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ORIGINALARBEITEN/ORIGINALS

Advanced method of including housing stiffness into calculation of gear systems

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Abstract

One of the central goals during the design of helical gear systems is the achievement of a well-distributed contact load in the gear mesh. An equal load distribution is a key factor for a high load carrying capacity, the economic use of materials and a long lifetime. Mesh misalignment can be caused by tooth deflections, manufacturing deviations or elastic deformation of the shaft-bearing system and the gearbox housing. Those deformations have to be taken into account during the design process of adequate tooth-flank geometry. Elastic deformations of gearbox housings can be significant, especially in the case of automotive applications with aluminium cases. This paper presents an advanced method of including housing stiffness into the calculation of gear systems. A validation of the approach is carried out by comparing the calculated deformations with measurements of a static test rig of a hypoid gearbox.

Many calculation programs offer the opportunity to analyse the deformation behaviour of the shaft-bearing-housing system. Most of the components in these programs are described by analytic approaches. However, components that are geometrically more complex, like the housing or planet carriers cannot be represented as easily as that by analytic expressions. There are several alternatives to take into account the elasticity of those objects. One way is to model the stiffness of the bores using imported stiffness matrices. These matrices contain the elasticity of the bores itself as well as crossover influences between the bearings. The reduced stiffness matrices may be the result of a static reduction of the geometry using the finite element method (FEM). As state of the art, the reduction is mostly carried out at the centre points of the bearing bores. The proposed advanced method uses the static reduction of geometries on several points at the bores, distributed over the circumference. This approach offers a more detailed modelling of the elastic behaviour of complex geometries within the analytic deformation calculation of gear systems. To validate the advanced approach, the calculation results of the elastic deflections of the shaft-bearing-housing system is compared with measurements of a static test rig. In the course of these comparisons, the influence of different modelling methods of gearbox housings on the accuracy of the calculation results is discussed.

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Erweiterte Methode zur Einbeziehung von Gehäusesteifigkeiten in die Berechnung von Getriebesystemen

Zusammenfassung

Eines der zentralen Ziele bei der Auslegung von Stirnradgetrieben ist die Erzielung einer gleichmäßigen Lastverteilung im Zahneingriff. Eine gleichmäßige Lastverteilung ist ein Schlüsselfaktor für eine hohe Tragfähigkeit, den wirtschaftlichen Einsatz von Materialien und eine lange Lebensdauer. Abweichungen im Zahneingriff können durch Zahnverformung, Fertigungsabweichungen oder elastische Verformungen des Wellenlagersystems und des Getriebegehäuses verursacht werden. Diese Verformungen müssen bei der Auslegung einer geeigneten Zahnflankengeometrie berücksichtigt werden. Elastische Verformungen von Getriebegehäusen können erheblich sein, insbesondere bei Automobilanwendungen mit Aluminiumgehäusen. In diesem Beitrag wird eine fortschrittliche Methode zur Einbeziehung der Gehäusesteifigkeit in die Berechnung von Getriebesystemen vorgestellt. Eine Validierung des Ansatzes erfolgt durch den Vergleich der berechneten Verformungen mit Messungen an einem statischen Prüfstand eines Hypoidgetriebes.

Viele Getriebeberechnungsprogramme bieten die Möglichkeit, das Verformungsverhalten des Systems Welle-Lager-Gehäuse zu analysieren. Die meisten Komponenten in diesen Programmen werden durch analytische Ansätze beschrieben. Geometrisch komplexere Bauteile wie das Gehäuse oder die Planetenträger können jedoch nicht so einfach durch analytische Ausdrücke dargestellt werden. Es gibt mehrere Alternativen, um die Elastizität dieser Objekte zu berücksichtigen. Eine Möglichkeit besteht darin, die Steifigkeit der Bohrungen durch reduzierte Steifigkeitsmatrizen zu modellieren. Diese Matrizen enthalten sowohl die Elastizität der Bohrungen selbst als auch Quereinflüsse zwischen den Lagern. Die reduzierten Steifigkeitsmatrizen können das Ergebnis einer statischen Reduktion der Geometrie mittels der Finite-Elemente-Methode (FEM) sein. Als Stand der Technik wird die Reduktion meist an den Mittelpunkten der Lagerbohrungen durchgeführt. Die vorgeschlagene fortschrittliche Methode nutzt die statische Reduktion von Geometrien an mehreren Punkten der Bohrungen, die über den Umfang verteilt sind. Dieser Ansatz bietet eine detailliertere Modellierung des elastischen Verhaltens komplexer Geometrien im Rahmen der analytischen Verformungsberechnung von Zahnradsystemen. Zur Validierung des fortschrittlichen Ansatzes werden die Berechnungsergebnisse der elastischen Verformungen des Welle-Lager-Gehäuse-Systems mit Messungen eines statischen Prüfstandes verglichen. Im Zuge dieser Vergleiche wird auch der Einfluss unterschiedlicher Modellierungsmethoden von Getriebegehäusen auf die Genauigkeit der Berechnungsergebnisse diskutiert.

