ON THE EFFECTIVE SPATIO-TEMPORAL RANK OF WIRELESS COMMUNICATION CHANNELS

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Abstract - We consider the following receiver architecture for a wireless communication link with one transmit and multiple receive antennas: Spatial and temporal rank reduction based on long-term (average) spatio-temporal channel properties followed by space-time processing using Maximum-Likelihood estimates of the channel coefficients. Receivers based on this architecture use e.g. a temporal or spatio-temporal Rake, beamspace processing, or eigenvalue decomposition. Their performance critically depends on a good choice of the spatio-temporal rank. We define the effective spatio-temporal rank based on the mean square error (MSE) of the receiver's reduced rank channel estimate. Thus, we can determine the optimum rank and get an analytical insight in the fundamental trade-offs involved, when designing such systems. Using this criterion we discuss under which conditions beamforming is optimal compared to diversity combining.

Keywords - Space-Time Rake, spread spectrum, optimum rank reduction, channel estimation, bias-variance trade-off, beamforming

I. INTRODUCTION

It is known that space-time processing improves communication quality and overall system performance [1]. Due to additional degrees of freedom in space-time processing more channel parameters have to be estimated, which increases the variance of the channel estimates. Thus, channel estimation can become a limiting factor for transmission quality in wireless communications, where only a few pilot symbols are available, as the channel is changing. For this reason reduced rank processing techniques were proposed [2], which reduce the number of relevant channel parameters. Reducing the rank leads to a smaller variance of the channel estimates.

There are two approaches to rank reduction: The first reduces the channel based on the instantaneous channel coefficients or their estimates, respectively [2]. In the sequel we consider the second approach, which exploits long-term channel properties, i.e. the fact that channel delays, angles of arrival, and average power change slowly compared to the complex fading amplitudes of the spatial or temporal taps. This average channel knowledge is given in the spatio-temporal channel correlation matrices [3]. Representatives are the temporal Rake [4], where the fingers are selected based on the average power of the temporal tap, the 2D Rake filter, and the Space-Time Eigenrake [3].

All mentioned receivers have to determine the optimum spatio-temporal rank of the channel, as their performance critically depends on this parameter [5]. An attempt of defining the spatial rank for rank reduction based on instantaneous channel properties was made in [6], which relies on results given by Scharf for the reduced rank approximation of a random vector [7]. This approach fails to explain the strong dependence of the optimum rank on the number of training symbols available for channel estimation. The question of the effective spatio-temporal rank as it is relevant for a communication system with pilot-assisted channel estimation is addressed in this contribution.

For the system model in Section II we give the optimum rank reduction scheme based on long-term channel properties (Section III). The performance of this reduced rank scheme can be described by the mean square error (MSE) of the Maximum-Likelihood (ML) reduced rank channel estimate (Section IV). The optimum rank is the result of a trade-off between the amount of neglected signal power and the variance of the channel estimates. This optimum trade-off leads to the definition of the effective spatio-temporal rank in Section V. We apply this notion to standard channels and conclude that beamforming is the method of choice for low SNR and few pilot symbols (=high speed of the mobile), whereas optimum combining should be applied in high SNR and low speed (many pilot symbols available for channel estimation) scenarios. Comparing the effective rank obtained from the MSE criterion with the effective rank based on the raw bit error rate (BER), we observe that the MSE criterion tends to underestimate the effective rank (Section VI). This results in lower numerical complexity of the short-term processing stage with little BER degradation, which is due to the small sensitivity of the BER to the rank at the effective rank of the communication channel (i.e. at the rank with minimum BER).

