SUM THROUGHPUT MAXIMIZATION IN QUALITY OF SERVICE CONSTRAINED MULTIUSER MIMO SYSTEMS BASED ON PERTURBATION ANALYSIS

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ABSTRACT

Sum throughput maximization under a power constraint constitutes an important optimization problem for designing physical layer algorithms. However it often leads to an unfair distribution of resources to the users. On the other hand, due to the nature of the capacity region of the multiuser MIMO broadcast channel, strict Quality of Service (QoS) constraints such as rate ratios might lead to severe losses in sum capacity compared to the maximum possible sum capacity given a power constraint. For this reason minimum rates for each user are introduced and an algorithm is presented which divides the users into two groups: in one group all users are served with their required minimum rates as providing them with further resources will lead to decreases in sum capacity. Putting these further resources into the users of the other group instead is more beneficial for sum capacity. That is why these users will be served with higher rates than their required minimum. This user classification is conducted based on perturbation analysis of convex optimization problems.

1. INTRODUCTION

Future wireless communication systems will be characterized by a wide variety of different user applications, such as video streaming, data transfer, voice etc. All of those have different requirements on the wireless links between the transmitter and the mobile users. Additionally from a network operator's point of view it is desired to maximize the total throughput in such a system. It is the transmitter's task to accomplish these desires as good as possible given a transmit power constraint. In this paper on the physical layer level the user requirements are formulated as minimum transmission rates, which are measured by channel capacity and which must be fulfilled for every user. After satisfying these constraints there are several possibilities how to distribute the remaining resources, such as transmit power or sub carrier in an Orthogonal Frequency Division Multiplexing (OFDM) system, to the users. One is a fair distribution, such that each user receives the same proportional rate gain compared to its minimum required rate. That leads to the so-called *rate balancing problem*, as for example described for a multiuser multiple-input multiple-output (MIMO) OFDM system in [1]. Alternatively one can maximize the weighted sum of the users' capacities with an appropriate choice of weights, as done for example in [2]. However these algorithms maximize sum capacity in a way that a predefined point on the boundary of the capacity region should be achieved and leave no room for maximization of sum capacity on the boundary of the capacity region itself. Therefore in favor of a higher system throughput we give up the fair distribution of additional resources and present an algorithm how to shift resources from the rate balancing solution from [1] such that system throughput is increased without violating the minimum rate requirements. Consequently only these users which can contribute most to sum capacity will receive more resources than required to satisfy their minimum required rates. The main focus of this paper will be to identify those users. For this purpose we utilize the sensitivity analysis of a perturbed convex optimization problem [3]. By means of sensitivity and perturbation analysis one can find out how sensitive a given convex objective function is with respect to perturbations in the constraints of the optimization problem, i.e. how a change of a certain constraint affects the objective function. An example for the application of such an analysis can be found in [4]. Therein overall transmit power is minimized, whereby the users' Minimum Square Errors (MSEs) must not fall below certain predefined values. In case these requirements cannot be satisfied with the currently available transmit power, sensitivity analysis is used to determine that user whose constraint should be relaxed in order to gain the largest reduction in transmit power.

An optimum solution to the problem of maximizing a weighted sum of users' rates with minimum rate constraints in a multiuser MIMO OFDM system has recently been found in [5]. However, the presented iterative algorithm exhibits a high computational complexity, even within one iteration. Furthermore the stated problem may lead to solutions, where one or a few users receive to many system resources such that they can transmit at higher rates than their applications can make use

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of. Our algorithm assures a fair allocation of resources among the users to be served with more than their required minimum rates and also allows the consideration of maximum transmission rates.

The outline of the paper is as follows. After introducing the system model in Section 2 we will shortly review the algorithm presented in [1], as it will be used during the solution of the presented problem. We will then continue by explaining the proposed algorithm in Section 4. The simulation results are shown in Section 5 before we conclude the paper in Section 6.

Notation: Bold lowercase and uppercase letters denote vectors and matrices, respectively, $(*)^{\mathrm{H}}$ is the Hermitian of a matrix or vector and by $\boldsymbol{I}_n$ we refer to the $n \times n$ identity matrix.

