PHYSICAL LAYER CHARACTERIZATION OF MIMO SYSTEMS BY MEANS OF MULTIOBJECTIVE OPTIMIZATION

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ABSTRACT

A method is presented for efficiently describing the capabilities of the physical layer in a MIMO communication system. Such a description is required in systems that determine the optimum operation point based on information exchange between layers. We characterize the physical layer by means of efficient MSE tuples. Such efficient tuples are found by sampling the boundary of the MSE region. A particularly simple sampling algorithm is derived. We show how the distance between subsequent samples can be directly controlled, allowing us to provide a characterization of the physical layer that is both compact and representative.

1. INTRODUCTION

A key challenge in the design of future wireless communication systems is the efficient provision of a multitude of different applications such as voice, data and real-time video. Different applications have different requirements. Optimum performance can only be achieved if the choice of the operation point in each layer takes into account the properties of the different applications. This observation leads to the concept of cross-layer optimization. One possible approach to cross-layer optimization is to use a global decision function that jointly optimizes the parameters of all layers for determining the overall optimum operation point. Using a global decision function requires full transparency between layers. In this work, we follow a different approach. We focus on signal processing at the physical layer of a wireless *multiple-input multiple-output* (MIMO) communication system. We investigate to what extent physical layer processing can contribute to finding the overall optimum operation point in a cross-layer optimization context.

The physical layer processes the symbol streams created by upper layers. If knowledge about the properties of the upper layers is not available, a common approach is to choose a physical layer operation point based on knowledge available at the physical layer only. An example is the joint design of linear transmit and receive filters based on a *sum of mean squared error* (SMSE) criterion [1, 2]. In [1, 2], the sum of the MSEs of the different data streams is minimized, which does not take into account the properties of the upper layers. To overcome this limitation, layer interaction is necessary. One possibility is to have the upper layers formulate QoS targets, which are then incorporated into the filter design (e.g., [3]). However, this approach may not provide the optimum solution if the upper layers offer some flexibility with respect to the QoS targets.

Instead of a top-down interaction in terms of QoS targets, we propose a bottom-up interaction in terms of a set of operation points that the physical layer can provide (for a detailed treatment of bottom-up optimization based on layer descriptions, see [4]). Using the MSEs of the data streams as performance measure, a description of all possible filter settings is given by the achievable MSE region. However, we seek a characterization that is as compact as possible while not excluding the optimum solution. Thus, we are only interested in those points on the boundary of the MSE region where the MSE of one data stream can only be decreased by increasing the MSE of at least one other data stream. The set of these points is denoted as efficient set; *multiobjective optimization* (MOO) is the mathematical framework for finding such efficient sets. We propose to characterize the physical layer by a finite subset of the efficient set. There exist a number of methods to compute efficient points (cf. [5]). We show that for a MIMO system with jointly optimized linear precoders and decoders, the computation of efficient points is particularly simple. We exploit this simplicity to compute a finite number of efficient points that represents a good discretization of the efficient set.

Notation: $\boldsymbol{I}_N$ is the $N \times N$ identity matrix, $\boldsymbol{e}_k$ is the kth column of $\boldsymbol{I}_N$, $\mathbf{1}_B$ is a vector of B ones; $\mathrm{E}[\cdot]$, $\|\cdot\|_{\mathrm{F}}$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote expectation, Frobenius norm, transposition, and conjugate transposition, respectively.

2. SYSTEM MODEL

We consider the wireless MIMO system depicted in Fig. 1, consisting of a linear transmit filter $\boldsymbol{P} \in \mathbb{C}^{N \times B}$, a fre-

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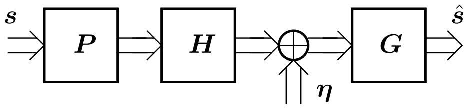

Fig. 1. Linear MIMO System

quency flat-fading channel $\boldsymbol{H} \in \mathbb{C}^{M\times N}$ and a linear receive filter $\boldsymbol{G} \in \mathbb{C}^{B\times M}$, where B, M and N denote the number of data streams, receive antennas and transmit antennas, respectively. The number of streams is chosen such that $B \leq \operatorname{rank}(\boldsymbol{H})$.

