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CHANNEL ESTIMATION AND EQUALIZATION FOR GSM WITH MULTIPLE ANTENNAS

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ABSTRACT

We present a complete space-time demodulation scheme for the Global System for Mobile Communications (GSM) assuming two antennas at the receiver. The scheme includes multipath channel estimation using a blind method (Cross-Relation (CR) with Maximum In-band Energy (MIE) method), two semi-blind methods (combination of the training sequence with either the CR method alone or the CR with MIE method), and a method based on the training sequence alone. For the equalization we apply a zero-forcing equalizer, which equalizes only over the band of the Gaussian Minimum Shift Keying (GMSK) signal. The demodulation of the equalized GMSK signal is done by an implementation of the Viterbi-Algorithm employing decision metrics based on the amplitude of the equalized signal. The simulation results show that the semi-blind scheme which is based on the maximum in-band energy method is superior to the other three methods and therefore outperforms the approach of solely exploiting the training sequence. The success of these methods shows that it's not necessary to apply the linear approximation of the GMSK signal in the case of multiple antennas to effect blind and semi-blind channel identification. Thus, these multiple antenna based algorithms are applicable to non-linear modulation schemes as well as standard linear modulation schemes.

1. INTRODUCTION

Blind equalization methods [1, 2] usually presume a linear modulation representation [3] of the desired signal. We use channel estimation schemes, which only need the information of the bandwidth of the signal and the position of the training symbols in the burst, if training based methods are used. Although we use a GMSK signal, which is non-linear, we are able to make a sample-spaced estimation of the multipath channel with an estimate of the maximum delay spread determined through a-priori experimental tesing. We then design equalization filters using this estimation to obtain the transmitted GMSK signal, which is demodulated by a Viterbi Algorithm demodulator [5]. This paper is organized as follows. In Section 2 we describe the GMSK modulation scheme and the channel model. Then we focus on the maximum in band energy (MIE) method [2] as an example for blind channel estimation in Section 3. This method is based on the cross relation method [1]. Both blind channel estimation schemes (MIE and cross relation) are used in combination with the training symbols in our semi-blind algorithms. The methods which are based on the training sequence (LS method and the semi-blind methods) are explained in Section 4. In Section 5 and 6 we present a method to equalize a signal inside a band and the implementation of the Viterbi Algorithm, respectively. Finally, Section 7 shows the simulation results for the BER with respect to the SNR.

2. MODULATION AND CHANNEL MODEL

The bit sequence is modulated by using an integrated pulse response of a gaussian lowpass filter as the phase function for the Frequency Shift Keying (FSK). The resulting non-linear GMSK baseband signal is

$$y(t) = \exp\left[j\frac{\pi}{2} \sum_{k=-\infty}^{\infty} s_k \psi(t - kT_0) \right]$$

where $s_k \in \{-1, +1\}$ is the transmitted binary data, T_0 is the symbol time, and

$$\psi(t) = \int_{-\infty}^{t} q(\tau - 2T_0) d\tau$$

is the phase function. The integrand $q(t)$ is the convolution of the gaussian pulse

$$g(t) = B\sqrt{\frac{2\pi}{\ln 2}} \exp\left[-\frac{2\pi^2 B^2 t^2}{\ln 2} \right]$$

and the rectangular NRZ pulse

$$\mathrm{rect}(t) = \begin{cases} 1/T_0, & |t| \le T_0/2 \\ 0, & |t| > T_0/2. \end{cases}$$

Note that we set the GMSK parameter $BT_0 = 0.3$ (like in GSM systems). Hence, we set the time-shift of $q(t)$ in the integral to $2T_0$, because this yields an approximately causal pulse $\psi(t)$. Figures 1 and 2 show plots of the phase function and the GMSK spectrum for $BT_0 = 0.3$, respectively. For later purposes we assume that the phase function is zero for times smaller than 0 and one for times greater than $4T_0$. However, for the implementation of the Viterbi Algorithm, we assume that $\psi(t)$ only changes between T_0 and $3T_0$.

