ITERATIVE THP TRANSCEIVER OPTIMIZATION FOR MULTI-USER MIMO SYSTEMS BASED ON WEIGHTED SUM-MSE MINIMIZATION

Amine Mezghani, Raphael Hunger, Michael Joham, and Wolfgang Utschick

Seventh IEEE International Workshop on Signal Processing Advances for Wireless Communications (SPAWC 2006)

Cannes, France

July 2nd–5th, 2006

Munich University of Technology
Associate Institute for Signal Processing
http://www.nws.ei.tum.de

ITERATIVE THP TRANSCEIVER OPTIMIZATION FOR MULTI-USER MIMO SYSTEMS BASED ON WEIGHTED SUM-MSE MINIMIZATION

Amine Mezghani, Raphael Hunger, Michael Joham, and Wolfgang Utschick

Institute for Circuit Theory and Signal Processing
Munich University of Technology, 80290 Munich, Germany
e-mail: {mezghani, hunger, joham, utschick}@nws.ei.tum.de

ABSTRACT

We address the joint optimization of transmitter and receivers for a multi-user *multiple-input multiple-output* (MIMO) *broadcast channel* (BC) system under the assumption of perfect *channel state information* (CSI) at both, transmitter and receivers. *Tomlinson Harashima precoding* (THP) is employed for inter-user interference presubtraction and the *mean square error* (MSE) is minimized. Since the downlink problem is difficult to handle, we formulate an equivalent uplink problem by exploiting the duality between THP and *decision feedback equalization* (DFE). We present an iterative solution, which delivers suboptimum transmit and receive matrices as well as a suboptimum precoding order. The performance of the algorithm is studied theoretically and experimentally.

1. INTRODUCTION

In a downlink scenario, the receivers are decentralized and can be equipped with a single antenna (multi-user MISO system) or even with multiple antennas (multi-user MIMO system) each. The decentralization implies the necessity of pre-equalization at the transmitter. Linear pre-processing was considered in [1] based on the *minimum MSE* (MMSE) and *zero-forcing* (ZF) criteria, where the receivers were restricted to apply the same scalar weight. Jointly optimizing the linear transmit and receive matrices for the *Quality of Service* (QoS) and sum-MSE criteria was considered in [2, 3, 4] for the MISO case and was extended to the MIMO case in [5, 6, 7, 8].

Nonlinear precoding strategies clearly improve the performance compared to linear pre-processing and may achieve the full capacity [9]. THP with restricted receivers was derived using a sum-MSE approach in [10]. A QoS based joint optimization of THP transceivers was considered in [11] (MISO) and [12] (MIMO) for fixed precoding order. However, the sum-MSE based optimization of THP is very difficult and up to now, only solutions based on an exhaustive search exist [13].

Our contribution is a solution for the weighted sum-MSE minimization for the downlink with nonlinear beamforming. For simplicity, only the inter-user interference is presubtracted nonlinearly, whereas the streams of each user are jointly precoded linearly. The design is performed in a dual uplink model, which offers a better mathematical structure, by iteratively solving the necessary first order KKT conditions for local optimality. We cannot prove the global optimality of such a solution since the problem is non-convex but we show through simulations in Section 7 that the algorithm delivers excellent solutions in practice. Our paper is organized as follows: In Section 3 we prove the duality between DFE and THP in terms of MSE for general feedback matrices (contrary to previous proofs). Sections 4 and 5 deal with the iterative solution of the weighted sum-MSE optimization using a gradient projection approach [14]. The precoding order problem is discussed in Section 6. Finally, we present simulation results in Section 7.

2. SYSTEM MODEL AND NOTATION

We consider a K-users MIMO BC system with an M antennas transmitter and the kth receiver has N_k antennas (see Fig. 1). The channel matrix of user k is $\boldsymbol{H}_k^{\mathrm{H}} \in \mathbb{C}^{N_k \times M}$, $k = 1, \ldots, K$, and the vector $\boldsymbol{s}_k \in \mathbb{C}^{B_k}$ comprises the B_k uncorrelated unit variance symbols of user k which are assumed to be uncorrelated with other users' symbols. The vectors $\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_K$ denote zero-mean white noise with variance σ_n^2 for each component. In addition to the feedforward filters $\bar{\boldsymbol{P}}_k \in \mathbb{C}^{M \times B_k}$, the transmitter is extended with a $modulo$ device M($\bullet$) and a spatial feedback filter $\bar{\boldsymbol{F}}$, where for $k > j$, $\bar{\boldsymbol{F}}_{k,j} \in \mathbb{C}^{B_{\pi_k} \times B_{\pi_j}}$. The users are successively precoded with order π, i.e., user π_k sees the interference caused by the users $\pi_{k+1}, \ldots, \pi_K$. The decentralized receive filter of user k is $\bar{\boldsymbol{G}}_k \in \mathbb{C}^{B_k \times N_k}$. For a shorter notation, we introduce the matrix products

$$\bar{\boldsymbol{A}}_{k,j} := \bar{\boldsymbol{G}}_{\pi_k} \boldsymbol{H}_{\pi_k}^{\mathrm{H}} \bar{\boldsymbol{P}}_{\pi_j} \quad \text{and} \quad \boldsymbol{A}_{k,j} := \boldsymbol{G}_{\pi_k} \boldsymbol{H}_{\pi_j} \boldsymbol{P}_{\pi_j}. \tag{1}$$