II. SYSTEM MODEL

Consider a direct-sequence spread-spectrum communication link with one transmit and N_r receive antenna elements. The received signal is sampled at the chip rate. A discrete-time baseband channel model (1) with a tapped delay line of L taps and white complex Gaussian noise $\boldsymbol{n}_r[t]$ is employed, which models all intra- and intercell-interference. $c[t]$ is the transmitted chip sequence.

$$\boldsymbol{r}[t] = \sum_{\ell=1}^{L} \boldsymbol{h}[\ell] c[t-\tau_\ell] + \boldsymbol{n}_r[t] \quad \in \mathbb{C}^{N_r \times 1} \qquad (1)$$

Equivalent Channel Model: For our derivations we assume that a spreading sequence with perfect autocorrelation properties is used, such that the receiver can separate all temporal paths perfectly. Thus, the channel can be defined by an equivalent flat channel model with $M = L\,N_r$ signals:

$$\boldsymbol{x}[t] = \boldsymbol{h}s[t] + \boldsymbol{n}[t] \quad \in \mathbb{C}^{M\times 1}. \tag{2}$$

In the sequel t is the integer time index w.r.t. a symbol period. $s[t]$ denotes the transmitted QAM-symbol sequence with average power $P_s = \mathrm{E}[|s[t]|^2]$, which is temporally uncorrelated for simplicity. $\boldsymbol{x}[t]$ contains the received and despread spatial and temporal signal components. $\boldsymbol{n}[t]$ is the additive stationary zero-mean complex Gaussian random process with correlation matrix $\boldsymbol{R}_n[k] = \mathrm{E}[\boldsymbol{n}[t+k]\,\boldsymbol{n}[t]^{\mathrm{H}}] = \delta[k]\,\sigma_n^2\,\mathbf{1}$. ($\delta[k]$ is the Kronecker function, $\mathbf{1}$ the unity matrix.) The spatio-temporal channel vector $\boldsymbol{h} = [\boldsymbol{h}[1]^{\mathrm{T}}, \boldsymbol{h}[2]^{\mathrm{T}}, \cdots, \boldsymbol{h}[L]^{\mathrm{T}}]^{\mathrm{T}}$ is a random vector statistically independent of $\boldsymbol{n}[t]$ with correlation matrix

$$\boldsymbol{R}_h = \mathrm{E}[\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}] = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{H}}. \tag{3}$$

Eqn. (3) gives the eigenvalue decomposition of $\boldsymbol{R}_h$ with a unitary matrix $\boldsymbol{U}$ containing the eigenvectors and a diagonal matrix $\boldsymbol{\Lambda} = \mathrm{diag}\{[\lambda_1, \lambda_2, \cdots, \lambda_M]\}$ of the eigenvalues λ_i with $\lambda_i \geq \lambda_{i+1}$. Refer to [8] for a discussion about the estimation of $\boldsymbol{R}_h$.

Now, we reduce the spatio-temporal rank of the channel and received signal, respectively, to R dimensions with the matrix $\boldsymbol{W} \in \mathbb{C}^{M\times R}$ and obtain the reduced rank version of the signal

$$\boldsymbol{y}[t] = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{x}[t] \quad \in \mathbb{C}^{R\times 1} \tag{4}$$

and the channel vector $\boldsymbol{h}_{\mathrm{red}}$

$$\boldsymbol{h}_{\mathrm{red}} = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{h} \quad \in \mathbb{C}^{R\times 1}. \tag{5}$$

For processing and estimation we consider a block of N symbols $\boldsymbol{s} \in \mathbb{C}^{N\times 1}$ and write the system model as

$$\boldsymbol{X} = \boldsymbol{h}\boldsymbol{s}^{\mathrm{T}} + \boldsymbol{N} \quad \in \mathbb{C}^{M\times N} \tag{6}$$

$$\boldsymbol{Y} = \boldsymbol{W}^{\mathrm{H}}\boldsymbol{X} \quad \in \mathbb{C}^{R\times N} \tag{7}$$

with $\boldsymbol{N} = [\boldsymbol{n}[1], \boldsymbol{n}[2], \cdots, \boldsymbol{n}[N]]$ and $\boldsymbol{X}$, $\boldsymbol{Y}$ defined accordingly (Figure 1).

