# Linearly Prefiltered OFDM Based on Long-Term Properties of the Channel Covariance Matrix

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***Abstract*— In this article we introduce an improved OFDM scheme in the frame of linearly precoded OFDM, based on the long-term properties of the channel. The channel covariance matrix is computed based on the ITU channel model and we show that the propagation delays represent long-term properties which leads to correlation between several subchannels. The long-term channel covariance matrix can be fed back to the transmitter and the extra knowledge used for prefiltering. We propose a scheme in which the symbols are linearly prefiltered with the eigenvectors of the channel covariance matrix corresponding to the significant eigenvalues and use multistream detection at the receiver side. The results of the simulations showed that eigenpreprocessing with adaptive bit and power loading and LMMSE detection at the receiver side provides a significantly lower bit error rate, especially at high signal to noise ratios compared with other types of linear prefiltering that do not use knowledge about the channel covariance matrix.**

## I. Introduction

The OFDM system has emerged as a good alternative to mitigate the effects of frequency selectivity in wideband mobile communication systems. The use of a Cyclic Prefix (CP) for preventing inter-block interference is known to be equivalent to multiple flat fading parallel transmission channels in the frequency domain [1]. The drawback of this technique is the loss of diversity, so the symbols which are associated to subcarriers in fade cannot be recovered anymore, leading to poor performance of uncoded OFDM. Several coding schemes have been proposed to make OFDM more robust to frequency fades. The most common are convolutional coding, Trellis Coded Modulation (TCM), Turbo-codes, block codes (e.g. Reed-Solomon, BCH). This systems have shown a good reliability, which have helped OFDM become a part of several standards and makes it a strong candidate for the $4^{\text{th}}$ generation of mobile communications [2].

Another method to reduce the number of errors due to subchannels in fade is the combination of the multicarrier (MC) with the spread spectrum (SS) technique. From the point of view of the performances in the single user case, [3] and [4] present results of schemes that imply the linear precoding of the symbols before transmission. It is shown that this technique is robust against the frequency fades of the channel and it can perform even better than COFDM.

Our idea is to introduce the long-time properties of the channel in a new prefiltering scheme, advantage that will lead to better performance. We will show how the covariance matrix can be used to obtain uncorrelated streams and propose a prefiltering matrix based on adaptive bit and power loading of the eigenmodes of the long-term channel covariance matrix. A similar approach is presented in the spatial domain in [5], [6], [7].

In Section II we present the OFDM system, in Section III we compute the long-term covariance matrix and show its properties, followed by our approach to prefiltering in Section IV, simulation results in Section V and some conclusions in Section VI.

## II. System Model

We start from the standard uncoded OFDM system and assume that the use of cyclic prefix (CP) both preserves the orthogonality of the tones and eliminates intersymbol interference (ISI) between consecutive OFDM symbols. The channel is assumed to be slowly fading, approximated constant during one OFDM symbol. The number of tones in the system is $N$ which makes the effective symbol length $T = NT_{\mathrm{S}}$, where $T_{\mathrm{S}}$ is the sampling interval. The length of the cyclic prefix is $T_{\mathrm{G}} = LT_{\mathrm{S}}$ and the total symbol length is $T_{\mathrm{t}} = T + T_{\mathrm{G}}$. Since the duration of the impulse response of the channel is by assumption shorter than the cyclic prefix, we can describe the system as a set of parallel Gaussian channels with correlated attenuation $h_k$ [1],

$$h_k = H(j2\pi\frac{k}{N}), k = 0..N-1, \tag{1}$$

where $H(j\omega) \triangleq \mathcal{F}\{h(\tau, t_0)\}$ is the channel transfer function at time $t_0$, i.e. the Fourier transform of the channel impulse response.

The spreaded OFDM [3] or the linearly precoded OFDM [4] propose to modify this scheme by linearly precoding the transmitted symbol with a unitary matrix, $\boldsymbol{A}$, and apply multistream detection at the receiver side, Figure 1.

