On Strategies of Multiuser MIMO Transmit Signal Processing

Ruly Lai-U Choi, Michel T. Ivrlač, Ross D. Murch, and Wolfgang Utschick

Abstract—**In this letter, we introduce five different strategies of linear transmit signal processing for multiuser multiple-input multiple-output (MIMO) systems and provide performance comparisons in terms of maximum throughput in both uncorrelated and correlated channels when the number of transmit antennas is much larger than the number of receive antennas. It is shown that the multiuser MIMO schemes are preferable to time-division multiple-access (TDMA)-based MIMO schemes, hence demonstrating the power of multiuser MIMO signal processing. Our work also indicates possibilities for future research in finding efficient suboptimal algorithms. As an example, we show that our multiuser MIMO decomposition scheme can improve the maximum throughput compared to TDMA-based MIMO schemes for large number of transmit antennas or high transmit power.**

Index Terms—**Multiple-input multiple-output (MIMO) systems, multiuser decomposition, multiuser MIMO, transmit single processing.**

I. Introduction

In RECENT years, wireless multiple-input multiple-output (MIMO) systems with multiple antennas employed at both the transmitter and receiver have gained attention because of their promising improvement in terms of capacity [1], [2]. However, there has been only limited work on multiuser MIMO (MU-MIMO) systems in downlink communications. The downlink capacity of such a MU-MIMO system is still an open question [3], even though recently certain progress has been made in this area (e.g., [5] and [6], which elaborate on the concept of "dirty paper writing" introduced in [4]). Also, it is still not clear that whether applying MU-MIMO processing is better than time-division multiple-access (TDMA)-based MIMO schemes.

In this letter, we attempt to show that applying MU-MIMO processing is better than TDMA based schemes and also demonstrate some useful MU-MIMO methods when the number of transmit antennas are much larger than the number of receive antennas. Based on the configuration proposed in [7] and [8], in which linear transmit preprocessing is employed at the transmitter, we introduce different strategies of transmit signal processing for MU-MIMO systems. Five strategies are discussed, including: 1) max-min mutual information MU-MIMO with single-user coding, 2) max-min mutual information MU-MIMO with multiuser coding; 3) MU-MIMO decomposition with power allocation; 4) MU-MIMO decomposition with equal power allocation; and 5) TDMA-based MIMO schemes (TDMA-MIMO). Performance in terms of capacity and maximum throughput is investigated and compared among the different strategies for uncorrelated channels and correlated channels in the situation when the number of transmit antennas is much larger than the number of receive antennas at each user. It turns out that the max-min mutual information MU-MIMO schemes (both with single user coding and multiuser coding) are preferable to TDMA-MIMO scheme in all cases, hence demonstrating the power of MU-MIMO signal processing. This also indicates possibilities for future research in finding efficient suboptimal algorithms. As an example, we show that the MU-MIMO decomposition scheme can improve the maximum throughput compared to the TDMA-MIMO scheme for large number of transmit antennas or high transmit power.

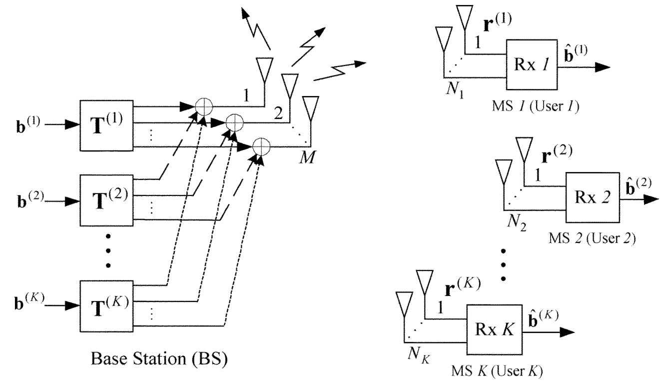

Fig. 1. System configuration of a multiuser MIMO system.

The structure of this letter is as follows. In Section II, the system model is introduced. Then, Section III describes the strategies, while Section IV provides sample numerical results. Finally, Section V concludes this work.