With these definitions, the estimate $\hat{\boldsymbol{s}}_{\pi_k}$ of user π_k reads as

$$\begin{aligned}
\hat{\boldsymbol{s}}_{\pi_k} &= \big(\bar{\boldsymbol{A}}_{k,k}\boldsymbol{u}_k + \textstyle\sum_{j\neq k} \bar{\boldsymbol{A}}_{k,j}\boldsymbol{u}_j + \bar{\boldsymbol{G}}_{\pi_k}\boldsymbol{\eta}_{\pi_k}\big) \bmod \tau \\
&= \big(\bar{\boldsymbol{A}}_{k,k}\boldsymbol{u}_k + \textstyle\sum_{j\neq k} \bar{\boldsymbol{A}}_{k,j}\boldsymbol{u}_j + \bar{\boldsymbol{G}}_{\pi_k}\boldsymbol{\eta}_{\pi_k} \\
&\qquad - \textstyle\sum_{j<k} \bar{\boldsymbol{F}}_{k,j}\boldsymbol{u}_j + \textstyle\sum_{j<k} \bar{\boldsymbol{F}}_{k,j}\boldsymbol{u}_j\big) \bmod \tau \\
&= \big(\boldsymbol{s}_{\pi_k} - \boldsymbol{u}_k + \bar{\boldsymbol{A}}_{k,k}\boldsymbol{u}_k + \textstyle\sum_{j\neq k} \bar{\boldsymbol{A}}_{k,j}\boldsymbol{u}_j \\
&\qquad - \textstyle\sum_{j<k} \bar{\boldsymbol{F}}_{k,j}\boldsymbol{u}_j + \bar{\boldsymbol{G}}_{\pi_k}\boldsymbol{\eta}_{\pi_k}\big) \bmod \tau,
\end{aligned} \tag{2}$$

since $(\sum_{j<k} \bar{\boldsymbol{F}}_{k,j}\boldsymbol{u}_j) \bmod \tau = (\boldsymbol{s}_{\pi_k} - \boldsymbol{u}_k) \bmod \tau$. When we neglect the *modulo-loss*, the error vector $\boldsymbol{e}_{\pi_k}^{\mathrm{DL}} = \hat{\boldsymbol{s}}_{\pi_k} - \boldsymbol{s}_{\pi_k}$ is

$$\boldsymbol{e}_{\pi_k}^{\mathrm{DL}} = (\bar{\boldsymbol{A}}_{k,k} - \mathbf{I})\boldsymbol{u}_k + \bar{\boldsymbol{G}}_{\pi_k}\boldsymbol{\eta}_{\pi_k} + \sum_{j\neq k} \bar{\boldsymbol{A}}_{k,j}\boldsymbol{u}_j - \sum_{j<k} \bar{\boldsymbol{F}}_{k,j}\boldsymbol{u}_j. \tag{3}$$

Assuming that the entries of $\boldsymbol{u}_i$ have unit variance ($\tau = \sqrt{6}$) and are uncorrelated, and $\boldsymbol{u}_i$ and $\boldsymbol{u}_k$ are mutually uncorrelated for $i \neq k$ [15], the MSE $\varepsilon_{\pi_k}^{\mathrm{DL}} = \mathrm{E}[\|\boldsymbol{e}_{\pi_k}^{\mathrm{DL}}\|_2^2]$ can be written as

$$\begin{aligned}
\varepsilon_{\pi_k}^{\mathrm{DL}} = \mathrm{tr}\Big[&(\bar{\boldsymbol{A}}_{k,k} - \mathbf{I})(\bar{\boldsymbol{A}}_{k,k} - \mathbf{I})^{\mathrm{H}} + \textstyle\sum_{j>k} \bar{\boldsymbol{A}}_{k,j}\bar{\boldsymbol{A}}_{k,j}^{\mathrm{H}} \\
&+ \textstyle\sum_{j<k} (\bar{\boldsymbol{A}}_{k,j} - \bar{\boldsymbol{F}}_{k,j})(\bar{\boldsymbol{A}}_{k,j} - \bar{\boldsymbol{F}}_{k,j})^{\mathrm{H}} + \sigma_n^2 \bar{\boldsymbol{G}}_{\pi_k}\bar{\boldsymbol{G}}_{\pi_k}^{\mathrm{H}}\Big].
\end{aligned} \tag{4}$$

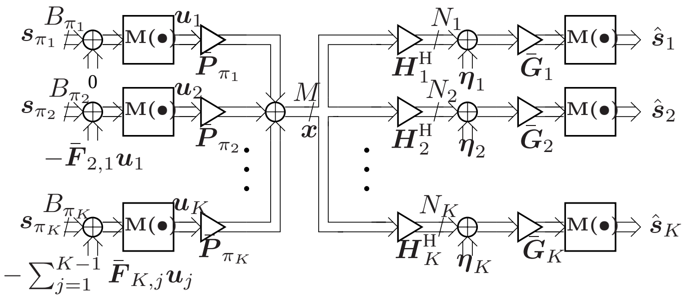

Fig. 1. Downlink with nonlinear transmit beamforming.

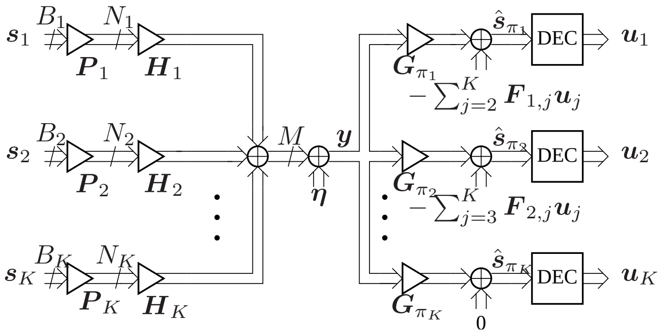

Fig. 2. Equivalent uplink with nonlinear receive beamforming.