The resulting GMSK signal $y(t)$ is then transmitted over an unknown static multipath channel $h_i(t)$ which may be expressed as

$$h_i(t) = \sum_{p=1}^{P} g_{ip} \delta(t - \tau_p),$$

where g_{ip} and τ_p are the complex gain and delay, respectively, and $\delta(t)$ is the Dirac delta function. The delay τ_p is uniformly spread over the interval $[0, \tau_{max}]$. Thus, the received signal of the i-th antenna element is

$$x_i(t) = h_i(t) * y(t) + w_i(t), \quad i = 1, \ldots, M,$$

where $w_i(t)$ is the additive white gaussian noise at the i-th antenna. The received signal of N_s symbols is sampled at l times per symbol time T_0, so the data sequence used in the following development

$$x_i[n] = x_i(\frac{T_0}{l} n), \quad n = 0, \ldots, lN_s - 1.$$

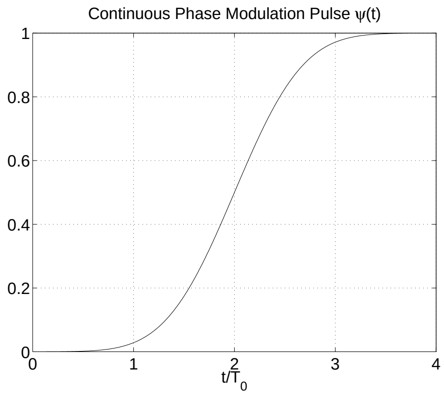

Figure 1. Phase Function $\psi(t)$.

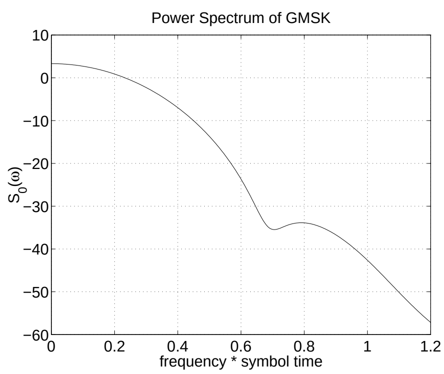

Figure 2. Power Spectrum S_0

3. MAXIMUM IN BAND ENERGY METHOD FOR BLIND CHANNEL ESTIMATION

In [1] Xu, Liu, Tong, and Kailath presented the cross relation method for blind channel estimation. The cross relation (CR) gives the following equation for the noiseless case

$$h_i(t) * x_k(t) = h_k(t) * x_i(t),$$

where '*' denotes linear convolution. We try to approximate the unknown multipath channel h_i with

$$\hat{h}_i(t) = \sum_{m=0}^{N_h-1} \hat{h}_i[m]\delta(t - \frac{T_0}{l}m),$$

hence we can replace h_i by $\hat{h}_i$ in the CR. We set $N_h = \tau_{max} l/T_0$ in order to cover the whole delay spread. By using the sampled versions of the received signals we obtain

$$\sum_{m=0}^{N_h-1} \hat{h}_i[m]x_k[n-m] - \sum_{m=0}^{N_h-1} \hat{h}_k[m]x_i[n-m] = 0, \quad \forall n$$

We assume that only two antennas $i = 1, 2$ are used, since all the methods shown in this paper can be easily extended for more than two antennas. For the case of two antennas we can derive the following equation system

$$[\mathbf{X}_2, -\mathbf{X}_1]\hat{\mathbf{h}} = \mathbf{0}$$

with

$$\mathbf{X}_i = \begin{bmatrix} x_i[N_h-1] & x_i[N_h-2] & \dots & x_i[0] \\ x_i[N_h] & x_i[N_h-1] & \dots & x_i[1] \\ \vdots & \vdots & \ddots & \vdots \\ x_i[lN_s-1] & x_i[lN_s-2] & \dots & x_i[lN_s-N_h] \end{bmatrix}$$

and

$$\hat{\mathbf{h}} = \left[\hat{h}_1[0], \dots, \hat{h}_1[N_h-1], \hat{h}_2[0], \dots, \hat{h}_2[N_h-1]\right]^T.$$

The solution for this overdetermined equation system is the "smallest" right singular vector of $[\mathbf{X}_2, -\mathbf{X}_1]$.