II. System Model

The configuration of the multiuser MIMO system is shown in Fig. 1, where M antennas are located at the base station (BS) and N_k antennas are located at the kth mobile station (MS). We consider a system with K MS's or users and $M > \sum_{i=1}^{K} N_i - \min\{N_k, k = 1, \ldots, K\}$. Note that K is the number of users that we would like to serve simultaneously. It may or may not be equal to the total number of users in a practical system, which may be much larger than K and, therefore, other multiple access techniques are used as well. At the BS, the signals are processed before transmission. Let $\mathbf{b}^{(k)}$ represent the $L_k \times 1$ transmit data symbol vector for user k, where L_k is the number of parallel data symbols transmitted simultaneously for user k ($k = 1, \ldots, K$). This data symbol vector is passed through the transmit preprocessing, which is characterized by

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the preprocessing matrix $\mathbf{T}^{(k)}$, a $M \times L_k$ matrix that takes in L_k nonzero values and outputs M terms. Each of the M output terms is transmitted by a transmit antenna.

We assume that the channel is flat fading and denote the MIMO channel to user k as $\mathbf{H}^{(k)}$, which is a $N_k \times M$ matrix, whose (i,j)th element is the complex gain from the jth transmit antenna to the ith receive antenna for user k. Also, we assume instantaneous channel state information at the transmitter. At the receiver of user k, N_k receive antennas are used to receive the L_k data symbols and the received signals can be written by a vector of length N_k, which is given by

$$\mathbf{r}^{(k)} = \mathbf{H}^{(k)} \sum_{i=1}^{K} \mathbf{T}^{(i)} \mathbf{b}^{(i)} + \mathbf{n}^{(k)} \tag{1}$$

where the noise $\mathbf{n}^{(k)}$ is an $N_k \times 1$ vector, whose elements are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and variance σ_{n}^2.

III. MULTIUSERMIMO STRATEGIES

Based on the system configuration described in Section II, five strategies are introduced and discussed in this section. All strategies assume complete knowledge of the channel at the transmitter.

A. *Max-Min Mutual Information MU-MIMO With Single-User Coding Scheme*

In this max-min mutual information MU-MIMO with single-user coding scheme, our primary objective is to select the nonzero K preprocessing matrices, $(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)})$, for the K users such that the minimum mutual information among the K users is maximized. It is assumed that the users' signals are encoded and decoded independently and, therefore, single user coding is applied. This assumption of single-user coding provides a benchmark for the performance of other MU-MIMO transmit signal processing schemes, which aim at decoupling the multiuser MIMO channel into several single-user MIMO channels. In this scheme, the problem statement can be expressed as

$$\begin{aligned} &\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)}\right) \\ &\quad = \underset{\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)}\right)}{\arg} \max\min \left\{ I^{(k)}, k = 1, \cdots, K \right\} \\ &\quad \text{s.t.} \sum_{k=1}^{K} \operatorname{trace}\left(\mathbf{T}^{(k)} \mathbf{T}^{(k)^{\mathrm{H}}}\right) = P_{\mathrm{T}} \end{aligned} \tag{2}$$

where the superscript $^{\mathrm{H}}$ represents the complex conjugate transpose operator. The constraint states that the average total transmit power of all the users is limited and is equal to P_{T}. $I^{(k)}$ is the mutual information of user k (mutual information between $\mathbf{b}^{(k)}$ and $\mathbf{r}^{(k)}$), which is given by

$$\begin{aligned} I^{(k)} = \log_2 \det \Bigg(\mathbf{I} + \mathbf{T}^{(k)^{\mathrm{H}}} \mathbf{H}^{(k)^{\mathrm{H}}} \Bigg[\sigma_{\mathrm{n}}^2 \mathbf{I} + \mathbf{H}^{(k)} \\ \times \sum_{i=1, i \neq k}^{K} \mathbf{T}^{(i)} \mathbf{T}^{(i)^{\mathrm{H}}} \mathbf{H}^{(k)^{\mathrm{H}}} \Bigg]^{-1} \mathbf{H}^{(k)} \mathbf{T}^{(k)} \Bigg). \end{aligned} \tag{3}$$

