OUT-OF-BAND RADIATION IN MULTICARRIER SYSTEMS: A COMPARISON

Leonardo G. Baltar, Dirk S. Waldhauser and Josef A. Nossek
Munich University of Technology
Institute for Circuit Theory and Signal Processing
Arcisstrasse 21, 80290 Munich, Germany
{baltar,waldhauser,nossek}@nws.ei.tum.de

Abstract OFDM systems suffer from high out-of-band radiation. Consequently, they require methods reducing those spectral out-of-band components. Because of adjustable frequency confinement, filter bank based multicarrier systems allow for a lower out-of-band radiation. This paper compares an OFDM system employing one out-of-band reduction method with a filter bank based multicarrier system (FBMC).

Keywords: Multicarrier, OFDM, filter banks, out-of-band radiation, spectrum analysis

1. Introduction

Multicarrier systems have been considered as one of the most promising modulation solutions for future wireless communication systems due to their robustness against multipath propagation and the efficient use of bandwidth of the transmission channel. Orthogonal frequency-division multiplexing (OFDM) systems provide this efficiency but suffer from high out-of-band radiation originated from the sidelobes of the modulated subcarriers. Therefore, either a spectral guard band between adjacent services, zero input subcarriers or some kind of out-of-band reduction method need to be employed.

The use of spectrum guard bands or zero input subcarriers result in an undesired loss in the scarce spectrum resource.

Recently, new methods for out-of-band energy reduction in OFDM multicarrier systems have been proposed for the application of overlay systems. They are either based on the use of certain subcarriers as so-called cancellation carriers (CCs) [1] at the ends of the OFDM signal spectrum, on the employment of weighting coefficients at each subcarrier input [2] or on multiple choice sequences [3]. However, those methods result in an increase in bit error ratio (BER), a loss in bandwidth efficiency, an increased peak-to-average power ratio (PAPR), additional signaling overhead and/or an increased processing complexity at the transmitter and receiver. Multicarrier systems anyway suffer from increased PAPR values compared to single-carrier systems [4].

One alternative to the conventional OFDM system are filter bank based multicarrier systems (FBMC) or transmultiplexer (TMUX) systems. A TMUX based on exponentially modulated filter banks has the advantage of reduced implementation complexity by the use of polyphase decompositions and the Fast Fourier Transform (FFT) [5]. The stopband attenuation of the TMUX prototype filter determines the out-of-band energy of the modulated signal and can, therefore, be adjusted very flexibly in accordance with the requirements. The inevitable prototype filter lengths L with $L > M$, where M is the number of subcarriers, only allow for orthogonality in so-called orthogonally multiplexed QAM (OQAM) systems [6] or Modified DFT (MDFT) filter banks [7].

In this paper we compare the performance of an OFDM system employing the aforementioned cancellation carriers technique for the reduction of out-of-band radiation with a multicarrier system based on a Modified Discrete Fourier Transform transmultiplexer (MDFT-TMUX) [7] in the context of current and future 3GPP specifications [8, 9]. We show how the data throughput can be increased by the employment of an FBMC without substantially increasing complexity and latency while still fitting into the specified spectrum mask. That increase has two origins: there is no need of a prefix and the number of occupied subcarriers can be greater than the recommended.

In Section 2 we describe the filter bank based multicarrier system and present the spectrum modeling for both FBMC and conventional OFDM with and without cyclic prefix. Some spectrum examples are shown in Section 3 along with complexity and latency analysis. We summarize the results and draw some conclusions in Section 4.

2. System model

First, we briefly present the basics of multicarrier systems based on digital filter banks and in the sequel we describe the spectrum models adopted in the simulations.

2.1 Filter bank based multicarrier systems

There are three basic differences between the FBMC and the conventional OFDM system: no inclusion of a (cyclic) prefix; the complex input symbols have their real and imaginary parts interleaved, resulting in what is called OQAM; and there is a filtering step after the complex modulation of each sub-channel, also called polyphase network.

As mentioned before the best choice for a FBMC is the one where a prototype filter is modulated by complex exponentials. The prototype is designed in a way that adjacent subcarriers overlap, but remain orthogonal, and in non-neighboring subcarriers the attenuation guarantees negligible interference. The prototype can be, for example, a truncated root raised cosine filter (RRC) with length L and roll-off ρ. With this kind of prototype intersymbol interference (ISI) is also eliminated, provided that an OQAM stage is included [6, 7].

