AN EFFICIENT APPROXIMATION OF THE OFDMA OUTAGE PROBABILITY REGION

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ABSTRACT

A block-fading multi-user OFDMA downlink can be characterized by its outage probability region. For application in wireless resource allocation, a computationally efficient approximation of the outage region is required. The outage region is fully characterized by its Pareto efficient boundary. As a result, a finite set of well-distributed Pareto efficient samples provides the desired efficient approximation. The method of proper equality constraints (PEC) is employed to compute efficient points. In addition, an algorithm for choosing the equality constraints is presented, which provides a good distribution of samples and ensures fast convergence of the non-convex PEC optimization.

1. INTRODUCTION

We investigate the outage probability region of a block-fading multi-user downlink with orthogonal multiple access. Outage probability is the appropriate information-theoretic performance metric in a block-fading channel if a codeword spans only a small number of channel realizations and if the channel statistics, but not the channel realization, are known at the transmitter [1]. In a single-user system, for a given rate, a unique outage probability value results. In multi-user systems, however, available resources at the transmitter can be shifted between users. For each resource allocation, an outage probability tuple results, which contains the outage probabilities of all users for this particular resource allocation. Varying about all feasible power allocations yields the set of all achievable outage probability tuples – the outage probability region. This is in full analogy to the capacity region of the non-fading (static) broadcast channel [2].

Recently, the capacity region of the (non-fading) MIMO broadcast channel has received wide attention (see, e.g., [3]. Computing the outage probability (loosely speaking) requires averaging over the statistics of the channel, which makes the problem of computing the outage probability region even more involved than computing the capacity region. Recent work on outage probability has focused on single-user MISO and MIMO systems (see, e.g., [4]). In this paper, we consider a comparably simple multi-user system model, with single antenna receivers and transmitter, and the users being separated by an orthogonal multiple access scheme. Our interest for such a system arose in the context of OFDMA systems. Therefore, we consider two different types of resources to be allocated: transmit power (continuous resource) and subcarriers (discrete resource).

In general, it is not possible to find an analytical description of the outage probability region. As a result, we can only compute an approximation of the outage probability region by numerically computing a finite number of achievable outage probability tuples. From an information theoretic viewpoint, the computational complexity for computing an approximation of the outage probability region is of minor interest. In the last decade, however, near-capacity coding techniques have emerged, making information theoretic performance metrics a candidate for implementation in wireless systems. In order to base resource allocation in a wireless multiuser system on the outage probability region, computationally efficient algorithms for computing a good approximation are required. In the following, we develop an algorithm for approximating the outage probability region that provides a good approximation with only a limited number of samples and a low computational complexity per sample.

The approximated outage region provides a characterization of the coded physical layer in terms of achievable outage probability tuples. In a wireless multi-user communication system, a second instance is needed to decide which of the achievable tuples actually to use for transmission, i.e., which tuple results in best system performance. The outage region provides this instance with an abstracted view of the coded physical layer. This encapsulation property forms the basis for a modular cross-layer optimization [5].

The proposed algorithm is based on the observation that computing an efficient approximation of an achievable region is closely related to the mathematical theory of multiobjective optimization [6]. As motivated in Section 3, the outage region is approximated by a set of tuples that have the property of being Pareto efficient. In Section 4, we discuss how to compute a single Pareto efficient sample by applying the method of "proper equality constraints" [7] to the outage-based resource allocation problem. In Section 5, an algorithm for achieving a good distribution of samples is presented, and an approximated outage region for a sample three user system is shown.

