

# DATA DETECTION AND CHANNEL ALLOCATION ON THE UPLINK OF AN SDMA MOBILE RADIO SYSTEM

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## ABSTRACT

On the uplink of an SDMA mobile radio system  $K > 1$  users are transmitting in the same frequency band and time slot. These signals are received at the base station with an array of  $M \geq K$  antennas. In this paper a complete base station receiver structure to jointly detect the bits transmitted by the  $K$  user will be presented. To guarantee good receiver performance a DOA-sensitive channel allocation scheme will be described. The feasibility of the concept is proved by extensive simulations in realistically modelled 3D mobile radio channels.

## 1. DATA MODEL

Throughout this paper the symbol  $|\cdot|$  designates the magnitude of a complex number. Vectors are always considered to be column vectors and are denoted by lower case bold faced letters. Matrices are designated by upper case bold faced letters. The symbol  $(\cdot)^H$  indicates the complex conjugate transposition of a matrix or vector. The symbol  $\|\cdot\|_F$  designates the Froebenius norm of a matrix.

We assume an SDMA mobile radio system operating in a number of different time and/or frequency slots like the one described in [1] and [9]. In each slot  $K$  mobile users have to be spatially separated by means an  $M$  element antenna array at the base station.

The bandwidth of all baseband signals received from the users is considered to be much smaller than the reciprocal of the maximum time that a planar radio wave needs to cover the length of the array (narrow-band approximation).

Each user  $k$  produces a baseband signal  $s_k(t)$  on the uplink. Without loss of general validity the baseband signal is subject to the constraint  $E\{|s_k(t)|^2\} = 1$ .

Each signal reaches the base station through a high number  $Q_k$  of propagation paths produced by diffractions and scatterings [2] [3]. Each path  $q = 1 \dots Q_k$  impinging on the antenna  $m = 1 \dots M$  can be characterized by the time delay  $\tau_{k,m,q}$  and the time-varying complex amplitude

$$b_{k,m,q}(t) = A_{k,m,q} e^{j(f_{k,m,q}t + \phi_{k,m,q})}. \quad (1)$$

The complex amplitude  $b_{k,m,q}(t)$  incorporates the transmission factor  $A_{k,m,q}$ , the phase shift  $\phi_{k,m,q}$  and the Doppler frequency  $f_{k,m,q}$ .

The summation of all signals transmitted by the  $K$  users yields the uplink receive signal for the antenna  $m$  in baseband notation as follows:

$$x_m(t) = n(t) + \sum_{k=1}^K \sqrt{P_k} \sum_{q=1}^{Q_k} b_{k,m,q} s_k(t - \tau_{k,m,q}). \quad (2)$$

$P_k$  denotes the RF power corresponding to the signal transmitted by the user  $k$  whereas  $n(t)$  designates a random AWGN process<sup>1</sup> of variance  $\sigma^2$ .

Let us now assume bursty transmission with  $L$  binary symbols per burst and user. Low user velocities provided the time-varying complex amplitudes  $b_{k,m,q}(t)$  can be approximated by burstwise constant complex amplitudes  $b_{k,m,q}$ . Sampling the signal  $s_k(t)$  at the symbol rate  $1/T$  over the duration of one burst yields the complex valued sequence  $s_{k,1} \dots s_{k,L}$ .

Considering a limited channel memory  $C \cdot T$ , the time delays  $\tau_{k,m,q}$  can be replaced by discrete delays  $\tau_{min} + (c-1) \cdot T$ , ( $c = 1 \dots C$ ). Then, the discrete representation of the signal  $x_m(t)$  is given by the sequence  $x_{m,1} \dots x_{m,L}$

$$x_{m,l} = n_{m,l} + \sum_{c=1}^C \sqrt{P_k} \sum_{q=1}^{Q_k} b_{k,m,q} s_{k,l+1-c_{k,m,q}}. \quad (3)$$

<sup>1</sup>The process is assumed "white" within the bandwidth of the signal  $x_m(t)$ .

