Comparison of Filter Bank Based Multicarrier Systems with OFDM

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Abstract— **Multicarrier systems are favored for contemporary and future, both stationary and mobile, communication systems. They promise a high bandwidth efficiency and at the same time the capability to cope with frequency selective (radio) channels. Unfortunately, the peak-to-average power ratio (PAPR) is increased at the transmitter of multicarrier systems compared to single carrier systems. Filter bank based multicarrier systems, i. e. transmultiplexer filter banks, provide a much better spectral shaping of the subcarriers than orthogonal frequency division multiplexing (OFDM). This leads to several advantages against OFDM systems, which are highlighted in this contribution. Particularly, the PAPR performance of filter bank based multicarriers systems and OFDM are compared.**

I. INTRODUCTION

Multicarrier systems provide optimum adaptability to the time and frequency selectivity of propagation channels, which simplifies their equalization. This is very attractive for mobile communication channels which are subject to multipath propagation and vary frequently with time.

Multicarrier systems allow for an adaptation to the frequency response of the channel by using different modulation alphabets and power allocation for the respective subcarriers. In this way an approximation to the waterfilling solution can be achieved and the available bandwidth can be used very efficiently.

On the one hand OFDM is a very popular special case of a multicarrier system. It is used in many current standards because of its efficient implementation with Fast Fourier Transforms (IFFT and FFT) and its "simple" equalization. This equalization is realized by a time-domain guard interval, the cyclic prefix (CP) and a simple complex scaling for each subcarrier. This holds true as long as the CP exceeds the effective channel impulse response, which incorporates the pulse shaping filters at the transmitter and the receiver and the propagation channel. But the bandwidth efficiency is reduced up to 25%.

On the other hand filter bank based multicarrier systems with complex modulation provide a better spectral shaping of the subbands. This can be utilized for simplifying the equalization of intersymbol (ISI) and intercarrier interference (ICI) when no cyclic prefix is used — in order to maintain the high bandwidth efficiency multicarrier systems can provide — as shown in, e. g. [1], [2].

One major drawback of multicarrier systems is the increase of the PAPR compared to single carrier systems. This increase is the result of the superposition of a large number of statistically independent subchannels which are able to constructively sum up to high peaks [3].

The problem is that practical transmission systems are peak-power limited and show nonlinear characteristics which cause spectral widening of the transmit signal.

In the literature, there are several PAPR reduction techniques like amplitude clipping, partial transmit sequence, selected mapping, active constellation extension, etc. [4] gives a comprehensive overview of the used techniques. The main objective of this work is to show the behavior of OFDM and MDFT filter bank based multicarrier systems in the context of the resulting PAPR distribution.

II. BLOCK STRUCTURE OF THE MULTICARRIER SYSTEMS

Fig. 1 shows a very efficient implementation of the synthesis part of a filter bank based multicarrier system with M subcarriers, where the subcarrier filters $H_m(z)$ are complex modulated versions of a chosen prototype filter $H_0(z)$, i. e. $H_m(z) = H_0(W_M^m z)$, $m = 0, 1, \ldots, M - 1$, $W_M = \exp(-\mathrm{j}\,2\pi/M)$. The prototype filter $H_0(z) = \sum_{m=0}^{M-1} z^{-m} H_{0,m}(z^M)$ is decomposed into its polyphase components $H_{0,m}(z^M)$. This step is then combined beneficially with the complex modulation of the subcarrier filters in order to separate an inverse Discrete Fourier Transform (IDFT) with subsequent polyphase filtering. With M is equal to a power of 2 the IDFT can be computed as inverse Fast Fourier Transform (IFFT) and the length of the polyphase components is reduced by the factor M compared to the prototype filter $H_0(z)$ and its complex modulated versions $H_i(z)$, for $i = 1, 2, \ldots, M - 1$. The IFFT and the polyphase filters operate at the low symbol rate. This means a reduction in the computations by a factor of M^2 for the additional complexity of the IFFT operation compared to the original complex modulated filter bank structure.

OFDM can be recognized as a special case of this efficiently implemented complex modulated filter bank depicted in Fig. 1 with $H_{0,m}(z^M) = 1$. This corresponds to a prototype filter $H_0(z) = 1 + z^{-1} + \cdots + z^{-(M-1)}$.

