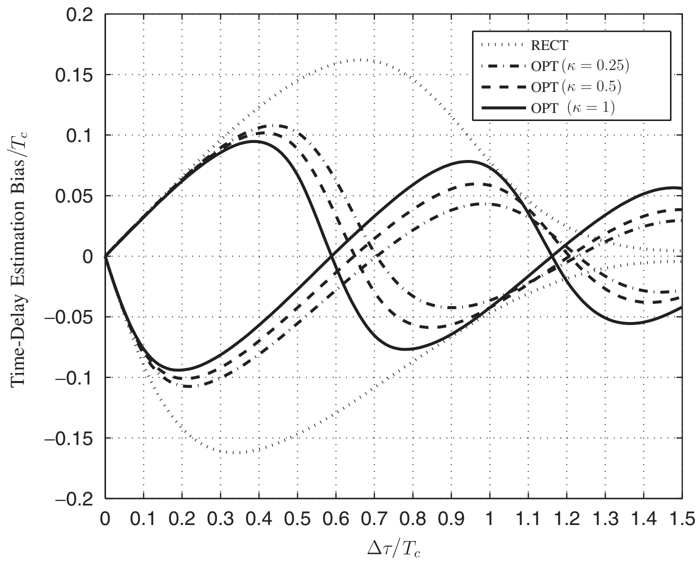

Figure 5. Multipath error envelope for different pulses.

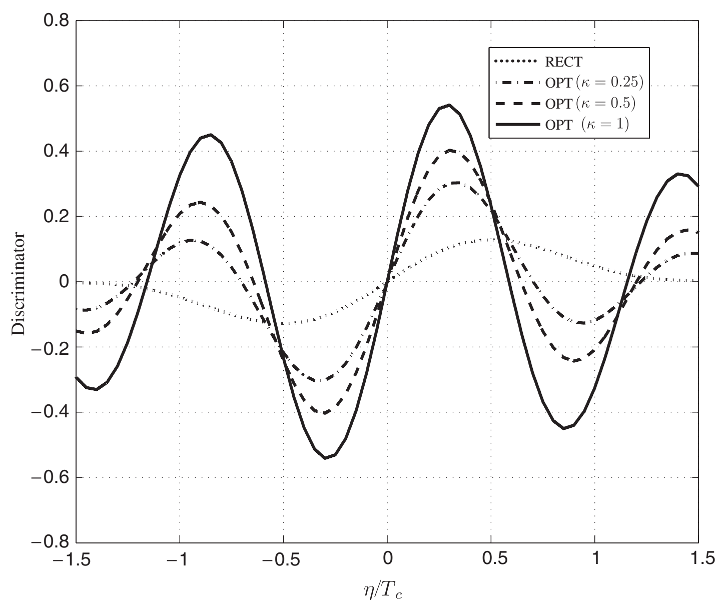

Figure 6. Normalized early–late discriminator *S*-curve.

narrow correlator [12] with an early–late correlator spacing of 0.1 chip. The maximum biases occur if the reflective multipath is either in phase or out of phase with the LOSS. Figure 5 depicts the so-called multipath error envelope. Here, one can observe that for smaller κ, the maximum biases get not only slightly larger for small $\Delta\tau/T_c$ but also smaller for larger $\Delta\tau/T_c$. In Figure 6, the normailzed early–late discriminator output of a DLL is plotted for the different chip pulse shapes *versus* η which gives the difference of delay of the received signal to the local reference code. We assume a narrow correlator [12] with early–late correlator spacing of 0.1 chip. The signals with different chip pulse shapes are normalized in power. In Figure 6, one can observe that for $\kappa=1$ we get the steepest slope of the discriminator S-curve at the tracking point ($\eta=0$) which results in the smallest standard deviation σ_τ given in Figure 4. However, the S-curve offers ambiguous and stable tracking points at $|\eta/T_c|>1$ which have to be avoided. This problem also occurs for the Galileo BOC(1, 1) signal [13]. The smaller the chosen value of κ, the less crucial these false tracking points get. Thus, κ represents the trade-off between the different performance requirements. The smaller the selected κ, the better the acquisition and multipath performance and the more robust the tracking. On the other hand the closer κ is to 1, the smaller the standard deviation σ_τ becomes.

5. CONCLUSION

In this work, we proposed a systematic methodology in order to design optimum chip pulse shapes $p(t)$ for a PN sequence which are absolutely bandlimited, their energy mainly concentrated within the chip duration, and which minimize the variance of the time-delay estimation error σ_τ^2 subject to constraints concerning the shape of the autocorrelation function $R(\varepsilon)$. The shaping threshold κ typifies the trade-off between the different performance requirements as the shape of the autocorrelation function directly affects tracking accuracy, tracking robustness, acquisition performance, and multipath performance.

The results clearly show that the optimum chip pulse shapes obtained by the proposed optimization method compared with the conventionally used bandlimited rectangular chip pulse shape significantly reduce the standard deviation of the tracking error and the multipath error simultaneously. Regarding tracking robustness which is given by the shape of the discriminator S-curve of a DLL and good acquisition performance for which there should advantageously be small local extrema v_i $(i\in\mathbb{N})$ of $R(\varepsilon)$, a trade-off can be accomplished by adjusting κ.

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