From (4), we see that the MSE $\varepsilon_k^{\mathrm{DL}}$ of user k in the downlink depends on all precoding matrices $\boldsymbol{P}_j^{\mathrm{DL}}$ which are strongly coupled by the sum power constraint. Thus, we formulate an equivalent uplink problem with a better mathematical structure in the next section, where the precoders are decoupled and only a joint power constraint has to be considered.

3. NON-LINEAR DOWNLINK/UPLINK DUALITY

Fig. 2 shows the dual uplink system (*multiple access channel*, MAC) with DFE. We note that the users are decoded in the reverse order compared to the downlink, which is necessary for the duality. In other words, user π_K is decoded first and user π_1 last. Assuming that the signals from users $\pi_K, \ldots, \pi_{k+1}$ have correctly been decoded, i.e., $\boldsymbol{u}_j \triangleq \boldsymbol{s}_{\pi_j}$ for $j > k$, the estimate $\hat{\boldsymbol{s}}_{\pi_k}$ can be expressed as

$$\hat{\boldsymbol{s}}_{\pi_k} = \boldsymbol{A}_{k,k}\boldsymbol{s}_{\pi_k} + \boldsymbol{G}_{\pi_k}\boldsymbol{\eta} + \sum_{j\neq k}\boldsymbol{A}_{k,j}\boldsymbol{u}_j - \sum_{j>k}\boldsymbol{F}_{k,j}\boldsymbol{u}_j, \quad (5)$$

and the MSE $\varepsilon_{\pi_k}^{\mathrm{UL}} = \mathrm{E}[\|\hat{\boldsymbol{s}}_{\pi_k} - \boldsymbol{s}_{\pi_k}\|_2^2]$ can be derived easily:

$$\begin{aligned}\varepsilon_{\pi_k}^{\mathrm{UL}} = \mathrm{tr}\Big[&(\boldsymbol{A}_{k,k}-\mathbf{I})(\boldsymbol{A}_{k,k}-\mathbf{I})^{\mathrm{H}} + \textstyle\sum_{j<k}\boldsymbol{A}_{k,j}\boldsymbol{A}_{k,j}^{\mathrm{H}} \\ &+ \textstyle\sum_{j>k}(\boldsymbol{A}_{k,j}-\boldsymbol{F}_{k,j})(\boldsymbol{A}_{k,j}-\boldsymbol{F}_{k,j})^{\mathrm{H}} + \sigma_n^2\boldsymbol{G}_{\pi_k}\boldsymbol{G}_{\pi_k}^{\mathrm{H}}\Big].\end{aligned} \quad (6)$$

Theorem 1 *Given the dimensions $B_1, \ldots, B_K$ of the user-symbol vectors, the MIMO BC channel and the dual MIMO MAC channel achieve the same user-wise* MSE *region, when using THP for the MIMO BC and DFE for the MIMO MAC under a fixed sum power constraint P_{tr}, if the modulo-loss of THP and the error propagation of the DFE are neglected.*

Proof. To prove the MSE duality between the uplink and downlink, we show that for any set of precoders $\boldsymbol{P}_k$, receivers $\boldsymbol{G}_k$, and feedback matrix $\boldsymbol{F}$ describing the uplink system and achieving certain MSE values, there exists at least one set of linear precoders and receivers for the dual BC channel that achieves the same MSE values under the same sum power and vice versa (see the next two subsections).

3.1. Uplink to Downlink Transformation

Similar to the duality of the linear system in [8], the transformation is based on switching the role of precoders and receivers and scaling them with K strictly positive constants:

$$\bar{\boldsymbol{P}}_k = \alpha_k\boldsymbol{G}_k^{\mathrm{H}}, \;\; \bar{\boldsymbol{G}}_k = \frac{1}{\alpha_k}\boldsymbol{P}_k^{\mathrm{H}}, \;\; \bar{\boldsymbol{F}}_{k,j} = \frac{\alpha_{\pi_j}}{\alpha_{\pi_k}}\boldsymbol{F}_{j,k}^{\mathrm{H}}. \quad (7)$$

Setting $\varepsilon_{\pi_k}^{\mathrm{UL}} = \varepsilon_{\pi_k}^{\mathrm{DL}}$ yields a following linear equation system:

$$\boldsymbol{T}_{\mathrm{N}}\begin{bmatrix}\alpha_{\pi_1}^2 \\ \vdots \\ \alpha_{\pi_K}^2\end{bmatrix} = \sigma_n^2\begin{bmatrix}\mathrm{tr}(\boldsymbol{P}_{\pi_1}\boldsymbol{P}_{\pi_1}^{\mathrm{H}}) \\ \vdots \\ \mathrm{tr}(\boldsymbol{P}_{\pi_K}\boldsymbol{P}_{\pi_K}^{\mathrm{H}})\end{bmatrix}, \quad (8)$$

where

$$\boldsymbol{T}_{\mathrm{N},k,i} = \begin{cases}-\mathrm{tr}\Big[(\boldsymbol{A}_{i,k}-\boldsymbol{F}_{i,k})(\boldsymbol{A}_{i,k}-\boldsymbol{F}_{i,k})^{\mathrm{H}}\Big] & \text{if } i<k, \\ -\mathrm{tr}\Big[\boldsymbol{A}_{i,k}\boldsymbol{A}_{i,k}^{\mathrm{H}}\Big] & \text{if } i>k. \\ \mathrm{tr}\Big[\sigma_n^2\boldsymbol{G}_{\pi_k}\boldsymbol{G}_{\pi_k}^{\mathrm{H}}\Big] - \sum_{j\neq k}\boldsymbol{T}_{\mathrm{N},j,k} & \text{if } i=k,\end{cases} \quad (9)$$

