Comparison Of Two DOA Tracking Implementations For SDMA

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ABSTRACT

Utilizing adaptive antenna arrays at the base stations of next generation mobile communication systems has been proposed as a promising approach to meet future requirements, e.g. spatio-temporal filtering techniques benefit from the detailed knowledge of the directional channel parameters. Unfortunately, in parameter estimation the computation of the signal subspace often turns out to be the most time-consuming part. However, the computational complexity of the subspace estimation can be significantly reduced by means of tracking. To this end, we propose a new technique which is merely based on the projector representation of the subspace. The presented results are based on the DSP implementation of two algorithms for tracking the azimuthal and elevation angles of impinging wavefronts in an alternating mobile communications system.

I. INTRODUCTION

Third and fourth generation mobile radio communications systems will be characterized by a growing number of more sophisticated services including multi-media applications [1, 2, 3]. This leads to increased data rates as well as the asymmetric distribution of the traffic concerning the uplink and downlink connection. Adaptive antennas can be deployed to meet the future high spectral and quality requirements of the demanding 3G/4G mobile communication systems. Adaptive antennas exploit the inherent spatial diversity structure of the mobile radio channel, provide antenna gain, and enable interference suppression [4].

The performance of spatio-temporal downlink processing is substantially based on channel estimation including the estimation of spatial characteristics of the mobile radio channel. Furthermore channel estimation has to meet real-time requirements in order to cope with the time-varying scenarios in mobile communications systems. To this end, the use of tracking algorithms often offers a crucial alternative to standard methods in parameter estimation and system identification.

This work is especially devoted to tracking algorithms for the estimation of signal subspaces. Whereas the tracking of the orthonormal basis of the signal subspace is subject of a number of papers in the past, the immediate tracking of the unique projection matrix of the signal subspace has not been proposed yet. In [5] the author presents a thorough overview of most of the adaptive algorithms for subspace tracking. Thus, the subspace tracking techniques can be grouped into three families: (1) classical eigenvalue decomposition (EVD) and singular value decomposition (SVD) methods modified for use in adaptive processing, (2) variations of rank-one updating algorithms, and (3) algorithms that consider the eigenvalue/singular value decomposition as a constrained or unconstrained optimization problem which can be adaptively processed by means of gradient based methods.

In this work, we extend the variety of approaches by a tracking method which directly operates on the unique projection matrix onto the signal subspace instead of tracking its orthonormal basis. This new projection based algorithm is motivated by the PAST algorithm [5]. The PAST belongs to the type of methods which consider the EVD/SVD as an optimization problem. It has recently been subject of a real-time implementation of algorithms for a GSM base station [6]. Consequently, the presented simulation results provide a comparison between both algorithms in terms of computational complexity and estimation error.

II. SYSTEM STRUCTURE

The performance evaluation of the proposed algorithm is based on the DOA (direction of arrival) estimation by means of ESPRIT [7]. Hereby, the signal subspace estimation has been of particular interest, since this step of ESPRIT-like methods generally turns out to be the most time-consuming part. In order to get close to real systems, a DSP served as the hardware processor. Implementation is done on the C6701 32-bit floating point DSP from TEXAS INSTRUMENTS with a clock rate of 150 MHz [8]. The supported very long instruction word technique (VLIW) enables the DSP to address its multiple hardware units simultaneously and to carry out up to eight instructions per clock cycle. The source code is completely written in C and represents an implementation of 2D-Unitary ESPRIT [9]. Thus, two parameters of each of the impinging wavefronts at a uniform rectangular antenna array (URA) can be resolved, the azimuth φ and the elevation ϑ angles of the DOAs.

