

On the Equivalence of Prerake and Transmit Matched Filter

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Abstract

Similar to linear receive processing we assume for linear transmit processing that the transmitter completely knows the channel impulse responses to the different receivers for linear transmit processing. Contrary to linear receive processing where three basic types, namely matched filter, zero-forcing filter, and Wiener filter, are well researched and understood, for linear transmit processing only the zero-forcing variant has been identified. The prerake concept seems to be a good candidate for the transmit matched filter, since it simply moves the receive channel matched filter to the transmitter.

We will show that this intuitive system design for transmit processing is equivalent to matched filtering for receive processing.

1 Introduction

Since uplink and downlink share the same frequency band in *time division duplex* (TDD) systems the transmission in uplink and downlink can be described by the same channel impulse response, if the coherence time of the channel is small enough [1]. Due to this full reciprocity of the channel for uplink and downlink transmission the base station (BS) can estimate the channel impulse responses for downlink transmission to all mobile stations (MSs) during uplink reception. Note that this channel estimation is also necessary for uplink demodulation and detection. Therefore, we can presume that the BS knows all downlink channel impulse responses. This knowledge can be used for *transmit processing, pre-equalization, or precoding*, i. e. processing the signal before transmission to combat the deteriorating effects of the channel.

The main advantage of transmit processing is the possibility to simplify the receivers, i. e. MSs. To exploit this advantage we assume the simplest possible receiver: a filter matched to the signal waveform, i. e. the pulse shaping filter for a *time division multiple ac-*

cess (TDMA) system or the code for a *code division multiple access* (CDMA) system.

Because the transmitter (BS) has no influence on the noise at the MSs, the most intuitive approach for transmit processing is a transmission filter which removes all *intracell interference* at the MSs. Liu et. al. [2] proposed this *zero-forcing* pre-equalization in a TDD TDMA system for the single user and multiuser case, whereas Baier et. al. [3] and Joham et. al. [4] applied this concept to TDD CDMA multiuser systems. Vojčić et. al. [5] showed that zero-forcing precoding results from the *minimum mean square error* criterion for the detector signal at the receiver and Kowalewski et. al. [6] examined the influence of channel estimation and change of the channel impulse responses due to the time separation of uplink and downlink.

Esmailzadeh et. al. [7] introduced the *prerake* concept for single user systems which follows from the intuitive idea to move the channel matched filter (rake, e. g. [8]) from the receiver to the transmitter. The reasons given in [7] for this movement of the channel matched filter was the linearity of the system, the commutation property of linear convolution, and

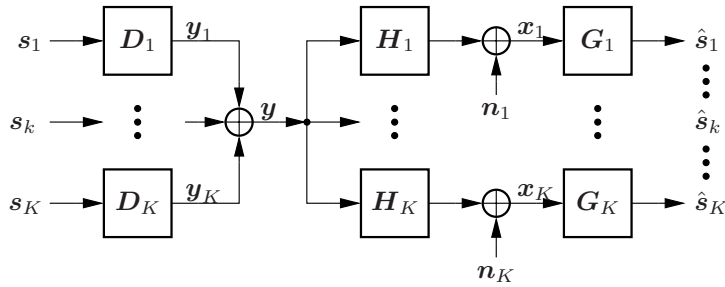


Figure 1: Downlink Receive Processing

the need for simple MSs. In [9], Esmailzadeh et. al. extended the prerake to the multiuser case where each user's signal is filtered by the respective channel matched filter before they are summed up and transmitted.

For linear receive processing, three basic filter types, *matched filter*, *zero-forcing filter*, and *Wiener filter*, are well known [10]. The matched filter tries to deal with noise, the zero-forcing filter mitigates the intracell interference, and the Wiener filter finds a trade-off between noise and intracell interference. However, for linear transmit processing, only the zero-forcing filter type has been derived by utilizing the same criterion as for receive processing, i. e. interference suppression.

In this paper, the *transmit matched filter* (TxMF) is derived by exploiting similarities between transmit and receive processing. The relationship of the transmit matched filter to the transmit zero-forcing filter (TxZF) is investigated and analogies to receive processing are shown.