The cross relation method has a big disadvantage: the cross relation is also fulfilled if the estimated channels have highpass characteristics. The received signal is zero outside the bandwidth of the transmitted signal. If the $\hat{h}_i(t)$ are highpass, then the convolution will also be zero inside the bandwidth, thus the difference is zero over the whole band.

To avoid these bad estimates Zoltowski and Tseng [2] proposed the Maximum in Band Energy method (MIE). The idea is to use a solution out of the nullspace of $[\mathbf{X}_2, -\mathbf{X}_1]$, which maximizes the energy inside the bandwidth of the transmitted signal to avert trivial highpass estimates.

We collect the "smallest" right singular vectors of $[\mathbf{X}_2, -\mathbf{X}_1]$ in the matrix $\mathbf{V}$, then $\hat{\mathbf{h}}$ can be expressed as $\hat{\mathbf{h}} = \mathbf{V}\boldsymbol{\beta}$. To find a proper linear combination $\boldsymbol{\beta}$ we maximize the in-band energy of $\hat{\mathbf{h}}$ with

$$\max_{\beta} \frac{\sum_i \text{in-band energy of channel } i}{\sum_i \text{total energy of channel } i}.$$

This formulation leads to the following expression

$$\max_{\boldsymbol{\beta}} \frac{\boldsymbol{\beta}^H \mathbf{V}^H \mathbf{B} \mathbf{V} \boldsymbol{\beta}}{\boldsymbol{\beta}^H \boldsymbol{\beta}}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{F}_{GSM} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{GSM} \end{bmatrix},$$

where

$$\mathbf{F}_{GSM} = \int_{-\omega_0}^{\omega_0} S_0(\omega)\mathbf{f}_h(\omega)\mathbf{f}_h^H(\omega)d\omega,$$

$\mathbf{f}_h(\omega) = [1, e^{j\omega}, \dots, e^{j(N_h-1)\omega}]^T$, $S_0(\omega)$ is the power frequency spectrum of the GMSK signal, and ω_0 is the 20 dB bandwidth ($\omega_0 \approx 0.5/T_0$) of $S_0(\omega)$ (see Figure 2, $S_0(\omega)$ is computed numerically [4]).

In order to maximize the in-band energy, we choose $\boldsymbol{\beta}$ as the "largest" right singular vector of $\mathbf{V}^H\mathbf{B}\mathbf{V}$. The desired channel estimation is then $\hat{\mathbf{h}} = \mathbf{V}\boldsymbol{\beta}$. Note that the blind channel estimation has the ambiguity of one unknown scalar, because one can multiply the right singular vectors with a scalar and divide the left singular vectors by the same scalar and the SVD will remain correct.

4. TRAINING SEQUENCE BASED CHANNEL ESTIMATION METHODS

Because the GSM standard includes a 26 bit long training sequence in the middle of the burst, we investigated the usage of methods which take advantage of the training bits. The first method relies only on the training bits and solves a least squares problem (LS method). The other two methods are semi-blind methods which combine the knowledge of the training symbols with the cross relation method and with the maximum in band energy method.

If we use the $N_t = 26$ training bits, we have to take into account that the GMSK signal has "memory". All previously sent bits have influence on the current phase. However, after four symbol times the phase portion according to a particular bit has the constant value $\pm\pi/2$ (see Figure 1). Thus, we do not use the samples of the first three symbol times of the training sequence, because we do not know the values of the prior bits, hence our actual training sequence is three bits shorter. So we know the transmitted sequence and the received signal (to within a $e^{jn\pi/2}$ ambiguity) of the remaining $N_t - 3$ bits of the training sequence. That leads to the following least squares equation system:

$$\mathbf{Y}\hat{\mathbf{h}} = \begin{bmatrix} \mathbf{Y}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_t \end{bmatrix} \hat{\mathbf{h}} = \mathbf{x}_t,$$

where the known transmitted data is collected in the convolution matrix

$$\mathbf{Y}_t = \begin{bmatrix} y_t[N_h+3l-1] & y_t[N_h+3l-2] & \dots & y_t[3l] \\ y_t[N_h+3l] & y_t[N_h+3l-1] & \dots & y_t[3l+1] \\ \vdots & \vdots & \ddots & \vdots \\ y_t[N_t l-1] & y_t[N_t l-2] & \dots & y_t[N_t l-N_h] \end{bmatrix}$$

