A Modified MMSE Receiver for Quantized MIMO Systems

Amine Mezghani, Mohamed-Seifeddine Khoufi and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Munich University of Technology, 80290 Munich, Germany
E-Mail: {Mezghani, Khoufi, Nossek}@nws.ei.tum.de

***Abstract*— We investigate the joint optimization of quantizer and linear receiver for the quantized multi-input multi-output (MIMO) channel. Our approach is based on a *mean square error* (MSE) criterion, taking into account the effects of quantization. An essential aspect of our derivation is that we do not make the assumption of uncorrelated white quantization errors. The performance of the modified filter as well as the effects of quantization are studied theoretically and experimentally. Furthermore, we provide a lower bound on the capacity of this channel. Through simulation, we demonstrate the usefulness of our approach.**

I. Introduction

Most of the contributions on receiver design for multiple-input multiple-output (MIMO) systems assume that the receiver has access to the channel data with infinite precision. In practice, however, a quantizer is applied to the receive signal, so that the channel measurements can be processed in the digital domain. The reliance on high-resolution analog-to-digital converters (ADCs) easily becomes unjustified as soon as we have to do with MIMO channels [1]. In this case, the needed ADCs to fulfill this assumption are expensive and even no more feasible. In fact, in order to reduce circuit complexity and save power and area, low resolution ADCs have to be employed [2]. Therefore, the proposed receiver designs do not necessarily have good performance when operating on quantized data in a real system. In [1] and [3], the effects of quantization are studied from an information theoretical point of view. In [4], the authors examined these effects experimentally by using a standard Zero-Forcing filter at the receiver. In this paper, we are interested in modifying the well-known linear *minimum mean square error* (MMSE) receiver (or Wiener filter) taking into account the presence of the quantizer. Under the choice of an optimal uniform/non-uniform scalar quantizer we evaluate the resulting MSE between the estimated and the transmitted symbols and we minimize it with a linear filter. Thereby, no assumption of uncorrelated white quantization errors is made. Through simulation, we compare the new filter to the conventional Wiener filter in terms of uncoded BER. In our model we assume perfect channel state information (CSI) at the receiver, which can be obtained even with coarse quantization [5].

Our paper is organized as follows. Section II describes the system model and notational issues. In Section III, we discuss the properties of the chosen Quantizer, then we derive the optimal receiver in section IV. In Section V, we deal with the effects of quantization on the MSE. Next, we provide a lower bound on the channel capacity in section VI, based on this MSE approach; and we also examine the effects of quantization on the capacity at low and high SNR values. Finally, we present some simulation results in section VII.

II. System Model And Notation

We consider a point to point MIMO Gaussian channel where the transmitter employs M antennas and the receiver has N antennas. Fig. 1 shows the general form of a quantized MIMO system, where $\boldsymbol{H} \in \mathbb{C}^{N\times M}$ is the channel matrix. The vector $\boldsymbol{x} \in \mathbb{C}^{M}$ comprises the M transmitted symbols with zero-mean and covariance matrix $\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}]$. The vector $\boldsymbol{\eta}$ refers to zero-mean complex circular Gaussian noise with covariance $\boldsymbol{R}_{\eta\eta} = \mathrm{E}[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}]$. $\boldsymbol{y} \in \mathbb{C}^{N}$ is the unquantized channel output:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{\eta}. \tag{1}$$

In our system, the real parts $y_{i,R}$ and the imaginary parts $y_{i,I}$ of the receive signals y_i, $1 \leq i \leq N$, are each quantized by a b-bit resolution uniform/non-uniform scalar quantizer. Thus, the resulting quantized signals read as:

$$r_{i,l} = Q(y_{i,l}) = y_{i,l} + q_{i,l},\ l \in \{R, I\},\ 1 \leq i \leq N, \tag{2}$$

where $Q(\cdot)$ denotes the quantization operation and $q_{i,l}$ is the resulting quantization error.

The matrix $\boldsymbol{G} \in \mathbb{C}^{M\times N}$ represents the receive filter, which delivers the estimate $\hat{\boldsymbol{x}}$:

$$\hat{\boldsymbol{x}} = \boldsymbol{G}\boldsymbol{r}. \tag{3}$$

Our aim is to choose the quantizer and the receive matrix $\boldsymbol{G}$ minimizing the MSE $= \mathrm{E}[\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2^2]$, taking into account the quantization effect. Throughout this paper, $r_{\alpha\beta}$ denotes $\mathrm{E}[\alpha\beta^*]$.