2. SYSTEM MODEL

We consider a multi-user system with a single transmitter sending data to K non-cooperating single-antenna receivers. The channels to the users are assumed to be frequency flat Rayleigh block-fading. An *Orthogonal Frequency Division Multiple Access* (OFDMA) scheme with N_c subcarriers is applied to separate the users. The OFDMA mode is introduced to add additional degrees of freedom in the resource allocation. We employ the usual representation of an OFDM system as a set of parallel subchannels (e.g., [8]). Let $N_k \in \mathbb{N}_{0,+}$ denote the number of subcarriers allocated to user k. Moreover, let δ_k denote the transmit power allocated to user k. The available transmit power at the transmitter is limited to E_{tr}. Due to the fact that the channels are frequency flat, δ_k is distributed equally on all subcarriers allocated to user k. The received signal on the n-th subcarrier allocated to user k is given by

$$y_{k,n} = \sqrt{\frac{\delta_k}{N_k}} h_k x_{k,n} + \eta_{k,n},$$

where $x_{k,n}$ denotes the unit norm transmit symbol, and $\eta_{k,n} \sim \mathcal{CN}(0, \sigma_k^2)$ is additive white Gaussian noise, assumed to have equal variance on all subcarriers of user k. According to the Rayleigh block-fading assumption, the channels $h_k \sim \mathcal{CN}(0, \rho_k)$ are assumed to be constant during one coded OFDM block and independent from block to block [1]. The transmitter knows the average channel gains ρ_k, but not the actual channels h_k, while the receivers are assumed to have perfect knowledge of their respective h_k.

3. PROBLEM SETUP

For each user, a data stream with rate R_k (in bits per OFDM symbol) is transmitted. Due to the limited channel knowledge of the transmitter, this rate may be higher than the channel capacity offered by the set of subcarriers allocated to user k, resulting in a channel outage. While the computation of the outage probability is rather involved for OFDM over frequency selective channels [8], for frequency flat channels it is simply the outage probability of a single subcarrier for a rate requirement of $\frac{R_k}{N_k}$, i.e., the overall rate R_k is "divided" among the allocated subcarriers. For a resource allocation $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_K)$ and $\boldsymbol{N} = (N_1, \ldots, N_k)$, the outage probability ε_k of user k is given by:

$$\varepsilon_k(\boldsymbol{\delta}, \boldsymbol{N}) = 1 - \exp\left(-\left(2^{\frac{R_k}{N_k}} - 1\right)\frac{\sigma_k^2 N_k}{\delta_k \rho_k}\right). \qquad (1)$$

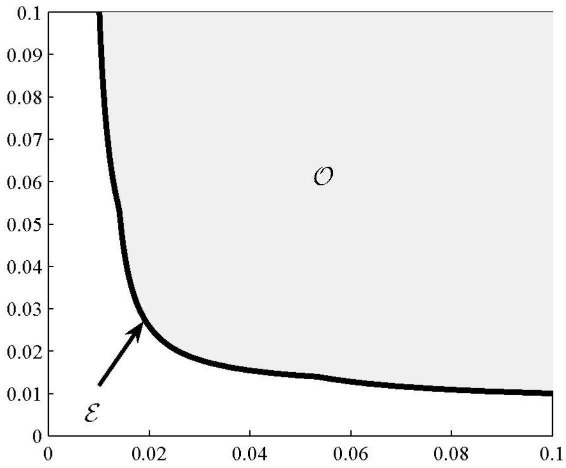

Figure 1. Outage Probability Region $\mathcal{O}$ and Efficient Set $\mathcal{E}$

The resulting outage probabilities for all users are collected in a tuple $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_K)$. The set of feasible resource allocations is given by

$$\mathcal{G} = \{(\boldsymbol{\delta}, \boldsymbol{N}) \in \mathbb{R}_{0,+} \times \mathbb{N}_{0,+} : \|\boldsymbol{\delta}\|_1 \leq E_{\text{tr}}, \|\boldsymbol{N}\|_1 \leq N_c\}.$$

The achievable outage region $\mathcal{O}$ is obtained (theoretically) by computing $\boldsymbol{\varepsilon}(\boldsymbol{\delta}, \boldsymbol{N})$ for all feasible resource allocations. For the case of $K = 2$ users, a sample outage region is shown in Fig. 1.