After recalling the receive matched filter and the prerake concept in Section 3 and 4, respectively, we explain our reasoning by discussing the equivalence of receive noise power and transmit power in Section 5. The derivation of the transmit matched filter is given in Section 6 where we will also show that the TxMF is a generalization of the prerake.

2 System Model

Due to the slot structure of the TDD transmission signal we can collect all M_k symbols $s_k^{(m)}$ of MS k for one slot in the zero mean \mathbb{C}^{M_k} vector

$$\mathbf{s}_k = [s_k^{(0)}, \dots, s_k^{(M_k-1)}]^T, \quad k = 1, \dots, K, \quad (1)$$

with the covariance matrix $\mathbf{R}_{s_k} = \text{E}[\mathbf{s}_k \mathbf{s}_k^H] \in \mathbb{C}^{M_k \times M_k}$. For downlink receive processing, the modulation operation $\mathbf{D}_k \in \mathbb{C}^{N_c \times M_k}$ is *a priori* known to the receiver (cf. Figure 1) and the receiver has to design an appropriate filter \mathbf{G}_k to recover the desired signal \mathbf{s}_k from the channel output \mathbf{H}_k . Contrary, in downlink transmit processing systems, we try to find a linear transformation $\mathbf{P}_k \in \mathbb{C}^{N_c \times M_k}$ which maps the

symbols \mathbf{s}_k to the portion $\mathbf{y}_k \in \mathbb{C}^{N_c}$ of the transmit signal $\mathbf{y} \in \mathbb{C}^{N_c}$ dedicated to MS k (cf. Figure 2) where N_c denotes the number of chips in one slot. Thus, the resulting transmit signal for all K MSs reads as

$$\mathbf{y} = \sum_{k=1}^K \mathbf{y}_k = \sum_{k=1}^K \mathbf{P}_k \mathbf{s}_k. \quad (2)$$

As depicted in Figures 1 and 2, the transmit signal \mathbf{y} propagates over K different channels \mathbf{H}_k to the respective MS. The channel matrix $\mathbf{H}_k \in \mathbb{C}^{N_c \times N_c}$ is a convolution matrix and therefore, \mathbf{H}_k is a Toeplitz matrix (e.g. [4]). Note that the channel model can be easily extended to the multiple transmit antenna case where \mathbf{H}_k is block Toeplitz (e.g. [3, 11]). The channel output is perturbed by zero mean Gaussian noise $\mathbf{n}_k \in \mathbb{C}^{N_c}$ with the covariance matrix $\mathbf{R}_{n_k} = \text{E}[\mathbf{n}_k \mathbf{n}_k^H] \in \mathbb{C}^{N_c \times N_c}$ and the received signal \mathbf{x}_k is filtered with $\mathbf{D}_k^H \in \mathbb{C}^{M_k \times N_c}$ which is matched to the signal waveform to obtain the estimate $\hat{\mathbf{s}}_k \in \mathbb{C}^{M_k}$ of the transmitted symbols \mathbf{s}_k dedicated to MS k :

$$\hat{\mathbf{s}}_k = \mathbf{D}_k^H \mathbf{x}_k = \mathbf{D}_k^H \mathbf{H}_k \mathbf{y} + \mathbf{D}_k^H \mathbf{n}_k. \quad (3)$$

Note that the transmitter has to know the matched filtering \mathbf{D}_k^H *a priori* to be able to find an appropriate transmit filter \mathbf{P}_k .

3 Receive Matched Filter

For linear receive processing (cf. Figure 1), the receiver has to know the modulation filter $\mathbf{D}_k \in \mathbb{C}^{N_c \times M_k}$ and the channel \mathbf{H}_k to compute the receive filter $\mathbf{G}_k \in \mathbb{C}^{M_k \times N_c}$. Consequently, the transmit signal can be written as

$$\mathbf{y} = \sum_{k=1}^K \mathbf{D}_k \mathbf{s}_k \quad (4)$$

and the estimate of the transmitted symbols for MS k can be expressed as

$$\hat{\mathbf{s}}_k = \mathbf{G}_k \mathbf{H}_k \mathbf{y} + \mathbf{G}_k \mathbf{n}_k. \quad (5)$$