If the prototype filter $H_0(z)$ is chosen as a square root raised cosine (RRC) filter which pairwise satisfies the Nyquist criterion, the channels can be realized in phase
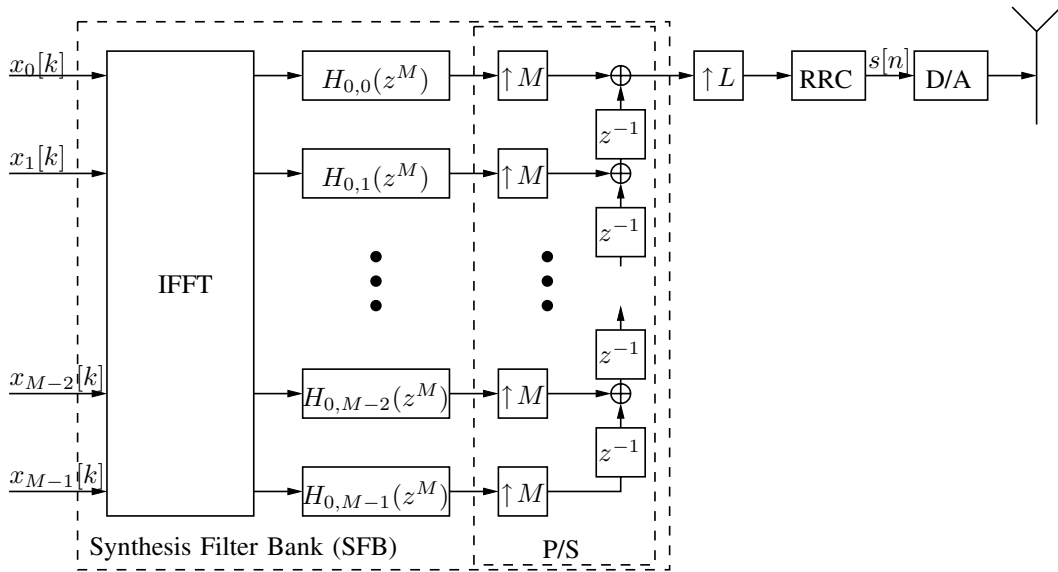
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Fig. 1. Efficient Implementation of the Synthesis Part of a Complex Modulated Filter Bank

quadrature [5] in order to eliminate interchannel interference caused by the spectral overlap of the frequency responses of the subband filters, cf. Fig. 2. This structure is also called orthogonally multiplexed QAM system ([6]) or modified DFT transmultiplexer (MDFT TMUX) ([7]).

III. COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION (CCDF) OF THE PAPR

A. OFDM Case

Assuming that the vectors $\boldsymbol{x}[k] = [x_0[k], x_1[k], ..., x_{M-1}[k]]^{\mathrm{T}}$ are transmitted, where $x_m[k]$ are M complex modulated independent data symbols corresponding to the k-th block, the complex time-domain transmit signal is generated by serializing the $\boldsymbol{s}_k = [s_k[0], s_k[1], ..., s_k[M-1]]^{\mathrm{T}}$ blocks, where

$$s_k[n] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} x_m[k] \exp\left(\frac{\mathrm{j} 2\pi mn}{M}\right), \qquad (1)$$
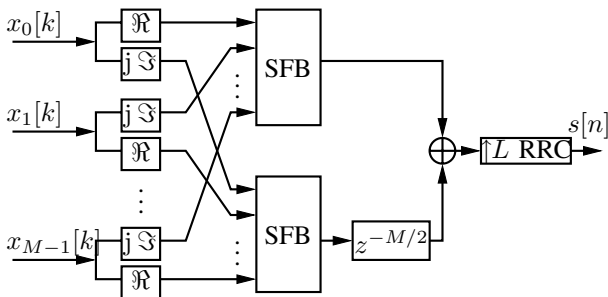

Fig. 2. Efficient Offset-QAM Implementation

for $0 \leq n \leq M - 1$ and $-\infty \leq k \leq \infty$. The PAPR for the k-th transmitted OFDM block is defined as

$$\mathrm{PAPR}_k = \frac{\max\limits_{0 \leq n \leq M-1} |s_k[n]|^2}{\mathrm{E}\left[|s_k[n]|^2\right]} \approx \frac{\max\limits_{0 \leq n \leq M-1} |s_k[n]|^2}{\frac{1}{M-1} \sum\limits_{k=0}^{M-1} |s_k[n]|^2}, \quad (2)$$

where the variance $\mathrm{E}\left[|s_k[n]|^2\right]$ has been replaced by its unbiased estimator. The absolute PAPR value can also be defined in practice, but then it would represent the upper bound that occurs with very low probability, which actually decreases exponentially with M [8].