The optimum coherent combiner under the assumptions from above is a maximum ratio combiner, which uses the estimates of the instantaneous channel $\hat{\boldsymbol{h}}_{\mathrm{red}}$ (short-term processing, Figure 1)

$$\hat{\boldsymbol{s}}^{\mathrm{T}} = \hat{\boldsymbol{h}}_{\mathrm{red}}^{\mathrm{H}}\boldsymbol{Y}. \tag{8}$$

ML Channel Estimator: If the sequence of N transmitted symbols is a pilot sequence known to the receiver, the ML channel estimator for the reduced channel coefficients is given by

$$\hat{\boldsymbol{h}}_{\mathrm{red}} = \frac{1}{N\,P_s}\boldsymbol{Y}\boldsymbol{s}^*. \tag{9}$$

This estimator is minimum variance and unbiased w.r.t. $\boldsymbol{h}_{\mathrm{red}}$, but biased w.r.t. estimation of $\boldsymbol{h}$

$$\hat{\boldsymbol{h}}_{\mathrm{red}} = \boldsymbol{h}_{\mathrm{red}} + \frac{1}{N\,P_s}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{N}\boldsymbol{s}^*. \tag{10}$$

Note, that perfect delay estimates were assumed above, when despreading the temporal paths, as they change slowly and, thus, can be estimated reliably averaging over a long period. Furthermore, the channel order L is assumed to be known.

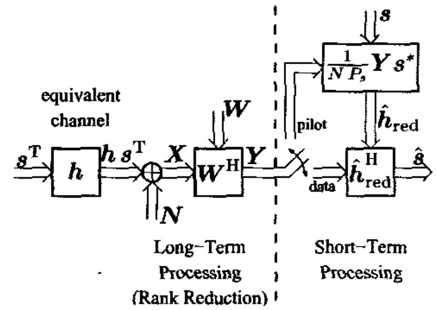

Fig. 1. Receiver concept exploiting long-term and short-term properties of the wireless communication channel to improve channel estimation and reduce computational complexity. Pilot and data symbols are time-multiplexed.

III. Optimum Rank Reduction Based on Long-Term Channel Properties

For rank reduction we rely on the observation that the temporal and spatial channel properties, i.e. the delays and the directions of arrival, change slowly compared to the (fast fading) complex amplitudes in $\boldsymbol{h}$. These spatio-temporal long-term properties can be described by the correlation matrix $\boldsymbol{R}_h$, which contains the "average" information about the channel. Thus, $\boldsymbol{R}_h$ can be estimated very accurately, when averaging over many blocks [3].

As the interference $\boldsymbol{n}[t]$ is white, we aim to maximize the signal power in $\boldsymbol{y}[t]$ for a given Rank R, i.e. $\max_{\boldsymbol{W}} \mathrm{E}[\|\boldsymbol{W}^{\mathrm{H}}\boldsymbol{h}\,s[t]\|_2^2]$ [5]. This is stated in the following theorem, which is a consequence of the Poincaré separation theorem (see [9] for a proof).

Theorem: *The optimum rank reducing transformation $\boldsymbol{W} \in \mathbb{C}^{M\times R}$ for the system in Eqn. (6) with noise correlation matrix $\boldsymbol{R}_n[k] = \delta[k]\,\sigma_n^2\,\mathbf{1}$, which solves*

$$\max_{\boldsymbol{W}} \ \mathrm{trace}\left\{\boldsymbol{W}^{\mathrm{H}}\boldsymbol{R}_h\boldsymbol{W}\right\} \quad \text{s.t.} \quad \mathrm{rank}\,\boldsymbol{W} = R \ \text{ and } \ \boldsymbol{W}^{\mathrm{H}}\boldsymbol{W} = \mathbf{1}, \tag{11}$$

is given by

$$\boldsymbol{W} = \boldsymbol{U}_{\mathrm{red}}\boldsymbol{Q} \quad \text{with} \quad \boldsymbol{U}_{\mathrm{red}} = \boldsymbol{U}[\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_R] \quad \text{and} \quad \boldsymbol{Q}\boldsymbol{Q}^{\mathrm{H}} = \mathbf{1},\ \boldsymbol{Q} \in \mathbb{C}^{R\times R}. \tag{12}$$