Since the DSP is not part of a real communication system, the input data of 2D-Unitary ESPRIT is generated by MATLAB. Thereby the transmission medium is supposed to be isotropic and linear. The noise is modeled as a complex, zero-mean white Gaussian process. Under the assumptions of narrowband and farfield signals,

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Table 1: Numerical methods used in the Unitary ESPRIT implementation

Step of Unitary ESPRIT	Numerical method
Signal Subspace Estimation	Modified Jacobi method, PAST, or projector tracking
Solution of Invariance Equations	Least Squares algorithm
Eigenvalue Calculation + Pairing	QR algorithm

the complex baseband measurements of the kth antenna element can be described as

$$x_k(t) = \sum_{i=1}^{d} \xi_k \cdot s_i(t) e^{-j\frac{2\pi}{\lambda_c}\langle e(\varphi_i,\vartheta_i), r_k\rangle} + n_k(t), \quad (1)$$

$1 \leq k \leq M$, where the (φ_i, ϑ_i) refer to the DOA parameters of the ith wavefront and $e(\varphi_i, \vartheta_i)$ and r_k denote the unit vector steering in direction of (φ_i, ϑ_i), and the coordinate vector of the kth antenna element within the array of size M. The $n_k(t)$ is the additive white gaussian noise and ξ_k is the complex response of the kth sensor element. The d indicates the number of impinging wavefronts. The $s_i(t)$ are the complex envelopes of the impinging signals, which are BPSK-modulated with no oversampling. Calculating the x_k for all M sensor elements and collecting the data over N snapshots yields the data matrix X serving as input for Unitary ESPRIT.
2D-Unitary ESPRIT itself can be divided into three major steps [9]. At first, the real-valued signal subspace is estimated. Subsequently, two independent invariance equations are formed and solved. Finally, the eigenvalues of the solution matrices of the second step lead to the desired DOA parameters.
Table 1 lists the numerical methods used when implementing each step of Unitary ESPRIT. Subject to performance comparison are the three signal subspace estimation schemes. In order to save runtime, the Jacobi Method has been modified [10]. The basic idea is to concentrate the data relevant for the signal subspace in a preprocessing step and then perform the Jacobi method with a smaller matrix.

III. PROJECTOR TRACKING

In the PAST algorithm (projector approximation subspace tracking), the cost function

$$J(W) = E\{\|x - WW^T x\|_2^2\}, \quad (2)$$

$W \in \mathbb{R}^{M\times d}$, with the matrix-valued argument W is considered [5, 11]. The x and the R_{xx} denote the receive vector of the antenna array and the covariance matrix of X, respectively. It can be shown that $W_{\text{opt}} = E_S Q$ is a global minimizer of $J(W)$. In that case, the column space of E_S spans the desired real-valued signal subspace [5]. The $Q \in \mathbb{R}^{d\times d}$ denotes an arbitrary orthogonal matrix.
Now consider a matrix P_W which defines the projection onto the real-valued signal subspace: $P_W = E_S E_S^T = E_S Q Q^T E_S^T = E_S Q (E_S Q)^T = W_{\text{opt}} W_{\text{opt}}^T$. This motivates to replace WW^T in $J(W)$ by the matrix argument $P \in \mathbb{R}^{M\times M}$ yielding a new cost function in terms of P [12, 13]

$$J(P) = E\{\|x - Px\|_2^2\}, \quad (3)$$

$P \in \mathbb{R}^{M\times M}$. It can be shown that $P_{\text{opt}} = E_S E_S^T$ is a global minimizer of $J(P)$ with the first d columns of P_{opt} spanning the desired signal subspace [12]. Computing P_{opt} leads to a constrained minimization problem, as P is a projector and has to maintain its rank d. Unlike in PAST, a brute force approach to the tracking of the projection matrix by applying a matrix calculus to the scalar objective function with matrix argument fails since the non-full rank property of the projector prohibits gradient based approaches [12].
An update algorithm which maintains the rank of P is the update via rotations of the form

$$P_k = Q P_{k-1} Q^T, \quad (4)$$

where Q is an orthogonal matrix and k denotes the index of the iteration step [13]. The data of each slot serves for a single iteration step. Therefore, k also denotes the slot index.
The Q can now be expressed as a product of elementary rotation matrices $Q_{pq}(\phi_{pq})$. Each of them is completely characterized by a single parameter, the elementary rotation angle ϕ_{pq}, with ϕ being the vector of all elementary rotation angles ϕ_{pq}.
The optimization problem now is to find a ϕ such that $J(Q(\phi) P Q^T(\phi))$ attains a global minimum. Note that $J(P)$ is parameterized by ϕ. This advantageous parameterization is the main difference to the PAST algorithm. The solution to this optimization problem leads to a gradient decent method: for each slot k, the gradient $\nabla_\phi J$ of $J(\phi)$ with respect to ϕ is calculated. Consequently, the calculation of the incremental rotation angles $\Delta\phi_{pq}$ can be written as [13]