We aim at a computationally efficient approximation of the outage region. Given an achievable outage tuple, we know that any tuple for which at least one user has higher outage is also achievable. Thus, the outage region is fully characterized by the set of the *smallest* achievable outage tuples, i.e., by those tuples on the boundary of $\mathcal{O}$ for which the outage probability of one user can only be further decreased by increasing the outage probability of at least one other user. The latter property is just the definition of Pareto efficiency [6]. Accordingly, the smallest tuples constitute the so-called efficient set $\mathcal{E}$. See Fig. 1 for an illustration of the efficient set.

Computing the efficient set corresponds to solving a multiobjective optimization (MOO) problem [6]:

$$\mathcal{E} = \min_{(\boldsymbol{\delta}, \boldsymbol{N}) \in \mathcal{G}} \boldsymbol{\varepsilon}(\boldsymbol{\delta}, \boldsymbol{N}). \qquad (2)$$

Note that in contrast to single-objective optimization, the solution of a MOO problem is in general given by a set.

Finding an analytical solution to problem (2) is not possible. Therefore, *we seek a good approximation of $\mathcal{E}$ by a finite number of efficient tuples $\boldsymbol{\varepsilon}$.*

4. PARETO OPTIMAL RESOURCE ALLOCATION

There exists a wide variety of methods for solving MOO problems [9]. In this paper, we employ the method of *proper* *equality constraints* (PEC) [7] for determining a set of efficient outage tuples. From Fig. 1 it is clear that the outage region is non-convex. In contrast to other MOO methods, such as the popular weighted sum method [9], the PEC method is applicable to problems with a non-convex achievable region.

In the PEC method, all but one objectives are constrained to equal a predefined value, while the remaining objective is minimized under these additional equality constraints. Without loss of generality, we choose ε_1 as the objective to be minimized, and add the constraints $\varepsilon_k = \varepsilon'_k, \forall k > 1$. In the context of PEC, the constraints $\varepsilon_k = \varepsilon'_k$ are denoted as *parametric constraints*. In the example from Fig. 1, the PEC optimization corresponds to minimizing ε_1 on the intersection between $\mathcal{O}$ and the horizontal line $\varepsilon_2 = \varepsilon'_2$. If the intersection is not empty, an efficient tuple results. If it is empty, i.e., the parametric constraints are too demanding, no solution to the PEC optimization exists. In the following, we assume that a PEC solution exists. A strategy for choosing feasible parametric constraints is discussed in Section 5. For a general MOO problem, it is also possible that a PEC solution is not efficient, due to "folds" in the achievable region [7]. For problem (2), however, all PEC solutions are efficient. An intuition that the outage region contains no folds is given by Fig. 1, a rigorous proof is provided in [10].

With $\boldsymbol{\varepsilon}_{2:K} = (\varepsilon_2, \ldots, \varepsilon_K)$ and $\boldsymbol{\varepsilon}' = (\varepsilon'_2, \ldots, \varepsilon'_K)$, the PEC problem reads as

$$\min_{\boldsymbol{\delta}, \boldsymbol{N}} \varepsilon_1 \quad \text{s.t.} \quad (\boldsymbol{\delta}, \boldsymbol{N}) \in \mathcal{G}, \boldsymbol{\varepsilon}_{2:K} = \boldsymbol{\varepsilon}'.$$

Next, we state that a necessary condition for a resource allocation to be optimum is that it uses all available resources:

$$\boldsymbol{\varepsilon}(\boldsymbol{\delta}, \boldsymbol{N}) \in \mathcal{E} \Rightarrow \|\boldsymbol{\delta}\|_1 = E_{\text{tr}} \wedge \|\boldsymbol{N}\|_1 = N_{\text{c}}. \tag{3}$$

The proof is straightforward: Consider a resource allocation that does not use all available resources and the corresponding outage tuple $\tilde{\boldsymbol{\varepsilon}}$. Pick one user k with $N_k > 0$ and add all remaining resources to this user. This will reduce the outage probability of this user without affecting the outage probabilities of the other users. Thus, $\tilde{\boldsymbol{\varepsilon}}$ is not Pareto efficient.