2. SYSTEM MODEL

We consider the downlink of a multi-user MIMO system with a transmitter with M_{Tx} antennas and K users with M_{Rx} receive antennas at each user. The channel matrix of user k on carrier n is denoted as $\boldsymbol{H}_{k,n}$. Receivers as well as the transmitter have full channel state information (CSI). We employ an OFDM system with C carrier and the transmit power is limited to P_{Tx}. The k-th user's minimum rate requirement is denoted as $R_{k,\mathrm{min}}$ We thereby make the assumption that unconstrained sum capacity maximizing algorithms such as SESAM [6] do not meet these requirements.

3. SESAM WITH RELATIVE RATE CONSTRAINTS

The main idea of the Successive Encoding Successive Allocation Method (SESAM) [6] is to decompose the MIMO broadcast channel into a set of virtually decoupled scalar interference-free sub channels. Each of these sub channels results from assigning a transmit beamforming vector to a user and applying a matched filter, which is capacity conserving in this case, at the corresponding receiver. Total suppression of multiuser interference is achieved as follows: Let $\pi(1)$ be the user to be encoded first on a certain carrier n and $\boldsymbol{v}_{\pi(1),n}$ its corresponding transmit beamforming vector. Then the channels $\boldsymbol{H}_{k,n}$ are projected into the nullspace of the beamforming vector by the operation

$$\boldsymbol{H}_{k,n,2} = \boldsymbol{H}_{k,n}(\boldsymbol{I}_{M_{\mathrm{Tx}}} - \boldsymbol{v}_{\pi(1),n}\boldsymbol{v}_{\pi(1),n}^{\mathrm{H}}).$$

User Allocation is then continued with the projected channel matrices $\boldsymbol{H}_{k,n,2}$, whereas after each user allocation the channels are again projected into the nullspace of all beamforming vectors. Due to the properties of the projections up $\min(M_{\mathrm{Tx}}, M_{\mathrm{Rx}})$ sub channels can result at the same time and at the same frequency. That assures that users $\pi(i)$ do not interfere with users $\pi(j)$ to be encoded later, whereas $j > i$. Interference from previously encoded users is suppressed by Dirty Paper Coding (DPC).

The optimum user allocation depends on the analyzed problem. For unconstrained maximization of sum capacity (see [6]) the user with the strongest principal singular value of the channel matrix $\boldsymbol{H}_{k,n}$ in the first dimension and the user with the strongest principal singular value of the projected channel matrices $\boldsymbol{H}_{k,n,i}$ in the following dimensions is chosen. In OFDM systems the algorithm can be run on each sub carrier in parallel. Power is allocated to all sub channels according to waterfilling.

In [1] the rate balancing problem is considered, wherein relative rate constraints ρ_k are introduced for each user. That leads to the following optimization problem:

$$\begin{aligned} &\max \sum_{k=1}^{K} R_k(p_{\ell,k}) \\ \text{s.t. } &\frac{R_k}{R_{\text{ref user}}} = \rho_k, \quad \forall k = 1, ..., K, k \neq \text{ref user} \\ &\sum_{k=1}^{K}\sum_{\ell=1}^{n_k} p_{\ell,k} = P_{\mathrm{Tx}}, \quad p_{\ell,k} \geq 0, \end{aligned} \tag{1}$$

wherein $p_{\ell,k}$ and n_k denote the power allocated to the ℓ-th sub channel of user k and the number of subhannels allocated to user k. For simplicity reasons we index the sub channels allocated to a certain user by $\ell = 1, ..., n_k$ regardless on which carrier and in which spatial dimension the sub channels are assigned. "ref user" is an arbitrarily chosen reference user.Note that here we use a slightly different for the rate balancing problem than in [1]. It can be easily shown that by choosing $\gamma = R_{\text{ref user}}$ in [1] the two problems are equivalent. (1) is divided into a sub channel allocation followed by a power allocation. The former is again conducted successively, whereas in each spatial dimension it works as follows: first the number of sub carriers each user will receive in the current dimension is determined. This number depends on the rate ratios ρ_k and the single user rates each user would achieve if it occupied all carriers. Secondly the actual allocation user to sub carrier is performed such that the previously computed numbers of sub carriers are achieved and as little sum capacity as possible is lost compared to the allocation according to the strongest principal singular value. Given a certain sub channel allocation finally power allocation is conducted to fulfill the rate ratios. For details the reader is referred to [1].