The transmitted data symbol vectors $\boldsymbol{s}$ are assumed to be uncorrelated and to have unit power, i.e. $\mathrm{E}\left[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\right] = \boldsymbol{I}_B$. The estimated data symbol vector $\hat{\boldsymbol{s}}$ is given by

$$\hat{\boldsymbol{s}} = \boldsymbol{G}\boldsymbol{H}\boldsymbol{P}\boldsymbol{s} + \boldsymbol{G}\boldsymbol{\eta},$$

where $\boldsymbol{\eta}$ is complex AWGN with $\boldsymbol{R}_{\boldsymbol{\eta}} = \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}\right]$. The MSE of the k-th data stream is given by

$$\sigma^2_{\varepsilon_k} = \mathrm{E}\left[|s_k - \hat{s}_k|^2\right]. \qquad (1)$$

3. MULTIOBJECTIVE OPTIMIZATION

In this section, we summarize the basic concepts of multiobjective optimization (e.g. [6]). The minimization of a vector-valued cost function $\boldsymbol{f} : \mathbb{C}^N \to \mathbb{R}^B$ (with $B > 1$) is considered. In order to make a statement about the optimality of a point $\boldsymbol{f}(\boldsymbol{x})$, an order relation on $\mathbb{R}^B$ is required [6]. We use the following definition of an order "$\preceq$":

$$\boldsymbol{y}^1 \preceq \boldsymbol{y}^2 \quad \Longleftrightarrow \quad y^1_k \leq y^2_k \quad \forall k = 1,\ldots,K.$$

Note that "$\preceq$" is a partial order, i.e. there exist elements $\boldsymbol{y}^1, \boldsymbol{y}^2$ for which neither $\boldsymbol{y}^1 \preceq \boldsymbol{y}^2$ nor $\boldsymbol{y}^2 \preceq \boldsymbol{y}^1$ is true.

Let $\mathcal{G}$ and $\boldsymbol{f}(\mathcal{G})$ denote the feasible set and the image of the feasible set under $\boldsymbol{f}$, respectively. Now assume that there exist points $\boldsymbol{y}^1, \boldsymbol{y}^2 \in \boldsymbol{f}(\mathcal{G})$ that are smaller (with respect to $\preceq$) than any $\boldsymbol{y} \in \boldsymbol{f}(\mathcal{G})$ they can be compared to, but for which neither $\boldsymbol{y}^1 \preceq \boldsymbol{y}^2$ nor $\boldsymbol{y}^2 \preceq \boldsymbol{y}^1$ is true. There is no reason for differentiating between $\boldsymbol{y}^1$ and $\boldsymbol{y}^2$, and thus both are considered valid solutions of the minimization. Based on this generalized concept of optimality, two sets are defined: The efficient set

$$\mathcal{E} = \{\boldsymbol{y} \in \boldsymbol{f}(\mathcal{G}) \mid \nexists \boldsymbol{y}' \in \boldsymbol{f}(\mathcal{G}) \setminus \boldsymbol{y} : \boldsymbol{y}' \preceq \boldsymbol{y}\},$$

which contains all the points that are smaller than all points they can be compared to, and the corresponding Pareto set

$$\mathcal{P} = \{\boldsymbol{x} \in \mathcal{G} \mid \boldsymbol{f}(\boldsymbol{x}) \in \mathcal{E}\}.$$

Points $\boldsymbol{x} \in \mathcal{P}$ are called Pareto optimal. Based on these two sets, we can define the operators $\min$ and argmin for multiobjective optimization as follows:

$$\min_{\boldsymbol{x}\in\mathcal{G}} \boldsymbol{f}(\boldsymbol{x}) \doteq \mathcal{E}, \quad \operatorname*{argmin}_{\boldsymbol{x}\in\mathcal{G}} \boldsymbol{f}(\boldsymbol{x}) \doteq \mathcal{P}.$$