The required transmit power for user k, $k > 1$, to fulfill the parametric constraint $\varepsilon_k = \varepsilon'_k$, given a number of N_k subcarriers, is given by

$$\delta_k(N_k, \varepsilon'_k) = -\left(2^{\frac{R_k}{N_k}} - 1\right) \frac{\sigma_k^2 N_k}{\rho_k \ln(1 - \varepsilon'_k)}. \tag{4}$$

With $\boldsymbol{N}_{2:K} = (N_2, \ldots, N_K)$ and (3), we can write

$$\delta_1(\boldsymbol{N}_{2:K}, \boldsymbol{\varepsilon}') = E_{\text{tr}} - \sum_{k=2}^{K} \delta_k(N_k, \varepsilon'_k), \quad \text{and} \tag{5}$$

$$N_1(\boldsymbol{N}_{2:K}) = N_{\text{c}} - \sum_{k=2}^{K} N_k. \tag{6}$$

With (5), (6) and (4), the PEC problem is transformed into the minimization of ε_1 with respect to the subcarriers allocated to the users $2, \ldots, K$:

$$\min_{\boldsymbol{N}_{2:K}} \varepsilon_1(\boldsymbol{N}_{2:K}) \quad \text{s.t.} \quad \boldsymbol{N}_{2:K} \in \mathcal{G}_{\boldsymbol{\varepsilon}'},$$

with

$$\varepsilon_1(\boldsymbol{N}_{2:K}) = \varepsilon_1\big(\delta_1(\boldsymbol{N}_{2:K}, \boldsymbol{\varepsilon}'), N_1(\boldsymbol{N}_{2:K})\big), \quad \text{and}$$
$$\mathcal{G}_{\boldsymbol{\varepsilon}'} = \{\boldsymbol{N}_{2:K} \in \mathbb{N}_{0,+}^{K-1} : \delta_1(\boldsymbol{N}_{2:K}, \boldsymbol{\varepsilon}') \geq 0, \|\boldsymbol{N}_{2:K}\|_1 \leq N_{\text{c}}\}.$$

The function $\varepsilon_1(\boldsymbol{N}_{2:K})$ is non-convex. Despite this non-convexity, the optimum subcarrier allocation for the parametric constraint $\boldsymbol{\varepsilon}'$ can be determined based on local gradient information. Consider the real-valued extension of $\mathcal{G}_{\boldsymbol{\varepsilon}'}$:

$$\bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'} = \{\boldsymbol{N}_{2:K} \in \mathbb{R}_{0,+}^{K-1} : \delta_1(\boldsymbol{N}_{2:K}, \boldsymbol{\varepsilon}') \geq 0, \|\boldsymbol{N}_{2:K}\|_1 \leq N_{\text{c}}\}.$$

We have the following result:

Theorem 4.1. *On* $\operatorname{int} \bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$, $\boldsymbol{\nabla}\varepsilon_1(\boldsymbol{N}_{2:K}) = 0$ *has at most one solution. If a solution* $\hat{\boldsymbol{N}}$ *exists,* $\varepsilon_1(\hat{\boldsymbol{N}})$ *is the unique minimum of* $\varepsilon_1(\boldsymbol{N}_{2:K})$ *on* $\operatorname{int} \bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$.