$$\Delta\phi_{pq} = -2\delta \left(P_{k-1}^{(q)} R_{xx,k,(p)} - P_{k-1}^{(p)} R_{xx,k,(q)} \right), \quad (5)$$

with $(\bullet)^{(i)}$ denoting the ith row vector and $(\bullet)_{(i)}$ denoting the ith column vector of a matrix. The update of P can be computed with

$$P_k = Q_{pq}(\Delta\phi_{pq}) P_{k-1} Q_{pq}(-\Delta\phi_{pq}), \quad (6)$$

$\forall \;\; 1 \leq p \leq M-1, p+1 \leq q \leq M$, (cf. [13]). The initialization of P is equally to a matrix which is sparse and only contains entries (e.g. ones) in its first d diagonal elements and zeros elsewhere. Alternatively, the

initial computation of $\boldsymbol{E}_S$ is provided by means of the eigenvalue decomposition based on the received input data from the first slot and $\boldsymbol{P}_0 = \boldsymbol{E}_S \boldsymbol{E}_S^T$.
Although the projector tracking within Unitary ESPRIT requires only real-valued data processing, the approach is also applicable in complex-valued scenarios by considering rescaling factors [13].

IV. PERFORMANCE EVALUATION

The performance of the 2D-Unitary ESPRIT implementation is evaluated by means of cycle counts during program execution. Figure 1 shows the total execution time of 2D-Unitary ESPRIT and its decomposition into the particular steps in a scenario with $d = 4$ impinging wavefronts and $N = 140$ snapshots subject to the number of antenna elements M in the URA. It is evident that the signal subspace estimation is the most time-consuming part, especially when large URAs are used. In the following the results of the execution time comparison of the three signal subspace estimation methods, namely the modified Jacobi method, PAST, and projector tracking, are presented. Table 2 lists an assessment of the computational complexity of the schemes subject to M, N, and d. Note that PAST directly operates on the real-valued data matrix, whereas the two other methods need the estimation of the covariance matrix $\widehat{\boldsymbol{R}_{xx}}$. Therefore the number of calculations needed for the computation of $\widehat{\boldsymbol{R}_{xx}}$ has to be added to the execution times of Jacobi and projector tracking in order to get comparable results. The Figures 2 and 3 illustrate the results of DSP simulations. Execution times of the subspace estimation subject to M are shown in Figure 2, subject to N in Figure 3, respectively. As easily can be seen, the tracking schemes are able to reduce the computation time of the signal subspace significantly compared to the modified Jacobi method. With respect to each other, either PAST or projector tracking achieves a better performance, depending on the number of antenna elements and snapshots, respectively. PAST works faster for small N or large M, whereas projector tracking achieves a better result for large N or small M. This is due to the different dependency of the computational complexity on the parameters M and N (c.f. Table 2).

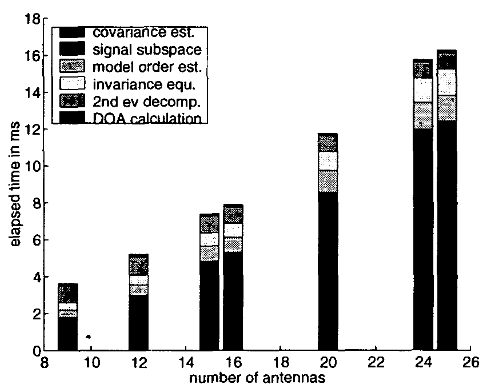

Figure 1: Execution time of 2D-Unitary ESPRIT and its composition subject to the number of antenna elements M

Table 2: Computational complexity of the signal subspace estimation schemes

Est. scheme	real add.	real mult.
Calc. of $\widehat{\boldsymbol{R}_{xx}}$	$(2N-1)\frac{M(M+1)}{2}$	$NM(M+1)$
Mod. Jacobi	$\mathcal{O}(M^3)$	$\mathcal{O}(M^3)$
PAST	$6MNd$	$6MNd$
Proj. tracking	$3M^2(M-1)$	$3M^2(M-1)$