4. MSE-OPTIMUM FILTER DESIGN

We aim at characterizing the physical layer by a set $\mathcal{M} \subset \mathcal{E}$ of efficient MSE tuples $\boldsymbol{m} = \left[\sigma^2_{\varepsilon_1},\ldots,\sigma^2_{\varepsilon_K}\right]^{\mathrm{T}}$. In order to find this set, we first determine the set of Pareto optimal filters $\boldsymbol{P}$ and $\boldsymbol{G}$.

$$\tilde{\mathcal{P}}_{\mathrm{I}} = \operatorname*{argmin}_{(\boldsymbol{P},\boldsymbol{G})} \boldsymbol{m}(\boldsymbol{P},\boldsymbol{G}) \quad \text{s.t.} \quad \|\boldsymbol{P}\|^2_{\mathrm{F}} \leq E_{\mathrm{tr}}. \qquad (2)$$

In the following, in order to emphasize our focus on the MSE of each individual data stream, we denote problem (2) as the *individual MSE* (IMSE) problem

Consider the eigenvalue decomposition of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}_{\boldsymbol{\eta}}\boldsymbol{H}$:

$$\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}_{\boldsymbol{\eta}}\boldsymbol{H} = \begin{pmatrix}\boldsymbol{V}_B & \tilde{\boldsymbol{V}}\end{pmatrix}\begin{pmatrix}\boldsymbol{\Lambda}_B & \boldsymbol{0}\\ \boldsymbol{0} & \tilde{\boldsymbol{\Lambda}}\end{pmatrix}\begin{pmatrix}\boldsymbol{V}_B & \tilde{\boldsymbol{V}}\end{pmatrix}^{\mathrm{H}},$$

where the diagonal matrix $\boldsymbol{\Lambda}_B$ contains the B largest eigenvalues of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}_{\boldsymbol{\eta}}\boldsymbol{H}$ arranged in decreasing order.

For the SMSE criterion, it is well-known that the optimum filters perform a decomposition of the channel matrix into so-called eigenmodes, i.e. the original channel $\boldsymbol{H}$ is converted into B parallel scalar subchannels whose gains are given by the B largest eigenvalues. The question now is whether eigenmode transmission is also optimum for the IMSE problem. We first restate a result from [7]:

Theorem 1. *Let* $(\boldsymbol{P}_I, \boldsymbol{G}_I)$ *be a Pareto optimal point of* (2). *Then there exists a vector* $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_B]^{\mathrm{T}}$ *with*

$$\alpha_k \geq 0 \quad \textit{and} \quad \boldsymbol{1}^{\mathrm{T}}_B\boldsymbol{\alpha} = 1 \qquad (3)$$

such that $(\boldsymbol{P}_I, \boldsymbol{G}_I)$ *is a Karush-Kuhn-Tucker point of the* scalar *optimization problem*

$$\min_{(\boldsymbol{P},\boldsymbol{G})} \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{m} \quad \textit{s.t.} \quad \|\boldsymbol{P}\|^2_F \leq E_{tr}. \qquad (4)$$

The objective function in (4) is a weighted sum of MSEs (WMSE). Joint linear filter design under the WMSE criterion is investigated in [3]. Theorem 1 allows us to apply the results derived in [3] to the IMSE problem. In particular, in [3], it is shown that the optimum strategy under the WMSE criterion is again eigenmode transmission.

Theorem 2. *For each Pareto optimal pair* $(\boldsymbol{P}_I, \boldsymbol{G}_I)$ *of* (2) *it holds that* $\boldsymbol{P}_I$ *and* $\boldsymbol{G}_I$ *can be written as*

$$\boldsymbol{P}_I = \boldsymbol{V}_B\boldsymbol{\Pi}\boldsymbol{\Phi} \quad \textit{and} \qquad (5)$$
$$\boldsymbol{G}_I = \boldsymbol{\Psi}\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{V}^{\mathrm{H}}_B\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}_{\boldsymbol{\eta}}, \qquad (6)$$

with diagonal $B \times B$ *matrices* $\boldsymbol{\Phi}$ *and* $\boldsymbol{\Psi}$ *and a permutation matrix* $\boldsymbol{\Pi}$.