Different criteria are possible to design the receive filter \mathbf{G}_k , e.g. interference suppression leads to a zero-forcing constraint and minimizing the *mean square error* generates a Wiener solution for \mathbf{G}_k . However, the

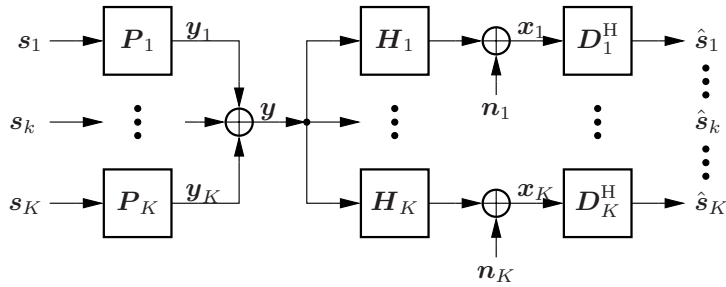


Figure 2: Downlink Transmit Processing

receive matched filter maximizes the *signal to noise ratio* (SNR) at the output of the receive filter \mathbf{G}_k . Usually, the receive matched filter is first derived for white Gaussian noise \mathbf{n}_k via Cauchy-Schwarz inequality (e. g. [12]) and then generalized to the colored noise case. We choose a different way by maximizing the signal portion in $\hat{\mathbf{s}}_k$ induced by the desired signal \mathbf{s}_k and keeping the power of the noise component $\mathbf{G}_k \mathbf{n}_k$ in $\hat{\mathbf{s}}_k$ constant:

$$\mathbf{G}_k = \arg \max_{\mathbf{G}} \gamma_k(\mathbf{G}) \text{ s. t. : } \mathbb{E} \left[\|\mathbf{G} \mathbf{n}_k\|_2^2 \right] = E_{n_k}, \quad (6)$$

where

$$\gamma_k(\mathbf{G}) = \text{Re} \left(\mathbb{E} \left[\mathbf{s}_k^H \hat{\mathbf{s}}_k \right] \right) = \text{Re} \left(\text{tr} \left(\mathbf{G} \mathbf{H}_k \mathbf{D}_k \mathbf{R}_{s_k} \right) \right) \quad (7)$$

and $\mathbb{E} \left[\|\mathbf{G} \mathbf{n}_k\|_2^2 \right] = \text{tr} \left(\mathbf{G} \mathbf{R}_{n_k} \mathbf{G}^H \right)$. The desired signal in $\hat{\mathbf{s}}_k$ is selected by correlation with \mathbf{s}_k and to end up with a $\hat{\mathbf{s}}_k$ which is in-phase with the desired signal \mathbf{s}_k we only consider the real part of the correlation. The optimization in Equation (6) leads to

$$\mathbf{G}_k = \alpha_k \mathbf{R}_{s_k} \mathbf{D}_k^H \mathbf{H}_k^H \mathbf{R}_{n_k}^{-1}, \quad (8)$$

which is the *generalized matched filter* [13] with the noise whitener $\mathbf{R}_{n_k}^{-1/2}$ and $\mathbf{R}_{n_k}^{-1} = \mathbf{R}_{n_k}^{-1/2, H} \mathbf{R}_{n_k}^{-1/2}$, the channel matched filter \mathbf{H}_k^H , and the waveform matched filter \mathbf{D}_k^H . Since we designed a matched filter for the whole transmitted sequence \mathbf{s}_k for MS k , also the correlations between the symbols are taken into account. The factor α_k is just a scaling factor and can be dropped.

4 Prerake

If the noise \mathbf{n}_k at the receiver input and the transmitted symbols \mathbf{s}_k are white, i. e. the respective covariance matrices \mathbf{R}_{n_k} and \mathbf{R}_{s_k} are weighted identity matrices, the receive matched filter is a channel matched filter (rake) \mathbf{H}_k^H followed by a correlator \mathbf{D}_k^H . Esmailzadeh et. al. [7] proposed to move the channel matched filter to the transmitting BS to simplify the MS. Therefore, the resulting transmit filter for prerake processing reads as

$$\mathbf{P}_k = \beta_k \mathbf{H}_k^H \mathbf{D}_k. \quad (9)$$

The transmitted symbols \mathbf{s}_k are first modulated by \mathbf{D}_k and then filtered by the relocated receive matched filter \mathbf{H}_k^H . The validity of this transition was explained by the commutation property of linear convolution and in [7] an analytical proof for the equality of rake and prerake for a single user system is given, if the factor β_k is chosen to keep the transmitted energy per symbol constant, hence,

$$\beta_k = \frac{N_c}{\sqrt{\text{tr} \left(\mathbf{H}_k^H \mathbf{H}_k \right)}}. \quad (10)$$