Alternatively the signal received at the antenna  $m$  can be expressed as

$$x_{m,l} = n_{m,l} + \sum_{k=1}^K \sum_{c=1}^C h_{k,m,c} s_{k,l+1-c} \quad \forall l = 1 \cdots L, \quad (4)$$

with  $\mathbf{h}_{k,m}^T = (h_{k,m,1} \cdots h_{k,m,C})$  denoting the sampled channel impulse response corresponding to the channel between the user  $k$  to the antenna  $m$ .

In order to get (4) in matrix notation we will define the  $M \times L$  array data matrix  $\mathbf{X}$ , the  $M \times L$  AWGN matrix  $\mathbf{N}$ , the  $M \times (K \cdot C)$  channel impulse response matrix  $\mathbf{H}$  and the  $(K \cdot C) \times L$  user data matrix  $\mathbf{S}$ .

The matrices  $\mathbf{X}$  and  $\mathbf{N}$  are composed of the array samples  $x_{m,l}$  and the noise samples  $n_{m,l}$ , respectively. The channel impulse responses from all users to all antennas are arranged in the matrix  $\mathbf{H}$  as follows:

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}_{1,1}^T & \cdots & \mathbf{h}_{K,1}^T \\ \vdots & & \vdots \\ \mathbf{h}_{1,M}^T & \cdots & \mathbf{h}_{K,M}^T \end{pmatrix}. \quad (5)$$

The matrix  $\mathbf{S}$  results from stacking the Toeplitz matrices  $\mathbf{S}_1 \cdots \mathbf{S}_K$  which are defined as

$$\mathbf{S}_k = \begin{pmatrix} s_{k,1} & \cdots & s_{k,C} & \cdots & s_{k,L} \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & s_{k,1} & \cdots & s_{k,L-C+1} \end{pmatrix}. \quad (6)$$

Finally (4) can be expressed by the system equation

$$\mathbf{X} = \mathbf{N} + \sum_{k=1}^K \mathbf{X}_k = \mathbf{N} + \mathbf{H}\mathbf{S}, \quad (7)$$

with  $\mathbf{X}_k$  denoting the part of the signals caused by the user  $k$ .

## 2. TRANSMITTER STRUCTURE

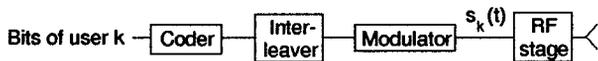


Figure 1: Transmitter

Each user  $k$  is equipped with a transmitter transforming his bits into the corresponding RF signal as depicted in fig. 1. In compliance with the GSM specifications [4] the coder is a convolutional coder with rate  $r = 0.5$ . The coded bits are interleaved with memory 8 and modulated according to the GMSK modulation scheme. The baseband signal  $s_k(t)$  is then passed to the RF stage and the mobile antenna.

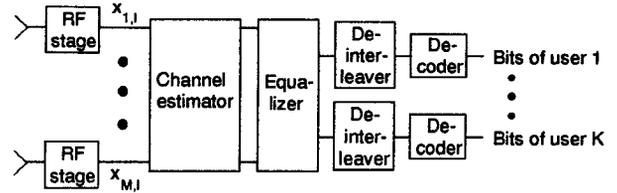


Figure 2: Receiver

## 3. RECEIVER STRUCTURE

At the base station each antenna  $m$  is equipped with an RF unit producing the baseband samples  $x_{m,l}$ . These samples are arranged in the array data matrix  $\mathbf{X}$  and fed to the multi-user data detector. As depicted in fig. 2 the detector consists of a channel estimator, a hard-output equalizer, a deinterleaver and a Viterbi decoder.