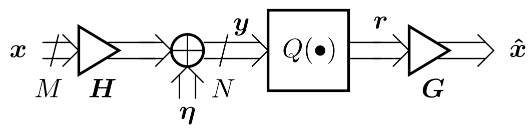

Fig. 1. Quantized MIMO System

III. Optimal Quantizer

Each quantization process is given a distortion factor $\rho_q^{(i,l)}$ to indicate the relative amount of quantization noise generated, which is defined as follows

$$\rho_q^{(i,l)} = \frac{\mathrm{E}[q_{i,l}^2]}{r_{y_{i,l}y_{i,l}}}, \tag{4}$$

where $r_{y_{i,l}y_{i,l}} = \mathrm{E}[y_{i,l}^2]$ is the variance of $y_{i,l}$ and the distortion factor $\rho_q^{(i,l)}$ depends on the number of quantization bits b, the quantizer type (uniform or non-uniform) and the probability density function of $y_{i,l}$. Note that the signal-to-quantization noise ratio (SQNR) has an inverse relationship with regard to the distortion factor

$$\mathrm{SQNR}^{(i,l)} = \frac{1}{\rho_q^{(i,l)}}. \tag{5}$$

In our system, the uniform/non-uniform quantizer design is based on minimizing the *mean square error* (distortion) between the input $y_{i,l}$ and the output $r_{i,l}$ of each quantizer. In other words, the SQNR values are maximized. Under this optimal design of the scalar finite resolution quantizer, whether uniform or not, the following equations hold for all $0 \leq i \leq N$, $l \in \{R, I\}$ [6], [7], [8]:

$$\mathrm{E}[q_{i,l}] = 0 \tag{6}$$
$$\mathrm{E}[r_{i,l}q_{i,l}] = 0 \tag{7}$$
$$\mathrm{E}[y_{i,l}q_{i,l}] = -\rho_q^{(i,l)} r_{y_{i,l}y_{i,l}}. \tag{8}$$

Obviously, Eq. (8) follows from Eqs (4) and (7). For the uniform quantizer case, Eq. (6) holds only if the probability density function of $y_{i,l}$ is even.

Under multipath propagation conditions and for large number of antennas the quantizer input signals $y_{i,l}$ are approximately Gaussian distributed and thus, they undergo nearly the same distortion factor ρ_q, i.e., $\rho_q^{(i,l)} = \rho_q \ \forall i \forall l$. Furthermore, the optimal parameters of the uniform as well as the non-uniform quantizer and the resulting distortion factor ρ_q for Gaussian distributed signal are tabulated in [7] for different bit resolutions b. Recent research work on optimally quantizing the Gaussian source can be found in [9], [10], [11].

Now, let $q_i = q_{i,R} + jq_{i,I}$ be the complex quantization error. Under the assumption of uncorrelated real and imaginary part of y_i, we easily obtain:

$$\begin{aligned} r_{q_iq_i} &= \mathrm{E}[q_iq_i^*] = \rho_q r_{y_iy_i}, \\ r_{y_iq_i} &= \mathrm{E}[y_iq_i^*] = -\rho_q r_{y_iy_i}. \end{aligned} \tag{9}$$

For the uniform quantizer case, it was shown in [11], that the optimal quantization step Δ for a Gaussian source decreases as $\sqrt{b}2^{-b}$ and that ρ_q is asymptotically well approximated by $\frac{\Delta^2}{12}$ and decreases as $b2^{-2b}$.

On the other hand, the optimal non-uniform quantizer achieves, under high-resolution assumption, approximately the following distortion [12]

$$\rho_q \approx \frac{\pi\sqrt{3}}{2} 2^{-2b}. \tag{10}$$

This particular choice of the (non-)uniform scalar quantizer minimizing the distortion between $\boldsymbol{r}$ and $\boldsymbol{y}$, combined with the receiver of the next section, is also optimal with respect to the total MSE between the transmitted symbol vector $\boldsymbol{x}$ and the estimated symbol vector $\hat{\boldsymbol{x}}$, as we will see later.