The modulations can be implemented via a DFT. With this modification, the polyphase components of the protoype filter are placed after each output of the DFT, instead of filtering each subchannel. In this way, an efficient implementation is obtained.

Figure 1 depicts an efficient structure of an MDFT synthesis filter bank. More efficient structures for the MDFT filter bank exist [10], but this topic is out of the scope of this work.

It is worth mentioning that, if the protoype has length M and all coefficients are equal to one, the OQAM stage can be eliminated and the conventional OFDM modulator is obtained.

2.2 Power spectrum density (PSD)

The total instantaneous spectral density of the signal at the output of a general multicarrier modulator results from the sum of the spectral densities of each ℓ-th subcarrier and it is described in the normalized angular frequency domain ω, for $0 \leq \omega < 2\pi$, by

$$S_k(\omega) = \sum_{\ell=0}^{M-1} |H_\ell(\omega)|^2 S_{x_{\ell,k}}(\omega), \quad -\infty \leq k \leq \infty \tag{1}$$

where $x_{\ell,k}$ is the complex QAM symbol modulating the ℓ-th subcarrier at the k-th time instant and $S_{x_{l,k}}(\omega)$ its corresponding spectrum density. This holds true because of the reasonable assumption of uncorrelated input symbols $x_{\ell,k}$, which will be justified because of coding and interleaving in practical systems. We define two types of shaping filters $H_\ell(\omega)$ for each subcarrier:

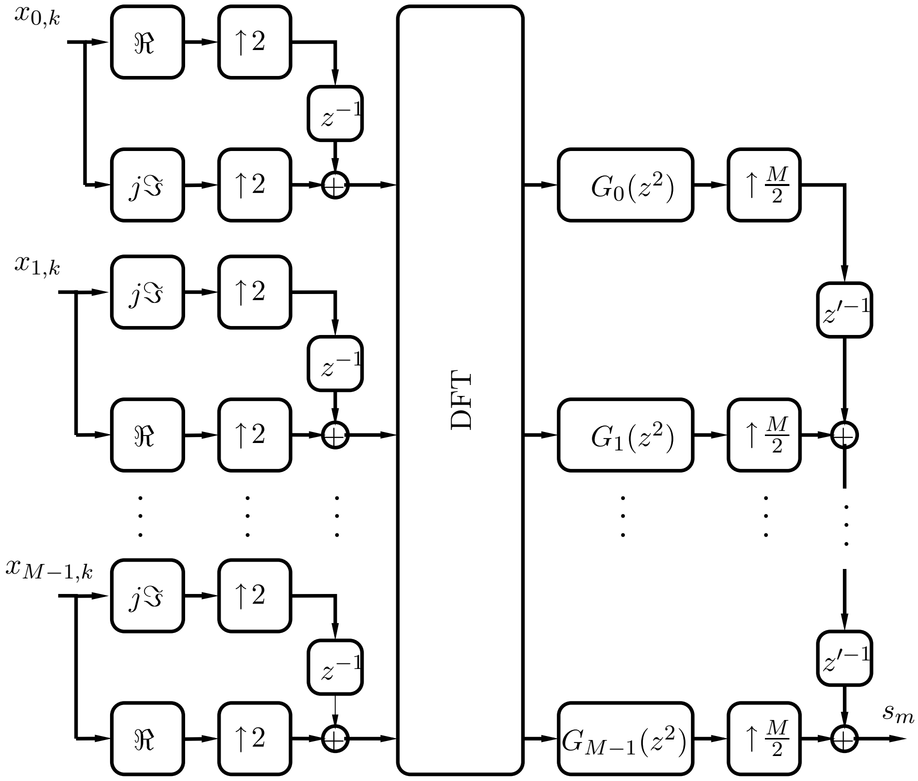

Figure 1. Synthesis Part of an MDFT Filter Bank, where z'^{-1} represents a delay in the output symbol rate

$$H_\ell(\omega) = \begin{cases} E_\ell(\omega) & : \text{ OFDM system,} \\ F_\ell(\omega) & : \text{ FBMC system.} \end{cases} \tag{2}$$

It can be demonstrated that the amplitude of the Fourier transform of the ℓ-th subchannel in the conventional OFDM system is given by the Dirichlet kernel

$$E_\ell(\omega) = \frac{\sin\left(M\left(\frac{\omega}{2} - \frac{\pi\ell}{M}\right)\right)}{M \sin\left(\frac{\omega}{2} - \frac{\pi\ell}{M}\right)}, \quad \ell = 0, ..., M-1. \tag{3}$$

The insertion of the cyclic prefix (CP) modifies (3) according to

$$\tilde{E}_\ell(\omega) = \frac{\sin\left((M + L_{\mathrm{CP}})\left(\frac{\omega}{2} - \frac{\pi\ell}{M}\right)\right)}{M \sin\left(\frac{\omega}{2} - \frac{\pi\ell}{M}\right)}. \tag{4}$$

We can conclude from (4) that the cyclic prefix will change the bandwidth of each subchannel resulting in ripples in the final spectrum [11].