Note that (3) is derived under the assumption that the elements of $\mathbf{b}^{(k)}$ $(k = 1, \ldots, K)$ are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unity variance and by using

$$I^{(k)} = H\left(\mathbf{r}^{(k)}\right) - H\left(\mathbf{r}^{(k)} | \mathbf{b}^{(k)}\right) = H\left(\mathbf{r}^{(k)}\right) - H\left(\tilde{\mathbf{n}}^{(k)}\right) \tag{4}$$

where $\tilde{\mathbf{n}}^{(k)} = \mathbf{H}^{(k)} \sum_{i=1, i\neq k}^{K} \mathbf{T}^{(i)} \mathbf{b}^{(i)} + \mathbf{n}^{(k)}$ is the interference plus noise vector, and $H(\mathbf{x})$ is the differential entropy defined as $H(\mathbf{x}) = -\mathrm{E}[\log_2(f_{\mathbf{x}}(\mathbf{x}))]$, in which $f_{\mathbf{x}}(\mathbf{x})$ is the probability density function of the random vector $\mathbf{x}$. The second equality in (4) is valid because we assume all signals and noise are independent. Also, notice that the optimization problem in (2) consists of $\sum_{k=1}^{K} 2ML_k$ real variables since $\mathbf{T}^{(k)}$ is a $M \times L_k$ complex matrix. We discuss the numerical solution for this nonlinear optimization problem in Section IV.

B. *Max-Min Mutual Information MU-MIMO Scheme With Multiuser Coding*

Mutual information can be increased, if the encoding of the users' signals is done in a cooperative manner. That is, a multiuser coding is performed. Even though the channel capacity of such an unrestricted multiuser MIMO channel is still an open problem, a lower bound on the achievable channel capacity is known, which uses the "dirty paper" result from [4]. Consider the received signal of user k from (1), we can obtain $\mathbf{r}^{(k)} = \mathbf{H}^{(k)} \mathbf{T}^{(k)} \mathbf{b}^{(k)} + \mathbf{v}^{(k)} + \mathbf{n}^{(k)}$, where $\mathbf{v}^{(k)} = \mathbf{H}^{(k)} \sum_{i=1, i\neq k}^{K} \mathbf{T}^{(i)} \mathbf{b}^{(i)}$ represents the Gaussian multiuser interference. If the transmitter has complete knowledge of the interference term $\mathbf{v}^{(k)}$, the channel capacity is the same as if $\mathbf{v}^{(k)}$ were not present. This result is known as the "dirty paper" result. As $\mathbf{v}^{(k)}$ is determined by the channel matrices and the user's encoded signals, this result can be exploited by a multiuser code design. For the sake of simplicity, we will restrict the discussion to the case of $K = 2$ users in the following of this subsection. Since the optimum design for the code of the first user requires already full knowledge of the code of the second user, and *vice versa*, the channel capacity is still an open problem. However, a suboptimum solution can be obtained [5], [6] by designing the code for the first user independently from the second user, and apply the "dirty paper" result to the code of the second user only. In this way, the mutual information is given by

$$\begin{aligned} I^{(1)} &= \log_2 \det \Bigg(\mathbf{I} + \mathbf{T}^{(1)^{\mathrm{H}}} \mathbf{H}^{(1)^{\mathrm{H}}} \\ &\qquad \times \left[\sigma_{\mathrm{n}}^2 \mathbf{I} + \mathbf{H}^{(1)} \mathbf{T}^{(2)} \mathbf{T}^{(2)^{\mathrm{H}}} \mathbf{H}^{(1)^{\mathrm{H}}} \right]^{-1} \\ &\qquad \times \mathbf{H}^{(1)} \mathbf{T}^{(1)} \Bigg) \\ I^{(2)} &= \log_2 \det \left(\mathbf{I} + \frac{1}{\sigma_{\mathrm{n}}^2} \mathbf{T}^{(2)^{\mathrm{H}}} \mathbf{H}^{(2)^{\mathrm{H}}} \mathbf{H}^{(2)} \mathbf{T}^{(2)} \right) \\ I &= \min\left\{ I^{(1)}, I^{(2)} \right\}. \end{aligned} \tag{5}$$

Since the labeling of users is arbitrary, one can also achieve mutual information

$$\begin{aligned}\widetilde{I}^{(2)} &= \log_2 \det\Big(\mathbf{I} + \mathbf{T}^{(2)^{\mathrm{H}}}\mathbf{H}^{(2)^{\mathrm{H}}} \\ &\qquad \times \Big[\sigma_{\mathrm{n}}^2\mathbf{I} + \mathbf{H}^{(2)}\mathbf{T}^{(1)}\mathbf{T}^{(1)^{\mathrm{H}}}\mathbf{H}^{(2)^{\mathrm{H}}}\Big]^{-1} \\ &\qquad \times \mathbf{H}^{(2)}\mathbf{T}^{(2)}\Big) \\ \widetilde{I}^{(1)} &= \log_2 \det\left(\mathbf{I} + \frac{1}{\sigma_{\mathrm{n}}^2}\mathbf{T}^{(1)^{\mathrm{H}}}\mathbf{H}^{(1)^{\mathrm{H}}}\mathbf{H}^{(1)}\mathbf{T}^{(1)}\right) \\ \widetilde{I} &= \min\left\{\widetilde{I}^{(1)}, \widetilde{I}^{(2)}\right\}. \end{aligned} \tag{6}$$