Obviously, $\boldsymbol{T}_{\mathrm{N}}$ is a strictly (column) diagonally dominant real-valued matrix[1], so it is non-singular ($|\boldsymbol{T}_{\mathrm{N}}| > 0$); moreover, it has strictly positive diagonal entries and negative off-diagonal entries, thus all entries of the inverse matrix $\boldsymbol{T}_{\mathrm{N}}^{-1}$ are non-negative and the diagonal entries of $\boldsymbol{T}_{\mathrm{N}}^{-1}$ are strictly positive.[2] Summing up all equations of the system, we get:

$$\sum_{k=1}^{K}\mathrm{tr}(\bar{\boldsymbol{P}}_k\bar{\boldsymbol{P}}_k^{\mathrm{H}}) = \sum_{k=1}^{K}\mathrm{tr}(\alpha_k^2\boldsymbol{G}_k\boldsymbol{G}_k^{\mathrm{H}}) = \sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{P}_k\boldsymbol{P}_k^{\mathrm{H}}). \quad (10)$$

We see that there is always a strictly positive[3] solution vector $[\alpha_1^2, \ldots, \alpha_K^2]^{\mathrm{T}}$, such that the uplink system can be transformed into an equivalent downlink system with the same individual MSEs and by using the same sum power (due to Equation 10).

3.2. Downlink to Uplink Transformation

Conversely, it can be shown by the same reasoning that every downlink system can be transformed into an equivalent uplink system with the same individual MSEs and with the same sum-power P_{tr}. The uplink transceivers are obtained through:

$$\boldsymbol{P}_k = \bar{\alpha}_k\bar{\boldsymbol{G}}_k^{\mathrm{H}}, \;\; \boldsymbol{G}_k = \frac{1}{\bar{\alpha}_k}\bar{\boldsymbol{P}}_k^{\mathrm{H}}, \;\; \boldsymbol{F}_{k,j} = \frac{\bar{\alpha}_{\pi_j}}{\bar{\alpha}_{\pi_k}}\bar{\boldsymbol{F}}_{j,k}^{\mathrm{H}}. \quad (11)$$

The scalars $\bar{\alpha}_1^2, \ldots, \bar{\alpha}_K^2$ satisfy following system of equations:

$$\bar{\boldsymbol{T}}_{\mathrm{N}}\begin{bmatrix}\bar{\alpha}_{\pi_1}^2 \\ \vdots \\ \bar{\alpha}_{\pi_K}^2\end{bmatrix} = \sigma_n^2\begin{bmatrix}\mathrm{tr}\left(\bar{\boldsymbol{P}}_{\pi_1}\bar{\boldsymbol{P}}_{\pi_1}^{\mathrm{H}}\right) \\ \vdots \\ \mathrm{tr}\left(\bar{\boldsymbol{P}}_{\pi_K}\bar{\boldsymbol{P}}_{\pi_K}^{\mathrm{H}}\right)\end{bmatrix}, \quad (12)$$

where the strictly (column) diagonally dominant real valued matrix $\bar{\boldsymbol{T}}_{\mathrm{N}}$ has a similar structure as $\boldsymbol{T}_{\mathrm{N}}$ in (9). Thus, a strictly positive solution to (12) exists and the downlink can be transformed to an uplink with the same sum power.

[1] $\boldsymbol{T}_{\mathrm{N},k,k} > \sum_{j\neq k}|\boldsymbol{T}_{\mathrm{N},j,k}| \quad \forall k.$

[2] Consider the explicit formula of the adjoint matrix for the proof.

[3] We assume that all $\boldsymbol{P}_k \neq \mathbf{0}$, since we have to consider only the active users. For the other users, the duality is evident. Therefore, $\alpha_k > 0 \quad \forall k.$

4. OPTIMUM RECEIVER AND FEEDBACK MATRICES

To design the system, we first derive the optimum receive and feedback matrices, assuming the transmitter $\boldsymbol{P}_k$ and the precoding order π are fixed. Then, we deal with the difficult part, i.e., the derivation of the optimum transmit matrices. We can easily see from (6) that the optimum feedback matrices are

$$\boldsymbol{F}_{k,j} = \boldsymbol{G}_{\pi_k}\boldsymbol{H}_{\pi_j}\boldsymbol{P}_{\pi_j} = \boldsymbol{A}_{k,j}, \tag{13}$$

since they minimize every user's MSE ε_{π_k} separately.
Now, for given precoders $\boldsymbol{P}_k$ and precoding order π we can see in a similar way that the receivers $\boldsymbol{G}_k$ must be the MMSE receivers minimizing each MSE ε_k individually:

$$\boldsymbol{G}_k = \boldsymbol{P}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{T}_k, \tag{14}$$

where

$$\boldsymbol{T}_{\pi_k} = (\textstyle\sum_{j\leq k}\boldsymbol{H}_{\pi_j}\boldsymbol{P}_{\pi_j}\boldsymbol{P}_{\pi_j}^{\mathrm{H}}\boldsymbol{H}_{\pi_j}^{\mathrm{H}} + \sigma_n^2\mathbf{I})^{-1}. \tag{15}$$

The MSE of user k herewith becomes

$$\varepsilon_k = \mathrm{tr}\,[\boldsymbol{E}_k] = \mathrm{tr}(\mathbf{I} - \boldsymbol{P}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{T}_k\boldsymbol{H}_k\boldsymbol{P}_k). \tag{16}$$

where the error covariance matrix of user k is denoted by $\boldsymbol{E}_k$. Note that these optimum receivers and feedback matrices are independent of the cost function, as long as the cost is increasing in every user's MSE. In particular, sum-MSE minimization and QoS design, i.e., minimization of the transmit power under individual MSE constraints, satisfy this condition.