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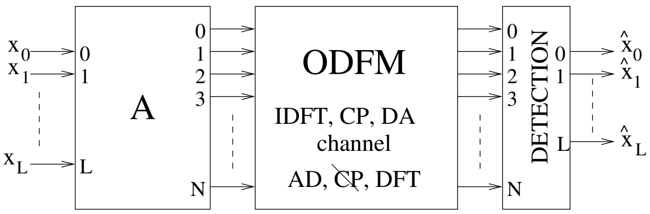


Fig. 1. Linearly prefiltered OFDM system model

We can write the input-output relation before the detector in matrix form as:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{A}\boldsymbol{x} + \boldsymbol{n}, \tag{2}$$

where: $\boldsymbol{y} \triangleq [y_0\ y_1 \ldots\ y_{N-1}]^{\mathrm{T}}$ is the received signal vector, $\boldsymbol{H} \triangleq \mathrm{diag}[H(j0)\ H(j2\pi\frac{1}{N})\ldots\ H(j2\pi\frac{N-1}{N})]$ is a diagonal matrix with entries equal to the subchannel coefficients in the frequency domain; $\boldsymbol{x} \triangleq [x_0\ x_1 \ldots\ x_L]^{\mathrm{T}}$ is the vector of the transmitted symbols, taken from an alphabet $\Psi$ and with powers $p_i$; $\boldsymbol{n} \triangleq [n_0\ n_1 \ldots\ n_{N-1}]^{\mathrm{T}}$ is a vector with the realizations of the additive white Gaussian noise.

In the sequel we consider the eigenvectors of the frequency domain channel covariance matrix to apply unitary prefiltering to OFDM.

## III. Long-term properties in OFDM

The sampled Channel Impulse Response (CIR) $\boldsymbol{h}^{(\mathrm{T})} \in \mathbb{C}^N$ is composed of $L$ paths with WSSUS attenuations, i.e. $\boldsymbol{h}^{(\mathrm{T})} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_h^{(\mathrm{T})})$ where $\boldsymbol{\Sigma}_h^{(\mathrm{T})} \triangleq \mathrm{diag}\left[\sigma_1^2, \ldots, \sigma_M^2\right]$, with $\sigma_{i_1}^2, \ldots, \sigma_{i_L}^2 \neq 0$ only if $\tau_{i_\ell} = i_\ell\ T_{\mathrm{s}}$ corresponds to any of the propagation delays of the CIR. Performing IFFT at the trasnmitter and FFT at the receiver side we can write the channel as a diagonal matrix $\boldsymbol{H}$, with correlated inputs $H\left(j2\pi\frac{k}{N}\right)$. We want to check the validity of the affirmation that the propagation path delays are long-term properties of the channel, i.e. the channel covariance matrix remains unmodified over a long enough period $t_{\max}$. Computing analytically the covariance matrix, we get that its general term depends on the propagation delay through the exponential: $e^{-j2\pi\tau/T_{\mathrm{S}}}$. In order that this general term not to be modified we must impose the constrain:

$$\frac{\Delta\tau_{\max}}{T_{\mathrm{S}}} \ll 1, \tag{3}$$

where $\Delta\tau_{\max}$ is the maximum variation of the propagation delays in the time interval $t_{\max}$. Considering a maximum speed of the mobile $v_{\max}$, the variation in propagation delay due to variation of distance will be:

$$\Delta\tau_{\max} = \frac{v_{\max}t_{\max}}{c}, \tag{4}$$

where $c$ is the speed of light.