Thus, the signal power in the reduced space of $\boldsymbol{y}[t]$ is

$$\mathrm{trace}\left\{\boldsymbol{W}^{\mathrm{H}}\boldsymbol{R}_h\boldsymbol{W}\right\} = \sum_{i=1}^{R}\lambda_i. \tag{13}$$

e_i is the i-th column of the unity matrix. The solution for $\boldsymbol{W}$ is not unique, as its columns are a unitary basis of the subspace spanned by the eigenvectors of $\boldsymbol{R}_h$ corresponding to the R largest eigenvalues. This fact can be exploited to employ projector tracking [10] to track the long-term channel structure, when it changes. Moreover, the transformation $\boldsymbol{W}$ has the following property:

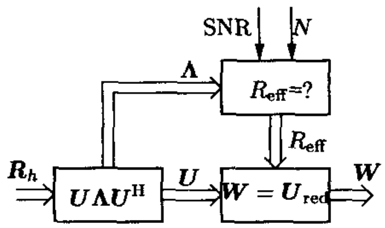

Fig. 2. Calculation of the matrix $\boldsymbol{W}$ for rank reduction based on the long-term (average) channel properties $\boldsymbol{R}_h$ and on the effective spatio-temporal rank R_{eff}.

Corollary: *If and only if $\boldsymbol{R}_h$ has algebraic rank R, Eqn. (12) yields a sufficient statistic for estimating the channel $\boldsymbol{h}$ or the symbols $s[t]$, i.e.*

$$\text{p}(\boldsymbol{x}[t]|\boldsymbol{y}[t];\boldsymbol{h},s[t]) = \text{p}(\boldsymbol{x}[t]|\boldsymbol{y}[t])$$

is independent from $\boldsymbol{h}$ and $s[t]$.

This follows from the fact that signal components of $\boldsymbol{x}[t]$ in the subspace orthogonal to span$\{\boldsymbol{W}\}$ are uncorrelated with $\boldsymbol{y}[t]$. As $\boldsymbol{n}[t]$ is Gaussian, they are statistically independent to $\boldsymbol{y}[t]$ and do not provide any additional information for estimating $\boldsymbol{h}$ or $s[t]$.

Receivers based on criterion (11):

- If we have a temporal channel ($N_r = 1$) and assume uncorrelated scattering, $\boldsymbol{R}_h$ is a diagonal matrix and $\boldsymbol{U} = \boldsymbol{1}$. Thus, we obtain the *temporal Rake* [4], which simply selects the R taps with largest average power.
- If the temporal channel order is $L = 1$, the mean of long-term processing is the *spatial Eigenrake*, which points R beams towards the impinging signal [8], [5]. If we choose $R = 1$, rank reduction is equivalent to beamforming. For $R = M$ we perform full rank processing, which is often referred to as diversity combining of the spatially sampled signal.
- The *spatio-temporal Eigenrake* [3] is a combination of the previous two receivers. The correlation matrix $\boldsymbol{R}_h$ of a spatio-temporal channel $\boldsymbol{h}$ with uncorrelated scattering is block diagonal

$$\boldsymbol{R}_h = \begin{bmatrix} \boldsymbol{R}_h[1] & & \\ & \ddots & \\ & & \boldsymbol{R}_h[L] \end{bmatrix} \quad (14)$$

and we select the strongest eigenvectors among all spatial correlation matrices $\boldsymbol{R}_h[\ell] = \text{E}[\boldsymbol{h}[\ell]\boldsymbol{h}[\ell]^{\text{H}}]$ of the temporal taps $\ell \in \{1, 2, \cdots, L\}$ as columns of $\boldsymbol{W}$.