Therefore, the minimum mutual information I_{DP} among the $K = 2$ users achievable by using the "dirty paper" argument is then given by

$$I_{\mathrm{DP}} = \max\{I, \widetilde{I}\}. \tag{7}$$

In this max-min mutual information MU-MIMO with multiuser coding scheme, our primary objective is to select the nonzero preprocessing matrices, ($\mathbf{T}^{(1)}$, $\mathbf{T}^{(2)}$), for the $K = 2$ users such that the minimum mutual information I_{DP} among the K users is maximized.

$$\begin{aligned}\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}\right) &= \underset{\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}\right)}{\arg} \max\{I_{\mathrm{DP}}\} \\ &\qquad \text{s.t.} \sum_{k=1}^{2} \operatorname{trace}\left(\mathbf{T}^{(k)}\mathbf{T}^{(k)^{\mathrm{H}}}\right) = P_{\mathrm{T}}. \end{aligned} \tag{8}$$

We discuss the numerical solution for this nonlinear optimization problem in Section IV.

C. MU-MIMO Decomposition With Power Allocation Scheme

In this scheme, our goal is to select the nonzero K preprocessing matrices, ($\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)}$), such that: 1) at the receiver of each MS, there is no interference from the other $K-1$ users; and 2) the minimum mutual information among the K users is maximized. The problem statement can be expressed as

$$\begin{aligned}&\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)}\right) \\ &= \underset{\left(\mathbf{T}^{(1)}, \mathbf{T}^{(2)}, \ldots, \mathbf{T}^{(K)}\right)}{\arg} \begin{cases} \mathbf{H}^{(1)} \sum\limits_{i=1, i\neq 1}^{K} \mathbf{T}^{(i)}\mathbf{b}^{(i)} = \mathbf{0} \\ \mathbf{H}^{(2)} \sum\limits_{i=1, i\neq 2}^{K} \mathbf{T}^{(i)}\mathbf{b}^{(i)} = \mathbf{0} \\ \qquad \vdots \\ \mathbf{H}^{(K)} \sum\limits_{i=1, i\neq K}^{K} \mathbf{T}^{(i)}\mathbf{b}^{(i)} = \mathbf{0} \\ \max\min\left\{I^{(k)}, k = 1, \cdots, K\right\} \end{cases} \\ &\quad \text{s.t.} \sum_{k=1}^{K} \operatorname{trace}\left(\mathbf{T}^{(k)}\mathbf{T}^{(k)^{\mathrm{H}}}\right) = P_{\mathrm{T}} \end{aligned} \tag{9}$$

where $I^{(k)}$ is the mutual information of user k, which can be given by (3). In this case, it can be simplified as

$$I^{(k)} = \log_2 \det\left(\mathbf{I} + \frac{1}{\sigma_{\mathrm{n}}^2}\mathbf{T}^{(k)^{\mathrm{H}}}\mathbf{H}^{(k)^{\mathrm{H}}}\mathbf{H}^{(k)}\mathbf{T}^{(k)}\right) \tag{10}$$

provided the solution exists. Also, it should be noted that $\mathbf{H}^{(k)} \sum_{i=1, i\neq k}^{K} \mathbf{T}^{(i)}\mathbf{b}^{(i)}$ represents the interference to user k due to the other $K-1$ users and, therefore, the first part of problem statement in (9) nulls all interference for each user, the solution of which is given by the multiuser MIMO Decomposition [7], [8]. That is, we may write the transmit preprocessing matrix as the product of a matrix $\mathbf{V}^{(k)}$ with $\mathbf{V}^{(k)^{\mathrm{H}}}\mathbf{V}^{(k)} = \mathbf{I}$, and a nonzero matrix $\mathbf{A}^{(k)}$. That is

$$\mathbf{T}^{(k)} = \mathbf{V}^{(k)}\mathbf{A}^{(k)}, \tag{11}$$

where $\mathbf{V}^{(k)}$ is chosen such that $\mathbf{H}^{(i)}\mathbf{V}^{(k)} = \mathbf{0}$ for $i \neq k$, and $\mathbf{A}^{(k)}$ can be chosen according to some criterion. The matrix $\mathbf{V}^{(k)}$ can be computed by singular value decomposition