Introducing in (4) in (3) it results the constrain:

$$\frac{\Delta\tau_{\max}}{T_{\mathrm{S}}} = \frac{v_{\max}t_{\max}}{cT_{\mathrm{S}}} \ll 1 \Rightarrow t_{\max} \ll \frac{cT_{\mathrm{S}}}{v_{\max}}. \tag{5}$$

For system parameters conform to actual and proposed wideband mobile communication systems (see Table I) and for a threshold of 1% in the phase variation, we can be confident in a $t_{\max} = 9\mathrm{ms}$. Equivalently, during $N_1 = t_{\max}/T_{\mathrm{t}} \approx 300$ OFDM symbols, the channel covariance matrix can be approximated constant. This will represent sufficient time to feed it back, or more exactly the eigenvectors.

| **PARAMETER** | **VALUE** |
|---|---|
| System bandwidth | $B$=10MHz |
| Sampling interval | $T_{\mathrm{S}}$=100ns |
| Carrier frequency | $f_0$=5GHz |
| Maximum delay | $\tau_{\max}$=2.5$\mu$S |
| Coherence bandwidth | $B_{\mathrm{C}}$=400KHz |
| Maximum speed | $v_{\max}$=120Km/h |
| Maximum Doppler frequency | $f_{\mathrm{D}_{\max}}$=600Hz |
| Coherence time | $T_{\mathrm{C}}$=1.6ms |
| OFDM symbol duration | $T_{\mathrm{O}}$=25.6$\mu$S |
| Guard interval duration | $T_{\mathrm{G}}$=3.1$\mu$S |
| Total OFDM symbol duration | $T_{\mathrm{t}}$=28.7 $\mu$S |
| Number subcarriers | $N = 256$ |
| Guard interval length | $L = 31$ |

TABLE I
RADIO CHANNEL AND SYSTEM PARAMETERS FOR A 4G SYSTEM

The channel is modeled based on the one defined by ITU in [8], using only the time domain of it, as we have only one antenna at the transmitter and one antenna at the receiver. We have computed the channel transfer function covariance matrix and the eigenvalues to extract the long-term properties of the channel. One realization of the covariance matrix and the corresponding eigenvalues are presented in Figure 2 and 3.

We can observe in Figure 2 the fact that several subchannel attenuations (not necessarily neighboring) are strongly correlated due to the approximately constant propagation delays and the ratio between these delays. We can also see from Figure 3 the fact that the covariance matrix is quite low ranked as we will show in the next section.

## IV. Eigenpreprocessing

Having the information of the channel long-term covariance matrix available at the transmitter, we can preprocess the transmitted signal to obtain uncorrelated streams.

We are looking for a precoding matrix $\boldsymbol{A}$ that transforms the channel into a set of uncorrelated subchannels, to obtain the