Proof. The gradient of ε_1 can be written as:

$$\boldsymbol{\nabla}\varepsilon_1 = f_1(\boldsymbol{N}_{2:K})\boldsymbol{g}(\boldsymbol{N}_{2:K}), \qquad \text{with}$$

$$f_1(\boldsymbol{N}_{2:K}) = \frac{\sigma_1^2}{\rho_1 \delta_1^2} \exp\left(\frac{-\sigma_1^2}{\rho_1 \delta_1}\Big(2^{\frac{R'_1}{N_1(\boldsymbol{N}_{2:K})}} - 1\Big) N_1(\boldsymbol{N}_{2:K})\right).$$

Note that in the interior of $\bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$, we have $\delta_1 > 0$ and $N_1 > 0$. The Hessian $\boldsymbol{\nabla}^2\varepsilon_1$ then computes as follows:

$$\boldsymbol{\nabla}^2\varepsilon_1 = f_1 \boldsymbol{J}(\boldsymbol{g}) + \boldsymbol{g}\boldsymbol{\nabla}^{\mathrm{T}} f_1,$$

where $\boldsymbol{J}(\boldsymbol{g})$ is the Jacobian of $\boldsymbol{g}$. It is positive definite, as

$$\boldsymbol{J}(\boldsymbol{g}) = \left(\delta_1 + 2^{\frac{R'_1}{N - \sum_{k=2}^{K} N_k}} \frac{(\ln 2 R'_1)^2}{(N - \sum_{k=2}^{K} N_k)^3}\right) \boldsymbol{I} + \boldsymbol{D} + \boldsymbol{A},$$

where $\boldsymbol{D}$ is a diagonal matrix with positive entries and $\boldsymbol{A}$ is a skew-symmetric matrix. Therefore at each point $\hat{\boldsymbol{N}}$, where $\boldsymbol{g}(\hat{\boldsymbol{N}}) = 0$, the Hessian $\boldsymbol{\nabla}^2\varepsilon_1$ is positive definite and consequently $\hat{\boldsymbol{N}}$ can only lead to a minimum of ε_1. If there was another solution $\tilde{\boldsymbol{N}}$ with $\boldsymbol{g}(\tilde{\boldsymbol{N}}) = 0$ and $\tilde{\boldsymbol{N}} \neq \hat{\boldsymbol{N}}$ in $\operatorname{int} \bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$, this point would also have to be a minimum, which constitutes a contradiction. Hence $\hat{\boldsymbol{N}}$ is, if existent, the only extremum on $\operatorname{int} \bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$. □

Now consider the case that the solution $\hat{\boldsymbol{N}}$ exists on $\operatorname{int} \bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$. In this case, the optimum (discrete) subcarrier allocation $\boldsymbol{N}_{\text{opt}}$ is determined by a simple discrete gradient descent method. The critical point in the optimization is choosing an initial subcarrier allocation $\boldsymbol{N}_{\text{init}}$. For the algorithm to succeed, it is required that

$$\varepsilon_1(\boldsymbol{N}_{\text{init}}, \boldsymbol{\varepsilon}') \in \operatorname{int} \mathcal{G}_{\boldsymbol{\varepsilon}'}. \tag{7}$$

Starting from a valid initialization, a gradient based method finds the unique minimum if care is taken that the iteration does not leave $\operatorname{int}\mathcal{G}_{\boldsymbol{\varepsilon}'}$. At each iteration, we simply de- or increment the number of subcarriers allocated to one of the user by one, depending on the gradient. Moreover, due to the discrete nature of $\boldsymbol{N}_{2:K}$ it is simple to detect those steps that would lead outside of $\operatorname{int}\mathcal{G}_{\boldsymbol{\varepsilon}'}$.

In the second case, i.e., no optimum exists on $\operatorname{int}\bar{\mathcal{G}}_{\boldsymbol{\varepsilon}'}$, the optimum subcarrier allocation lies on the boundary of $\mathcal{G}_{\boldsymbol{\varepsilon}'}$. However, every subcarrier allocation on the boundary of $\mathcal{G}_{\boldsymbol{\varepsilon}'}$ corresponds to the case that at least one user gets no resources, i.e., the outage probability of those users is equal to 1. As a result, we can circumvent this case by limiting our considerations to $\varepsilon_k \leq \varepsilon_{k,\max}$, with

$$\varepsilon_{k,\max} < 1.$$

While choosing $\varepsilon_k' \leq \varepsilon_{k,\max}$ is simple, it is not obvious which choices of $\boldsymbol{\varepsilon}'$ guarantee that $\varepsilon_1(\boldsymbol{N}_{\text{opt}}) < 1$. This issue is addressed in Section 5. The cases in which B users get no resources can be treated by removing these users from the optimization and considering a $K - B$ system.