$$\begin{bmatrix} \mathbf{H}^{(1)} \\ \vdots \\ \mathbf{H}^{(k-1)} \\ \mathbf{H}^{(k+1)} \\ \vdots \\ \mathbf{H}^{(K)} \end{bmatrix} = \begin{bmatrix} \widetilde{\mathbf{U}}^{(k)} & \mathbf{U}^{(k)} \end{bmatrix} \cdot \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{\mathbf{V}}^{(k)^{\mathrm{H}}} \\ \mathbf{V}^{(k)^{\mathrm{H}}} \end{bmatrix}. \tag{12}$$

From (12), we can see that the dimension of $\mathbf{V}^{(k)}$ is $M \times n_k$, where $n_k \geq \max\{0, M - \sum_{i=1, i\neq k}^{K} N_i\}$. Note that in order to guarantee $n_k > 0$, it is required $M > \sum_{i=1}^{K} N_i - \min\{N_k, k = 1, \ldots, K\}$, which is the sufficient condition for the existence of the nonzero $\mathbf{V}^{(k)}$ $(k = 1, \ldots, K)$ solution. In this way, it decomposes the multiuser MIMO system into K independent single-user MIMO systems. By letting $\widetilde{\mathbf{H}}^{(k)} = \mathbf{H}^{(k)}\mathbf{V}^{(k)}$, (10) can be rewritten as

$$I^{(k)} = \log_2 \det\left(\mathbf{I} + \frac{1}{\sigma_{\mathrm{n}}^2}\mathbf{A}^{(k)^{\mathrm{H}}}\widetilde{\mathbf{H}}^{(k)^{\mathrm{H}}}\widetilde{\mathbf{H}}^{(k)}\mathbf{A}^{(k)}\right). \tag{13}$$

The matrix $\mathbf{A}^{(k)}$ should be chosen according to [1] to maximize the mutual information subject to an average transmit power constraint trace$(\mathbf{A}^{(k)}\mathbf{A}^{(k)^{\mathrm{H}}}) = p_k$, where p_k is the transmit power assigned to user k. Finally, the optimization problem in (9) becomes power optimization according to

$$\max_{p_1, \cdots p_K} \min_{1 \leq k \leq K} I^{(k)} \text{ s.t. } \sum_{k=1}^{K} p_k = P_{\mathrm{T}}. \tag{14}$$

Notice that there are K real variables in this optimization scheme, which is much simpler than the scheme introduced in Section III-A. We refer this scheme as MU-MIMO decomposition with power allocation scheme. Note that the optimization of this scheme is nonlinear. We discuss numerical solution of this nonlinear optimization scheme in Section IV.

D. MU-MIMO Decomposition With Equal Power Allocation Scheme

To avoid the nonlinear optimization involved in computing the power allocation introduced in Section III-C, a simplified scheme is defined, which provides each user with equal power. That is, $p_k = P_{\mathrm{T}}/K$. We refer to this simplified scheme as MU-MIMO decomposition with equal power allocation scheme.

Note that this scheme requires no power allocation and an analytical solution can be obtained by choosing the matrix $\mathbf{A}^{(k)}$ according to [1] to maximize the mutual information subject to an average transmit power constraint $\mathrm{trace}(\mathbf{A}^{(k)}\mathbf{A}^{(k)^{\mathrm{H}}}) = P_{\mathrm{T}}/K$. However, for a particular channel realization, each user will have different mutual information with the minimum among the users being less than the MU-MIMO decomposition with power allocation scheme described in Section III-C.