Proof. Theorem 1, Lemmas 1 and 2 from [3]. □

Remark 1. The permutation $\boldsymbol{\Pi}$ determines which stream is transmitted over which eigenmode. In contrast to the SMSE problem, a parameter $\boldsymbol{\Pi}$ is required, as a fixed allocation of eigenmodes to streams cannot be assumed.

Remark 2. A popular method for solving MOO problems is the so-called weighting method (e.g., [6, 5]). The weighting method scalarizes a MOO problem by minimizing a weighted sum of the objectives, which, in our case, corresponds to the WMSE problem. As a result, we could also use the WMSE method to find Pareto optimal filters. However, in the following we argue that this approach is inferior.

Plugging Eqs. (5) and (6) into Eq. (1) and defining

$$d_k = \boldsymbol{e}_k^{\mathrm{T}} \boldsymbol{\Pi}^{\mathrm{T}} \boldsymbol{\Lambda}_B \boldsymbol{\Pi} \boldsymbol{e}_k, \tag{7}$$

the MSE of the k-th data stream is given by

$$\sigma_{\varepsilon_k}^2 = 1 - 2\,\mathrm{Re}\left\{\psi_k d_k \phi_k\right\} + |\psi_k|^2 d_k^2 |\phi_k|^2 + |\psi_k|^2 d_k, \tag{8}$$

where ϕ_k and ψ_k denote the k-th diagonal element of $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, respectively. With respect to $\boldsymbol{\Psi}$, we can minimize each MSE separately by choosing

$$\boldsymbol{\Psi} = \left(\boldsymbol{\Phi}\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{\Lambda}_B\boldsymbol{\Pi}\boldsymbol{\Phi}^{\mathrm{H}} + \boldsymbol{I}_B\right)^{-1}\boldsymbol{\Phi}^{\mathrm{H}}. \tag{9}$$

Plugging Eq. (9) into Eq. (8) yields

$$\sigma_{\varepsilon_k}^2 = 1 - (d_k|\phi_k|^2 + 1)^{-1} d_k |\phi_k|^2 = (d_k|\phi_k|^2 + 1)^{-1}. \tag{10}$$

Obviously, the MSE of the k-th user depends only on the absolute value of ϕ_k. Let the power allocation to the eigenmodes be denoted by $\boldsymbol{p} = [p_1, \ldots, p_B]^{\mathrm{T}}$, where $p_k = |\phi_k|^2$. The set of feasible power allocations is given by

$$\mathcal{T}_{\mathrm{p}} = \{\boldsymbol{p} \in \mathbb{R}_{0,+}^B : \mathbf{1}_B^{\mathrm{T}}\boldsymbol{p} \le E_{\mathrm{tr}}\}. \tag{11}$$

Moreover, let $\mathcal{T}_{\Pi}$ denote the set of $B \times B$ permutation matrices. Finally, let $\mathcal{T}$ denote the product set of $\mathcal{T}_{\mathrm{p}}$ and $\mathcal{T}_{\Pi}$. Now, the IMSE problem (2) can be rewritten as

$$\mathcal{P}_{\mathrm{I}} = \underset{(\boldsymbol{p},\boldsymbol{\Pi})}{\mathrm{argmin}}\, \boldsymbol{f}(\boldsymbol{p},\boldsymbol{\Pi}) \quad \text{s.t.} \quad (\boldsymbol{p},\boldsymbol{\Pi}) \in \mathcal{T},$$

with $f_k(\boldsymbol{p},\boldsymbol{\Pi}) = (d_k(\boldsymbol{\Pi})p_k + 1)^{-1}$. Next, define

$$\tilde{\boldsymbol{f}}(\boldsymbol{p},\boldsymbol{\Pi}) = \boldsymbol{\Pi}\boldsymbol{f}(\boldsymbol{p},\boldsymbol{\Pi}). \tag{12}$$

The k-th entry of $\tilde{\boldsymbol{f}}$ corresponds to the MSE of the data stream that is transmitted over the k-th eigenmode. Recall that the eigenmodes are ordered in decreasing order, thus $\tilde{f}_1$ corresponds to the MSE of the data stream that is transmitted over the strongest eigenmode.