5. CHOOSING THE PARAMETRIC CONSTRAINTS

For a parametric constraint $\boldsymbol{\varepsilon}' = (\varepsilon_2', \ldots, \varepsilon_K')$, a single efficient sample $\boldsymbol{\varepsilon} = (\varepsilon_1, \boldsymbol{\varepsilon}')$ is generated. An approximation of the efficient set is obtained by computing samples $\boldsymbol{\varepsilon}$ for different choices of $\boldsymbol{\varepsilon}'$. Clearly, the choice of the parametric constraints $\boldsymbol{\varepsilon}'$ determines the distribution of the samples, i.e., choosing $\boldsymbol{\varepsilon}'$ in a careful manner is crucial for the quality of the approximation.

Moreover, in the previous section it was shown that it is important to choose $\boldsymbol{\varepsilon}'$ such that the iterations of the subcarrier allocation optimization start in $\operatorname{int}\mathcal{G}_{\boldsymbol{\varepsilon}'}$.

In this section, we develop an iterative algorithm for determining a set of parametric constrains $\boldsymbol{\varepsilon}'$. In the following, superscript (j) is used to denote the j-th iteration. For simplicity of presentation, we first consider the case of $K = 2$ users. In this case, $\boldsymbol{\varepsilon}' = \varepsilon_2'$. The algorithm starts with

$$\varepsilon_2^{\prime,(0)} = \varepsilon_{2,\max} \quad \text{and} \quad N_{\text{init}}^{(0)} = \frac{N_{\text{c}}}{2}.$$

Setting ε_2' to the largest value of interest makes the maximum amount of transmit power available to user 1 (for the given subcarrier allocation). As a result, it is likely that (7) holds. If (7) does not hold for this choice of ε_2' and N_{init}, $\varepsilon_{2,\max}$ is increased until (7) holds. Alternatively, one could try more elaborate choices for N_{init} than simply dividing the subcarriers equally among users.

Next, the minimization from Section 4 is carried out, yielding an optimum subcarrier allocation $N_{\text{opt}}^{(0)}$.

At the j-th iteration, $j > 0$, ε_2' is decremented by $\Delta\varepsilon^{\prime,(j)}$:

$$\varepsilon_2^{\prime,(j)} = \varepsilon_2^{\prime,(j-1)} - \Delta\varepsilon^{\prime,(j)}.$$

The stepsize $\Delta\varepsilon^{\prime,(j)}$ is determined as follows: Assuming that $\Delta\varepsilon^{\prime,(j)}$ is small enough and that the optimum subcarrier allocation does not change, the increment in ε_1 is approximated by

$$\Delta\varepsilon_1^{(j)} \approx \alpha_2^{(j)} \Delta\varepsilon_2^{\prime,(j)},$$

with

$$\alpha_2^{(j)} = \left.\frac{\partial\varepsilon_1(N_{\text{opt}}^{(j-1)}, \varepsilon_2')}{\partial\varepsilon_2'}\right|_{\varepsilon_2' = \varepsilon_2^{\prime,(j-1)}}.$$

The distance d between two subsequent samples is given by

$$d = \sqrt{(\Delta\varepsilon_1^{(j)})^2 + (\Delta\varepsilon_2^{\prime,(j)})^2} \approx \Delta\varepsilon_2^{\prime,(j)}\sqrt{(\alpha_2^{(j)})^2 + 1}.$$