and the received samples are put in the vector $\mathbf{x}_t = [\mathbf{x}_{t1}^T, \mathbf{x}_{t2}^T]^T$, where

$$\mathbf{x}_{ti} = [x_i[N_{trpos}+3l], \dots, x_i[N_{trpos}+N_t l - N_h]]^T.$$

N_{trpos} is the number of the first received sample that belongs to the training sequence. The known GMSK modulated training sequence can be computed as follows:

$$y_t[n] = \exp\left(j\frac{\pi}{2} \sum_{m=0}^{N_t-1} t_m \psi\left(\frac{T_0}{l}n - mT_0\right)\right),$$

where $t_m \in \{-1,+1\}$ are the known training bits. Note that we have again an ambiguity of the resulting channel estimation, because we calculate the reference GMSK signal $y_t[n]$ without the unknown prior bits and each unknown bit is equal to a multiplication with $\pm j$. Hence, our estimation has to be multiplied with an unknown scalar out of $\{+1,-1,+j,-j\}$.

The solution of the least squares equation is the multiplication with the pseudoinverse of $\mathbf{Y}$, thus

$$\hat{\mathbf{h}} = \mathbf{Y}^\dagger \mathbf{x}_t = (\mathbf{Y}^H\mathbf{Y})^{-1}\mathbf{Y}^H\mathbf{x}_t.$$

The LS method can be extended by the cross relation method as presented by Li and Ding in [3]. We just combine the equations of the CR with the LS method and end up with the equation of the semi-blind algorithm

$$\mathbf{X}_s\hat{\mathbf{h}} = \begin{bmatrix} \mathbf{Y}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_t \\ \mathbf{X}_2 & -\mathbf{X}_1 \end{bmatrix} \hat{\mathbf{h}} = \mathbf{x}_s.$$

We can use all the samples of one burst for the matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ since the knowledge of the corresponding bits is not necessary. The vector $\mathbf{x}_s$ is the same as $\mathbf{x}_t$, besides the appropriate number of zeros has to be inserted at the end. We again solve the equation by using the pseudoinverse, thus $\hat{\mathbf{h}} = \mathbf{X}_s^\dagger \mathbf{x}_s$. Note again, that the solution for $\hat{\mathbf{h}}$ is not unique, the unknown scalar is taken out of $\{+1,-1,+j,-j\}$.

Because the simple combination of the CR with the training symbols doesn't improve the channel estimation (cf. Section 7), we decided to choose a more sophisticated approach. Joham, Utschick, Nossek, and Zoltowski [6] developed a semi-blind method which reduces the solution space by the cross relation method and then computes the channel estimation by solving the least squares equation of the training sequence in this reduced space. We here extend this approach by further reducing the solution space with the maximum in band energy method (MIE, [2]) in addition to the CR.

Recall from Section 3 the cross relation for two receiver antennas can be written

$$[\mathbf{X}_2, -\mathbf{X}_1]\hat{\mathbf{h}} = \mathbf{0}.$$

The MIE method exploits the knowledge that $\hat{\mathbf{h}}$ lies in the nullspace of $\mathbf{X}_b = [\mathbf{X}_2, -\mathbf{X}_1]$ and gives a channel estimation which is a linear combination of the "smallest" right singular vectors (collected in the matrix $\mathbf{V}$) of $\mathbf{X}_b$ which maximizes the energy of the channel estimation inside the bandwidth of the transmitted signal. The solution is the "largest" right singular vector of $\mathbf{X}_{mie} = \mathbf{V}^H\mathbf{B}\mathbf{V}$ (cf. Section 3).