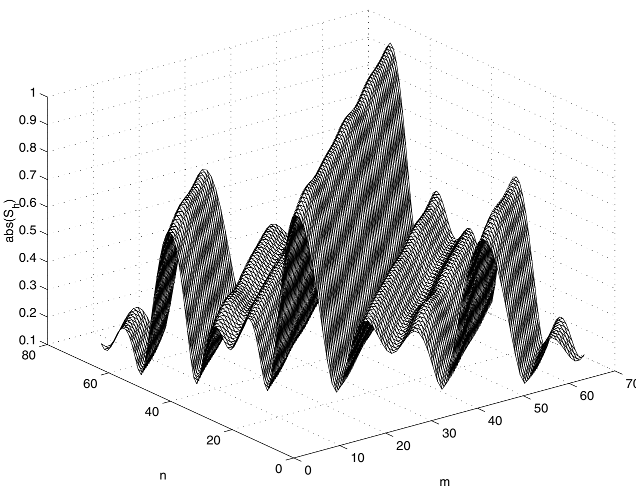


Fig. 2. Channel covariance matrix $\mathbf{\Sigma}_h^{(\mathrm{F})}$

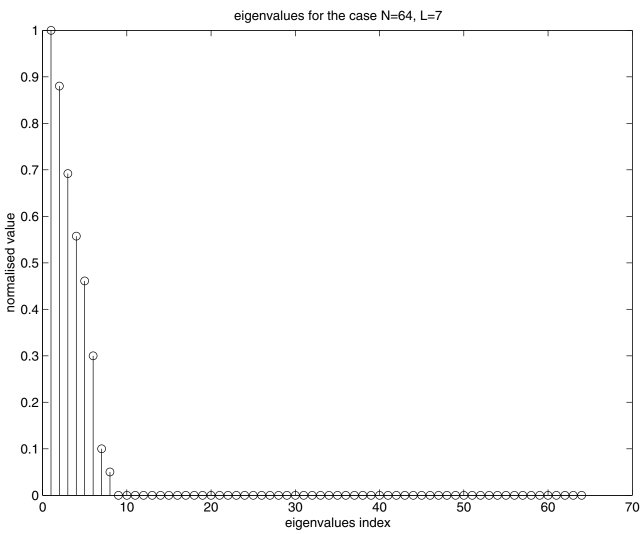


Fig. 3. Eigenvalues in the case $N = 64, L = 7$

maximum diversity achievable. This means that the covariance matrix of $\boldsymbol{h}_{\mathrm{A}}^{(\mathrm{F}),\mathrm{T}} \triangleq \mathbf{1}_{\mathrm{N}}^{\mathrm{T}} \boldsymbol{H}\boldsymbol{A}$,

$$\mathrm{E}\{\boldsymbol{h}_{\mathrm{A}}^{(\mathrm{F}),*}\boldsymbol{h}_{\mathrm{A}}^{(\mathrm{F}),\mathrm{T}}\} = \mathrm{E}\{\boldsymbol{A}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\mathbf{1}_{\mathrm{N}}\mathbf{1}_{\mathrm{N}}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{A}\} = \boldsymbol{A}^{\mathrm{H}}\mathbf{\Sigma}_{\mathrm{h}}^{(F)}\boldsymbol{A}, \tag{6}$$

is diagonal.[1] Writing the eigenvalue decomposition of the frequency domain long-term covariance matrix $\mathbf{\Sigma}_{\mathrm{h}}^{(F)}$ we get:[2]

$$\mathbf{\Lambda} = \boldsymbol{U}^{\mathrm{H}}\mathbf{\Sigma}_{\mathrm{h}}^{(F)}\boldsymbol{U} \tag{7}$$

and, by identification between equation (6) and (7), it results that $\boldsymbol{A}$ should be the eigenvector matrix of the channel.

[1] The all ones column vector of length $N$ is denoted by $\mathbf{1}_{\mathrm{N}}$.

[2] Introducing the unitary discrete Fourier transform matrix $\boldsymbol{F} \in \mathbb{C}^{N\times N}$, the matrix transformation $\mathbf{\Sigma}_{\mathrm{h}}^{(F)} = \mathrm{E}\{\boldsymbol{F}^{-1}\mathbf{\Sigma}_{\mathrm{h}}^{(T)}\boldsymbol{F}^{-\mathrm{H}}\}$ corresponds to the discrete transformation of the CIR between its representations in time and frequency domain, i.e. $\boldsymbol{h}^{(F)} = \boldsymbol{F}^{-1}\boldsymbol{h}^{(T)}$.

We will compute the rank of the matrix $\mathbf{\Sigma}_{\mathrm{h}}^{(F)}$ which is equal with the rank of the matrix $\boldsymbol{A}$ to see how many uncorrelated streams can be used. In [9] it is shown that the dimension of the space of an essentially time - and band - limited signal is about $2BT + 1$, where $B$ is the one-sided bandwidth of the signal and $T$ is the time interval of the signal. In our case the channel transfer function is considered bandlimited to $B = 1/2T_{\mathrm{S}}$, via transmit pulse shaping, and time limited to $T_0 = LT_s$, as the cyclic prefix exceeds the delay spread of the channel, so the dimension of the channel transfer function should be $2BT_0 + 1 = 2\cdot 1/(2T_s)\cdot LT_s + 1 = L + 1$. This fact was also observed in Figure 3 where the eigenvalues in a particular realization are shown.