4. ALGORITHM

We will first give a brief overview of the proposed algorithm: In Step 1 the users are classified into two groups: users in one group are served with their corresponding minimum rates, as further rate increases will be harmful for the total sum rate, while users of the other group share the remaining system resources for optimum system performance. In the second step the spatial dimensions on each sub carrier are assigned to the users. Finally in the third step the available transmit power is allocated to the resulting sub channels.

4.1. Step 1: User Classification

As simulation results showed in [1] the boundary of the capacity region achievable by the SESAM algorithm with relative rate constraints is nearly concave. Because we have furthermore excluded the case that the point of maximum sum capacity lies within the feasible region defined by the minimum rates, it is reasonable to assume that the optimum solution will lie on the boundary of the feasible region. That means that certain users will only be served with their corresponding minimum rates. In this step those users are determined. For this purpose we resort to the perturbation and sensitivity analysis of a convex optimization problem, which is explained in [3]. For our purposes we analyze an optimization problem which aims at achieving a certain point on the boundary of the capacity region. This point must lie within the feasible domain and by perturbation analysis we can determine the direction in which this point should be moved within the feasible domain in order to obtain a higher sum capacity. Optimization problem (1) is chosen for this aim, wherein the rate ratios are chosen according to:

$$\rho_k = \frac{R_{k,\text{min}}}{R_{\text{ref user, min}}}. \tag{2}$$

Solving Problem (1) with ρ_k according to (2) leads to transmission rates, which are either all below or above the required minimum rates. Hence this part of the algorithm can also be seen as a kind of feasibility test. For the further proceeding we assume that the original problem is feasible. The strategy which should be followed in case the transmit power is not enough to satisfy the minimum rate requirements is beyond the scope of this paper. As a convex optimization problem is needed for perturbation analysis we assume that the sub channel allocation remains fixed after it has been determined according to the algorithm in [1] given the rate ratios in (2). Hence in the following we consider only the power allocation problem.

$$\min_{p_{\ell,k}} \left(-\sum_{k=1}^{K} R_k(p_{\ell,k}) \right) = \tag{3}$$

$$= \min_{p_{\ell,k}} \left(-\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2) \right),$$

$$\text{s.t. } \frac{R_k}{R_{\text{ref user}}} = \rho_k, \quad \forall k = 1, \dots K, k \neq \text{ref user},$$

$$\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} p_{\ell,k} = P_{\text{Tx}}, \quad p_{\ell,k} \geq 0,$$

wherein $\lambda_{\ell,k}$ denotes the gain of the ℓ-th sub channel of user k. As we want to achieve the boundary of the capacity region, the power budget is fully exploited. We minimize the negative sum capacity in (3) in order to obtain a convex objective function. Obviously this is equivalent to maximizing sum capacity. By introducing "perturbation terms" u_k in the constraints, (3) reads as:

$$\min_{p_{\ell,k}} \left(-\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2) \right) \tag{4}$$

$$\text{s.t. } \frac{R_k}{R_{\text{ref user}}} = \rho_k + u_k, \quad \forall k = 1, \dots K, k \neq \text{ref user},$$

$$\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} p_{\ell,k} = P_{\text{Tx}}, \quad p_{\ell,k} \geq 0.$$

The perturbation terms u_k consider the fact that the constraints are no longer fixed to ρ_k but variable, whereby we are interested in finding rules for choosing u_k in a way that the maximum sum capacity will be increased. The Lagrange function of (4) then reads as:

$$\mathcal{L} = -\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2) + \tag{5}$$

$$+ \sum_{\substack{k=1 \\ k \neq \text{ref user}}}^{K} \nu_k \left(\frac{R_k}{R_{\text{ref user}}} - (\rho_k + u_k) \right) + \tag{6}$$

$$+ \eta \left(\sum_{k=1}^{K} \sum_{\ell=1}^{n_k} p_{\ell,k} - P_{\text{Tx}} \right) + \sum_{k=1}^{K} \sum_{\ell=1}^{n_k} \mu_{k,\ell} p_{\ell,k}$$

with the Lagrange multipliers ν_k, η and $\mu_{k,\ell}$. As for the optimization problem in (4) strong duality is assumed and the optimum sum capacity $C_{\text{sum,opt}} = \sum_{k=1}^{K} R_{k,\text{opt}}$ is differentiable with respect to u_k, the following relationships between the Lagrange multipliers ν_k and $C_{\text{sum,opt}}$ hold [3]:

$$\left. \frac{\partial C_{\text{sum,opt}}}{\partial u_k} \right|_{u_1,\dots,u_K=0} = \nu_k \quad \forall k = 1, \dots, K, k \neq \text{ref user}.$$