In our new semi-blind method we first compute the "smallest" right singular vectors of $\mathbf{X}_b$ and put them into $\mathbf{V}$, and constrain the solution of the channel estimation to the range of $\mathbf{V}$. This is similar to the method in [6]. To further improve the solution space, we follow the MIE approach and reduce the solution space a second time. To this end, we collect the "largest" right singular vectors of $\mathbf{X}_{mie}$ in the matrix $\mathbf{V}_{mie}$, and assume the wanted channel estimation lies in the range of $\mathbf{V}_{mie}$. Thus, we can write

$$\hat{\mathbf{h}} = \mathbf{V}_{mie}\mathbf{c}.$$

The least squares equation system which we encounter when we exploit the training sequence may then be expressed as follows:

$$\mathbf{Y}\hat{\mathbf{h}} = \mathbf{Y}\mathbf{V}_{mie}\mathbf{c} = \mathbf{Y}_{mie}\mathbf{c} = \mathbf{x}_t.$$

The solution is the pseudoinverse of $\mathbf{Y}_{mie}$. Hence, we get the semi-blind channel estimation

$$\hat{\mathbf{h}} = \mathbf{V}_{mie}\mathbf{Y}_{mie}^\dagger\mathbf{x}_t = \mathbf{V}_{mie}(\mathbf{Y}_{mie}^H\mathbf{Y}_{mie})^{-1}\mathbf{Y}_{mie}^H\mathbf{x}_t.$$

Again, the resulting $\hat{\mathbf{h}}$ has an ambiguity. Since we dropped the first three training symbols and the previously transmitted data bits are unknown, the estimation has to be multiplied with an unknown scalar out of $\{+1,-1,+j,-j\}$.

5. IN BAND EQUALIZATION

For the linear Equalization we use the zero forcing multichannel equalizer. If the two FIR channels do not share a common spectral null, two equalizing FIR filters, $g_1[n]$ and $g_2[n]$, each of length N_g, may be determined using the following optimization problem, in order to equalize the signal and suppress the noise:

$$\min_{\mathbf{g}_1,\mathbf{g}_2} E\left\{\left|\mathbf{g}_1^H\mathbf{n}_1 + \mathbf{g}_2^H\mathbf{n}_2\right|^2\right\} = \mathbf{g}_1^H\mathbf{g}_1 + \mathbf{g}_2^H\mathbf{g}_2,$$

$$\text{subject to: } \hat{h}_1[n] * g_1[n] + \hat{h}_2 * g_2[n] = \delta[n-D],$$

where $\mathbf{g}_1$ and $\mathbf{g}_2$ are $N_g \times 1$ vectors containing the equalizer coefficients and D is some delay. The $N_g \times 1$ vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are the noise samples at each antenna. Since we do not use a matched filter, we can assume, that the autocorrelation matrix of the noise $E\left\{\mathbf{n}_1\mathbf{n}_1^H\right\} = E\left\{\mathbf{n}_2\mathbf{n}_2^H\right\} = \sigma^2\mathbf{I}$ (white noise). By defining

$$\mathbf{g} = \left[\mathbf{g}_1^T, \mathbf{g}_2^T\right]^T \quad \text{and} \quad \hat{\mathbf{H}} = \left[\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2\right],$$

where the convolution matrix $\hat{\mathbf{H}}_i$ is shown below

$$\hat{\mathbf{H}}_i = \begin{bmatrix} \hat{h}_i[0] & 0 & \dots & 0 \\ \hat{h}_i[1] & \hat{h}_i[0] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \hat{h}_i[N_h-1] & \hat{h}_i[N_h-2] \\ 0 & \dots & 0 & \hat{h}_i[N_h-1] \end{bmatrix},$$

we get the solution for the equalizer filters

$$\mathbf{g} = \hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H)^{-1}\boldsymbol{\delta},$$

where $\boldsymbol{\delta}$ is a zero vector except a one at the D^{th} position. Since we oversample very much (i.e., $l = 5$ for the blind MIE method), the prementioned zero forcing equalizer has the problem that the channel estimation is very poor outside the bandwidth. Therefore, the noise gain is large, which leads to bad bit error rates after the demodulation. Hence, we use the following expression to equalize the signal only over the bandwidth:

$$\min_{G_1,G_2} \int_{-\omega_0}^{\omega_0} S_0(\omega) \left| \sum_{i=1}^{2} \hat{H}_i(\omega) G_i(\omega) - e^{-j\omega D} \right|^2 d\omega,$$