5. WEIGHTED SUM-MSE OPTIMIZATION

Our goal is to design the transceiver minimizing a *weighted* sum of the users' MSEs ε_k with positive scalars w_k:

$$\min_{\{\boldsymbol{P}_1,\ldots,\boldsymbol{P}_K\},\pi} \varepsilon_{\mathrm{tot}} = \sum_k w_k\varepsilon_k \quad \text{s.t.: } \sum_k \mathrm{tr}(\boldsymbol{P}_k\boldsymbol{P}_k^{\mathrm{H}}) \leq P_{\mathrm{tr}}. \tag{17}$$

In this section, we consider a fixed precoding order π. Using (16), the KKT conditions of the weighted sum-MSE minimization (17) read as:

$$\mu\boldsymbol{P}_{\pi_k} \stackrel{!}{=} \boldsymbol{H}_{\pi_k}^{\mathrm{H}}\Big[w_{\pi_k}\boldsymbol{T}_{\pi_k}(\sigma_n^2\mathbf{I} + \sum_{j<k}\boldsymbol{H}_{\pi_j}\boldsymbol{P}_{\pi_j}\boldsymbol{P}_{\pi_j}^{\mathrm{H}}\boldsymbol{H}_{\pi_j}^{\mathrm{H}})\boldsymbol{T}_{\pi_k} - \sum_{j>k} w_{\pi_j}\boldsymbol{T}_{\pi_j}\boldsymbol{H}_{\pi_j}\boldsymbol{P}_{\pi_j}\boldsymbol{P}_{\pi_j}^{\mathrm{H}}\boldsymbol{H}_{\pi_j}^{\mathrm{H}}\boldsymbol{T}_{\pi_j}\Big]\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}, \tag{18}$$

with the Lagrangian multiplier $\mu \geq 0$. If we multiply (18) for each π_k with $\boldsymbol{P}_{\pi_k}^{\mathrm{H}}$ from the left and take its trace, we observe that the weighting w_k satisfies nearly the same linear system of equations as the scalars α_k of the uplink/dowlink transformation except for a constant μ/σ_n^2 (cf. Eq. 8) [4]. We get the following relation:

$$\frac{\alpha_1^2}{w_1} \stackrel{!}{=} \frac{\alpha_2^2}{w_2} \stackrel{!}{=} \cdots \stackrel{!}{=} \frac{\alpha_K^2}{w_K}, \tag{19}$$

which also holds for the linear case, see [8]. Hence, the weights α_k^2 can be computed directly from the transmit power constraint, which is the big advantage of our duality. In particular, for the sum-MSE minimization ($w_k = 1$), all α_k are equal.

[4]Remember that $\boldsymbol{G}_k = \boldsymbol{P}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{T}_k$ (see Eq. 14).

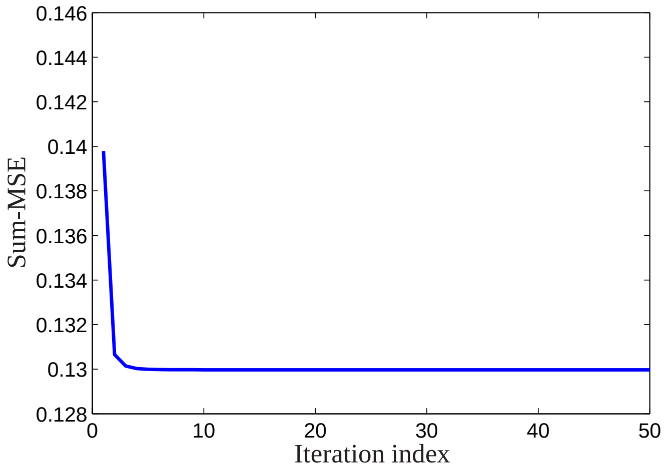

Fig. 3. Convergence of the MIMO weighted sum-MSE minimization algorithm; $M = 6$, $K = 3$, $N_k = 2$, $B_k = 2, w_k = 1\forall k$; SNR = 20dB.

In order to solve the constrained optimization problems, the standard unconstrained gradient algorithm can be modified to take into account the constraints. The modified gradient algorithm is called the *projected* gradient algorithm and its iteration is defined as follows [14]:

$$\boldsymbol{P}^{(\ell+1)} = [\boldsymbol{P}^{(\ell)} + \eta\boldsymbol{M}^{-1}\nabla f(\boldsymbol{P}^{(\ell)})]_{\perp}, \tag{20}$$

where ∇ corresponds to the matrix-valued nabla operator (*Jacobian* matrix) and $[.]_{\perp}$ denotes the projection operator onto the hypersphere with radius $\sqrt{P_{\mathrm{tr}}}$, η is the step size, and $\boldsymbol{M}$ represents a preconditioning matrix, which is chosen to be $\boldsymbol{M}^{-1} = \sqrt{\frac{P_{\mathrm{tr}}}{\|\nabla f\|_{\mathrm{F}}^2}}\mathbf{I}$ for simplicity. In this way, the speed of the algorithm becomes almost independent from the SNR.[5]

Algorithm 1 is the pseudo-code of the gradient projection iterative solution. The iteration is divided into two steps: the first one (lines 4 to 12) is the standard gradient iteration step and the second one (line 13) consists of a projection onto the constraint set. The convergence of this algorithm is proved by means of a descent argument [14]:

Theorem 2 *Suppose f is bounded below and Lipschitzian with the Lipschitz constant L, and $0 < \eta < 2/L$. The sequence generated by the gradient projection algorithm then converges. Furthermore, the limit point of this sequence satisfies the first order KKT optimality condition. In particular, if f is convex then the algorithm converges to the global minimum.*

Proof. See [14]. The parameter η ensures the convergence of the algorithm. Choosing $\eta = 1/d$ features excellent convergence properties (see Fig. 3), when d is initialized with 2 and is incremented, as the objective tends to increase (line 15).