E. TDMA-MIMO Scheme

In this TDMA-MIMO scheme, the K users are separated by time. That is, at a particular time instance, there is only one user that the BS is communicating to and all the transmit antennas and transmit power are used to transmit the data for that user. The advantage of this scheme is simplicity, but it cannot exploit performance gain from the other users. In this scheme, the mutual information of user k is given by

$$I^{(k)} = \frac{1}{K}\log_2 \det\left(\mathbf{I} + \frac{1}{\sigma_{\mathrm{n}}^2}\mathbf{T}^{(k)^{\mathrm{H}}}\mathbf{H}^{(k)^{\mathrm{H}}}\mathbf{H}^{(k)}\mathbf{T}^{(k)}\right) \quad (15)$$

where $\mathbf{T}^{(k)}$ is chosen to maximize $I^{(k)}$ subject to the total average transmit power (i.e., $\mathrm{trace}(\mathbf{T}^{(k)}\mathbf{T}^{(k)^{\mathrm{H}}}) = P_{\mathrm{T}}$). Note that the factor $1/K$ in (15) is because of the time division that each user only has $1/K$ of the time for transmission. Finally, note that the analytical solution for $\mathbf{T}^{(k)}$ can be obtained in this scheme and it is chosen according to [1] to maximize the mutual information subject to the average transmit power constraint $\mathrm{trace}(\mathbf{T}^{(k)}\mathbf{T}^{(k)^{\mathrm{H}}}) = P_{\mathrm{T}}$.

IV. Numerical Results

The strategies for the MU-MIMO systems introduced in the previous sections are now investigated by computer simulation. A system with six transmit antennas and two users (each with two receive antennas) is investigated, which we refer as to a 6 × (2,2) system. Two different channel models are used in the simulation: uncorrelated and semicorrelated [9]. The uncorrelated model is used for the situation when both the transmitter and receiver are in a rich scattering environment, while the semicorrelated model is used for the situation where only a few major scatterers are located near the transmitter. In the uncorrelated model, each element of the channel matrix is i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unity variance. In the semicorrelated scenario, we model $\mathbf{H}^{(k)} = \mathbf{G}^{(k)}\mathbf{D}^{(k)}$, where $\mathbf{G}^{(k)}$ is a $N_k \times Q$ matrix, whose elements are i.i.d. circularly symmetric complex Gaussian random variables with zero mean and unity variance. Here, Q denotes the number of independent departure propagation waves or directions of departure (DOD) and $\mathbf{D}^{(k)}$ is a matrix consisting of the steering vectors.

Due to the randomness of the channel matrix $\mathbf{H}^{(k)}$ ($k = 1, \cdots, K$), the mutual information is a random variable,

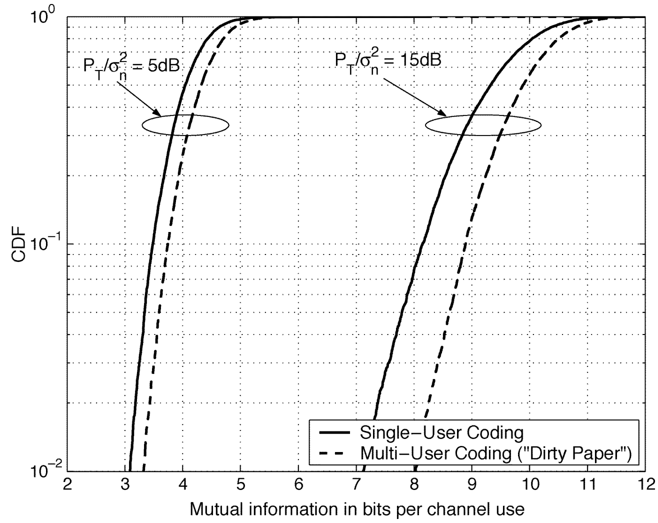

Fig. 2. Comparison of single-user versus multiuser coding. Capacity cdf for a 6 × (2,2) system with an uncorrelated channel is shown.

which can be described by its probability density function (pdf), $f_C(C)$. In this letter, we let C represent the minimum mutual information among the users. That is, $C = \min\{I^{(k)}, k = 1, \cdots, K\}$. The capacity cumulative probability function (cdf) is defined as $F_C(C) = \int_0^C f_C(x)dx$. The outage capacity for a particular outage probability, p, is defined as $C_{\mathrm{out}}(p) = F_C^{-1}(p)$, where $F_C^{-1}(p)$ denotes the inverse function of $F_C(C)$. Hence, $p = F_C(C_{\mathrm{out}}) = \Pr\{C < C_{\mathrm{out}}\}$, which means the probability of error-free decoding is $(1-p)$ when transmitting at the data rate of C_{out}. In this letter, the performance in terms of capacity cdf and maximum throughput (in bits/channel-use) is compared. The *maximum throughput* is defined as

$$\overline{R}_{\max} = \max_{C_{\mathrm{out}}} (1 - F_C(C_{\mathrm{out}}))\, C_{\mathrm{out}} \text{ bits/channel} - \text{use}. \quad (16)$$

The nonlinear optimization for the schemes described in III-A, III-B, and III-C is solved numerically. A sequential quadratic programming (SQP) method [10] is used. Modifications are made to the line search, where an exact merit function (see [11] and [12]) is used together with the merit function proposed by [13] and [14]. The line search is terminated when neither merit function shows improvement. Note that this algorithm may only give local solutions. So, the performance of these three nonlinear optimization schemes may be better than what we show in this letter. Nevertheless, our simulation results provide a reachable performance for these three schemes.