### 3.1. CHANNEL ESTIMATION

We suggest to execute the estimation of the channel impulse responses by means of orthogonal training sequences. We assume that all users  $1 \cdots K$  periodically transmit blocks of  $L_T$  a-priori known symbols. As a consequence a  $(K \cdot C) \times (L_T - C + 1)$  submatrix  $\mathbf{S}_T$  with a-priori known entries can be extracted from  $\mathbf{S}$ . With  $\mathbf{X}_T$  and  $\mathbf{N}_T$  denoting the corresponding array data submatrix and noise submatrix, respectively, the channel estimation problem can be put as

$$\mathbf{X}_T = \mathbf{N}_T + \mathbf{H}\mathbf{S}_T. \quad (8)$$

The least squares estimate  $\hat{\mathbf{H}}$  of the channel impulse response matrix can be computed by means of the Moore-Penrose inverse  $\mathbf{S}_T^+$  as follows

$$\hat{\mathbf{H}} = \mathbf{X}_T \mathbf{S}_T^+ = \mathbf{X}_T \mathbf{S}_T^H (\mathbf{S}_T \mathbf{S}_T^H)^{-1}. \quad (9)$$

Note that the calculation of  $\hat{\mathbf{H}}$  is not critical concerning real time or accuracy aspects because  $\mathbf{S}_T^+$  can be computed off-line. The design of the training sequences can be done off-line, too, aiming at optimal insensitivity of the channel estimation procedure to AWGN. Optimality in this sense is achieved by means of orthogonal training matrices  $\mathbf{S}_T$

$$\mathbf{S}_T \mathbf{S}_T^H = \rho \mathbf{I}, \quad \rho \in ]0, \infty[, \quad (10)$$

with  $\mathbf{I}$  denoting the  $(K \cdot C) \times (K \cdot C)$  identity matrix.

For GMSK-modulated signals perfect orthogonality of the training matrices cannot be achieved. Nevertheless channel estimation works for suboptimal training

matrices as well. E.g.: For  $C = 5$  and  $L_T = 26$  the following training sequences exhibit satisfying auto- and cross-correlation properties:

$$K = 1 \{ \text{"01000011101110100100001110"} \}, \quad (11)$$

$$K = 2 \left\{ \begin{array}{l} \text{"01000001111110101111010000"} \\ \text{"11111110101000110000011111"} \end{array} \right\}, \quad (12)$$

$$K = 3 \left\{ \begin{array}{l} \text{"1111111101000100001010111"} \\ \text{"11011010111100011001000111"} \\ \text{"0010000010011010111100000"} \end{array} \right\}. \quad (13)$$

### 3.2. EQUALIZATION

The equalization procedure can be interpreted as the estimation of the unknown user data submatrix  $\mathbf{S}_D$  by means of the known array data submatrix  $\mathbf{X}_D$  and the estimate  $\hat{\mathbf{H}}$  of the channel impulse matrix:

$$\mathbf{X}_D = \mathbf{N}_D + \hat{\mathbf{H}}\mathbf{S}_D. \quad (14)$$

The optimal bit error rate (BER) is yielded by a maximum likelihood sequence estimator (MLSE) which can be efficiently implemented with a Viterbi-like algorithm. Nevertheless, its computational complexity grows exponentially with the product  $K \cdot (C - 1)$  so that often it is necessary to resort to suboptimum algorithms like the M-algorithm [5], the vector decision feedback equalizer [6] or the minimum mean square error block decision feedback equalizer (MMSE-BDFE) [7].

### 3.3. SYNCHRONIZATION

We assume that by conventional techniques the SDMA system can provide a rough synchronization of the  $K$  transmitters and the base station receiver in a way that the difference of the base station sampling clock to the optimal clock is not more than half a bit duration.

We have tried to further improve the synchronization and, hence, the receiver by using a sampling rate which is  $R$  times the bit rate. At each antenna  $m$  the oversampled signal can be used to produce a set of  $2 \cdot R - 1$  sequences  $x_{m,l}(r)$  of length  $L$  with  $r = -R \dots + R$ . The sequence  $x_{m,l}(r)$  is equivalent to the signal  $x_m(t - r/(2 \cdot R \cdot T))$  sampled at the bit rate  $1/T$ .