IV. Optimal Receiver

The linear receiver $\boldsymbol{G}$ that minimizes the MSE

$$\mathrm{E}[\|\boldsymbol{\varepsilon}\|_2^2] = \mathrm{E}[\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2] = \mathrm{E}[\|\boldsymbol{x} - \boldsymbol{G}\boldsymbol{r}\|_2^2] \tag{11}$$

is given by

$$\boldsymbol{G} = \boldsymbol{R}_{xr}\boldsymbol{R}_{rr}^{-1}, \tag{12}$$

and the resulting MSE equals

$$\mathrm{MSE} = \mathrm{tr}\left(\boldsymbol{R}_{\varepsilon\varepsilon}\right) = \mathrm{tr}\left(\boldsymbol{R}_{xx} - \boldsymbol{R}_{xr}\boldsymbol{R}_{rr}^{-1}\boldsymbol{R}_{xr}^{\mathrm{H}}\right), \tag{13}$$

where $\boldsymbol{R}_{xr}$ reads as

$$\boldsymbol{R}_{xr} = \mathrm{E}[\boldsymbol{x}\boldsymbol{r}^{\mathrm{H}}] = \mathrm{E}[\boldsymbol{x}(\boldsymbol{y}+\boldsymbol{q})^{\mathrm{H}}] = \boldsymbol{R}_{xy} + \boldsymbol{R}_{xq}, \tag{14}$$

and $\boldsymbol{R}_{rr}$ can be expressed as

$$\boldsymbol{R}_{rr} = \mathrm{E}[(\boldsymbol{y}+\boldsymbol{q})(\boldsymbol{y}+\boldsymbol{q})^{\mathrm{H}}] = \boldsymbol{R}_{yy} + \boldsymbol{R}_{yq} + \boldsymbol{R}_{yq}^{\mathrm{H}} + \boldsymbol{R}_{qq}. \tag{15}$$

Our goal now is to determine the linear filter $\boldsymbol{G}$ as a function of the channel parameters and the quantization distortion factor ρ_q.

To this end, we derive all needed covariance matrices by using the fact that the quantization error q_i, conditioned on y_i, is statistically independent from all other random variables of the system.

First we calculate $r_{y_iq_j} = \mathrm{E}[y_iq_j^*]$ for $i \neq j$:

$$\begin{aligned} \mathrm{E}[y_iq_j^*] &= \mathrm{E}_{y_j}\big[\mathrm{E}[y_iq_j^*|y_j]\big] \\ &= \mathrm{E}_{y_j}\big[\mathrm{E}[y_i|y_j]\mathrm{E}[q_j^*|y_j]\big] \\ &\approx \mathrm{E}_{y_j}\Big[r_{y_iy_j}r_{y_jy_j}^{-1}y_j\mathrm{E}[q_j^*|y_j]\Big] \qquad (16) \\ &= r_{y_iy_j}r_{y_jy_j}^{-1}\mathrm{E}[y_jq_j^*] \\ &= -\rho_q r_{y_iy_j}. \qquad (17) \end{aligned}$$

In (16), we approximate the Bayesian estimator $\mathrm{E}[y_i|y_j]$ with the linear estimator $r_{y_iy_j}r_{y_jy_j}^{-1}y_j$, which holds with equality if the vector $\boldsymbol{y}$ is jointly Gaussian distributed. Eq. (17) follows from (9).

Summarizing the results of (9) and (17), we obtain:

$$\boldsymbol{R}_{yq} \approx -\rho_q\boldsymbol{R}_{yy}. \tag{18}$$

In a similar way, we evaluate $r_{q_iq_j}$ for $i \neq j$:

$$\begin{aligned} \mathrm{E}[q_iq_j^*] &= \mathrm{E}_{y_j}\big[\mathrm{E}[q_iq_j^*|y_j]\big] \\ &= \mathrm{E}_{y_j}\big[\mathrm{E}[q_i|y_j]\mathrm{E}[q_j^*|y_j]\big] \\ &\approx \mathrm{E}_{y_j}\Big[r_{q_iy_j}r_{y_jy_j}^{-1}y_j\mathrm{E}[q_j^*|y_j]\Big] \\ &= r_{y_jq_i}^*r_{y_jy_j}^{-1}\mathrm{E}[y_jq_j^*] \\ &= \rho_q^2r_{y_jy_i}^* = \rho_q^2r_{y_iy_j}, \end{aligned} \tag{19}$$

where we used Eq. (18) and (9). From (19) and (9) we deduce the covariance matrix of the quantization error:

$$\begin{aligned} \boldsymbol{R}_{qq} &\approx \rho_q\mathrm{diag}(\boldsymbol{R}_{yy}) + \rho_q^2\mathrm{nondiag}(\boldsymbol{R}_{yy}) \\ &= \rho_q\boldsymbol{R}_{yy} - (1-\rho_q)\rho_q\mathrm{nondiag}(\boldsymbol{R}_{yy}), \end{aligned} \tag{20}$$

with $\mathrm{diag}(\boldsymbol{A})$ denotes a diagonal matrix containing only the diagonal elements of $\boldsymbol{A}$ and $\mathrm{nondiag}(\boldsymbol{A}) = \boldsymbol{A} - \mathrm{diag}(\boldsymbol{A})$. Inserting the expressions (18) and (20) into Eq. (15), we obtain:

$$\boldsymbol{R}_{rr} \approx (1-\rho_q)(\boldsymbol{R}_{yy} - \rho_q\mathrm{nondiag}(\boldsymbol{R}_{yy})). \tag{21}$$