1 Introduction

Lightweight designs of gearbox components play an important role during the development of more efficient systems in the field of drive technology. Due to the large individual volume, gearbox housings offer a high potential to save weight, which is why they are often made of cast aluminium alloys in connection with a geometrically optimised structure. However, this may result in relatively soft housing structures and greater housing deflections under load [8]. Especially in applications with extreme load spectrums, like wind turbines, aircraft engines and marine propulsions, the casing deformations can reach the magnitude of the shaft bending or torsion [11]. Deflections of the bearing points due to housing elasticity lead to a deviation of the shaft position in the tooth contact from the nominal shaft position. These deflections cause an uneven load distribution in the gear mesh if they are not taken into account during the design process for the micro geometry. However, one key factor for reaching a high load factor, a long lifetime and a better acoustical behaviour is the achievement of a well-distributed contact load. There are several computer programs for calculating the deformations of the shaft-bearing system, such as RIKOR [10] and ROMAXDesigner [1, 2, 5, 6]. Harrison, Douglas et al. [6] investigate the effect of transmission housing flexibility on the gear mesh misalignment and transmission error. The study shows that the inclusion of the housing has a high influence on the gear tooth load distribution. They model the elasticity of the casing in ROMAXDesigner by coupling the shaft-bearing system with imported reduced stiffness matrices. Coultate [1, 2] describes an investigation of the impact of the gearbox housing and carrier on wind-turbine gearbox durability. He also describes the high impact of the model detail on the gear mesh misalignment in the sun-gear-to-planet-gear mesh. Kim et al. [7] are investigating the effect of dynamic stiffnesses of mechanical components on gear mesh misalignment. They are using a reduced stiffness matrix of the housing including dynamic stiffnesses along with the gear mesh frequency. They found that those dynamic stiffnesses can lead to different results in the design results than just using the static stiffnesses. Neubauer and Weinberger [10] present a new mechanical approach to computing the deformation behaviour of the shaft-bearing-housing system and the gear meshes in one system of equations in the program RIKOR. Weinberger and Glenk [12] develop a method to include reduced stiffness matrices of housings and planet carriers into the system. The centre point of the bearings are the reference points for the static condensation of the housing. Daffner, Otto et al. [3] validate the elastic deformations calculated by RIKOR with deformation measurements of a hypoid test gearbox. They use the described method to model the gearbox housing with reduced stiffness matrices. The study shows a higher accuracy in comparison between the calculation results and the measurements with consideration of the reduced stiffness matrix.

The goal of this study is to extend the method of including the housing flexibility into the calculation of gearbox systems in RIKOR. Instead of one point at the bearing centre point, the static reduction is carried out at several points at the drillings, distributed over their circumference. An iterative procedure searches for tensile forces between the housing bore and the bearing ring and deactivates them. Since in reality there are no tensile forces between the outer bearing ring and the bearing seat surface, a more accurate modelling of the elastic housing behaviour in RIKOR is expected. Through the advanced modelling method, an ovalization of the bores can be taken into account, which was not the case in the previous method. Furthermore, the proposed approach is validated by comparisons between the calculated deformation behaviour in RIKOR and the measurement results of the static hypoid test rig of Daffner, Otto et al. [3].

2 FEM model

Fig. 1 shows the model of the hypoid gearbox housing with four bearing seats. In the previous method, the centre points of the bearing bores act as masternodes for the static reduction [4] of the geometry. Rigid surface constraints (RBE2), which are represented by red lines in the picture, couple the slavenodes on the bearing seat surface with the master degrees of freedom (DOF). RBE2-elements as coupling between the housing structure and master nodes showed better performance than force constrains (RBE3) in comparison of calculated and measured deformations for gearbox housings in [12].

With the number of bearings $n_{bearings}$ and six DOFs (three translational and three rotational) at each masternode, a matrix with the dimension $m \times m$ and

$$m = 6 \cdot n_{\text{bearings}} \quad (1)$$

is the result of the reduction. That matrix can be included into the equation system of RIKOR to represent the housing elasticity. However, this method allows tensile forces between the outer ring and the bearing seats which are not possible in reality. A discretisation of the bearing seat with multiple *masternodes* at each bearing is one way of modelling the load-sharing between the outer ring and the seat more realistically. The reduction points are equally distributed over the circumference on the surface and are coupled with the nodes on the seat section of the housing. The dimension of the reduced stiffness matrix increases with the number of discretisation points n_{discr} to $\overline{m} \times \overline{m}$ with

$$\overline{m} = 6 \cdot n_{\text{bearings}} \cdot n_{\text{discr}} \quad (2)$$

Fig. 2 shows the coupling of eight masternodes with the slavenodes on each seat section. The reduction nodes are located at the centre of the sections.