In an FBMC system, we define the prototype coefficients as $h[n]$, with $n = 0, .., L-1$, where $L = KM + 1$ and K is the length of each polyphase components. We define the variable $\mathcal{L} = \frac{KM}{2}$ and assume that the prototype has even symmetry around the $\mathcal{L}$-th coefficient, this means that $h[n] = h[KM - n]$. The individual amplitude of the ℓ-th subcarrier is then given by

$$F_\ell(\omega) = \left[h\left[\mathcal{L}\right] + 2 \sum_{n=1}^{\mathcal{L}-1} h\left[\mathcal{L} - n\right] \cos\left(n \left(\omega - \frac{2\pi\ell}{M} \right) \right) \right]. \qquad (5)$$

Each $F_\ell(\omega)$ in (5) is equivalent to a frequency shifted version of the amplitude of the prototype filter.

3. Simulations

The Technical Specification Group for Radio Access Network of the 3GPP decided to focus the Long-Term Evolution feasibility study on multicarrier based downlink. Therefore, we will use the parameters recommended in the document [8], as it defines and describes the potential physical layer for evolved Universal Terrestrial Radio Access (E-UTRA).

The radio access has a hierarchical frame structure. Each radio frame has 10 ms and is composed by 20 subframes. The number of OFDM or FBMC symbols on each subframe depends on other parameters. The document specifies six different possible bandwidths (1.25, 2.5, 5, 10, 15, 20 MHz) and the subcarrier spacing $\Delta f = 15$ kHz is fixed regardless of the system bandwidth. The size of the FFT is chosen according to the desired bandwidth.

We will apply in our example the transmission bandwidth of 5 MHz, which corresponds to an FFT size of $M = 512$ for both OFDM and FBMC, where in both cases only 300 subcarriers are used, with the others having zero input symbols. The sampling frequency for that bandwidth is $f_\mathrm{s} = 7.68$ MHz.

For the conventional OFDM using cyclic prefix, a short or a long prefix is possible. When a short prefix is used, each subframe should have seven OFDM symbols, six of which have length $L_\mathrm{CP} = 36$ and one has $L_\mathrm{CP} = 40$. With these values, 93.33% of the subframe are used for data transmission.

The recommendation also foresees a longer prefix targeting multi-cell broadcast and very-large-cell scenarios. In this case, six OFDM symbols with $L_\mathrm{CP} = 128$ are filled into each subframe. With this value 80% of the subframe are used for data transmission.

For the FBMC case, seven and a half blocks of complex symbols (or fifteen blocks of pure real/complex symbols) compose each subframe.

Assuming an input with constant unitary spectral density ($S_{x_{l,k}}(\omega) = 1$) or, equivalently, uncorrelated symbols with unit energy, Figure 2 exhibits the power spectral densities of the FBMC with $K = 4$ and roll-off $\rho = 1$ and of the conventional OFDM without cyclic prefix. Besides that, it depicts the quadratic spectrum of one OFDM block employing the cancellation carriers method with $x_\ell = 1$, $\forall \ell$. The spectrum mask of the Universal Mobile Telecomunication System (UMTS) for a system with bandwidth of 5 MHz is drawn as a reference, where the multicarrier based E-UTRA physical layer with the same bandwidth has to fit into.

We used 2 CCs at each end of the spectrum for the optimization and didn't consider any power limitation, which means that the quadratic inequality constraint was not applied to the least squares problem [1]. We used 10 sidelobes at each side of the spectrum, which means that 10 samples in the optimization range were used [1].

If we loot at Figure 2, it is clear that the conventional OFDM without cyclic prefix or any method for reducing the out-of-band radiation does not fit into the specified mask.

We can also see that, when the FBMC or the OFDM with CCs is employed, they do not only fit into the mask, but also provide some room for further spectrum utilization. As a consequence, more subcarriers can be occupied, resulting in higher throughput and spectral efficiency.