To get more convenient expressions we replace $\hat{H}_i(\omega)$ and $G_i(\omega)$ by using

$$\hat{H}_i(\omega) = \mathbf{f}_h^H(\omega)\hat{\mathbf{h}}_i, \quad G_i(\omega) = \mathbf{f}_g^H(\omega)\mathbf{g}_i,$$

where

$$\mathbf{f}_h(\omega) = \left[1, e^{j\omega}, \ldots, e^{j(N_h-1)\omega}\right]^T, \ \mathbf{f}_g(\omega) = \left[1, \ldots, e^{j(N_g-1)\omega}\right]^T,$$

yielding

$$\min_{\mathbf{g}_1,\mathbf{g}_2} \left\{ \int_{-\omega_0}^{\omega_0} S_0(\omega) \left| \sum_{i=1}^{2} \mathbf{f}_h^H(\omega)\hat{\mathbf{h}}_i \mathbf{f}_g^H(\omega)\mathbf{g}_i - e^{-j\omega D} \right|^2 d\omega + \gamma \mathbf{g}^H \mathbf{g} \right\}.$$

The $N_h \times 1$ vector $\hat{\mathbf{h}}_i$ and the $N_g \times 1$ vector $\mathbf{g}_i$ contain the coefficients of the channel estimation and the equalization filters for each antenna, respectively. Note that we have added a term weighted by γ in order to reduce the noise gain.

After some calculation and the concatenation of $\mathbf{g}_1$ and $\mathbf{g}_2$ in $\mathbf{g}$ we end up with:

$$\min_{\mathbf{g}} \mathbf{g}^H \mathbf{F}_{hh} \mathbf{g} - \mathbf{f}_{hh}^H \mathbf{g} - \mathbf{g}^H \mathbf{f}_{hh} + \gamma \mathbf{g}^H \mathbf{g} + \text{const},$$

where

$$\mathbf{F}_{hh} = \begin{bmatrix} \mathbf{F}_{h_1h_1} & \mathbf{F}_{h_1h_2} \\ \mathbf{F}_{h_2h_1} & \mathbf{F}_{h_2h_2} \end{bmatrix}$$

and

$$\mathbf{f}_{hh} = \begin{bmatrix} \mathbf{F}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_s \end{bmatrix} \begin{bmatrix} \hat{\mathbf{h}}_1^* \\ \hat{\mathbf{h}}_2^* \end{bmatrix},$$

where $(\cdot)^*$ denotes the complex conjugate. With the definition of

$$\tilde{f}_s[n] = \int_{-\omega_0}^{\omega_0} e^{jn\omega} S_0(\omega) d\omega$$

the representation of $\mathbf{F}_s$ and $\mathbf{F}_{h_ih_k}$ is simplified as follows:

$$\mathbf{F}_s = \begin{bmatrix} \tilde{f}_s[-D] & \tilde{f}_s[1-D] & \ldots & \tilde{f}_s[N_h-1-D] \\ \tilde{f}_s[1-D] & \tilde{f}_s[2-D] & \ldots & \tilde{f}_s[N_h-D] \\ \vdots & \vdots & & \vdots \\ \tilde{f}_s[N_g-1-D] & \tilde{f}_s[N_g-d] & \ldots & \tilde{f}_s[N_g+N_h-2-D] \end{bmatrix}$$

and

$$\mathbf{F}_{h_ih_k} = \sum_{n=0}^{N_h-1} \sum_{m=0}^{N_h-1} \hat{h}_k[n]\hat{h}_i^*[m]\mathbf{F}[m-n],$$

where

$$\mathbf{F}[n] = \begin{bmatrix} \tilde{f}_s[n] & \tilde{f}_s[n-1] & \ldots & \tilde{f}_s[n-N_g+1] \\ \tilde{f}_s[n+1] & \tilde{f}_s[n] & \ldots & \tilde{f}_s[n-N_g+2] \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{f}_s[n+N_g-1] & \tilde{f}_s[n+N_g-2] & \ldots & \tilde{f}_s[n] \end{bmatrix}.$$