However, the rake and prerake systems exhibit different behaviors for the multi-user case. When orthogonal waveforms $\mathbf{D}_k, k = 1, \dots, K$, are utilized, i. e. $\mathbf{D}_k^H \mathbf{D}_\ell = \mathbf{0}$ for $k \neq \ell$, the prerake is worse than the rake, but the prerake can deal better with non-orthogonal waveforms than the rake [9].

5 Similarities of Linear Receive and Transmit Processing

To understand the similarities of receive and transmit processing we first have to discuss the sources of “noise” in a cellular mobile communications system. One type of noise is the thermal noise of the system components, e. g. amplifiers. In this work we focus on the second source of “noise”, namely intercell interference, i. e. signals from neighboring cells. Note that intercell interference can only be reduced by the BSs and MSs of the neighboring cells by decreasing transmit power.

The BS in the cell of interest is no source of noise at the MSs in the same cell, the BS generates intracell interference whose structure is known to the receiving MS, since it propagates through the same channel as the desired signal. Thus, intracell interference has the same dependence upon the channel as the desired signal.

The received signal can be divided into desired signal, intracell interference, and noise. As the receiver is passive, it cannot “avoid” intracell interference and noise. Since intracell interference propagates over the

same channel as the desired signal, receive processing has more information about intracell interference than about noise which makes it easier to deal with intracell interference than with noise. Usually, the receiver could remove intracell interference completely, whereas it can only minimize the effect of noise. On the other hand, the transmitting BS is active. It has to avoid intracell interference and noise for its cell and for the neighboring cells, respectively. The noise or intercell interference for the neighboring cells can only be reduced by decreasing transmit power, because the BS has no information about the impact of its transmitted signal on the neighboring cells, whereas intracell interference could be removed completely by suitable transmit processing. Therefore, we can see the **duality of noise** for receive processing and **transmit power** for transmit processing. Both have to be minimized, cannot be zero, and are a measure for the interactions between neighboring cells.

To demonstrate this duality, we recall the zero-forcing variant of receive processing which completely removes intracell interference and minimizes the noise power in the detection signal. Accordingly, transmit processing with a zero-forcing constraint has to minimize transmit power [3, 6, 4]:

$$\mathbf{P} = \arg \min_{\tilde{\mathbf{P}}} \mathbb{E} \left[\|\mathbf{y}\|_2^2 \right] \quad \text{s. t.: } \mathbf{D}^H \mathbf{H} \tilde{\mathbf{P}} = \mathbf{1}, \quad (11)$$

where $\mathbb{E} \left[\|\mathbf{y}\|_2^2 \right] = \text{tr} \left(\tilde{\mathbf{P}} \mathbf{R}_s \tilde{\mathbf{P}}^H \right)$, $\mathbf{P} = [\mathbf{P}_1, \dots, \mathbf{P}_K]$, and $\mathbf{R}_s = \text{diag}(\mathbf{R}_{s_1}, \dots, \mathbf{R}_{s_K})$. \mathbf{D} is defined in a similar way as \mathbf{R}_s and $\mathbf{H} = [\mathbf{H}_1^T, \dots, \mathbf{H}_K^T]^T$. The resulting TxZF can be written as

$$\mathbf{P} = \mathbf{H}^H \mathbf{D} \mathbf{B}^{-1}. \quad (12)$$

The transmitted symbols $\mathbf{s}_k, k = 1, \dots, K$ are transformed by \mathbf{B}^{-1} (e.g. [4]) before they are modulated and filtered by \mathbf{D}_k and \mathbf{H}_k^H , respectively.