The equalizer is supposed to operate on only one set  $x_{1,l}(r) \dots x_{M,l}(r)$  of sequences. The selection of the index  $r$  corresponding to the "optimally synchronized" set aims at causing the smallest channel model error due to the discretization of the delays from (2) to (3). Therefore, the channel estimator produces  $2 \cdot R - 1$  channel impulse response matrices  $\hat{\mathbf{H}}(r)$ .

That set  $x_{1,l}(r) \dots x_{M,l}(r)$  producing the smallest residual  $\|\mathbf{X}_T - \hat{\mathbf{H}}(r)\mathbf{S}_T\|_F$  will be passed to the equalizer because it is assumed to cause the lowest BER.

## 4. CHANNEL ALLOCATION

In an SDMA system an uplink channel allocation scheme has to take into account the directions of arrival (DOAs) of the users. To demonstrate this let us assume the following situation:  $K = 3$  users are transmitting in the same time/frequency slot with the wavelength  $\lambda$ . The base station is equipped with a  $\lambda/2$ -spaced uniform linear antenna array (ULA) with  $M = 12$  antennas. The three user-base radio channels are characterized by a single Rayleigh fading propagation path with the expectations  $E\{|b_{k,m,1}(t)|\} = 1$  and the time delays  $c_{k,m,1} = 1$  for all users  $k = 1 \dots 3$  and antennas  $m = 1 \dots 12$ . We also assume perfect power control and equal noise power  $N$  at the  $M$  antennas ( $P_k/N = \text{SNR}$ ). Since the three channels are not frequency selective,  $C = 1$  holds.

The array spacing is assumed to be small enough to render the fading processes  $b_{k,m,1}(t)$  corresponding to the same user  $k$  fully coherent [8]. Therefore, the column  $k$  of the  $12 \times 3$  channel impulse matrix  $\mathbf{H}$  is proportional to the steering vector

$$\mathbf{a}_k = (\Phi^1 \dots \Phi^M)^H, \quad \Phi = e^{-j\pi \sin \psi_k \cos \theta_k}, \quad (15)$$

with the azimuth and the elevation of the impinging wavefront denoted by  $\lambda$  and  $\psi_k$ , respectively.

Let us now state two extreme situations:

- (a) All users have exactly the same DOA at the base ( $\theta_k = 0^\circ$ ,  $\psi_k = 0^\circ$ ). Even if the fading processes  $b_{k,m,1}(t)$  at the same antenna  $m$  are statistically independent all columns of the channel impulse matrix  $\mathbf{H}$  are proportional to the same steering vector.  $\mathbf{H}$  only has rank 1.
- (b) The DOAs are chosen in a way that the steering vectors  $\mathbf{a}_k$  are orthogonal to each other ( $\theta_1 = 0^\circ$ ,  $\theta_2 = +30^\circ$ ,  $\theta_3 = -30^\circ$ ,  $\psi_k = 0^\circ$ ).  $\mathbf{H}$  has full rank 3.

The BERs for both cases (a) and (b) are depicted in fig. 3. The bad performance of the receiver in the case (a) urges the need for a DOA-sensitive channel allocation scheme to avoid "spatially unseparable" scenarios.

In the following we will present a procedure to check the "spatial separability" of an SDMA scenario. It was originally developed for downlink channel allocation [9] but (after slight modifications) turned out to be useful for the uplink case, too.