Afterwards, we get the covariance matrix $\boldsymbol{R}_{xq} = \mathrm{E}[\boldsymbol{x}\boldsymbol{q}^{\mathrm{H}}]$ in the following manner:

$$\begin{aligned} \mathrm{E}[\boldsymbol{x}\boldsymbol{q}^{\mathrm{H}}] &= \mathrm{E}_{\boldsymbol{y}}\big[\mathrm{E}[\boldsymbol{x}\boldsymbol{q}^{\mathrm{H}}|\boldsymbol{y}]\big] \\ &= \mathrm{E}_{\boldsymbol{y}}\big[\mathrm{E}[\boldsymbol{x}|\boldsymbol{y}]\mathrm{E}[\boldsymbol{q}^{\mathrm{H}}|\boldsymbol{y}]\big] \\ &\approx \mathrm{E}_{\boldsymbol{y}}\big[\boldsymbol{R}_{xy}\boldsymbol{R}_{yy}^{-1}\boldsymbol{y}\mathrm{E}[\boldsymbol{q}^{\mathrm{H}}|\boldsymbol{y}]\big] \\ &= \boldsymbol{R}_{xy}\boldsymbol{R}_{yy}^{-1}\mathrm{E}[\boldsymbol{y}\boldsymbol{q}^{\mathrm{H}}] \\ &= -\rho_q\boldsymbol{R}_{xy}. \end{aligned} \tag{22}$$

Thus, Equation (14) becomes

$$\boldsymbol{R}_{xr} \approx (1-\rho_q)\boldsymbol{R}_{xy}. \tag{23}$$

Summing up, we get from (21) and (23) the following expression for the Wiener filter (12) operating on quantized data:

$$\boldsymbol{G}_{\mathrm{WFQ}} \approx \boldsymbol{R}_{xy}(\boldsymbol{R}_{yy} - \rho_q\mathrm{nondiag}(\boldsymbol{R}_{yy}))^{-1}. \tag{24}$$

and for the resulting MSE, we get using (13)

$$\mathrm{MSE}_{\mathrm{WFQ}} \approx \mathrm{tr}\left[\boldsymbol{R}_{xx} - (1-\rho_q)\boldsymbol{R}_{xy}(\boldsymbol{R}_{yy} - \rho_q \mathrm{nondiag}(\boldsymbol{R}_{yy}))^{-1}\boldsymbol{R}_{xy}^{\mathrm{H}}\right]. \tag{25}$$

We obtain $\boldsymbol{R}_{yy}$ and $\boldsymbol{R}_{xy}$ easily from our system model:

$$\begin{aligned} \boldsymbol{R}_{yy} &= \boldsymbol{R}_{\eta\eta} + \boldsymbol{H}\boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}}, & (26)\\ \boldsymbol{R}_{xy} &= \boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}}. & (27) \end{aligned}$$

V. Effect of Quantization on the MSE

We first examine the first derivative of the $\mathrm{MSE}_{\mathrm{WFQ}}$ in (25) with respect to ρ_q:

$$\frac{\partial \mathrm{MSE}_{\mathrm{WFQ}}}{\partial \rho_q} = \mathrm{tr}\left[\boldsymbol{G}_{\mathrm{WFQ}}\mathrm{diag}(\boldsymbol{R}_{yy})\boldsymbol{G}_{\mathrm{WFQ}}^{\mathrm{H}}\right] > 0, \tag{28}$$

where $\boldsymbol{G}_{\mathrm{WFQ}}$ is given in (24). Therefore the $\mathrm{MSE}_{\mathrm{WFQ}}$ is monotonically increasing in ρ_q. Since we choose the quantizer to minimize the distortion factor ρ_q, our receiver and quantizer designs are jointly optimum with respect to the total MSE.