The solution for the equalization only over the bandwidth is then

$$\mathbf{g} = (\mathbf{F}_{hh} + \gamma \mathbf{I})^{-1} \mathbf{f}_{hh}.$$

6. VITERBI ALGORITHM

After we have equalized the received signal, we can demodulate the GMSK signal with a Viterbi Algorithm (VA) [5] based method to retrieve the transmitted bit sequence. As in the VA we have some states at each time step corresponding to the previous bit sequence. Our idea is to assign each possible combination of the three most recent bits to one state, thus we have eight states. From one time step to the next we drop the oldest bit and add a new one. Hence, only two transitions leave from each state at time step k (the two possible values of the newest bit) and only two transitions arrive at one state at time step $k+1$ (the two possible values of the bit which is dropped). Therefore, we have 16 transitions between two succeeding time steps. We can code each of the states per time step with the according three bits and the transitions are coded with four bits. Figure 3 depicts the structure of the trellis.

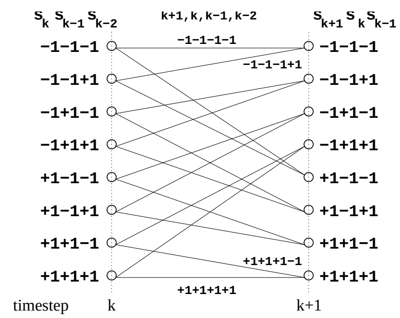

Figure 3. Viterbi Algorithm trellis

Since we use four bits for each transition and we pretend, that the phase function $\psi(t)$ (see Figure 1) only changes over two symbol times, it's sufficient to observe samples of only three symbol times, because outside this time period the phase changes also depend on other bits. One could claim that the GMSK signal has "memory", since all prior transmitted bits have an influence on the current value of the phase of the GMSK signal, and therefore we have to consider all previously sent bits. However, the "old" bits change the phase only by a multiple of $\pi/2$, thus the signal is just multiplied by a scalar out of $\{+1, +j, -1, -j\}$. To avoid this problem we multiply the received sequence with $\pm j$ according to the bit which was dropped after the last transition in the trellis.

To get a measure for the probability of each transition we make a least squares fit of the received samples over the three symbol times to a reference sequence, since we pretend to know the signal for these three symbol times. The reference sequences can be calculated before the VA is started and put in a list. The scalar α for which the difference $\mathbf{r} - \alpha\mathbf{y}$ is minimized, where $\mathbf{r}$ and $\mathbf{y}$ are the reference sequence and the equalized signal, respectively, is a value with a similar meaning as the logarithm of the probability of the transition in the trellis. As the VA minimizes the sum of $-\ln P$ over all transitions, we have to maximize the sum of all the transitions up to the states at the time instant k. So we take the incoming transition with the greater sum. After having decoded the N_s symbols of one burst, we encounter eight bit sequences, one for each state of the last time step. We then choose the state (bit sequence) with the greatest sum (greatest probability).

In the case of training sequence based channel estimation there still exists an unknown phase which results in an unknown scalar out of $\{+1, -1, +j, -j\}$. We observed that if the reference sequence is multiplied by j, but the unknown scalar is 1, then the algorithm gives a wrong result. But if we multiply the reference sequence by -1, then the resulting bit sequence is correct. Hence, we multiply the received signal with 1 and j and run two "parallel" VAs to solve this problem. At the end we take the bit sequence with the greater sum (greater probability).

The blind method delivers a channel estimation with a completely unknown phase. It's impossible to find the unknown phase value, but it's possible to reduce the problem to the same four discrete values as for the training sequence methods. First, we do a least squares fit to the reference sequences (each one is three symbol times long) as it was described for the decision process of our VA-implementation. For every timestep we choose the greatest α_i (which minimizes $\mathbf{r} - \alpha_i \mathbf{y}_i$) and compute $u = \sqrt{\left(\sum_{i=1}^{N_s} (-1)^{i-1} \alpha_i^2\right) / N_s}$, which is an estimate of the scalar to multiply the equalized signal by to derotate it.