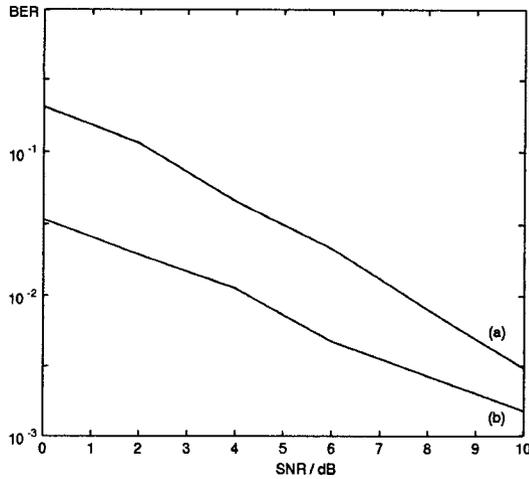


Figure 3: BERs for a scenario with identical (a) and orthogonal (b) steering vectors ( $K = 3$ ,  $M = 12$ )

- Estimate the spatial covariance matrices

$$\mathbf{C}_k = E\{\mathbf{X}_k \mathbf{X}_k^H\} \quad (16)$$

of all users  $k = 1 \dots K$ .

- For each matrix  $\mathbf{C}_k$  calculate a normalized eigenvector  $\mathbf{u}_k$  corresponding to its largest eigenvalue.
- Solve the real-valued linear  $K \times K$  system

$$\Psi \begin{pmatrix} v_1 \\ \vdots \\ v_K \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad (17)$$

$$\Psi = \begin{pmatrix} \mathbf{u}_1^H \mathbf{C}_1 \mathbf{u}_1 / 10^\alpha & \dots & -\mathbf{u}_K^H \mathbf{C}_1 \mathbf{u}_K \\ \vdots & \ddots & \vdots \\ -\mathbf{u}_1^H \mathbf{C}_K \mathbf{u}_1 & \dots & \mathbf{u}_K^H \mathbf{C}_K \mathbf{u}_K / 10^\alpha \end{pmatrix}.$$

- If (17) has a unique solution and if the numbers  $v_1 \dots v_K$  are all positive, the corresponding scenario will be considered spatially well separable.

A minimal value of 1 is recommended for the exponent  $\alpha$ . Increasing  $\alpha$  will result in a more restrictive spatial separability check.

## 5. SIMULATION RESULTS

The Schwabing district of the city of Munich, Germany, was chosen as the coverage area of an (imaginary) SDMA mobile radio cell. With an average building height of 5 stories this area measuring approx.  $20\text{km}^2$  can be considered a typical urban area in Europe. In this area 57 different locations representing 57

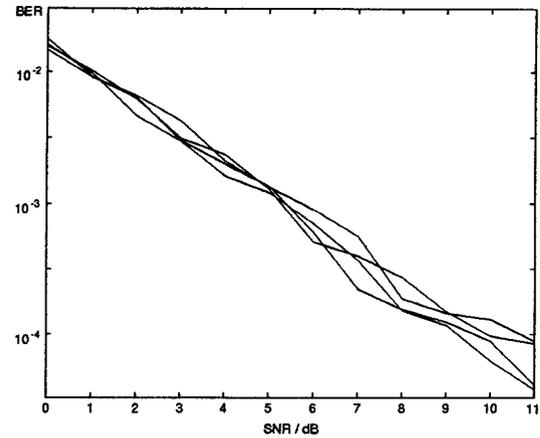


Figure 4: BERs for different oversampling factors  $R$  ( $K = 2$ ,  $M = 8$ )

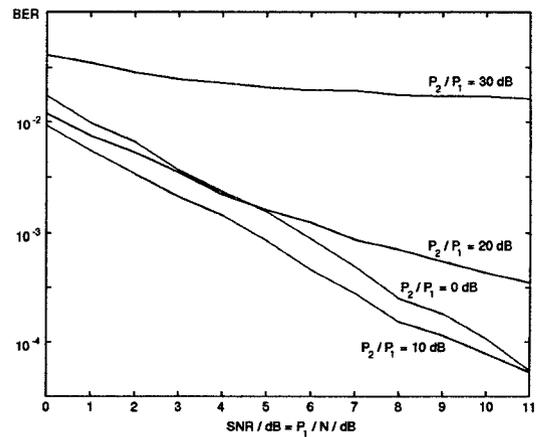


Figure 5: BERs for imperfect power control ( $K = 2$ ,  $M = 8$ )

different (imaginary) SDMA users were selected. Bursty transmission according to the GSM radio interface was assumed. At the base station the signals were received by means of a  $\lambda/2$  spaced ULA with  $M$  elements mounted to a rooftop  $26\text{m}$  above ground.