6. SUBOPTIMUM PRECODING ORDER

Problem (17) is very hard to tackle. A simultaneous or alternating optimization over $\boldsymbol{P}_k$ and π is impossible, since the first variable is continuous and the second one is discrete. Hence,

[5]The function is nearly flat for high SNR and the gradient becomes very small. Thus, the Jacobian has a small Frobenius norm, which makes this scaling important.

Algorithm 1 MIMO Weighted Sum-MSE Algorithm

Initialize: $\boldsymbol{P}_k^{(0)}(1:B_k, 1:B_k) \Leftarrow \sqrt{\frac{P_{\text{tr}}}{\sum B_k}}\mathbf{I}_{B_k}, \forall k$
$d \Leftarrow 2, \quad \ell \Leftarrow 0$
repeat
  $\ell \Leftarrow \ell + 1$
  $\boldsymbol{T}_{\pi_0} \Leftarrow \sigma_n^{-2}\mathbf{I}$
  for $k = 1, \ldots, K$ **do**
    $\boldsymbol{T}_{\pi_k} \Leftarrow \boldsymbol{T}_{\pi_{k-1}} - \boldsymbol{T}_{\pi_{k-1}}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}^{(\ell-1)}(\mathbf{I}_{B_{\pi_k}} + \boldsymbol{P}_{\pi_k}^{(\ell-1),\mathrm{H}}$
    $\times \boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}_{\pi_{k-1}}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}^{(\ell-1)})^{-1}\boldsymbol{P}_{\pi_k}^{(\ell-1),\mathrm{H}}\boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}_{\pi_{k-1}}$
  end for
  $\boldsymbol{S} \Leftarrow \mathbf{0}$
  for $k = K, \ldots, 1$ **do**
    $\boldsymbol{S} \Leftarrow \boldsymbol{S} + w_{\pi_k}\boldsymbol{T}_{\pi_k}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}^{(\ell-1)}\boldsymbol{P}_{\pi_k}^{(\ell-1),\mathrm{H}}\boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}_{\pi_k}$
    Gradient Update for $\boldsymbol{P}_k$ ($\forall k$):
    $\delta\boldsymbol{P}_{\pi_k}^{(\ell)} \Leftarrow \boldsymbol{H}_{\pi_k}^{\mathrm{H}}(w_{\pi_k}\boldsymbol{T}_{\pi_k} - \boldsymbol{S})\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}^{(\ell-1)}$
  end for
  $\forall k, \quad \delta\boldsymbol{P}_k^{(\ell)} \Leftarrow \sqrt{\frac{P_{\text{tr}}}{\sum \|\delta\boldsymbol{P}_k^{(\ell)}\|}}\delta\boldsymbol{P}_k^{(\ell)}$ (scaled gradient)
  $\forall k, \quad \boldsymbol{P}_k^{(\ell)} \Leftarrow \frac{1}{d}\delta\boldsymbol{P}_k^{(\ell)} + \boldsymbol{P}_k^{(\ell-1)}$
  $\forall k, \quad \boldsymbol{P}_k^{(\ell)} \Leftarrow \sqrt{\frac{P_{\text{tr}}}{\sum \|\boldsymbol{P}_k^{(\ell)}\|}}\boldsymbol{P}_k^{(\ell)}$ (Projection)
  if $\sum_k w_k \text{tr}(\boldsymbol{E}_k^{(\ell)}) > \sum_k w_k \text{tr}(\boldsymbol{E}_k^{(\ell-1)})$ **then**
    $d \Leftarrow d + 1, \quad \ell \Leftarrow \ell - 1$
  end if
until desired convergence accuracy for $\boldsymbol{E}_k$ is achieved
$\boldsymbol{G}_k^{\mathrm{DL}} \Leftarrow \frac{1}{\alpha_k}\boldsymbol{P}_k^{\mathrm{H}}, \quad \boldsymbol{P}_k^{\mathrm{DL}} \Leftarrow \alpha_k\boldsymbol{T}_k\boldsymbol{H}_k\boldsymbol{P}_k$
where $\alpha_k \Leftarrow \sqrt{\frac{w_k P_{\text{tr}}}{\sum \text{tr}(w_k \boldsymbol{P}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{T}_k\boldsymbol{H}_k\boldsymbol{P}_k)}}$

we propose to optimize the cost function with respect to π for given $\boldsymbol{P}_k$. However, we must choose $\boldsymbol{P}_k$ carefully, such that it does not influence the result. In order to assure a fair comparison between all possible permutations, we evenly allocate powers to the users, such that our solution depends on the system parameters (channels, weighting, and sum-power P_{tr}) as much as possible. Intuitively, we choose $\boldsymbol{P}_k$ as follows:

$$\boldsymbol{P}_k = \sqrt{\frac{P_{\text{tr}}}{\sum_k B_k}}\boldsymbol{V}_k \tag{21}$$

where $\boldsymbol{V}_k$ comprises the B_k dominant right singular vectors of $\boldsymbol{H}_k$. Herewith, we optimize the weighted sum-MSE w.r.t. the ordering π:

$$\min_{\pi}\sum_k w_{\pi_k}\text{tr}(\mathbf{I} - \boldsymbol{P}_{\pi_k}^{\mathrm{H}}\boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}_{\pi_k}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}). \tag{22}$$