Fig. 2 shows performance comparison of single user and multiuser ("dirty-paper") coding in an uncorrelated channel for $P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2 = 5$ and 15 dB. As expected, multiuser coding always leads to higher capacity than single-user coding. However, the difference is fairly small for medium transmit powers (e.g., $P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2 = 5$ dB) and becomes more significant only in the high transmit power region (e.g., $P_{\mathrm{T}}/\sigma_{\mathrm{n}}^2 = 15$ dB). Since the other evaluated MU-MIMO transmit processing schemes decouple the MU-MIMO channel in several single-user MIMO channels, we will consequently assume single-user coding in the following.

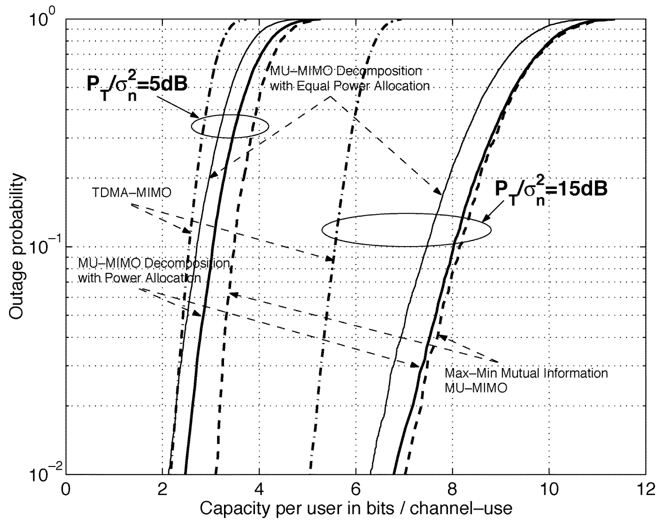

Fig. 3. Capacity cdf for a 6 × (2,2) system with an uncorrelated channel.

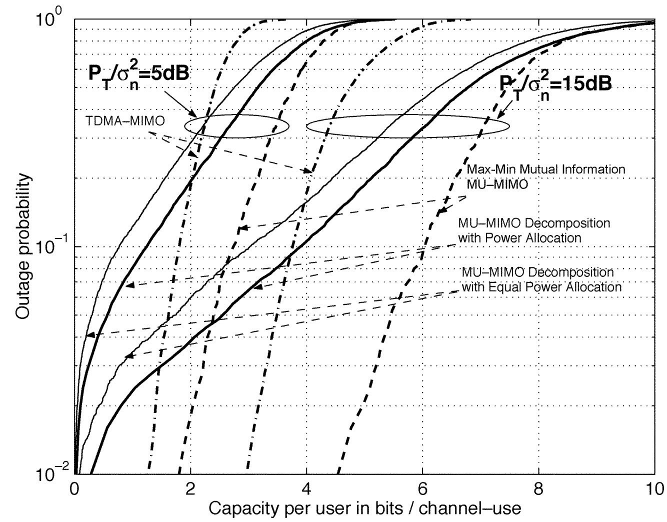

Fig. 5. Capacity cdf for a 6 × (2,2) system with a semicorrelated channel (2 DOD).

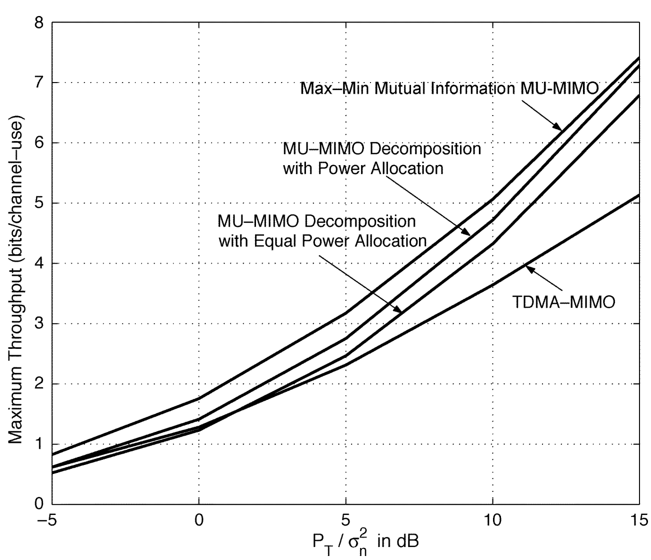

Fig. 4. Maximum throughput for a 6 × (2,2) system with an uncorrelated channel.