Now, we expand the MSE expression (25) into a Taylor series around $\rho_q = 0$ up to the order one:

$$\mathrm{MSE}_{\mathrm{WFQ}} \approx \mathrm{MSE}_{\mathrm{WF}} + \rho_q \mathrm{tr}\left[\boldsymbol{R}_{xy}\boldsymbol{R}_{yy}^{-1}\mathrm{diag}(\boldsymbol{R}_{yy})\boldsymbol{R}_{yy}^{-1}\boldsymbol{R}_{xy}^{\mathrm{H}}\right], \tag{29}$$

where $\mathrm{MSE}_{\mathrm{WF}} = \mathrm{MSE}_{\mathrm{WFQ}}|_{\rho_q=0}$ is the achievable MSE without quantization. Above equation gives the increase in MSE due to the quantization as a function of ρ_q and the channel parameters. It reveals also a residual error at higher SNR values, with is given by

$$\mathrm{MSE}_{\mathrm{res}} \approx \rho_q \mathrm{tr}\left[(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{H}}\mathrm{diag}(\boldsymbol{H}\boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}})\boldsymbol{H}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}\right], \tag{30}$$

since $\boldsymbol{R}_{xy}\boldsymbol{R}_{yy}^{-1} = \boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}}(\boldsymbol{R}_{\eta\eta} + \boldsymbol{H}\boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}})^{-1}$ converges to $(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{H}}$ as $\boldsymbol{R}_{\eta\eta}$ converges to $\boldsymbol{0}$, by means of the matrix inversion lemma.

Under a high resolution assumption, ρ_q is proportional to 2^{-2b}, therefore the residual MSE decreases exponentially with the number of quantization bits b.

VI. Lower Bound on the Mutual Information and the Capacity

In this section, we develop a lower bound on the mutual information rate between the input sequence $\boldsymbol{x}$ and the quantized output sequence $\boldsymbol{r}$ of the system in Fig. 1, based on our MSE approach. Generally, the mutual information of this channel can be expressed as [13]

$$I(\boldsymbol{x}, \boldsymbol{r}) = h(\boldsymbol{x}) - h(\boldsymbol{x}|\boldsymbol{r})). \tag{31}$$

Given $\boldsymbol{R}_{xx}$ under a power constraint $\mathrm{tr}(\boldsymbol{R}_{xx}) \leq P_{\mathrm{Tr}}$, we choose $\boldsymbol{x}$ to be Gaussian, which is not necessarily the capacity achieving distribution for our quantized system. Then, we can obtain a lower bound for $I(\boldsymbol{x}, \boldsymbol{r})$ (in bit/transmission) as

$$\begin{aligned} I(\boldsymbol{x}, \boldsymbol{r}) &= \log_2|\boldsymbol{R}_{xx}| - h(\boldsymbol{x}|\boldsymbol{r}) \\ &= \log_2|\boldsymbol{R}_{xx}| - h(\boldsymbol{x} - \hat{\boldsymbol{x}}|\boldsymbol{r}) \\ &\geq \log_2|\boldsymbol{R}_{xx}| - h(\underbrace{\boldsymbol{x} - \hat{\boldsymbol{x}}}_{\boldsymbol{\varepsilon}}) & (32) \\ &\geq \log_2\frac{|\boldsymbol{R}_{xx}|}{|\boldsymbol{R}_{\varepsilon\varepsilon}|} & (33) \end{aligned}$$

Since conditioning reduces entropy, we obtain inequality (32). On the other hand, The second term in (32) is upper bounded by the entropy of a Gaussian random variable whose covariance is equal to the error covariance matrix $\boldsymbol{R}_{\varepsilon\varepsilon}$ of the linear MMSE estimate of $\boldsymbol{x}$. Finally, we get using Eqs (25) and (27)

$$I(\boldsymbol{x}, \boldsymbol{r}) \gtrapprox -\log_2\left|\mathbf{I} - (1-\rho_q)\boldsymbol{R}_{xy}(\boldsymbol{R}_{yy} - \rho_q\mathrm{nondiag}(\boldsymbol{R}_{yy}))^{-1}\boldsymbol{H}\right|. \tag{34}$$

Now, considering the case of low SNR values, we get easily with

$$\boldsymbol{R}_{yy} \approx \boldsymbol{R}_{\eta\eta}, \tag{35}$$

and Eqs (34) and (27), the following first order approximation of the mutual information [1][2]

$$I(\boldsymbol{x}, \boldsymbol{r}) \gtrapprox (1-\rho_q)\mathrm{tr}[\boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\eta\eta}^{-1}\boldsymbol{H}]/\log(2). \tag{36}$$

Compared with the mutual information for the unquantized case, also at low SNR [14]

$$I(\boldsymbol{x}, \boldsymbol{y}) \approx \mathrm{tr}[\boldsymbol{R}_{xx}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\eta\eta}^{-1}\boldsymbol{H}]/\log(2), \tag{37}$$

the mutual information for the quantized channel degrades only by the factor $(1-\rho_q)$. For the spacial case $b = 1$, we have $\rho_q|_{b=1} = 1 - \frac{2}{\pi}$ (see [7]) and the degradation of the mutual information becomes

$$\lim_{\mathrm{SNR}\to 0} \frac{I(\boldsymbol{x}, \boldsymbol{y})}{I(\boldsymbol{x}, \boldsymbol{r})}\bigg|_{b=1} \approx \frac{2}{\pi}. \tag{38}$$

Using a different approach, we present in our paper [15] a similar result, and show that the above approximation is asymptotically exact.