Even with this simplification, the problem remains NP hard, as we must check all $K!$ possible permutations. To reduce complexity, we minimize each summand separately like in [10], i.e., π_k is chosen under the assumption that $\pi_{k+1}, \ldots, \pi_K$ are fixed. Since we do not necessarily have the same B_k for all users, every summand is divided by the number of streams:

$$\pi_k = \min_{i \notin \pi_{k+1},\ldots,\pi_K} w_i \text{tr}(\mathbf{I} - \boldsymbol{P}_i^{\mathrm{H}}\boldsymbol{H}_i^{\mathrm{H}}\boldsymbol{T}_{\pi_k}\boldsymbol{H}_i\boldsymbol{P}_i)/B_i. \tag{23}$$

For the MISO case, this suboptimum precoding order is equivalent to MMSE V-BLAST [16]. From simulations we observe that this solution mostly delivers the optimum precoding order. Algorithm 2 shows the detailed precoding order optimization, where the matrices $\boldsymbol{T}_{\pi_k}$ are computed successively (line 5).

Algorithm 2 Precoding Order Algorithm

1: **Initialize:** $\boldsymbol{P}_k(1:B_k, 1:B_k) \Leftarrow \sqrt{\frac{P_{\text{tr}}}{\sum B_k}}\boldsymbol{V}_k, \forall k$, where $\boldsymbol{V}_k = \text{SVD}(\boldsymbol{H}_k)$
2: $\boldsymbol{T} = (\sum_j \boldsymbol{H}_j\boldsymbol{P}_j\boldsymbol{P}_j^{\mathrm{H}}\boldsymbol{H}_j^{\mathrm{H}} + \sigma_n^2\mathbf{I})^{-1}$
3: **for** $k = K, \ldots, 1$ **do**
4:   $\pi_k = \min_{i \notin \pi_{k+1},\ldots,\pi_K} w_i\text{tr}(\mathbf{I} - \boldsymbol{P}_i^{\mathrm{H}}\boldsymbol{H}_i^{\mathrm{H}}\boldsymbol{T}\boldsymbol{H}_i\boldsymbol{P}_i)/B_i$
5:   $\boldsymbol{T} \Leftarrow \boldsymbol{T} + \boldsymbol{T}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k}(\mathbf{I}_{B_{\pi_k}} - \boldsymbol{P}_{\pi_k}^{\mathrm{H}}$
  $\times \boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}\boldsymbol{H}_{\pi_k}\boldsymbol{P}_{\pi_k})^{-1}\boldsymbol{P}_{\pi_k}^{\mathrm{H}}\boldsymbol{H}_{\pi_k}^{\mathrm{H}}\boldsymbol{T}$
6: **end for**

7. SIMULATION RESULTS

In our channel model, the entries of $\boldsymbol{H}_k$ are complex-valued realizations of independent zero-mean Gaussian random variables, each having the same variance $\mathrm{E}[|h_{k,n,m}|^2] = 1$. The uncoded *bit error rate* (BER) results were averaged over 1000 channel realizations, where 100 16QAM modulated symbols were transmitted per realization.

In Fig. 4 we compare the new THP transceiver with the TxWF-THP of [10], in which all users apply the same scalar at the receivers. We choose a MISO system (one antenna per user), where 3 users are served by only two transmit antennas, since we expect a big improvement of the BER performance in this case. In fact, the curve of the TxWF exhibits an error floor. This is due to the fact that we allocate the same scalar receiver for all users, and therefore nearly the same power to each of them. Since the channel is rank deficient, the SINR of the first precoded user cannot grow arbitrarily large. However, the new sum-MSE THP transceiver allows a strongly uneven power allocation, which enables each user to get more power than subsequent users, and thus an (nearly) arbitrary high SINR.

Fig. 5 compares our sum-MSE THP transceiver with existing MIMO THP approaches, in particular the block diagonal THP approach (also known as ZF-THP) [17]. We enhanced the performance of ZF-THP by allowing for a non-unitary precoder. Note that the algorithm in [17] requires that the number of transmit antennas must be greater than or equal to the total number of receive antennas in order to satisfy the null-space criterion. Therefore, we chose a system with 6 transmit antennas and 3 users with 2 receive antennas and 2 streams each. Not surprisingly, sum-MSE THP clearly outperforms ZF-THP. Fig. 5 shows also some linear processing approaches such as linear ZF from [18] and sum-MSE minimization from [8] (see also [7]). Obviously, the higher the SNR, the better the performance of the new THP system compared to linear transceivers.

8. CONCLUSION

We addressed the problem of jointly designing THP transmitters and receivers for a multi-user MIMO system. Thanks to a user-wise MSE duality between THP and DFE, we formulated a weighted sum-MSE minimization problem in the uplink and solved the KKT conditions iteratively using a gradient projection method, which has good convergence properties compared to alternating optimization of transmitter and receivers. We examined the precoding order problem and derived a suboptimum solution. Our THP transceiver, which has a low complexity structure, outperforms all existing solutions and offers excellent performance in rank deficient systems.