Trade-off: As long-term properties of the channel change slowly, the complexity of determining $\boldsymbol{W}$ is negligible. The rank R determines the numerical complexity of the short-term processing stage (maximum ratio combining, Figure 1) as well as the performance of the communication link [5]. Communication quality over flat fading channels is determined by the SNR at the output of the combiner Eqn. (8) and the variance of the channel estimator. On the one hand the signal power in the subspace orthogonal to span$\{\boldsymbol{W}\}$ is neglected for a given rank. It is given by $\sum_{R+1}^{M} \lambda_i$. On the other hand the variance of the estimated channel coefficients is reduced, as only $R \leq M$ parameters have to be estimated. We conclude: i) There is a rank $R = R_{\text{eff}}$, which results in best performance, i.e. the optimum trade-off between neglected signal power and estimation variance. ii) If complexity is a major issue, we want to choose the smallest rank R, which still gives satisfactory performance.

IV. Mean Square Error of Channel Estimation

To describe the trade-off discussed in the previous section analytically we introduce the mean square error (MSE) of the channel estimator (Eqn. 9) w.r.t. the channel $\boldsymbol{h}$. It measures the average error of the receiver's instantaneous channel knowledge compared to the "real" channel $\boldsymbol{h}$, which the receiver could potentially exploit completely for $R = M$.

$$\text{MSE} = \frac{1}{P_{\text{h}}}\text{E}[\|\boldsymbol{h} - \boldsymbol{W}\hat{\boldsymbol{h}}_{\text{red}}\|_2^2] \quad (15)$$

The MSE is normalized by $P_{\text{h}} = \text{trace}\{\boldsymbol{R}_h\} = \sum_{i=1}^{M} \lambda_i$ to normalize the channel power. For unitary columns in $\boldsymbol{W}$ Eqn. (15) yields

$$\text{MSE} = \underbrace{\frac{1}{P_{\text{h}}}\sum_{i=R+1}^{M}\lambda_i}_{\text{Bias}} + \underbrace{\frac{R}{N\gamma}}_{\text{Variance}} . \quad (16)$$

The SNR γ is defined as it would be obtained at the output of a MRC with perfect channel knowledge $\hat{\boldsymbol{h}} = \boldsymbol{h}$ and no rank reduction, i.e. full-rank $R = M$ preprocessing,

$$\gamma = \frac{\text{E}[\|\boldsymbol{h}\, s[t]\|_2^2]}{\text{E}[\|\boldsymbol{n}[t]\|_2^2]/M} = \frac{P_{\text{h}} P_{\text{s}}}{\sigma_{\text{n}}^2}. \quad (17)$$

The MSE of the channel estimator (16) has two terms: The *first term* is the fraction of the signal power, which was neglected when reducing the channel rank to R dimensions. This is the *bias* of the estimator, i.e. a systematic error, and depends on the eigenvalue spectrum of $\boldsymbol{R}_h$ and the rank R. It increases with smaller R. The estimation *variance* in the *second term* increases linearly with the number of parameters R to estimate. The slope is given by the SNR and the length of the pilot sequence. We expect that the larger the SNR at the receiver and the more pilot symbols we use, the larger we should choose the rank R.

V. The Effective Spatio-Temporal Rank

In this section we first define the effective channel rank and apply the notion of the effective rank to two channels, which are often discussed in the literature. In particular we address the question whether to do beamforming or optimum combining at the receiver.

The correlation matrix $\boldsymbol{R}_h$ of typical communication channels has full algebraic rank $R = M$, although its eigenvalues decrease with a rather steep slope (c.f. Figure 3). As discussed above the optimum rank for receiver processing based on long-term channel properties is in the interval $1 \leq R \leq M$. We define the *effective spatio-temporal rank* of the channel as the channel rank or the number of channel dimensions, which are relevant for the receiver in order to achieve optimum performance.

Definition: *Assume a single-input/multiple-output system as defined in Eqn. (2) with rank reduction according to Eqn. (4) and (11) and a ML-estimator to estimate the channel coefficients (Eqn. 9). The effective Rank R_{eff} of this system is defined as the Rank R, which achieves optimum transmission quality.*

Choosing the MSE (Eqn. 15) as a measure for receiver performance, the effective rank is given by

$$R_{\text{eff}} = \arg\min_{R} \text{MSE}(\gamma, N, R, M, \Lambda). \qquad (18)$$

Using this definition we also can define *optimality regions* of beamforming and full rank processing (diversity combing) for a given channel with the parameters $\{\Lambda,\ M\}$: The optimality region is the set of parameters $\{\gamma,\ N\}$, for which $R_{\text{eff}} = 1$ (beamforming) and $R_{\text{eff}} = M$ (optimum combining), respectively. Now we apply this notion to two typical wireless channels, characterized by their eigenvalue spectrum.