Hence if $\nu_k > 0$, the maximum sum capacity will increase for $u_k > 0$, i.e. the relative rate requirement ρ_k can be further increased. On the other hand for users with $\nu_k < 0$ a reduction of their rate requirements is necessary for an increase in sum capacity. Applied to the original problem with minimum rate constraints it is very likely that constraining those users with $\nu_k < 0$ on their minimum rates will gain a benefit in sum capacity compared to the SESAM algorithm with relative rate constraints applied so far. After computing the Lagrange multipliers ν_k as shown in the Appendix, we therefore conduct the following user classification:

- Users with $\nu_k < 0$ are put into the so-called "looser" group, which will be referred to as **Group L** in the following. They will be only served with the corresponding minimum required rates.
- The remaining users are subsumed into the "winner" **Group W**, i.e. during the next steps they will gain

more resources than in the current step of the algorithm. Consequently their individual rates will increase then. In the following we will maintain the relative rate constraints from (2) within this group for reasons that will become clearer later.

Remarks:

1. Although there are only $K-1$ constraints in (4) and accordingly only $K-1$ Lagrange multipliers relevant for our analysis, a classification of all users is possible. As shown in the Appendix, the sign of ν_k is independent of the choice of reference user. Therefore also the reference user, to which no Lagrange multiplier can be assigned in the current optimization problem, can be clearly classified without repeating the whole optimization with another reference user.

2. For the perturbed optimization problem we considered in (4) the new constraints introduced above imply that $u_i = 0$ for the users in Group W and $u_i < 0$ for the users in Group L. That is because their own rates are reduced compared to the unperturbed problem and the rate of the reference user, which can, without loss of generality, be assumed to belong to Group W, is very likely to increase. Trying to achieve $u_i > 0$ for the users in Group W, as suggested by the perturbation analysis, would discriminate the reference user.

3. The perturbation analysis is only valid near the optimum solution obtained with the initial constraints ρ_k from (2). That is not only because $C_{\text{sum, opt}}$ is usually a non monotonic function of all u_k and therefore the signs of the partial derivatives do not remain constant within the feasible domain. Furthermore the optimum sub channel allocation is likely to change within the whole feasible domain. To take these suboptimalities into account an iterative extension to our algorithm is presented in Section 4.4.

4.2. Step 2: Sub Channel Allocation

As in [6], the sub channel allocation is performed successively, i.e. we start in the first spatial dimension with the sub carrier allocation, project the users' channels into the null space of the already used subspaces and continue until no spatial dimensions are left. In each dimension first the number of sub carriers each user can occupy is determined. For the users in Group L the number of sub carriers $N_{i,k}$ the k-th user gains in the i-th dimension is computed according to:

$$N_{i,k} = \frac{R_{k,\text{min}}}{R_k^{(1)}} N_{i,k}^{(1)}.$$

Thereby $N_{i,k}^{(1)}$ denotes the number of sub carriers the k-th user has occupied in the spatial dimension i after running the SESAM algorithm with relative rate constraints during Step 1. $R_k^{(1)}$ is the total rate user k would achieve with the resource allocation determined in Step 1. The remaining sub carriers

$$N - \sum_{k \in \text{Group L}} N_{i,k}$$

are distributed amongst the users in Group W in the same way as the total number of sub carriers is distributed to the users in the SESAM algorithm with relative rate constraints. The necessary rate requirements are again chosen according to Equation (2), whereby the reference user is an arbitrary user of Group W. The actual allocation sub carrier to the users is then performed as in [1].