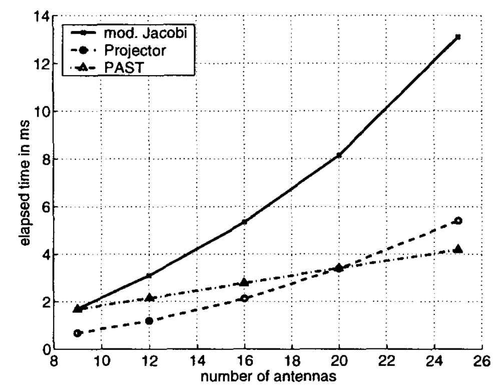

Figure 2: Execution time of the modified Jacobi method, PAST and projector tracking, depending on the number of antennas M

In Table 3 the scenario used for performance evaluation is presented.
With respect to the accuracy of the estimated DOAs, the RMSE has been taken as a measure of accuracy. It reads

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum \left\| \begin{bmatrix} \boldsymbol{\vartheta}_{\text{est}} \\ \boldsymbol{\varphi}_{\text{est}} \end{bmatrix} - \begin{bmatrix} \boldsymbol{\vartheta} \\ \boldsymbol{\varphi} \end{bmatrix} \right\|^2}, \quad (7)$$

where n denotes the number of averaging trials. The $\boldsymbol{\vartheta}_{\text{est}}$ $\{\boldsymbol{\varphi}_{\text{est}}\}$ and $\boldsymbol{\vartheta}$ $\{\boldsymbol{\varphi}\}$ are the estimated and the actual vectors of angles of the instantaneous DOAs, respectively. In most scenarios (different SNR and change of DOAs per slot) the RMSE difference of projector tracking and PAST is less than 0.1, as the right diagram of Figure 4 shows. Compared to subspace estimation via eigenvalue decomposition and without the use of forgetting factor techniques, both tracking schemes achieve more accurate results in scenarios with low SNRs (cf. left diagram of Figure 4). This is due to the fact that both methods inherently perform averaging as the subspace is tracked. Because noise is modeled as zero-mean, its influence is reduced. Note that tracking accuracy decreases in scenarios with rapid changes of the DOAs from slot to slot. However, in both schemes accuracy depends to a great extend on a carefully chosen step size.

Table 3: Scenario for comparing the signal subspace estimation schemes

Scenario			
URA with $M_y = M_x = 4$ *(variable)*			
$N = 140$ snapshots *(variable)*			
$d = 3$ impinging wavefronts			
100 slots			
10 averaging trials *(100 × 10 in Figure 4)*			
change of the DOAs from slot to slot: 0.1° *(0.0° ... 0.2° in Figure 4)*			
from (initial slot):	$\vartheta = [15° \quad 40° \quad 75°]$	$\varphi = [-40° \quad 0° \quad 40°]$	
to (last slot):	$\vartheta = [25° \quad 50° \quad 65°]$	$\varphi = [-30° \quad -10° \quad 50°]$	
BPSK modulation with noise-modeling as additional white gaussian			
SNR = 9 dB *(−5 dB ... 30 dB in Figure 4)*			
no spatial smoothing			

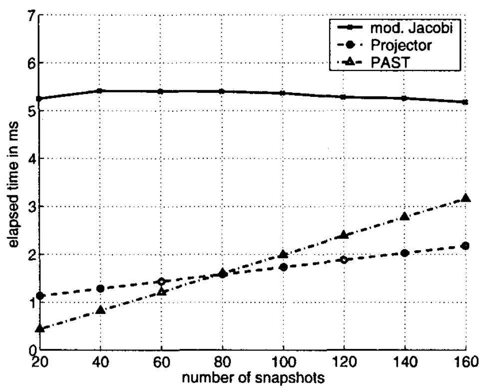

Figure 3: Execution time of the modified Jacobi method, PAST and projector tracking, depending on the number of snapshots N

V. CONCLUSIONS

We presented projector tracking as a new approach for subspace tracking. In order to evaluate the performance of projector tracking, we successfully integrated it in a 2D-Unitary ESPRIT implementation on a DSP. From our results we conclude that

- the signal subspace estimation is the most time-consuming part of parameter estimation
- the investigated tracking schemes PAST and projector tracking significantly reduce the computational complexity of subspace estimation
- depending on the scenario either PAST or projector tracking achieves the better performance
- in realistic scenarios with $N \gg M$ projector tracking works faster compared to PAST at comparable accuracy.