6 Transmit Matched Filter

The receive matched filter maximizes the signal portion due to the desired signal at the filter output (cf. Section 3) and additionally keeps the noise power constant. Consequently, the TxMF also maximizes the desired signal portion γ_k (cf. Equation 6) at the output of the receive filter \mathbf{D}_k^H , but has to constrain the transmit power (cf. Section 5). Therefore, we have to introduce a constraint which sets the transmit power of every transmitted signal \mathbf{y}_k to a MS specific value E_{s_k} :

$$\mathbf{P}_k = \arg \max_{\mathbf{P}} \gamma_k(\mathbf{P}), \quad \text{s. t.: } \mathbb{E} \left[\|\mathbf{y}_k\|_2^2 \right] = E_{s_k}, \quad (13)$$

where

$$\gamma_k(\mathbf{P}) = \text{Re} \left(\text{tr} \left(\mathbf{D}_k^H \mathbf{H}_k \mathbf{P} \mathbf{R}_{s_k} \right) \right) \quad (14)$$

and $\mathbb{E} \left[\|\mathbf{y}_k\|_2^2 \right] = \text{tr} \left(\mathbf{P} \mathbf{R}_{s_k} \mathbf{P}^H \right)$. The resulting transmit filter of the optimization in Equation (13) can be expressed as

$$\mathbf{P}_k = \sqrt{\frac{E_{s_k}}{\text{tr} \left(\mathbf{H}_k^H \mathbf{D}_k \mathbf{R}_{s_k} \mathbf{D}_k^H \mathbf{H}_k \right)}} \mathbf{H}_k^H \mathbf{D}_k \quad (15)$$

which is very similar to the prerake in Equation (9). The different factor β_k of the TxMF compared to the one of the prerake in Equation (10) follows from the optimization for the whole transmitted sequence \mathbf{s}_k in (13).

The TxMF in Equation (15) includes a channel matched filter \mathbf{H}_k^H like the receive matched filter (cf. Equation 8). Moreover, the modulation filter \mathbf{D}_k is a transmit filter matched to the receive filter \mathbf{D}_k^H . Equation (12) reveals the analogy of transmit and receive zero-forcing filter. The receive zero-forcing filter is a matched filter bank followed by a transformation which removes intracell interference (e.g. [10]), whereas the TxZF first transforms the transmitted symbols with \mathbf{B}^{-1} and then performs matched filtering by $\mathbf{H}^H \mathbf{D}$.

7 Comparison of TxMF & TxZF

Two systems have been utilized to compare the TxMF and the TxZF.

The first system is an artificial one with white Gaussian symbols $s_k^{(m)}$, i.e. \mathbf{s}_k is complex multivariate normal distributed with a covariance matrix $\mathbf{R}_{s_k} = \sigma_{s_k}^2 \mathbf{1}$. Only one user is active ($K = 1$), $M_k = 100$ symbols per slot are transmitted, and the results are the mean of 1000 different scenarios. The channel is modelled by a matrix \mathbf{H}_1 whose elements are complex normal distributed and have a variance $\sigma_h^2 = 1$. Moreover, the receiver performs no signal processing, i.e. $\mathbf{D}_1^H = \mathbf{1}$, and the noise \mathbf{n}_1 is Gaussian with a covariance matrix $\mathbf{R}_{n_1} = \sigma_n^2 \mathbf{1}$ (cf. Figure 2). To be able to compute the *mean square error* (MSE) $\mathbb{E} \left[\|\mathbf{s}_1 - \hat{\mathbf{s}}_1\|_2^2 \right]$ the received signal \mathbf{x}_1 has to be weighted with the scalar $\mathbf{x}_1^H \mathbf{s} / \|\mathbf{x}_1\|_2^2$ to correct the amplitude which has been distorted during transmission. The resulting MSEs versus the ratio of the transmitted power $E_{\text{tr}} = \mathbb{E} \left[\|\mathbf{y}\|_2^2 \right]$ and the noise variance σ_n^2 for the TxMF and the TxZF are shown in Figure 3. Obviously, the TxMF outperforms the TxZF for low transmit power and the TxZF is superior for low noise power, because the TxMF only maximizes the received power and neglects intracell interference and the TxZF only removes intracell interference. This system is especially difficult for the TxZF, because it has no additional degrees of freedom to minimize the transmit power. Nevertheless, the result in Figure 3 motivates to find a *transmit*

Wiener filter (TxWF) which has a similar behavior as the receive Wiener filter, i.e. for low SNR the TxWF is equivalent to the TxMF and for high SNR it converges to the TxZF. Hence, the TxWF would lead to a MSE like the TxMF for high noise power and decreasing MSE for rising transmit power as the TxZF.