4.3. Step 3: Power Allocation

First power is assigned to the sub channels occupied by the users of Group L. The aim is thereby to satisfy the minimum rate requirements with as little transmit power as possible. Mathematically this optimization problem reads as:

$$\min_{p_{\ell,k}} \left(\sum_{k \in \text{Group L}} \sum_{\ell=1}^{n_k} p_{\ell,k} \right) \tag{7}$$

$$\text{s.t.} \sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2) = R_{k,\text{min}}, \quad \forall k \in \text{Group L},$$

$$p_{\ell,k} \geq 0.$$

As in the SESAM algorithm with relative constraints [1], the solution to (7) is "waterfilling with user specific waterlevels η_k" [1]:

$$p_{\ell,k} = \max\left\{0, \left(\eta_k - \frac{1}{\lambda_{\ell,k}^2}\right)\right\}.$$

Considering the rate constraints in (7), the waterlevels compute according to:

$$\eta_k = 2^{\frac{R_{k,\text{min}} - \sum_{\ell=1}^{\hat{n}_k} \log_2(\lambda_{\ell,k}^2)}{\hat{n}_k}},$$

whereby $\hat{n}_k \leq n_k$ denotes the number of sub channels with $p_{\ell,k} > 0$ and it is assumed that the $\lambda_{\ell,k}^2$ are arranged in descending order from $\ell = 1$ to $\ell = n_k$. As $\hat{n}_k$ is unknown, the powers $p_{\ell,k}$ have to be determined iteratively: First the waterlevels η_k are computed with $\hat{n}_k = n_k$. Then the corresponding powers $p_{\ell,k}$ are determined. With those $\hat{n}_k$ can be updated and new waterlevels can be calculated. If $\hat{n}_k$ remains constant from one iteration to another, the algorithm has converged.

Afterwards the remaining power is distributed to the users in Group W. Applying pure waterfilling over the corresponding sub channels is optimum for sum capacity but does not guarantee compliance with the minimum rate requirements. Using the relative requirements from (2) it becomes very unlikely that these constraints are not met, as more resources than in step 1 are now available for the users in Group W. This choice also provides fairness within Group W. Hence the power allocation problem reads as

$$\min_{p_{\ell,k}} \left(- \sum_{k \in \text{Group W}} \sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2) \right)$$

$$\text{s.t.} \frac{\sum_{\ell=1}^{n_k} \log_2(1 + p_{\ell,k}\lambda_{\ell,k}^2)}{\sum_{\ell=1}^{n_{\text{ref user}}} \log_2(1 + p_{\ell,\text{ref user}}\lambda_{\ell,\text{ref user}}^2)} = \rho_k$$

$$\forall k \in \text{Group W}, k \neq \text{ref user}, \quad p_{\ell,k} \geq 0.$$

Again this optimization problem is identical to the power allocation problem described in [1]. Consequently it can be solved by applying the iterative algorithm from [1].

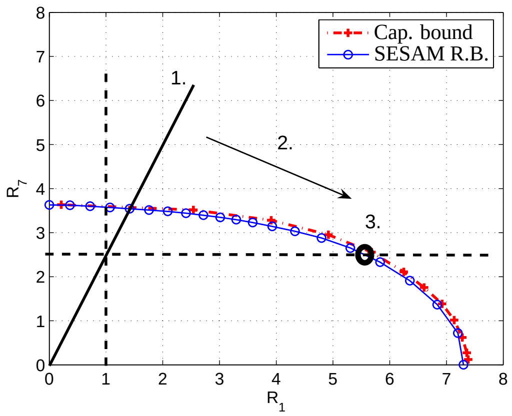

Fig. 1. Illustration of the proposed algorithm for a two user scenario

For a two user scenario the algorithm is visualized in Figure 1 and we selected users 1 and 7 from the scenario described in Section 5. The feasible domain is characterized by $R_{1,\text{min}} = 1$ bit / sub carrier and $R_{7,\text{min}} = 2.5$ bit / sub carrier. First a point on the boundary is achieved with the rate balancing algorithm from [1] denoted as "SESAM R.B." , which lies very close to the theoretical boundary, as also shown in Figure 1. Secondly the direction is determined by the user classification, to which boarder of the feasible domain one must move to increase sum capacity. Thirdly resources are reallocated to achieve this point.