In Figure 3 $N = 330$ subcarriers were used for the FBMC and $N = 328$ for the conventional OFDM with and without CCs, instead of $N = 300$.

With those new numbers of occupied subcarriers, an increase of 10% in the total throughput can be achieved for the FBMC and of 9.33% for the OFDM employing CCs.

We can notice in both spectrum examples that the OFDM system employing CCs presents strong ripples near the spectrum borders and around the DC subcarrier.

3.1 Complexity

In this section we consider the complexity of the signal generation only at the transmitter side. If we incorporate the modification proposed in [10] into the structure of Figure 1, and assume that M is a power of 2, the number of "flops" (floating-point operations) per output sample for the FBMC is given by

$$\mathsf{flops}_{\mathsf{FBMC}} = \mathsf{flops}_{\mathsf{FFT}} + 2M(4K + 1)$$

where $\mathsf{flops}_{\mathsf{FFT}}$ is the number of flops of the FFT and its current value is exhibited in [12]. If we use the values from the examples presented before

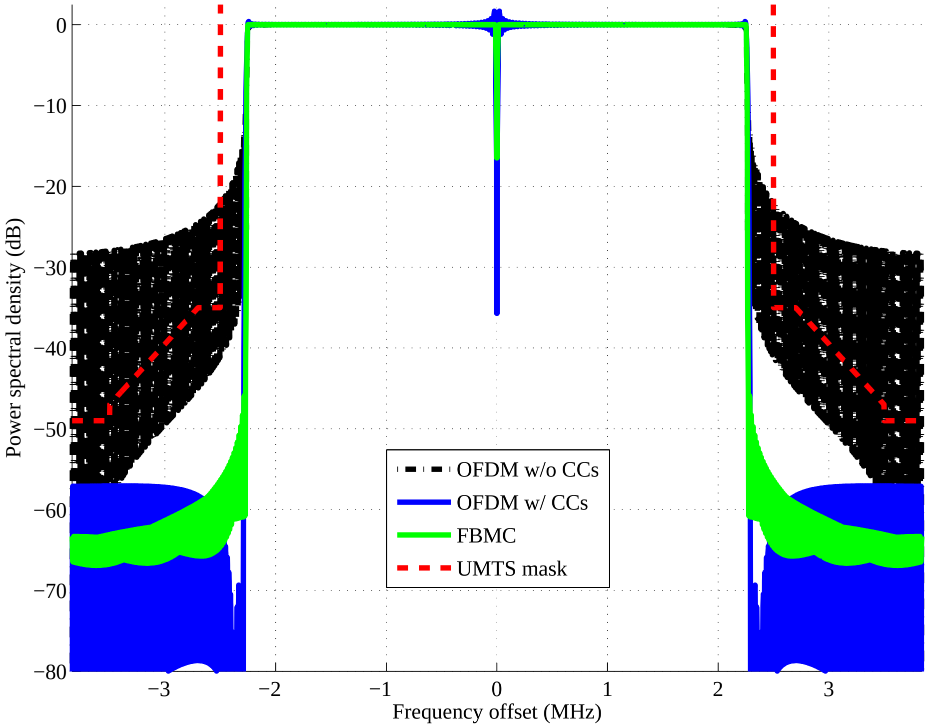

Figure 2. PSDs of FBMC and OFDM without CCs and squared frequency response of OFDM with CCs and input data $x_{\ell,k} = 1$. In all cases $N = 300$ subcarriers are active.

($K = 4$ and $M = 512$), the FBMC transmitter will need around twice the number of flops needed by conventional OFDM. Furthermore, if the prototype is designed to provide perfect reconstruction, the polyphase component pairs ℓ and $\ell + \frac{M}{2}$ can be efficiently realized using lattice structures [10].

When the CCs technique is incorporated into conventional OFDM, there is also an increase in complexity. Only to calculate the spectrum samples in the optimization range [1] for each input block, the same complexity as in the FBMC case is reached. But it still remains the computational burden for calculating the CCs coefficients. The latter will depend on the adoption or not of the quadratic constraint, and on the used method for solving the least squares problem.