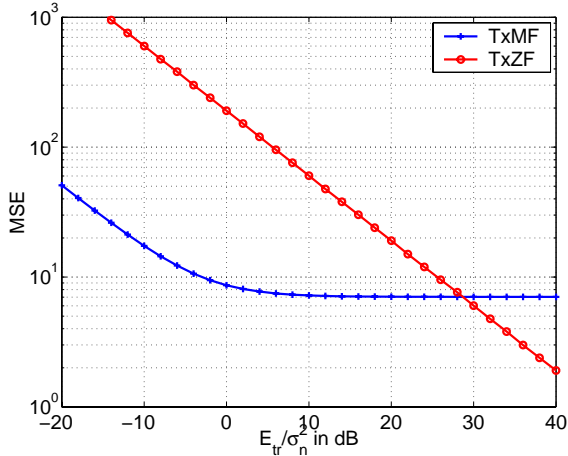


Figure 3: First system: MSE of TxMF and TxZF

The second example is a TDD-CDMA system where all users employ orthogonal codes with a spreading factor of 4 and $M_k = 64$ QPSK symbols per slot and user are transmitted, thus, one slot consists of $N_c = 256$ chips. Since CDMA is assumed, the k -th MS decorrelates the received signal \mathbf{x}_k with \mathbf{D}_k^H which is block diagonal. The channel impulse response has 5 taps whose variances are $\sigma_h^2 = 1$. Therefore, the channel matrices \mathbf{H}_k are Toeplitz or block Toeplitz for one or two transmit antennas at the BS, respectively. Again, the transmitted symbols \mathbf{s}_k and

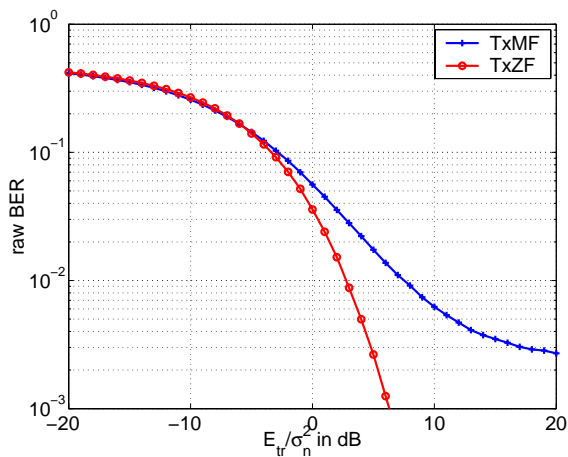


Figure 4: Second system: BER of TxMF and TxZF for one antenna, single user

the noise \mathbf{n}_k are white, i.e. the respective covariance matrices $\mathbf{R}_{\mathbf{s}_k}$ and $\mathbf{R}_{\mathbf{n}_k}$ are weighted identity matrices.

First, we assumed that the BS is equipped with one antenna. Figures 4 and 5 show the results with $K = 1$ and $K = 4$ active users, respectively. We can observe that the raw *bit error rate* (BER) of the TxMF saturates for decreasing noise power in both cases and in Figure 5 we can find a cross-over point of TxMF and TxZF like in Figure 3. Again, this result motivates to search for a TxWF which combines the advantages of TxMF and TxZF. The results also question the ne-