Each point in the plots fig. 4 to 6 results from 100 simulation runs with 80 GSM bursts [4] per run. Each burst carried  $2 \cdot 58 = 116$  coded bits, whereas the training sequences (11) to (13) had the length  $L_T = 26$ .

In each simulation run  $K$  out of the 57 users were selected in a random manner. If these two users did not pass the separability check with  $\alpha = 1$ , the selection was repeated.

The signal  $x_m(t)$  received at the antenna  $m$  was computed according to (2) comprising all propagation paths between the user  $k$  and the array within an attenuation range of 30 dB. To get these paths a 3D ray tracing tool described in [10] was fed with a digitized

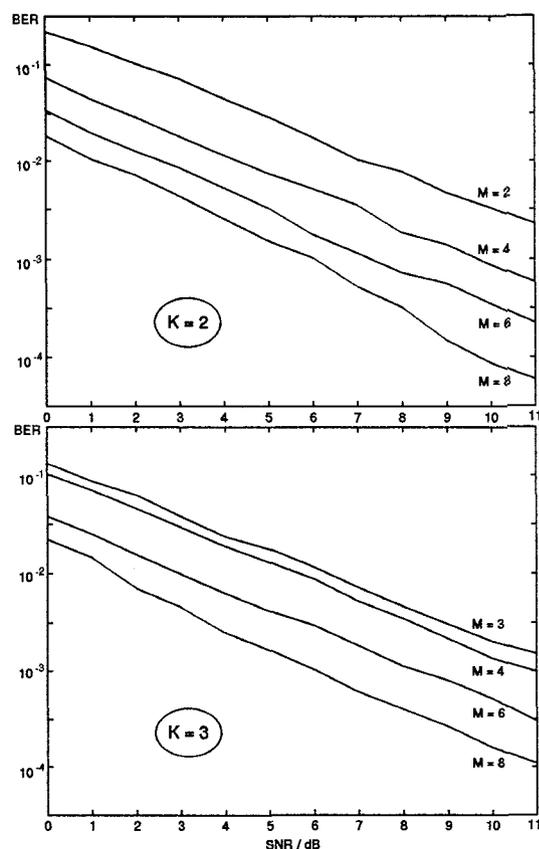


Figure 6: BERs for different numbers  $M$  of antennas

map of the city of Munich. Depending on the location of the user the tool produced the azimuths, elevations, delays and complex amplitudes corresponding to all relevant paths.

The number  $Q_k$  of relevant paths was in the range of 20 to 1000. The normalized length of the channel impulse response could be approximated by  $C = 3$ . The equalizer was implemented by means of a Viterbi-type algorithm operating with  $2^{K(C-1)}$  states.

## 6. CONCLUSIONS

We have presented a multi-antenna receiver structure for the uplink of an SDMA mobile radio system together with an appropriate channel allocation scheme. The performance of this new concept was analyzed by means of simulations in realistic urban multipath environments created by a 3D ray tracing tool. According to our simulation results adding SDMA features to a GSM type cellular mobile radio system can increase uplink capacity by a factor of  $K = 2$  for  $M \geq 5$  and  $K = 3$  for  $M \geq 7$  (see fig. 6).

Even though the receiver is near-far resistant (see

fig. 5), a DOA sensitive channel allocation scheme needs to be implemented (see fig. 3). Timing the bit sample clock with an accuracy better than half a bit duration does not significantly increase BER performance (see fig. 4).

## 7. REFERENCES

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