As it is well known from the literature and as shown in the simulations results, a PAPR evaluated at the symbol rate is rather optimistic. A better approximation for the continuous signal PAPR can be obtained by using a pulse shaped oversampled output. For this purpose an RRC filter is used after the oversampling.

The PAPR after the upsampling by a factor of L and pulse shaping is defined as

$$\mathrm{PAPR}_k \approx \frac{\max\limits_{0 \leq n \leq LM-1} |s_k[n]|^2}{\frac{1}{LM-1} \sum\limits_{k=0}^{LM-1} |s_k[n]|^2}. \qquad (3)$$

When $L = 1$, the symbol rate output is used. It is shown in the literature that oversampling with $L = 4$ gives sufficiently accurate results.

It can be noticed that the PAPR defined as above is a random variable. For this reason a statistical distribution must be taken into account, namely the CCDF is most frequently used in the literature. In [8] a theoretical approximation for the CCDF of the OFDM system has been derived. In this work, the CCDFs statistically obtained from numerical simulations are used instead of a theoretical approximation.

B. MDFT TMUX Case

Although in the MDFT filter bank based multicarrier system the concept of independent transmitted blocks is not strictly valid, in this work, the same PAPR evaluation method has been applied.

When the $s_k[n]$ samples from the transmitted signal are grouped into blocks of ML symbols, the same PAPR formula defined for the OFDM system in (3) can be applied.

IV. SIMULATION RESULTS

In the following simulations QPSK has been used for the MDFT filter bank system and the OFDM system as modulation alphabet for each of the subcarriers. The CCDF of the PAPR, i. e. the probability that the PAPR of the transmit signal exceeds a given threshold value, is simulated for $M = 64$ and $M = 256$ subcarriers, with and without pulse shaping with upsampling factor $L = 4$. The practical relevance of the PAPR without pulse shaping ($L = 1$) is not rated very high by the authors, the curves are supposed to show the influence of the pulse shaping on the PAPR results.

In the case of $M = 64$[1] and $M = 256$ subcarriers $M_0 = 12$ and $M_0 = 64$ subcarriers are zeroed out as frequency guard bands, respectively. This means that only 52 and 192 subcarriers are used for data transmission, respectively.

In Fig. 3, where $M = 64$ subcarriers are used, the PAPR-CCDF of the transmit signal of the MDFT TMUX is nearly identical with that of the OFDM system. This is true without pulse shaping and critical sampling ($L = 1$) as well as with RRC pulse shaping with roll-off factor $\rho_{ps} = 0.23$ and oversampling ($L = 4$).

If the number of subcarriers is increased from $M = 64$ to $M = 256$ the peak of the worst case is certainly increased, too. The PAPR of a QPSK OFDM signal with $L = 1$ is always upper bounded by M, where M is the number of subchannels (cf. [3]). Therefore, the probability of the PAPR
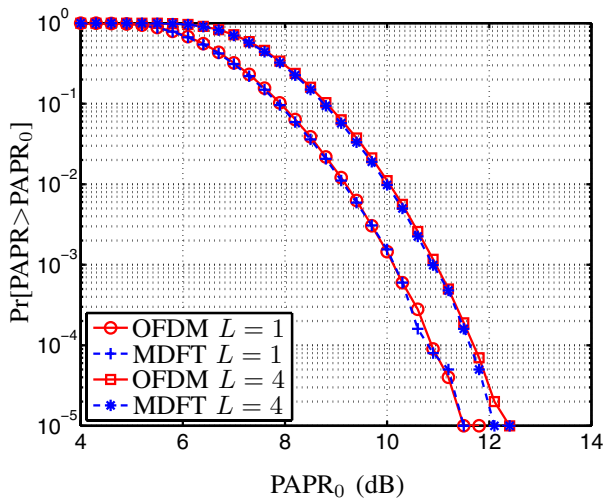
[1]IEEE Standards 802.11 and 802.16

to exceed a certain value is growing with M, and the CCDF curve in Fig. 4 is shifted to the right hand side compared to Fig. 3. Again the results for OFDM and the MDFT TMUX are nearly the same in both for $L = 1$ and pulse shaping with $L = 4$.