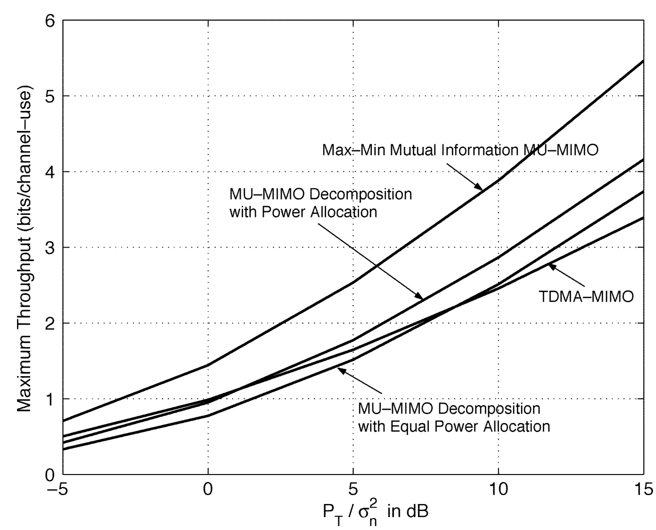

Fig. 6. Maximum throughput for a 6 × (2,2) system with a semi-correlated channel (2 DOD).

Figs. 3 and 4 provide performance comparisons in an uncorrelated channel. We can observe that the max-min mutual information MU-MIMO with single-user coding scheme has the best performance while the capacity and throughput of the TDMA-MIMO scheme is limited especially at large transmit power. The suboptimal scheme, MU-MIMO decomposition with power allocation, can obtain more than 3-dB gain in terms of transmit power at throughput of 5 bits/channel-use per user compared to the TDMA-MIMO scheme (see Fig. 4). A close observation reveals that the performance of the MU-MIMO decomposition with power allocation scheme is close to that of the max-min mutual information MU-MIMO with single-user coding scheme. It can be expected that the larger the number of transmit antennas is, the larger amount this decomposition scheme outperforms the MIMO-TDMA scheme and the closer the performance of this decomposition scheme approaches that of the max-min mutual information MU-MIMO scheme [9].

Figs. 5 and 6 give performance comparison in a semicorrelated channel with two directions of departure (or two major scatterers). That is, $Q = 2$. Similar to the uncorrelated channel case, we can observe that the max-min mutual information MU-MIMO with single-user coding scheme outperforms the MIMO-TDMA approach in terms of maximum throughput and outage capacity, except at very low outage probability. This is because the channel may not have enough degrees of freedom for the multiuser separation in this case. This fact has more severe effect on the MU-MIMO decomposition schemes because the interference-free decomposition may not be possible due to loss of numerical channel rank. For outage probability below 10%, the capacity degrades quickly to zero for all values of transmit power. However, a close observation of Fig. 6 shows that the MU-MIMO decomposition technique can provide higher maximum throughput in a large region of transmit power, which is interesting for practical use. This indicates possibilities for future research in finding efficient suboptimal algorithms. It also should be noted that the MU-MIMO decomposition with equal power scheme has inferior performance than the MIMO-TDMA scheme at low transmit power. This indicates that one has to pay special attention in designing multiuser processing and that sometimes the simplest approach may provide better performance.

V. Conclusion

In this letter, we have introduced five different strategies of transmit linear signal processing for multiuser MIMO systems and provided sample simulation comparisons in the situation when the number of transmit antennas are much larger than the number of receive antennas. It turns out that the multiuser MIMO schemes are preferable to TDMA based schemes, hence demonstrating the power of multiuser MIMO signal processing. This letter also indicates possibilities for future research in finding efficient suboptimal algorithms. As an example, we show the multiuser MIMO decomposition scheme can approach the max-min scheme in an uncorrelated channel for large number of transmit antennas or high transmit power.

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