4.4. Iterative Extension

While running the heuristic algorithm proposed in the previous section we moved from one point on the boundary of the capacity region to another on the basis of a local perturbation analysis at the first point. Clearly it can happen for a scenario with $K > 2$ users that at the second point the user classification done in Step 1 can no longer be justified, i.e. there are users in Group W, whose rate requirements should now also be reduced in order to increase sum capacity. On the other hand there might be users in Group L, which could contribute to a higher sum capacity without being forced to their minimum rate requirement. If the computational resources are available and the results of the above algorithm are dissatisfactory, we therefore propose an iterative application of the algorithm. In step 1 of each further iteration the Lagrange multipliers are determined the same way as described in the previous section, but with new requirements

$$\rho_k^{(n)} = \frac{R_k^{(n-1)}}{R_{\text{ref user}}^{(n-1)}}, \tag{8}$$

whereby $R_k^{(n-1)}$ denote the rates obtained through the previous iteration. If the signs of the Lagrange multipliers have changed compared to the last iteration, we adjust the user classification accordingly and repeat steps 2 and 3. Otherwise no further changes of the requirements in the proposed directions are possible and the algorithm has converged.
During an iterative application of the algorithm it is also possible to consider maximum rate requirements, i.e. it can be avoided that a user in Group W receives more capacity than its application can make use of. If in one iteration a user's rate is larger than the maximum it can exploit, that user is added to Group L with the corresponding maximum rate as requirement for the next iterations.

5. SIMULATION RESULTS

For the simulation results shown in Fig. 2 we used the measured MIMO indoor channels described in [7]. The transmitter is a uniform linear array (ULA) with $M_{\text{Tx}} = 4$ transmit antennas and the receivers are equipped with two antennas each. The OFDM system consists of $C = 1024$ sub carrier and the bandwidth is equal to 130 MHz. The receive SNR is 20 dB in the strongest channel. Figure 2 exhibits the individual rates with minimum rate requirements 0.1 bits/sub carrier for the odd and 0.2 bits/sub carrier for the even users. "SESAM minimum rates" denotes the proposed algorithm with one iteration. For comparison the algorithm from [1], denoted as "SESAM R.B.", and Time Division Multiple Access (TDMA) are also included in the figure. Those algorithms were required to serve the even users with twice the rate of the odd users. One can see that large gains in the individual rates as well as in sum capacity are possible, when minimum rate requirements are imposed on the users and the remaining resources can be distributed to the users without any constraints compared to scenarios, where, as in our case for TDMA and SESAM with relative rate constraints, strict requirements need to be fulfilled. In this scenario sum capacity increases by 72.5% compared to SESAM with relative rate constraints. This gain is achieved only with a readjustment of resources avoiding an iterative search for rate ratios or weights that achieve the point of maximum sum capacity within the feasible domain. Furthermore no time-sharing is required.

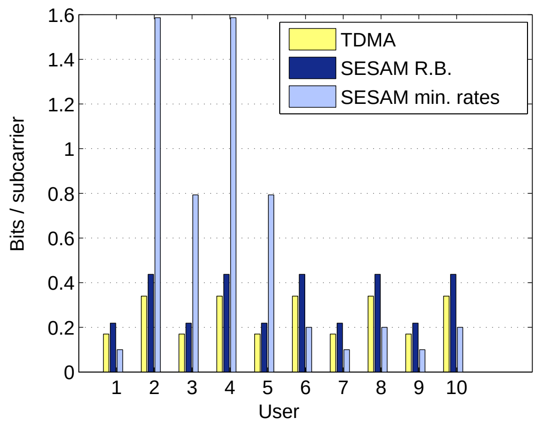

Fig. 2. Achievable transmission rates with the proposed algorithm after 1 iteration with measured indoor channels in an OFDM system with $C = 1024$ sub carrier, $M_{\text{Tx}} = 4$ transmit antennas and $M_{\text{Rx}} = 2$ antennas at each receiver, SNR=20dB.

Figure 3 exhibits the achievable rates when the algorithm is applied iteratively as proposed in Section 4.4. In this scenario the algorithm has converged after 2 iterations. Compared to the first iteration sum capacity is drastically increased by assigning user 10 also to Group L. The bars denoted as “boundary point” represent the point on the capacity region with the same rate ratios as the resulting rates from SESAM with minimum rates. As other SESAM based algorithms [6], [1], the boundary of the capacity region can be achieved closely. Certainly the resulting operation point makes only sense, if users 2, 3 and 4 can make use of these relatively high transmission rates. That is for example the case if they perform data transfer, which can then be finished earlier. But in contrast to [5], our algorithm delivers a valid solution after each iteration. Hence in case applications cannot make use of these high transmission rates, the algorithm can be stopped at any time. Furthermore, as already mentioned in Section 4.4, it allows the introduction of maximum rates.