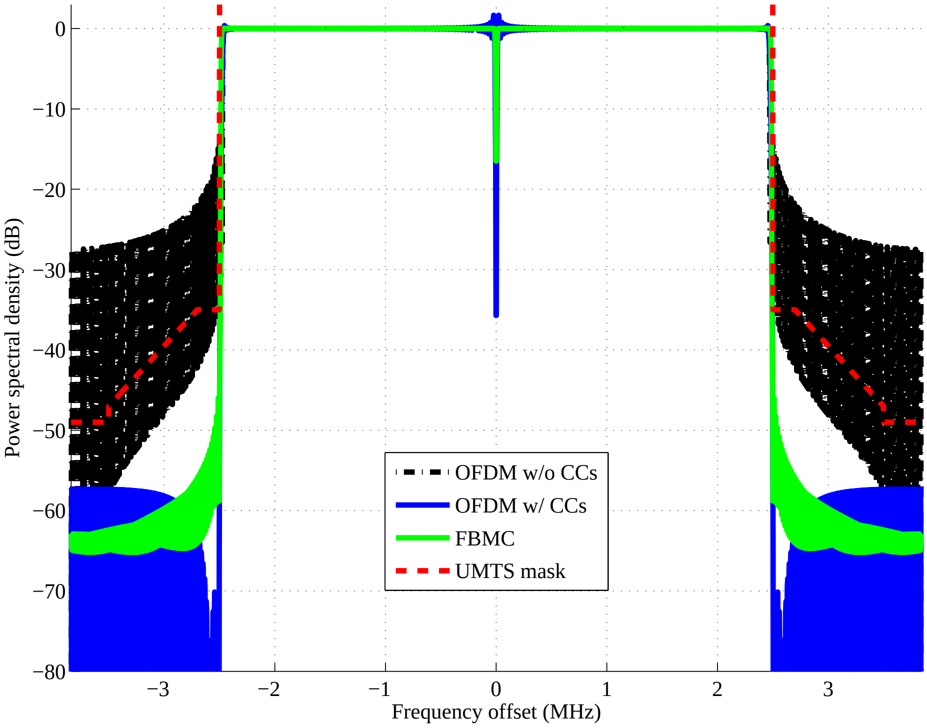

Figure 3. PSD of FBMC ($N = 330$) and OFDM without CCs ($N = 328$) and squared frequency response of OFDM with CCs and input data $x_{\ell,k} = 1$ ($N = 328$)

It is worth mentioning that in the FBMC, the same complexity exists at the receiver side, while for the OFDM with CCs, the receiver keeps the complexity of one FFT.

3.2 Latency

One of the drawbacks of employing an FBMC instead of a conventional OFDM, besides the increased complexity, is the increased latency. This latency is mainly caused by the filtering performed in the polyphase network. It can be demonstrated that the delay of the FBMC is given by $d_{\mathrm{FBMC}} = \frac{(K+1)M}{f_{\mathrm{s}}}$. The resulting latency for $K = 4$ and $M = 512$ is $d_{\mathrm{FBMC}} = 0.33$ ms. This delay added to the average delay of 4.0 ms generated by the adopted protocol architecture of LTE [8] still keeps the total user-plane delay below the recommended 5.0 ms.

When CCs are incorporated into conventional OFDM, some delay will also be inserted. But in this case, the delay will depend on the capabilities of the hardware employed to calculate the CCs.

4. Conclusions

We first briefly described the multicarrier system based on filter banks, then showed how the power spectral density for both FBMC and conventional OFDM can be modeled and explaned the effect of the cyclic prefix on the modeling of the latter.

The simulations were performed under the framework of the Long-Term Evolution recommendation from the 3GPP standardization group. We showed two examples of spectral densities: The first adopted the number of subcarriers found in the recommendations and a second used an increased number of active subcarriers for a more efficient spectrum occupation. We proved that if FBMC or OFDM with the cancellation carriers method is employed, more subcarriers can be occupied as defined in the 3GPP recommendations without exceeding the defined mask. If we combine this increase with the lack of prefix, the FBMC will achieve a gain of 17.15% or 35% in data throughput compared to the conventional OFDM system with the short or the long cyclic prefix, respectively.

We showed that the complexity is increased by a factor of two for both FBMC and OFDM with cancellation carriers when compared to conventional OFDM and that the increased latency in the FBMC resulting from polyphase filtering is acceptable and remains below the recommended latency.

Both FBMC and OFDM have the drawback of a high peak-to-average power ratio, but when the cancellation carriers method is included in OFDM, it becomes even higher.

As the FBMC system presents more degrees of freedom, the length of the polyphase components and the roll-off factor can be adjusted to keep the spectrum of the output signal at the transmitter into the specified mask for each regulated frequency band under consideration of allowed complexity and maximum latency.

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