Theorem 3. *If and only if for a pair* $(\boldsymbol{p},\boldsymbol{\Pi}) \in \mathcal{T}$

$$\mathbf{1}_B^{\mathrm{T}}\boldsymbol{p} = E_{tr} \qquad \textit{and} \tag{13}$$

$$\tilde{f}_1(\boldsymbol{p},\boldsymbol{\Pi}) \le \ldots \le \tilde{f}_B(\boldsymbol{p},\boldsymbol{\Pi}), \tag{14}$$

then $(\boldsymbol{p},\boldsymbol{\Pi})$ *is Pareto optimal.*

Proof. If $\mathbf{1}_B^{\mathrm{T}}\boldsymbol{p} < E_{\mathrm{tr}}$, we can add $\Delta p > 0$ to p_k, this will decrease f_k without affecting $f_j, j \neq k$, thus only $\boldsymbol{p}$ that satisfy $\mathbf{1}_B^{\mathrm{T}}\boldsymbol{p} = E_{\mathrm{tr}}$ can be Pareto optimal.

It suffices to consider $\boldsymbol{\Pi} = \boldsymbol{I}_B$ and a $\boldsymbol{\Pi}'$ that only exchanges two rows m and n, with $m \le n$. Given a $\boldsymbol{p}$ for which (13) and (14) hold, we show that it is not possible to construct a $\boldsymbol{p}'$ such that

$$\boldsymbol{f}(\boldsymbol{p}',\boldsymbol{\Pi}') \preceq \boldsymbol{f}(\boldsymbol{p},\boldsymbol{I}_B) \wedge \exists i : f_i(\boldsymbol{p}',\boldsymbol{\Pi}') < f_i(\boldsymbol{p},\boldsymbol{\Pi}).$$

First choose $p_i' = p_i, \forall i \neq m, n$ and $p_m' = \frac{\lambda_m}{\lambda_n} p_m$. This sets $f_i(\boldsymbol{p}',\boldsymbol{\Pi}') = f_i(\boldsymbol{p},\boldsymbol{\Pi}), \forall i \neq n$. By (14), $p_m \ge \frac{\lambda_n}{\lambda_m} p_n$. Moreover, $p_n' = p_n + p_m - p_m'$. Using these results, we can show that $f_n(\boldsymbol{p},\boldsymbol{I}_B) \le f_n(\boldsymbol{p}',\boldsymbol{\Pi}')$. Thus, we cannot construct a smaller $\boldsymbol{f}(\boldsymbol{p}',\boldsymbol{\Pi}')$.

Now, let $m < n$ and (14) does not hold for $(\boldsymbol{p}',\boldsymbol{\Pi}')$. We get $f_m(\boldsymbol{p},\boldsymbol{I}_B) < f_n(\boldsymbol{p},\boldsymbol{I}_B)$, thus $f_n(\boldsymbol{p},\boldsymbol{I}_B) < f_n(\boldsymbol{p}',\boldsymbol{\Pi}')$ and $\boldsymbol{f}(\boldsymbol{p},\boldsymbol{I}_B) \preceq \boldsymbol{f}(\boldsymbol{p}',\boldsymbol{\Pi}')$. Consequently, $(\boldsymbol{p}',\boldsymbol{\Pi}')$ cannot be Pareto optimal if (14) does not hold for $(\boldsymbol{p}',\boldsymbol{\Pi}')$. □

Remark 3. The result of Theorem 3 is rather intuitive: For optimality, the strongest eigenmode is allocated to the data stream with the smallest MSE, while the weakest eigenmode is allocated to the stream with the largest MSE.