Now assuming perfect CSI at the transmitter, the capacity of this channel can be expressed as [13]

$$C_{\mathrm{Q}} = \max I(\boldsymbol{x}, \boldsymbol{r}). \tag{39}$$

The maximization is performed over the input distribution of $\boldsymbol{x}$ under a power constraint $\mathrm{tr}(\boldsymbol{R}_{xx}) \leq P_{\mathrm{Tr}}$. Since the Gaussian distribution is not necessarily the capacity achieving distribution, we get from Eq. (34) the following lower bound on the capacity

$$C_{\mathrm{Q}} \gtrapprox \max_{\boldsymbol{R}_{xx}} -\log_2\left|\mathbf{I} - (1-\rho_q)\boldsymbol{R}_{xy}(\boldsymbol{R}_{yy} - \rho_q\mathrm{nondiag}(\boldsymbol{R}_{yy}))^{-1}\boldsymbol{H}\right|. \tag{40}$$

This optimization over $\boldsymbol{R}_{xx}$ is, unfortunately, intractable. As suboptimal solution for low SNR values (see Eq. (36)), we can employ the eigenmode transmission solution of the unquantized effective channel $\boldsymbol{R}_{\eta\eta}^{-1/2}\boldsymbol{H}$ (water-filling solution) given in [16] and [17]

$$\begin{aligned} \boldsymbol{R}_{xx} &= \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{H}} & (41)\\ \boldsymbol{\Sigma} &= (\mu\mathbf{I} - \boldsymbol{\Lambda}^{-1})_+, & (42) \end{aligned}$$

where μ is chosen to satisfy the transmit power constraint and $(x)_+$ denotes $\max(x, 0)$. The unitary matrix $\boldsymbol{V}$ and the diagonal matrix $\boldsymbol{\Sigma}$ are obtained from the eigenvalue decomposition of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\eta\eta}^{-1}\boldsymbol{H}$.

An other strategy consists of using a transmit zero-forcing scheme trying to pre-eliminate all the interference at the receiver, so that the symbols can be reliably distinguished each other at high SNR, even with few quantization bits; that is

$$\boldsymbol{R}_{xx} = \frac{P_{\mathrm{tr}}}{\mathrm{tr}[(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}]}(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}. \tag{43}$$

[1] We assume also that $\rho_q \ll 1$ (or $\boldsymbol{R}_{\eta\eta}$ is diagonal).
[2] Note that $\log|\mathbf{I} + \Delta\boldsymbol{X}| \approx \mathrm{tr}(\Delta\boldsymbol{X})$.

As the SNR go to ∞, and for the special case $M = N$, the capacity achieved by the zero-forcing scheme converges to [3]

$$\lim_{\text{SNR}\to\infty} C_Q^{\text{Tx-ZF}} = -\log_2|\rho_q \mathbf{I}| = -M\log_2(\rho_q) \sim 2Mb. \quad (44)$$

This means that the capacity increases linearly with the resolution b at high SNR. Intuitively, this is because the receiver can maximally distinguish $2Mb$ input symbols.

VII. Simulation Results

The performance of the modified Wiener filter for a 4- and 5-bit quantized output MIMO system (WFQ), in terms of BER averaged over 1000 channel realizations, is shown in Fig. 2 for a 10×10 MIMO system (QPSK), compared with the conventional Wiener filter (WF) and Zero-forcing filter (ZF). The symbols and the noise are assumed to be uncorrelated, that is $\boldsymbol{R}_{xx} = \sigma_x^2\mathbf{I}$ and $\boldsymbol{R}_{\eta\eta} = \sigma_\eta^2\mathbf{I}$. Hereby, the (pseudo-)SNR (in dB) is defined as

$$\text{SNR} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_\eta^2}\right). \quad (45)$$