Fig. 3 portrays the reduced stiffness matrix of the gearbox housing with translatoric and rotatoric DOF and eight reduction points at each bearing. The main diagonal submatrices represent the stiffness of the bearing seats itself, whereas lower and upper diagonal blocks show crossover influences between the bores and each reduction point on the seat surfaces. As this method includes more than one reduction node for each bearing seat, stiffness differences

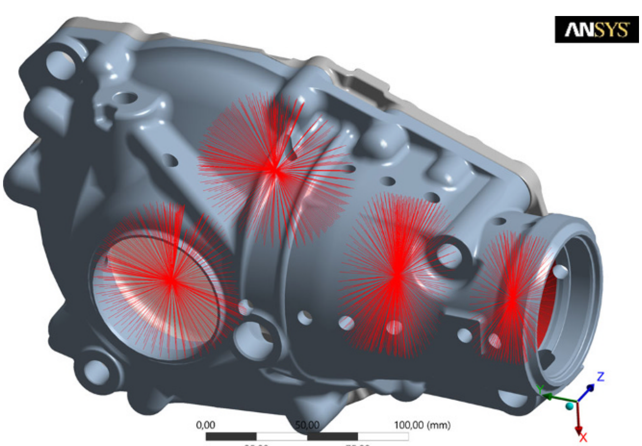

Fig. 1 Rigid surface constraint of the previous method

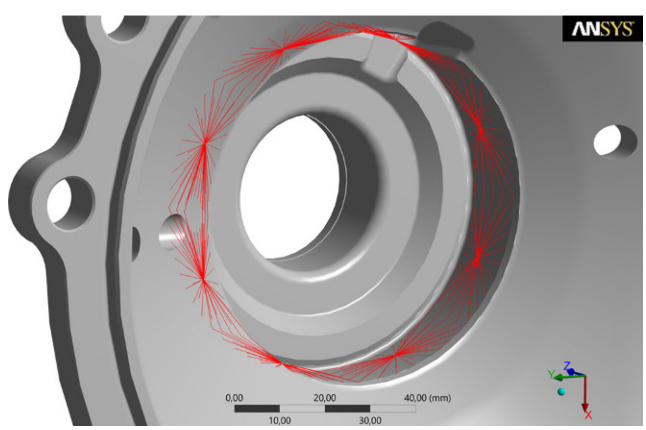

Fig. 2 Discretized static reduction of bearing seats

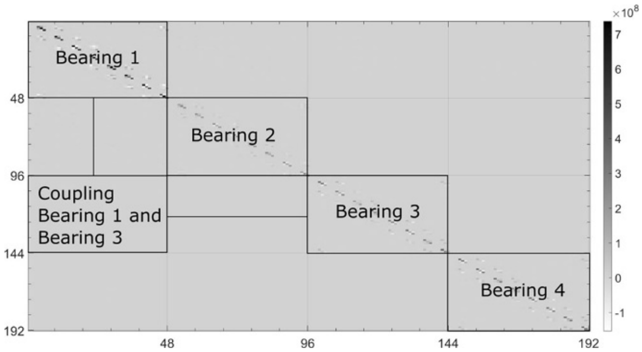

Fig. 3 Structure of the discretised reduced stiffness matrix

of the housing over the circumference can be modelled with a higher accurancy. Furthermore, it is possible to delete all the discretisation points of the housing which are loaded by tensile forces. Tensile forces between the bearing seats and outer rings are not possible, since there is no cohesive connection between the elements.

3 Iterative method to delete tensile forces

The deformation calculation in the program RIKOR [10] is based on a system of equations which couples translational and rotational displacements of rigid bodies with the forces and torques acting on them. Each gear element, such as shafts, bearings or the housing, consists of one or more rigid body and elastic links between them. This approach results in one main equation system of the whole gear system in the structure.

$$\boldsymbol{C} \cdot \boldsymbol{u} = \boldsymbol{F} \quad (3)$$

In this equation, the matrix C is the stiffness matrix of the system; vector u includes the DOF of the rigid bodies and F is the load vector. Each reduction point in the FEM model is in RIKOR modelled by a rigid body at the same position. Translational and rotational springs in radial and axial direction between the rigid bodies at the outer ring and the bearing seats transfer the forces and torques from the bearing to the bearing seat. These springs have a very high stiffness, as they should not bring additional elasticity into the system. The elasticity of the housing and crossover influences between the reduction points are included into the system by the importing of the values of the reduced stiffness matrix. Fig. 4 shows the described modelling of the discretised bearing seat with the translational coupling springs, whereas the graph does not show radial rotational springs and the axial springs.