7. MATLAB SIMULATIONS

In our simulations we used a channel with $P = 10$ unknown multipaths. The bit sequence and all channel parameters were chosen randomly for each Monte Carlo run except $g_{11} = 1$ and $\tau_1 = 0$. We assumed that the delay spread τ_{max} is about $10\mu s$, thus we set $\tau_{max} = 3T_0$ since $T_0 = 3.69\mu s$. The received signal $x(t)$ was sampled $l = 3$ times per T_0 except for the MIE, there we used $l = 3, 4, 5$. We exploited the whole GSM burst for the blind and semi-blind methods, therefore $N_s = 142$. The length N_h of the channel estimation $\hat{\mathbf{h}}_i$ was set to $3l$ to cover the whole delay spread. In the equalization algorithm we used $N_g = 1.5N_h$, $D = N_h$, and $\gamma = 1$.

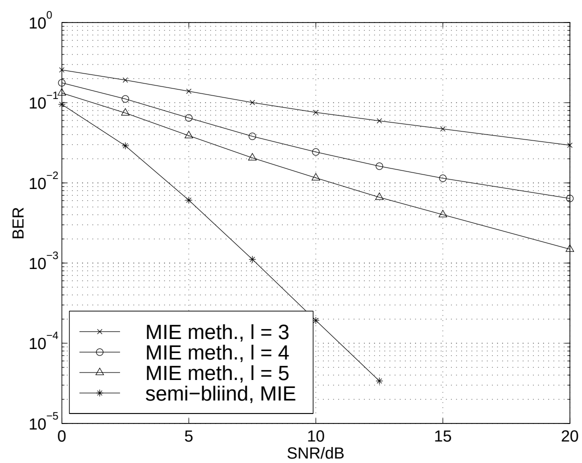

Figure 4. Maximum In Band Energy Method

Figure 4 shows the bit error rate (BER) for the MIE method of Zoltowski et al. [2]. The plots are the mean of 10000 Monte Carlo runs where we used an oversampling factor $l = 3, 4, 5$. The performance of the method improves steadily with increasing l, because the maximization inside the bandwidth is easier for the algorithm then. The MIE method is compared to the semi-blind method which combines the MIE method with the training sequence.

Figure 5 depicts the results for the channel estimation algorithms based on the training bits. As a reference we also show the BER for the AWGN channel. In this case no channel estimation and equalization is necessary, and the demodulation is performed by the Viterbi algorithm. The semi-blind approach of Li et. al. [3] is worse than the algorithm based only on the training data for low SNR. The explanation lies in the combination of the CR and the training data in one equation system. Because a least squares solution is used, the algorithm tries to reduce the error of both the CR and the training data based equation system. Thus, the performance can't be increased for low SNR since the CR is very sensitive to noise. We also show the results for

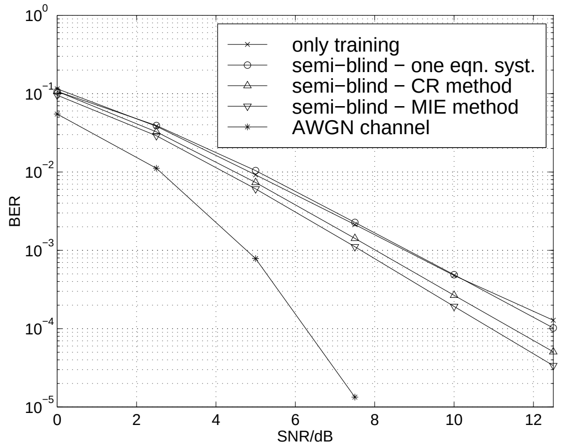

Figure 5. Training Sequence Based Methods

the semi-blind method of Joham et al. [6] which does not employ the MIE principle. Since the solution space is reduced by the cross relation method, this approach improves the LS method (only training symbols) for SNR values larger than 2.5 dB. The new semi-blind method presented in this paper performs better than any of the other methods shown. The explanation lies in the second step of the MIE method which averts bad channel estimates. We observe that the new semi-blind method has the same BER at a SNR lower than 9 dB as the LS method has at 10 dB. Thus, we gain more than 1 dB at a SNR of 10 dB.

8. CONCLUSION

A complete channel estimation and equalization scheme for GSM which doesn't use a linearized representation of the GMSK signal was presented. A blind, two semi-blind, and a training data algorithm were compared. The semi-blind algorithm which combines exploitation of the training sequence with the maximum in band energy method yielded improved performance relative the channel estimation based on the training sequence alone.

9. ACKNOWLEDGEMENT

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