Furthermore, the entries of $\boldsymbol{H}$ are complex-valued realization of independent zero-mean Gaussian random variables with unit variance. Clearly, the modified Wiener filter outperforms the conventional Wiener filter at high SNR. This is because the effect of quantization error is more pronounced at higher SNR values when compared to the additive Gaussian noise variance. Since the conventional Wiener filter converges to the ZF-filter at high SNR values and looses its regularized structure, its performance degrades asymptotically to the performance of the ZF-filter, when operating on quantized data. For comparison, we also plotted the BER curves for the WF and ZF filter, if no quantization is applied.
Fig. 3 illustrates the simulated MSE_{WFQ} compared with the closed-form expression (theoretical MSE_{WFQ}) from Eq. (25) for the same scenarios ($b = 4$ and 5), and averaged over 1000 channel realizations. As we can see, the approximation is very tight, which demonstrates the usefulness of our approach.
Fig. 4 shows, also for the same scenarios, the lower bounds on the average capacity (ergodic capacity) derived in the previous section compared to the unquantized case. Obviously, the eigenmode transmission strategy performs well at low SNR and closely to the unquantized system. Nevertheless, the zero-forcing transmit (TxZF) scheme achieves higher throughput at high SNR values, since the symbols can be more reliably distinguished at the receiver.

VIII. Conclusion

We addressed the problem of designing a linear MMSE receiver for MIMO channels with quantized outputs. Under an optimal choice of the quantizer, we provided an approximation for the mean squared error between the transmitted symbol and the received one. Then, we derived an optimized linear receiver, which shows better performance in terms of BER compared to the conventional Wiener filter. Moreover, our receiver does not present any extra complexity from the implementation point of view. We examined the capacity of such a system and proposed a lower bound on it. We also studied the effects of quantization on the MSE and the capacity.

[3] We use $\boldsymbol{R}_{yy} \approx \boldsymbol{H}\boldsymbol{R}_{xx}\boldsymbol{H}^{\text{H}}$ and Eq. (34).

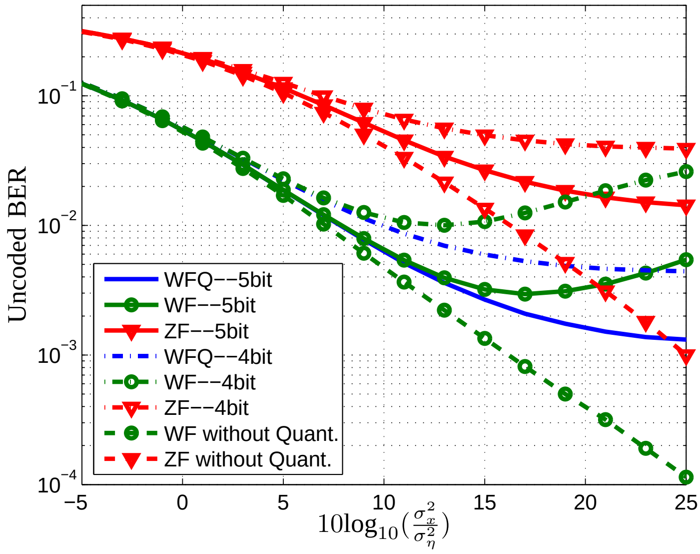

Fig. 2. WFQ vs. the conventional WF and ZF receivers, QPSK modulation with $M = 10$, $N = 10$, 4- ($\rho_q = 0.01154$) and 5- ($\rho_q = 0.00349$) bit uniform quantizer.

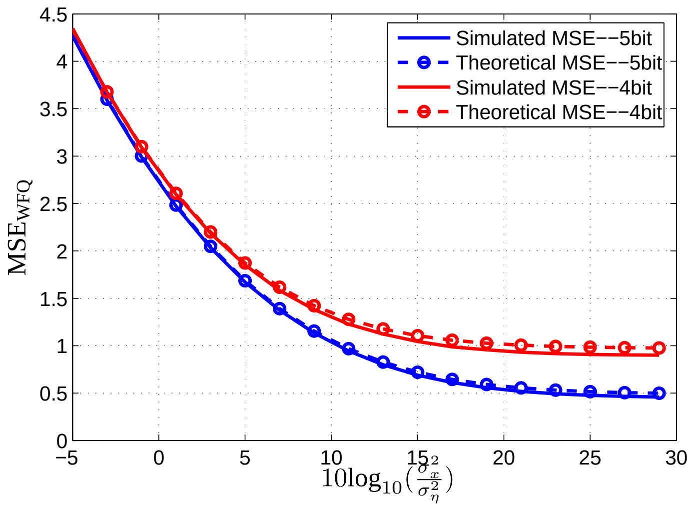

Fig. 3. Theoretical MSE_{WFQ} from Eq. (25) vs. simulated MSE_{WFQ}, $M = 10$, $N = 10$, 4- and 5-bit uniform quantizer.