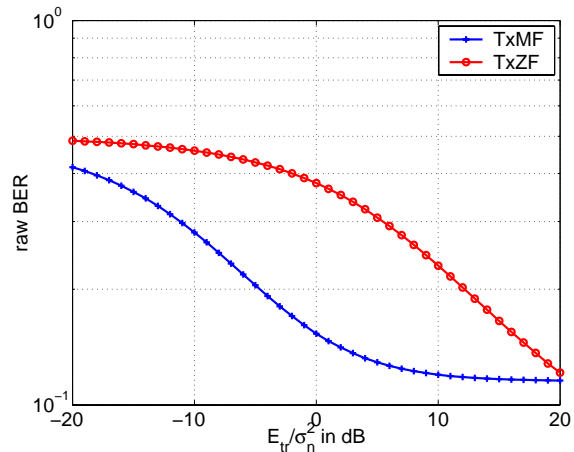


Figure 5: Second system: BER of TxMF and TxZF for one antenna, $K = 4$ users

cessity of the TxZF, if a raw or uncoded BER of 10 % is needed as for speech users. However, for data users which need a raw BER of 0.1 %, the TxMF is not applicable, because it saturates at higher BER values. When two transmit antennas are deployed at the BS, we get the results in Figures 6 and 7 for $K = 1$ and $K = 4$ users, respectively. The additional degrees

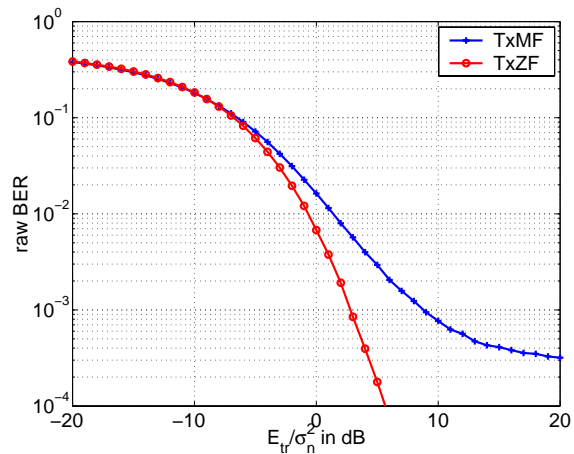


Figure 6: Second system: BER of TxMF and TxZF for two antennas, single user

of freedom help the TxZF to minimize the transmit power and the raw BER of the TxMF saturates at lower values compared to the single transmit antenna

case. Note that the necessary computational complexity for the TxMF is much lower than for the TxZF, because the inversion of the matrix \mathbf{B} is not needed whose dimension increases linearly with the number of transmitted symbols. The TxZF transforms the

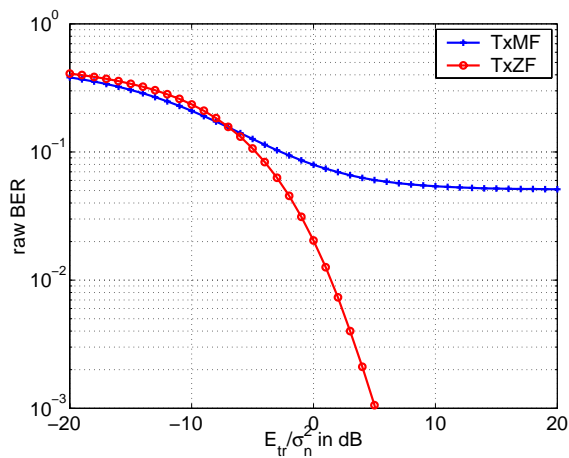


Figure 7: Second system: BER of TxMF and TxZF for two antennas, $K = 4$ users

transmitted symbols with \mathbf{B}^{-1} in addition to the operations of the TxMF.

8 Conclusions

We first have discussed the duality of receive noise power and transmit power. This duality helped to understand the optimization necessary to calculate the TxZF. The TxMF was derived with a maximization of the received power of the desired signal where the transmit power was kept constant. This optimization is equivalent to the one used to derive the receive matched filter, only the received noise power was replaced by the transmit power according to the duality between receive and transmit processing. The resulting TxMF is the prerake which is obtained by the transition of the channel matched filter from the receiver to the transmitter. Similar to receive processing where the zero-forcing filter is a matched filter bank followed by intracell interference cancellation, the TxMF is part of the TxZF. The TxZF first transforms the transmitted symbols and then passes them through a TxMF bank. The simulation results motivate to search for the third type of transmit filter, i. e. TxWF, which avoids the saturation of performance for high transmit power observed for the TxMF.

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