A result of the latest research has been that the projector tracking scheme can be modified for processing the antenna array data directly instead of operating on the covariance matrix. In that case, each snapshot receiver vector $\boldsymbol{x}$ increments/decrements the parameter of a single elementary rotation matrix separately and leads to an update of the projector matrix $\boldsymbol{P}$. The computation of the incremental rotation angle then reads as

$$\Delta\phi_{pq} = -2\delta\left(x_p \boldsymbol{P}^{(q)} - x_q \boldsymbol{P}^{(p)}\right)\boldsymbol{x} \qquad (8)$$

where the $x_{p(q)}$ denotes the $p(q)$th component of the receive vector. In each update step, only one elementary rotation is performed. The complexity for processing one slot of data consisting of N snapshots amounts to $8NM$ real multiplications and $4NM + 2N(M-1)$ real additions. Figure 5 shows the number of floating point operations (FLOPs) required in projector update with direct direct data processing divided by the FLOPs necessary when working with the covariance matrix in per cent subject to M and N. Especially for small N and/or large M, this modification promises a significant performance gain, however, the direct data processing version of projector tracking increases the estimation errors by up to a factor of 3, especially in noisy and fast changing scenarios.

Note that the snapshot-wise data processing is an inherent property of the PAST algorithm too[5], which unfortunately can not be exploited for the bursty data in slotwise structured data channels.

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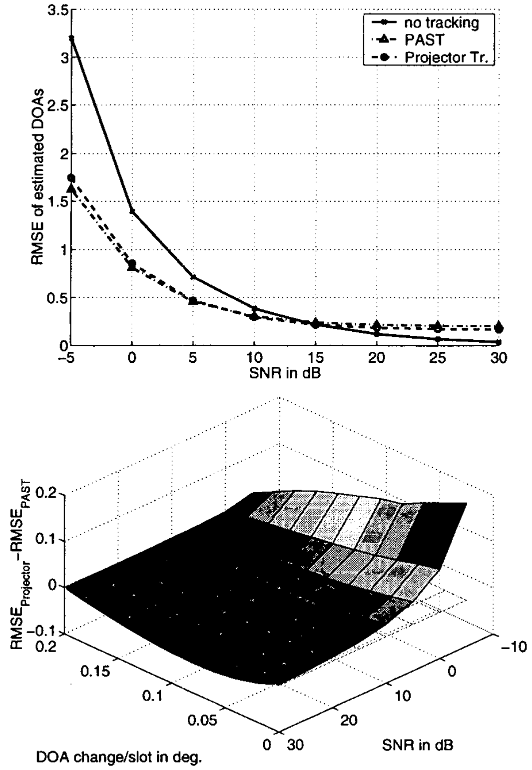

Figure 4: RMSE comparison of modified Jacobi EVD, projector tracking, and PAST

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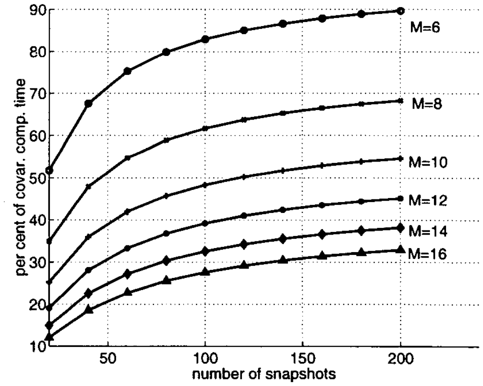

Figure 5: FLOPs of projector tracking with snapshot-wise data processing in per cent of the FLOPs needed for the covariance-based approach of projector tracking

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