In order to use only the significant modes for our filtering we have to truncate the matrix $\boldsymbol{A}$ to the $L + 1$ eigenvectors corresponding to the significant eigenvalues:

$$\boldsymbol{A}_{\mathrm{L}} \triangleq \boldsymbol{U}_{\mathrm{N}\times(\mathrm{L}+1)}, \tag{8}$$

where $\boldsymbol{U}_{\mathrm{N}\times(\mathrm{L}+1)}$ is the $N \times (L + 1)$ dimensional matrix composed of the first $L + 1$ columns of the eigenvector matrix, $\boldsymbol{U}$.

We have designed a filtering matrix $\boldsymbol{A}_{\mathrm{L}}$ which transfers the channel in a set of $L + 1$ uncorrelated subchannels. This result can be used in designing a scheme in which the frequency diversity is used.

Considering the fact that the eigenvectors are orthonormal, we can use the matrix $\boldsymbol{A}_{\mathrm{L}}$ to spread the transmitted symbols over the subcarriers. The received vector will be:

$$\boldsymbol{y}_A = \boldsymbol{H}\boldsymbol{A}_{\mathrm{L}}\boldsymbol{x}_{\mathrm{L}} + \boldsymbol{n}, \tag{9}$$

each received component being a linear combination of the transmitted symbols attenuated by the subchannel transfer coefficient $h_k$. The transmitted symbols, $\boldsymbol{x}_{\mathrm{L}}$ can be now defined accordingly to adaptive power and bit loading algorithms to be able to adapt to the channel conditions. To this end we will choose a variable constellation size for each eigenstream and will allocate the total power $P$ to the eigenstreams according to water filling based on the eigenvalues $\mathbf{\Lambda}_{\mathrm{L}}$. At the receiver side multi-stream detection is needed to take advantage of the diversity. Maximum likelihood detection is not a good candidate because its complexity is growing exponentially with $L$. This complexity is not manageable. We consider Channel Inversion (CI) and Linear Minimum Mean Squared Error (LMMSE) detection. The decision variable in the two cases, for equiprobable symbols and power allocation $\boldsymbol{P} = \mathrm{diag}[p_0, p_1, ..., p_{\mathrm{L}}]$, are given by the expressions:

$$\hat{\boldsymbol{x}}_{\mathrm{CI}} = \boldsymbol{A}_{\mathrm{L}}^{\mathrm{H}}\boldsymbol{H}^{-1}y_{\mathrm{A}}, \tag{10}$$

$$\hat{\boldsymbol{x}}_{\mathrm{LMMSE}} = \boldsymbol{P}^{1/2}\boldsymbol{A}_{\mathrm{L}}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}(\sigma_n^2\boldsymbol{I}_{\mathrm{N}} + \boldsymbol{H}\boldsymbol{A}_{\mathrm{L}}\boldsymbol{P}\boldsymbol{A}_{\mathrm{L}}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}})^{-1}\boldsymbol{y}_{\mathrm{A}}. \tag{11}$$

## V. Simulation Results

To obtain some basic information about the performances of the system, we have performed computer simulations to com-