Corollary 1. *Consider the permutation matrix* $\boldsymbol{\Pi} = \boldsymbol{I}_B$. *Let* $\boldsymbol{p}_0 \in \mathcal{T}_p$ *be chosen such that Eqs.* (13) *and* (14) *hold. Then all power allocations* $\boldsymbol{p}$ *given by* $\boldsymbol{p} = \boldsymbol{p}_0 + \Delta\boldsymbol{p}$, *with*

$$\Delta p_1 \ge \Delta p_2 \ldots \ge \Delta p_B, \tag{15}$$

$$\Delta p_k \ge -p_{0,k}, \quad \textit{and} \tag{16}$$

$$\mathbf{1}_B^{\mathrm{T}}\Delta\boldsymbol{p} = 0, \tag{17}$$

are Pareto optimal.

Proof. Eq. (14) holds for $\Delta\boldsymbol{p} = \mathbf{0}_B$. Eq. (15) ensures that Eq. (14) holds for $\Delta\boldsymbol{p} \neq \mathbf{0}_B$. Eqs. (16) and (17) ensure that $\boldsymbol{p} \in \mathcal{T}_{\mathrm{p}}$ and that Eq. (13) holds. □

Remark 4. If we choose $p_{0,k} = \lambda_k^{-1}\left(\sum_{i=1}^B \lambda_i^{-1}\right)^{-1} E_{\mathrm{tr}}$ in Corollary 1, equality holds in Eq. (14). Thus, with this choice for $\boldsymbol{p}_0$, we can generate all Pareto optimal power allocations $\boldsymbol{p}$ corresponding to $\boldsymbol{\Pi} = \boldsymbol{I}_B$ by varying $\Delta\boldsymbol{p}$.

Corollary 2. *Given a Pareto optimal power allocation* $\boldsymbol{p}$ *obtained with* $\boldsymbol{\Pi} = \boldsymbol{I}_B$, *a Pareto optimal power allocation* $\boldsymbol{p}'$ *corresponding to a permutation* $\boldsymbol{\Pi}'$ *can be obtained by simply permuting* $\boldsymbol{p}$, *i.e.* $\boldsymbol{p}' = \boldsymbol{\Pi}'\boldsymbol{p}$.

Remark 5. Corollaries 1 and 2 provide a very simple way for computing elements of the Pareto set $\mathcal{P}_{\mathrm{I}}$. In contrast, if the WMSE method is used for generating Pareto optimal points (cf. Remark 2), a WMSE problem has to be solved for each generated point. In particular, for each set of weights, the corresponding power allocation has to be computed iteratively [3]. Compared to Corollary 1, a much higher effort.