Uniform Eigenvalue Spectrum:
All eigenvalues of $\boldsymbol{R}_h$ are equal $\lambda_i = \sigma_{\text{h}}^2,\ \forall i \in \{1, 2, \cdots, M\}$. This is true if and only if the correlation matrix is a diagonal matrix

$$\boldsymbol{R}_h = \sigma_{\text{h}}^2 \mathbf{1}. \qquad (19)$$

Its eigenvectors form the columns of any unitary matrix $\boldsymbol{U}$: for $\boldsymbol{U} = \mathbf{1}$ rank reduction simply means selecting R signals in $\boldsymbol{x}[t]$. An example for this class of channels is a spatial channel ($L = 1$) with a large separation of the $M = N_{\text{r}}$ antenna elements at the receiver ("spatially uncorrelated channel").

Choosing $P_{\text{h}} = M\sigma_{\text{h}}^2$ we normalize the channel power $\text{trace}\{\boldsymbol{R}_h / P_{\text{h}}\} = 1$ and obtain

$$\begin{aligned} \text{MSE}(\gamma, N, R, M) &= M\frac{\sigma_{\text{h}}^2}{P_{\text{h}}} + R\left(\frac{1}{N\gamma} - \frac{\sigma_{\text{h}}^2}{P_{\text{h}}}\right) \\ &= 1 + R\left(\frac{1}{N\gamma} - \frac{1}{M}\right). \qquad (20) \end{aligned}$$

Two cases must be distinguished

1) $M < N\gamma$: The length of the pilot sequence and the SNR are large enough and criterion (20) suggests to perform full-rank processing (diversity combining), $R_{\text{eff}} = M$.
2) $M \geq N\gamma$: The channel estimates are so bad, as the SNR is small and the pilot sequence short, that "beamforming", i.e. selecting one arbitrary spatial/temporal path is optimum, $R_{\text{eff}} = 1$.

Exponential Eigenvalue Spectrum:
The exponentially decreasing spectrum of eigenvalues is a good match for many practical channels, e.g. a temporal channel with exponential power delay profile or a spatial channel.

$$\lambda_i = A\,e^{-i/\tau}. \qquad (21)$$

With Eqn. (21) and (16) we get

$$\text{MSE}(\gamma, N, R, M, \tau) = \frac{e^{-R/\tau} - e^{-M/\tau}}{1 - e^{-M/\tau}} + \frac{R}{N\gamma}. \qquad (22)$$

We take the first derivative to obtain a necessary condition for the effective rank

$$\frac{\text{d}}{\text{d}R}\text{MSE}(\gamma, N, R, M, \tau)\Big|_{R=R_{\text{eff}}} = 0. \qquad (23)$$

For an exponential eigenvalue spectrum the effective rank can now be given explicitly

$$R_{\text{eff}} = \tau \ln\left(\frac{\gamma N}{\tau\,(1 - e^{-M/\tau})}\right). \qquad (24)$$

R_{eff} has to be an integer in the interval $1 \leq R_{\text{eff}} \leq M$, i.e. criterion (22) has to be evaluated with R equal to the integers closest to the result of Eqn. (24). From this result we directly see the dependency of R_{eff} on the SNR, N, and the rate of decay τ of the eigenvalues λ_i.

VI. Comparison of the MSE and BER Criterion to Determine the Effective Rank

The effective rank can be determined according to different criteria. We chose the MSE of channel estimation (Eqn. 15, 18), as it gives an analytical insight in the fundamental trade-offs associated with reduced rank processing. Nevertheless, the raw bit error rate (BER) is more relevant to the performance of a radio communication link. We discuss to which extend the conclusions about the effective rank based on the MSE criterion can be applied and extended to the BER.