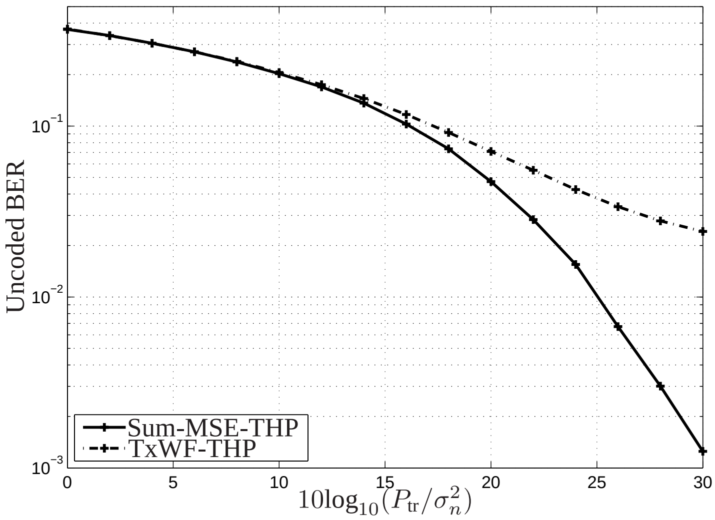

Fig. 4. TxWF-THP and sum-MSE-THP design (MISO); $M=2$, $K=3$.

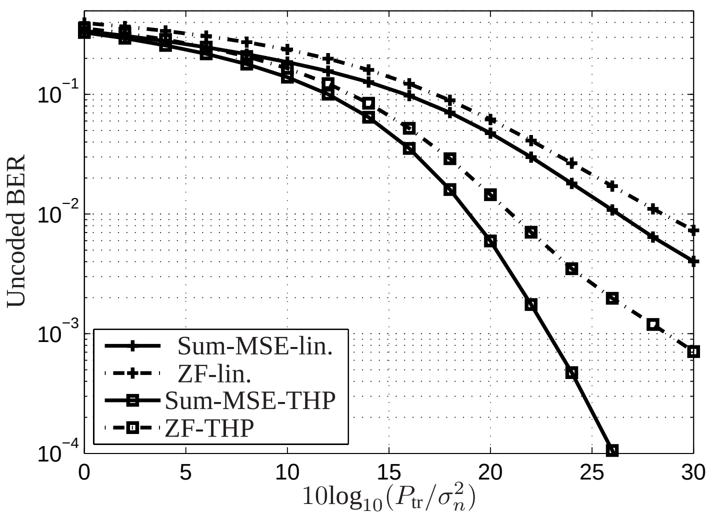

Fig. 5. ZF-THP, sum-MSE THP and linear designs (MIMO); $M=6$, $K=3$, $N_k=2$, $B_k=2, \forall k$;

9. REFERENCES

[1] M. Joham, W. Utschick, and J. A. Nossek, "Linear Transmit Processing in MIMO Communications systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700–2712, August 2005.

[2] M. Schubert and H. Boche, "Solution of the multi-user downlink beamforming problem with individual SINR constraints," *IEEE Trans. on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[3] S. Shi and M. Schubert, "MMSE transmit optimization for multi-user multi-antenna systems," *in ICASSP05, Philadelphia, USA*, March 2005.

[4] R. Hunger, W. Utschick, D. Schmidt, and M. Joham, "Alternating optimization for MMSE broadcast precoding," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Toulouse, France, May 2006.

[5] J. Chang, L. Tassiulas, and F. Rashid-Farrokhi, "Joint transmitter receiver diversity for efficient space division multiaccess," *IEEE Trans. On Wireless Communications*, vol. 1, no. 1, pp. 16–27, January 2002.

[6] M. Schubert, S. Shi, E. Jorswieck, and H. Boche, "Downlink Sum-MSE Transceiver Optimization for Linear Multi-User MIMO Systems," *in Asilomar CSSC, CA USA*, October 2005.

[7] A. Khachan, A. Tenenbaum, and R. S. Adve, "Linear processing for the downlink in multiuser MIMO systems with multiple data streams," *Submitted to IEEE International Conf. on Communications*, 2006.

[8] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver Design for Multi-user MIMO Systems," *Accepted for publication at ITG WSA*, Jan. 2006.

[9] N. Vishwanath, S. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of MIMO broadcast channel," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2658–2668, October 2003.

[10] M. Joham, J. Brehmer, and W. Utschick, "MMSE approaches to multiuser spatio-temporal Tomlinson-Harashima precoding," *in ITG SCC04*, Jan. 2004.

[11] M. Schubert and H. Boche, "Iterative Multiuser Uplink and Downlink Beamforming Under SINR Constraints," *IEEE Trans. on Signal Processing*, vol. 53, no. 7, pp. 2324–2334, July 2005.

[12] R. Doostnejad, T. J. Lim, and E. Sousa, "Precoding for the MIMO broadcast channels with multiple antennas at each receiver," in *in Proc. of The Johns Hopkins University Conf. on Information Sciences and Systems*, Baltimore, MD, Mar. 2005.

[13] M. Schubert and S. Shi, "MMSE Transmit Optimization with Interference Pre-Compensation," in *Proc. IEEE Vehicular Techn. Conf. (VTC) spring*, Stockholm, Sweden, May 2005.

[14] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and distributed computations*, Prentice-Hall, 1989.

[15] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*, John Wiley & Sons, 2002.

[16] K. Kusume, M. Joham, and W. Utschick, "MMSE Block Decision-Feedback Equalizer for Spatial Multiplexing with Reduced Complexity," in *Proc. Globecom 2004*, November 2004, vol. 4, pp. 2540–2544.

[17] V. Stankovic and M. Haardt, "Successive optimization Tomlinson-Harashima precoding (so thp) for multi-user MIMO Systems," in *Proc. ICASSP 2005*, March 2005, vol. 3, pp. 1117–1120.

[18] A. Bourdoux and N. Khaled, "Joint TX-RX optimisation for MIMO-SDMA based on a null-space constraint," *in Proc. of IEEE Vehicular Technology Conference (VTC-02 Fall), Vancouver, Canada*, pp. 171–174, Sept. 2002.