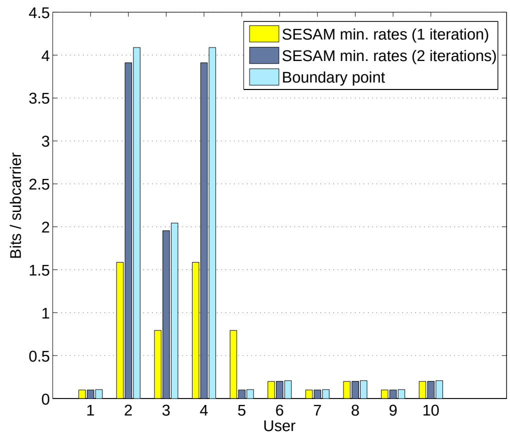

Fig. 3. Achievable transmission rates with iterative application of the proposed algorithm with measured indoor channels in an OFDM system with $C = 1024$ sub carrier, $M_{\text{Tx}} = 4$ transmit antennas and $M_{\text{Rx}} = 2$ antennas at each receiver, SNR=20dB.

6. CONCLUSIONS

In this paper we presented an algorithm which aims at maximizing sum capacity under minimum rate constraints. The main part constitutes a user classification which separates the users into two groups: one are served with their minimum required rates while the others receive additional system resources than required to satisfy their minimum rates. This classification is done on the basis of perturbation analysis of convex optimization problems, with which those users can be identified that contribute most to sum capacity.

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A. COMPUTATION OF THE LAGRANGE MULTIPLIERS FOR PERTURBATION ANALYSIS

In this appendix we will derive a formula for the computation of the Lagrange multipliers ν_k in the convex optimization problem (3). The optimum transmit power distribution can be obtained, as shown in [1], via a "waterfilling with user dependent water level ξ_k", i.e.

$$p_{\ell,k} = \max\left\{0, \left(\xi_k - \frac{1}{\lambda_{\ell,k}^2}\right)\right\}.$$

Inserting these solutions into the Karush-Kuhn-Tucker (KKT) conditions of (3) leads to:

$$\nu_k = (1 - \xi_k \eta \ln 2) R_{\text{ref user, opt}}, \quad \forall k \neq \text{ref user},$$

whereby $R_{\text{ref user, opt}}$ is the optimum rate of the reference user after running the SESAM algorithm with the relative rate requirements from (2). Furthermore we obtain from the KKT conditions:

$$\eta = \frac{1}{\xi_{\text{ref user}} \ln 2}\left(1 + \sum_{\substack{k=1\\k\neq\text{ref user}}}^{K} \frac{\nu_k \rho_k}{R_{\text{ref user, opt}}}\right).$$

Solving this linear system of equations results in

$$\nu_k = \frac{R_{\text{ref user, opt}}}{1 + \sum\limits_{\substack{i=1\\i\neq\text{ref user}}}^{K} \frac{\xi_i}{\xi_{\text{ref user}}} q_i} \left(\left(1 - \frac{\xi_k}{\xi_{\text{ref user}}}\right) + \sum_{\substack{i=1\\i\neq\text{ref user}}}^{K} \frac{\xi_k}{\xi_{\text{ref user}}} q_i \left(\frac{\xi_i}{\xi_k} - 1\right)\right).$$

Considering that $\rho_{\text{ref user}} = 1$ we obtain:

$$\nu_k = \frac{R_{\text{ref user, opt}}}{\sum\limits_{i=1}^{K} \xi_i \rho_i}\left(\rho_{\text{ref user}}(\xi_{\text{ref user}} - \xi_k) + \sum_{\substack{i=1\\i\neq\text{ref user}}}^{K} \rho_i(\xi_i - \xi_k)\right).$$

Further simplifications lead to:

$$\nu_k = \left(1 - \frac{\xi_k \sum_{i=1}^{K} \rho_i}{\sum_{i=1}^{K} \xi_i \rho_i}\right) R_{\text{ref user, opt}}$$

As all transmission rates are greater or equal to zero and the waterlevels ξ_k are independent of the choice of the reference user, the sign of ν_k is identical for all possible choices of the reference user.