Our simulations are based on the WCDMA uplink parameters: BPSK data and pilot symbols are transmitted. The length of the pilot sequence is 6 symbols per block. 6 pilot symbols correspond to $6 \cdot 256$ chips per slot in the WCDMA uplink. Corresponding to our assumptions in Section II perfect spreading sequences are used. To estimate the BER we simulate 10000 independent Rayleigh channels with 1000 data symbols per block.

We consider a spatio-temporal channel in a pico scenario [11] and $N_r = 8$ antenna elements at the receiver (base station). The eigenvalue profile in Figure 3 has two taps with an angular spread and strong spatial correlation at the final two taps ($M = 8 \cdot 4$). The MSE for typical SNR values (Figure 4) shows a distinct minimum at a rank R, which increases as the channel estimates improve with the SNR. Comparing the location of the minimum in Figure 4, i.e. the effective rank R_{eff} for a specific SNR, with the corresponding minimum of the BER (Figure 5), we observe a good correspondence at high SNR. The MSE criterion tends to underestimate the effective rank at lower SNR compared with the minimum of the BER, as the BER is less sensitive to estimation variance than to neglected signal power (bias). As the minimum in the BER is rather flat, underestimating the optimum rank does not affect performance much, but leads to a smaller complexity in short-term processing (channel estimation and MRC). Overall no more than $M/2$ fingers should be used in this scenario.

Furthermore, the effective rank R_{eff} increases with the number of pilot symbols N (c.f. Figure 6): When the channel changes slowly in time, more pilot symbols are available and a larger rank R should be chosen for optimum performance.

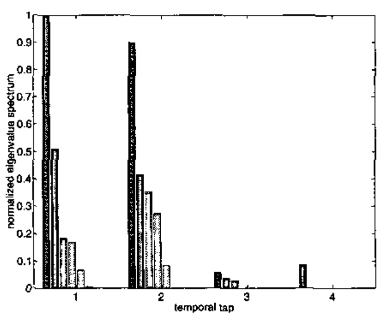

Fig. 3. The eigenvalue spectrum for each temporal tap of the spatio-temporal channel in a pico scenario as described in Section VI. The receiver uses $N_r = 8$ antenna elements.

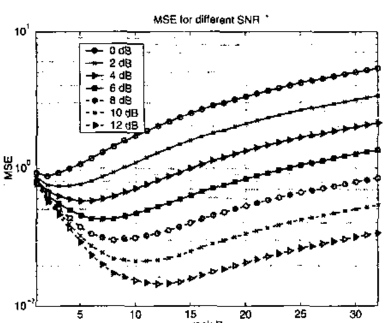

Fig. 4. The MSE of the reduced rank channel estimator ($N = 6$) for different SNR and the pico scenario (Figure 3).

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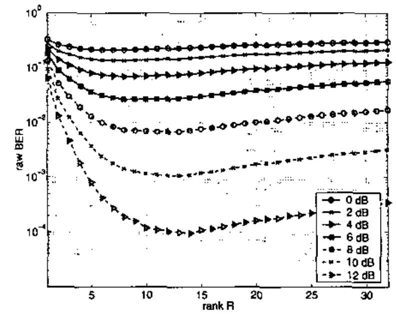

Fig. 5. The raw (uncoded) BER of the reduced rank system with ML channel estimation ($N = 6$) for different SNR and the pico scenario (Figure 3).

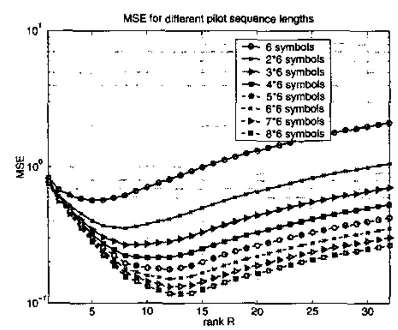

Fig. 6. The MSE of the reduced rank channel estimator for different lengths N of the pilot sequence and the pico scenario (Figure 3) for $\gamma = 4\,\text{dB}$.

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