Accordingly, in order to obtain a distance d between samples, $\Delta\varepsilon_2^{\prime,(j)}$ is set to

$$\Delta\varepsilon_2^{\prime,(j)} = \frac{d}{\sqrt{(\alpha_2^{(j)})^2 + 1}}.$$

For $\varepsilon^{\prime,(j)}$, the optimum subcarrier allocation $N_{\text{opt}}^{(j)}$ is determined, with

$$N_{\text{init}}^{(j)} = N_{\text{opt}}^{(j-1)}.$$

Due to the fact that the optimum subcarrier allocation usually does not change significantly between subsequent samples, using $N_{\text{opt}}^{(j-1)}$ as initialization results in fast convergence of the minimization. Moreover, a good initialization ensures that (7) holds, even for small ε_2'. The algorithm stops when $\varepsilon_1(N_{\text{opt}}^{(j)}, \varepsilon^{\prime,(j)}) \geq \varepsilon_{1,\max}$.

The extension to an arbitrary number K of users is straightforward. First, $\varepsilon_\ell' = \varepsilon_{\ell,\max}, \quad \forall \ell > 1$. Starting with $\ell = 2$, ε_ℓ' is decremented until $\varepsilon_1 \geq \varepsilon_{1,\max}$. If $\ell < K$, for each value of ε_ℓ', $\varepsilon_{\ell+1}'$ is first set to $\varepsilon_{\ell+1,\max}$ and then decremented until $\varepsilon_1 \geq \varepsilon_{1,\max}$. This recursive rule is applied to $\varepsilon_{\ell+1}'$ in the same manner.

For $K = 3$ users, an approximation of the efficient set for an example system is shown in Fig. 2. Note that the region is viewed from "behind" for a better visual impression. The sampling parameters are $\varepsilon_{k,\max} = 0.1$ and $d = 0.01$. The system parameters are given by $N_{\text{c}} = 512$, $R_1 = \frac{1}{3}N_{\text{c}}$, $R_2 = R_3 = N_{\text{c}}$, $\rho_k = 1$, $\sigma_k^2 = \sigma^2$, and

$$10\log_{10}\frac{E_{\text{tr}}}{K\sigma^2 N_{\text{c}}} = 15\text{dB}.$$

The circles in Fig. 2 correspond to the samples computed by the algorithm. The samples are connected by lines to indicate how the algorithm explores the efficient set. It starts with $\varepsilon_2' = \varepsilon_3' = 0.1$, then first decrements ε_3', then a single decrement in ε_2', and so on. On average 5.6 iterations per sample were needed. While the distribution of samples is generally