References

[1] J. A. Nossek and M. T. Ivrlač, "Capacity and coding for quantized MIMO systems," in *Intern. Wireless Commun. and Mobile Computing Conf. (IWCMC)*, Vancouver, Canada, July 2006, pp. 1387–1392, invited.

[2] R. Schreier and G. C. Temes, "Understanding Delta-Sigma Data Converters," *IEEE Computer Society Press*, 2004.

[3] M. T. Ivrlač and J. A. Nossek, "Challenges in Coding for Quantized MIMO Systems," in *IEEE International Symposium on Information Theory*, Seattle, U.S.A., July 2006, pp. 2114–2118.

[4] B. Murray and I. B. Collings, "AGC and Quantization Effects in a Zero-Forcing MIMO Wireless System," in *Proc. of the IEEE Int. Vehicular Technology Conf. (VTC) - Spring*, Melbourne, Australia, May 2006.

[5] M. T. Ivrlač and J. A. Nossek, "On Channel Estimation in Quantized MIMO Systems," *Accepted for publication at ITG WSA*, February 2007.

[6] N. S. Jayant and P. Noll, *Digital Coding of Waveforms*, Prentice-Hall, 1984.

[7] J. Max, "Quantizing for Minimum Distortion," *IEEE Trans. Inf. Theory*, vol. 6, no. 1, pp. 7–12, March 1960.

[8] J. G. Proakis, *Digital Communications*, McGraw Hill, New York, third edition, 1995.

[9] Naofal Al-Dhahir and John M. Cioffi, "On the Uniform ADC Bit

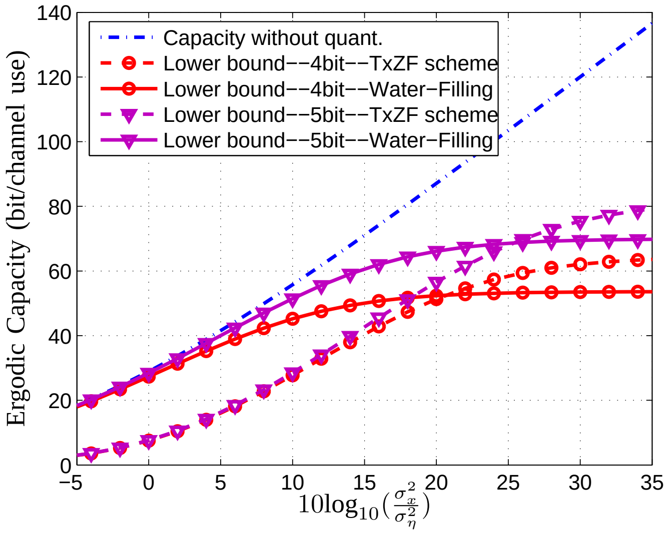

Fig. 4. Lower bounds on the ergodic capacity of the quantized MIMO system, $M = 10$, $N = 10$, 4- and 5-bit uniform quantizer.

Precision and Clip Level Computation for a Gaussian Signal," *IEEE Transactions on Signal Processing*, vol. 44, no. 2, February 1996.

[10] Sangsin Na and David L. Neuhoff, "On the Support of MSE-Optimal, Fixed-Rate, Scalar Quantizers," *IEEE Trans. Inform. Theory*, vol. 47, no. 7, pp. 2972–2982, November 2001.

[11] Dennis Hui and David L. Neuhoff, "Asymptotic Analysis of Optimal Fixed-Rate Uniform Scalar Quantization," *IEEE Trans. Inform. Theory*, vol. 47, no. 3, pp. 957–977, March 2001.

[12] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*, Kluwer Academic Publishers, Dordrecht, Niederlande, first edition, 1992.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley and Son, New York, 1991.

[14] S. Verdú, "Spectral Efficiency in the Wideband Regime," *IEEE Trans. Inform. Theory*, vol. 48, no. 6, pp. 1319–1343, June 2002.

[15] A. Mezghani and J. A. Nossek, "On Ultra-Wideband MIMO Systems with 1-bit Quantized Outputs: Performance Analysis and Input Optimization," *Submitted for publication at ISIT*, June 2007.

[16] G. Raleigh and J. Cioffi, "Spatio-temporal coding for wireless communications," *IEEE Transactions on Communications*, vol. 46, no. 3, pp. 357–366, March 1998.

[17] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx Beamforming Design for Multicarrier MIMO channels: A Unified Framework for Convex Optimization," *IEEE Trans. on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.