Fig. 5 depicts the results for the PAPR of a MDFT TMUX with RRC polyphase filters, where ρ_{fb} is taken from the set $\{0.25, 0.5, 0.75\}$.

The CCDF of the PAPR values is almost identical for these choices of ρ_{fb} at least for the same polyphase component length $K = 10$, which has been used in the simulations.

V. POWER SPECTRAL DENSITY

Current communication systems have to restrict the power spectral density of the transmit signal to accurately specified spectrum masks. The power spectral density of the transmit signal is investigated for OFDM with and without cyclic prefix and the MDFT TMUX. Both systems are setup with $M = 64$ subcarriers and a frequency guard band of $M_0 = 12$ subcarriers. The CP for OFDM transmission is chosen as 25% of the number of subcarriers of an OFDM symbol and, therefore, $N_{cp} = 16$.

The RRC pulse shaping filter which has to limit the bandwidth of the transmit signal and simultaneously avoid intersymbol interference has a roll-off factor $\rho_{ps} = 0.23$ and operates at an oversampling rate of $L = 4$.

Fig. 6 shows that the transmit signal of the MDFT TMUX after pulse shaping with an RRC ($\rho_{ps} = 0.23$) is sharply limited in its bandwidth. The reason is the spectral shaping of the subchannels by RRC filters with roll-off $\rho_{fb} = 0.5$. For practical spectrum masks the subsequent pulse shaping filter is only required to eliminate the periodic repetitions, not to accomplish the attenuation in the transition region. The PSD of the OFDM transmit signal without CP in contrast lacks a strict limitation and requires, therefore, the attenuation of the pulse shaping filter in the transition region.

In order to compensate for the loss in data rate, OFDM with CP is assumed to work at the increased sampling rate
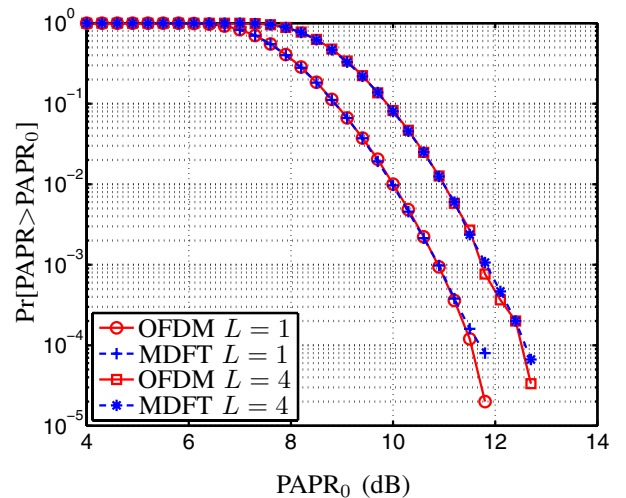
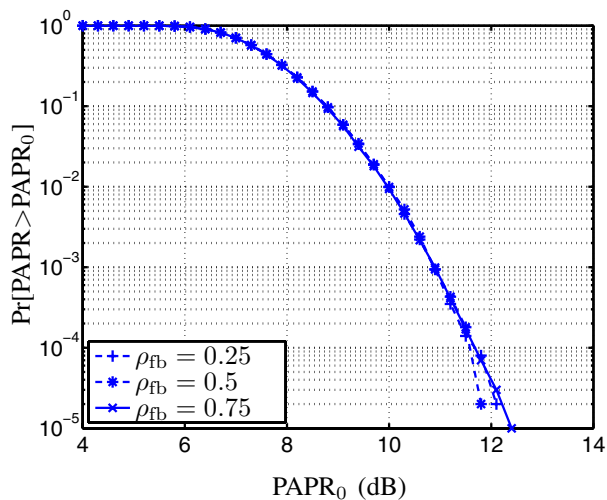Fig. 3. OFDM and MDFT with $M = 64$, $M_0 = 12$, $\rho_{ps} = 0.23$