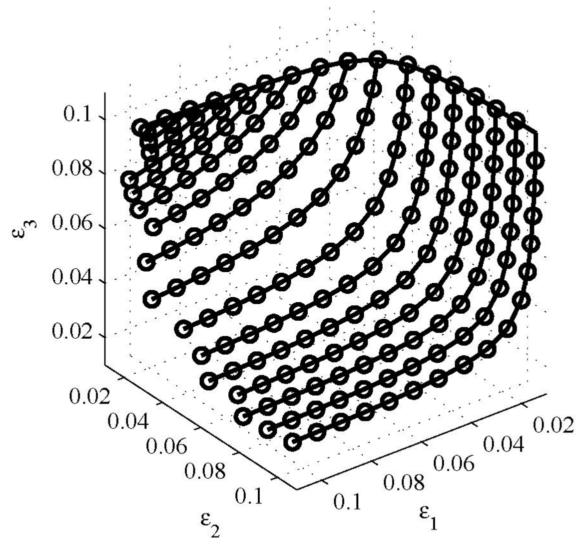

Figure 2. Approximation of $\mathcal{O}$ for $K = 3$ users

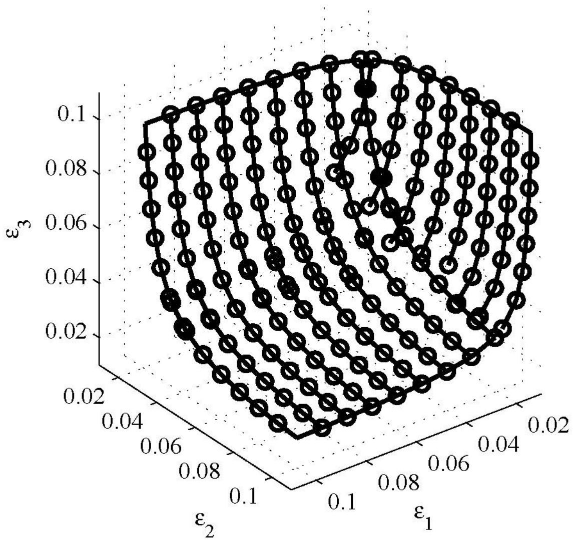

Figure 3. Approximation of $\mathcal{O}$ by 3 subsets

good, for small values of ε'_2 an undesirable accumulation results. The distance between neighboring trajectories is small, due to the fact that small changes in ε'_2 and ε'_3 result in large changes in ε_1. We can conclude that it is desirable to choose the least sensitive objective as the target objective in the PEC problem. This leads to the idea of computing K subsets of $\mathcal{E}$, with $\varepsilon_t, t = 1, \ldots, K$, as the target objective for the t-th subset and then taking the union of those subsets as the approximation of the outage region. In order to minimize the overlap between the subsets, a modified stopping criterion is needed. A small sensitivity of the target objective ε_t is desired, therefore ε'_ℓ is not incremented further if

$$\left|\frac{\partial \varepsilon_t}{\partial \varepsilon'_\ell}\right| \geq T,$$

where $T > 1$ determines the overlap of the subsets. Note that this approach depends on a specific property of problem (2) (the sensitivity of ε_t increases when decreasing ε'_ℓ). The resulting approximation for a threshold value of $T = 2$ for the example system is shown in Fig. 3. The modified algorithm avoids an accumulation of samples and provides a good overall distribution.

6. CONCLUSIONS

A method to approximate of the outage probability region of an OFDMA multi-user downlink was presented. The approximation is obtained by sampling the efficient boundary of the region. The sampling algorithm consists of two components: a non-convex discrete optimization, and an algorithm that a) controls the distribution of samples and b) ensures feasible parametric constraints. The proposed method achieves a good distribution of samples on the efficient set.

7. REFERENCES

[1] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, no. 2, pp. 359–378, May 1994.

[2] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley & Sons, Inc., 1991.

[3] S. Vishwanath, N. Jindal, and A.J. Goldsmith, "Duality, achievable rates, and sum-rate capacity of gaussian MIMO broadcast channels," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2658–2668, Oct. 2003.

[4] H. Boche and E. A. Jorswieck, "Outage probability of multiple antenna systems: optimal transmission and impact of correlation," in *Proc. IEEE Int. Zurich Seminar Comm.*, pp. 116–119, Sept. 2004.

[5] J. Brehmer and W. Utschick, "Modular cross-layer optimization based on layer descriptions," in *Proc. Wireless Pers. Multimedia Comm. Symp. (WPMC 2005)*, Aalborg, Denmark, Sept. 2005.

[6] Y. Sawaragi, H. Nakayama, and T. Tanino, *Theory of Multiobjective Optimization*, Academic Press, Orlando, 1985.

[7] J.G. Lin, "Multiple-objective problems: Pareto-optimal solutions by method of proper equality constraints," *IEEE Trans. Automat. Contr.*, vol. 21, pp. 641–650, Oct. 1976.

[8] J. Zheng and S. L. Miller, "Performance analysis of coded ofdm systems over frequency-selective fading channels," in *Proc. IEEE Global Telecomm. Conf. (GLOBECOM 2003)*, vol. 3, pp. 1623–1627, 2003.

[9] R. T. Marler and J. S. Arora, "Survey of multi-objective optimization methods for engineering," *Structural and Multidisciplinary Optimization*, vol. 26, no. 6, pp. 369–395, 2004.

[10] C. Guthy, "Pareto optimal physical layer descriptions of multiuser, multiantenna and multicarrier systems," Diploma thesis, Munich University of Technology, 2005.