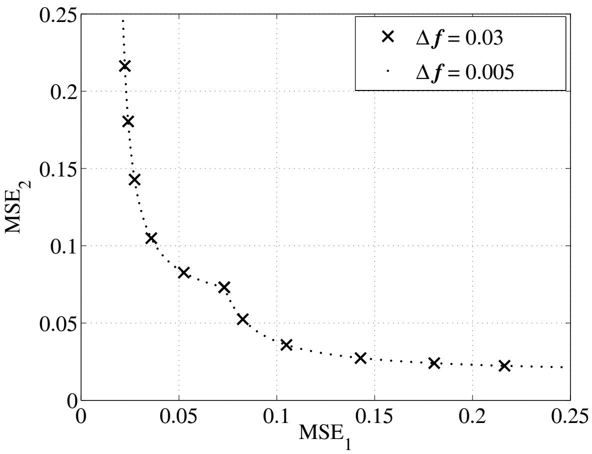

Fig. 2. Sampled Efficient Set

5. PHYSICAL LAYER CHARACTERIZATION

The previous section provided us with a simple way for generating elements of the Pareto set $\mathcal{P}_\mathrm{I}$. As mentioned before, we seek a representation of the physical layer in terms of efficient MSE tuples. Given the Pareto set $\mathcal{P}_\mathrm{I}$, the efficient set is given by $\mathcal{E}_\mathrm{I} = \boldsymbol{f}(\mathcal{P}_\mathrm{I})$. Both $\mathcal{P}_\mathrm{I}$ and $\mathcal{E}_\mathrm{I}$ contain an infinite number of elements. We use a finite subset $\mathcal{M} \subset \mathcal{E}_\mathrm{I}$ to characterize the physical layer. The characterization $\mathcal{M}$ is generated by first choosing a set $\mathcal{P}' \subset \mathcal{P}_\mathrm{I}$ of Pareto optimal $(\boldsymbol{p}, \boldsymbol{\Pi})$ and then computing $\mathcal{M} = \boldsymbol{f}(\mathcal{P}')$. Clearly, it is desirable to choose the elements in $\mathcal{P}'$ in such a way that $\mathcal{M}$ well characterizes $\mathcal{E}_\mathrm{I}$.

In order to get a good discretization of $\mathcal{E}_\mathrm{I}$, we propose a simple rule for $\Delta \boldsymbol{p}$ that achieves a nearly constant distance between subsequent samples in the objective space. According to Eq. (17), $\Delta \boldsymbol{p} \in \mathrm{null}(\mathbf{1}_B^\mathrm{T})$. Let $\boldsymbol{Q} \in \mathbb{R}^{B \times B-1}$ be an orthonormal basis of $\mathrm{null}(\mathbf{1}_B^\mathrm{T})$. For each of the $B-1$ columns $\boldsymbol{q}_i$ of $\boldsymbol{Q}$, a candidate power increment is given by $\Delta \boldsymbol{p}_i = \boldsymbol{q}_i \delta_i$. If the absolute value of δ_i is small enough, the distance between $\boldsymbol{f}(\boldsymbol{p})$ and $\boldsymbol{f}(\boldsymbol{p}+\Delta \boldsymbol{p}_i)$ can be well approximated by

$$|\Delta \boldsymbol{f}| = \|\boldsymbol{J}_{\boldsymbol{f}}(\boldsymbol{p}, \boldsymbol{I}_B)\boldsymbol{q}_i \delta_i\|_2 \,,$$

where $\boldsymbol{J}_{\boldsymbol{f}}$ denotes the (diagonal) Jacobian matrix of $\boldsymbol{f}$. Accordingly, a distance $|\Delta \boldsymbol{f}|$ between subsequent points in the objective space can be obtained by choosing

$$|\delta_i| = \|\boldsymbol{J}_{\boldsymbol{f}}(\boldsymbol{p}, \boldsymbol{I}_B)\boldsymbol{q}_i\|_2^{-1} \, |\Delta \boldsymbol{f}|. \qquad (18)$$

Fig. 2 shows subsets of the efficient set $\mathcal{E}_\mathrm{I}$ for a system with $B = 2$ data streams and a random channel realization. The first subset (dots in Fig. 2) was created with $\Delta \boldsymbol{f} = 0.005$, the second subset (crosses) used a larger distance $\Delta \boldsymbol{f} = 0.03$. It can be observed that the proposed algorithm can provide a nearly equidistant sampling of the efficient set for the case $B = 2$. Notably, the MSE region is non-convex, due to the fact that we can switch eigenmodes between streams.

Remark 6. Eq. (18) shows the significance of Theorem 3: Building on the theoretical results from Section 4, the properties of the generated efficient points can be *directly controlled*. This stands in clear contrast to the WMSE method, which does not provide direct control in the objective space. As a result, it is in general not possible to determine a set of weights $\boldsymbol{\alpha}$ that leads to an equidistant distribution of samples [5]. Another drawback of the weighting method is that it can only generate all efficient points if the efficient set lies on the boundary of a convex set – but our example just demonstrated that the MSE region is non-convex.

6. CONCLUSIONS

We introduced the concept of characterizing the physical layer in a MIMO system by a set of efficient points. Based on the theory of multiobjective optimization, an algorithm for sampling the efficient set was developed. The algorithm has low complexity and provides direct control in the objective space. The approach presented here represents one building block in a system that is optimized based on layer descriptions (cf. [4]).

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