Fig. 4. OFDM and MDFT with $M = 256$, $M_0 = 64$, $\rho_{ps} = 0.3$

Fig. 5. MDFT with $M = 64$, $M_0 = 12$

$f_{\mathrm{cp}} = 1.25 f_0 = 1.25/T$, where $z = \exp(pT)$. Neglecting that the shape of the power spectral density will change when the CP is inserted, the PSD of OFDM with CP is only a stretched version of the PSD of OFMD without CP as depicted in Figs. 6 and 7. This means that the bandwidth is increased by 25%.

VI. CONCLUSION

The simulation results in Section IV have shown that the CCDF for an MDFT TMUX and an OFDM system are almost identical. This is true independent of the number of subcarriers and also with respect to subsequent RRC pulse shaping.

The OFDM power spectral density is not so nicely bandlimited because of the low stopband attenuation of the trivial subband filters. Therefore, OFDM requires a larger system bandwidth for the same data rate. This fact is even exaggerated by applying a CP. Incorporating a CP and keeping the data rate constant leads to another increase in the required bandwidth as shown in Figs. 6 and 7.
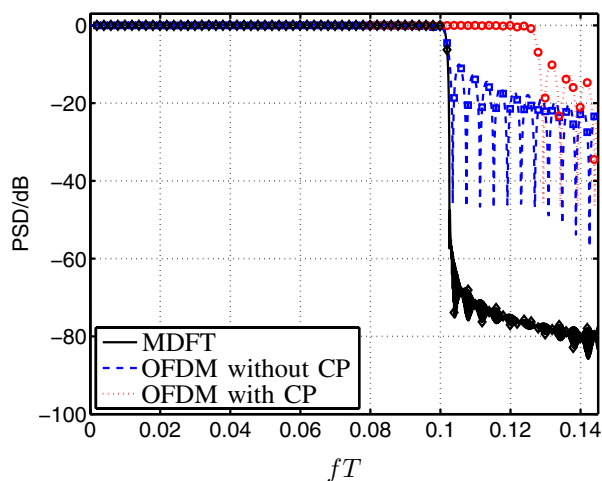
Another possibility to obtain equal data rates with the same bandwidth requirements for the MDFT TMUX and the OFDM system is to increase the modulation alphabet and/or the code rate of the channel codes in the OFDM system.

In all of these cases avoiding increased (uncoded or coded, respectively) bit error probabilities means to use higher transmit power for the OFDM system. Higher transmit power leads not necessarily to higher PAPR values, because the average power increases, too. But definitely it results in higher peak-power values, which are the ones which really hurt.

Therefore, an MDFT TMUX based transmitter has advantages regarding spectrum confinement and peak-power values compared to an OFDM based transmitter. On the other hand, the receiver will be more complex because of the necessary equalizer.

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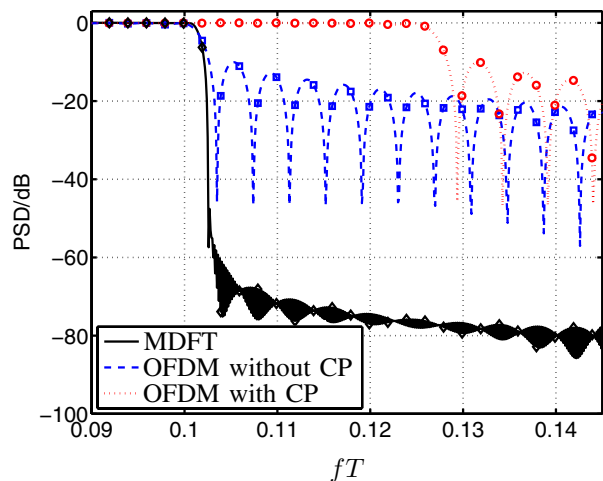Fig. 6. Power Spectral Density of MDFT vs. OFDM (w/o CP)

Fig. 7. Zoom into Power Spectral Density of MDFT vs. OFDM (w/o CP)