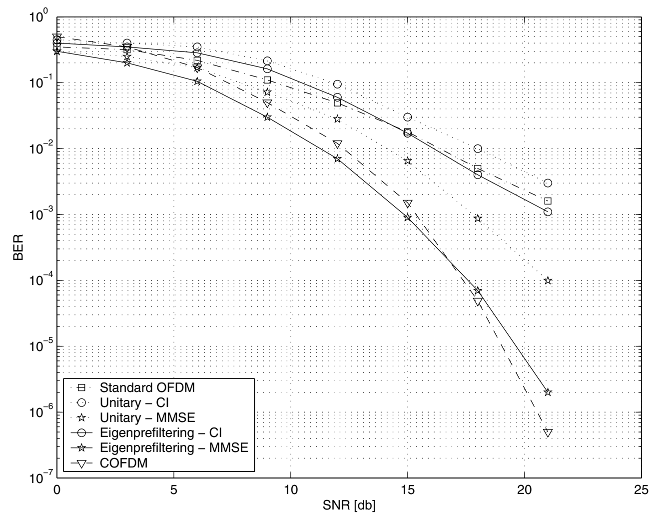


Fig. 4. Raw BER comparison for linearly prefiltered OFDM using long-term properties (eigenpreprocessing) and not (unitary matrix)

pare the newly developed scheme with the classical OFDM system and to highlight the benefits of using the long-term channel covariance matrix for transmitter processing. We have supposed perfect channel estimation at the receiver side and used the ITU channel model [8] presented in section III.

Our investigations followed two directions: one to highlight the benefits of using long-term properties at the transmitter side and the second to compare our system with one using COFDM.

To have a fair comparison we have aimed at the same bit throughput in all the compared setups. We have chosen the design parameters for our system to obtain the same rate as a convolutionally coded OFDM with code rate 1/2 punctured to 3/4 and using BPSK modulation on all the subcarriers. The power and bit allocation algorithm must assure an equivalent load of our system ($N_1 = 3/4N$ bits per OFDM symbol with the same total transmit power), constrain that is similar with the one solved by the Hughes-Hartogs algorithm presented in [10]. We have used Quadrature Amplitude Modulation (2, 4, 16, 64, 256QAM) and quantized transition power, to get closer to a real system based on receiver feedback information.

We have simulated also an equivalent linear prefiltered scheme that uses an arbitrary unitary matrix for prefiltering to put into evidence the advantages of the long-term information. In this case, too, we have used a low ranked matrix and increased the modulation order to have the same conditions and loaded all the streams with the same power and the same number of bits.

The results of the simulation are presented in Figure 3.

In the case of the random unitary prefiltering with LMMSE detection we see a faster decreasing slope of the Bit Error Rate (BER) with the Signal to Noise Ratio (SNR) compared with standard OFDM and a better performance due, only, to the spreading of the symbols on several subchannels and use of the frequency diversity as in [3], [4]. The channel inversion detection case is even worse due to the noise increase and its spreading over several symbols. The eigenprocessing system with LMMSE detection performs much better as it uses the channel eigenmodes to transmit the symbols. The decreasing slope of the BER with the SNR is much higher, as the maximum frequency diversity is used and adaptive power and bit loading optimize the transmission scheme.

Compared with convolutionally coded OFDM, we see that at low SNR our proposed system outperforms COFDM. There is a cross point at around 17dB from where COFDM shows a lower BER. The advantage of the eigenpreprocessing is the fact that it is a linear processing much easier to implement than the nonlinear coding. We see that even in these conditions the performances of the linear scheme is better than that of the nonlinear one for low SNR.

## VI. Conclusions

In this paper we have presented a linearly prefiltered OFDM scheme which profits from the long-term properties of the channel. We have showed that for wideband mobile communications systems the propagation delays represent long-term properties and for a long enough period the properties of the channel covariance matrix are directly related to the propagation delay profile.

Based on the availability of the eigenvectors of the covariance matrix at the transmitter side, via e.g. feedback, the eigenmodes of the channel can be used to transmit the symbols, resulting in maximum diversity.

To test our scheme we have compared it to an equivalent prefiltering scheme without using the long term knowledge and to a COFDM scheme using convolutional codes, providing the same throughput. We could conclude that the use of the long-term information at the transmitter brings a large performance increase, making the eigenprefiltering OFDM similar in performance as COFDM, still using only linear processing. At low SNR, the eigenpreprocessing OFDM even outperformed COFDM.

As a future work, it should be interesting to see the evolution of the cross point in the BER curve for several code rates and throughput.

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