Following the solution of the system, vector u contains the displacement values for each rigid body. Due to bearing forces and the modelled stiffness of the bearing seat, a displacement of the outer bearing ring occurs. Fig. 5 shows the displacements of the coupling points at the bearing seat segments because of a bearing force $F_{bearing}$ in the radial plane. Necessarily, tensile forces appear at some of the discretisation points. These points have to be detected by an analysation of the deformation condition of the bearing seat or rather of the radial translational spring. The deformation condition of the radial translational spring can be analysed through the displacement vectors of the rigid bodies. Therefore, Fig. 6 shows, by way of example, the positions of the mentioned rigid bodies before and after an iteration step. The vectors a_s and b_s are the displacements of the coupling points during the iteration. The vector components $a_{\parallel,s}$ and $b_{\parallel,s}$ are that part of the displacement in the direction of the spring, which couples the bearing ring and the seat.

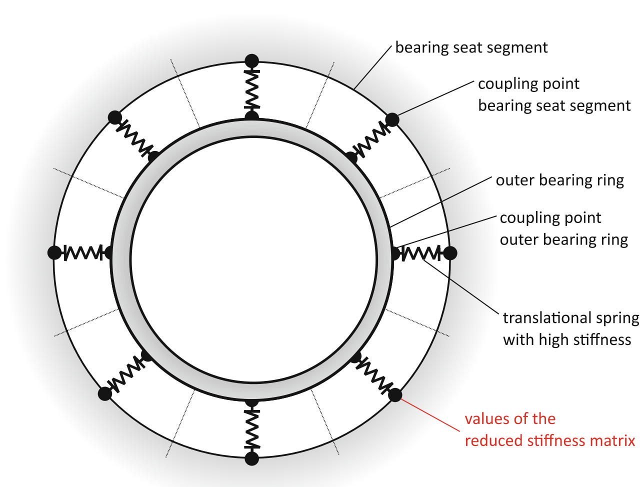

Fig. 4 Schematic visualization of the modeling of discretised bearing seats and coupling with the outer bearing ring in RIKOR

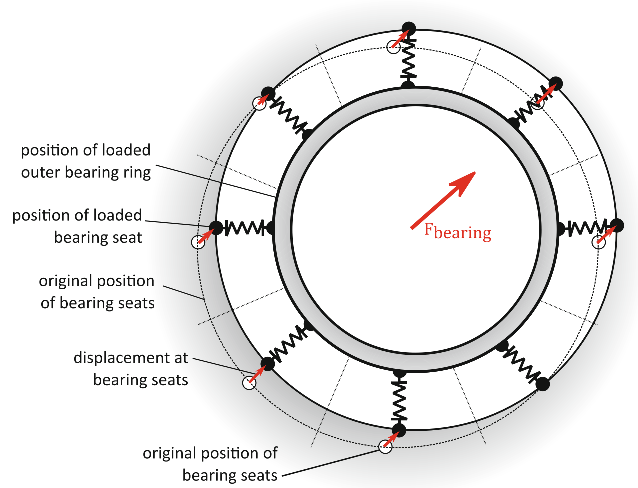

Fig. 5 Displacement of the coupling points at the bearing seats and of the outer bearing ring due to a bearing force $F_{bearing}$

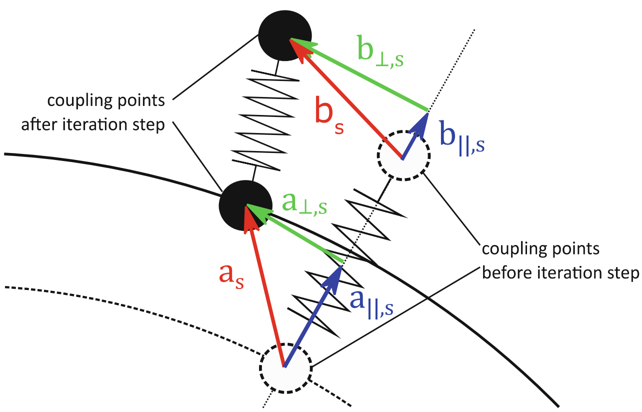

Fig. 6 Displacements components of the coupling points at the outer bearing ring and the bearing seat before and after an iteration step

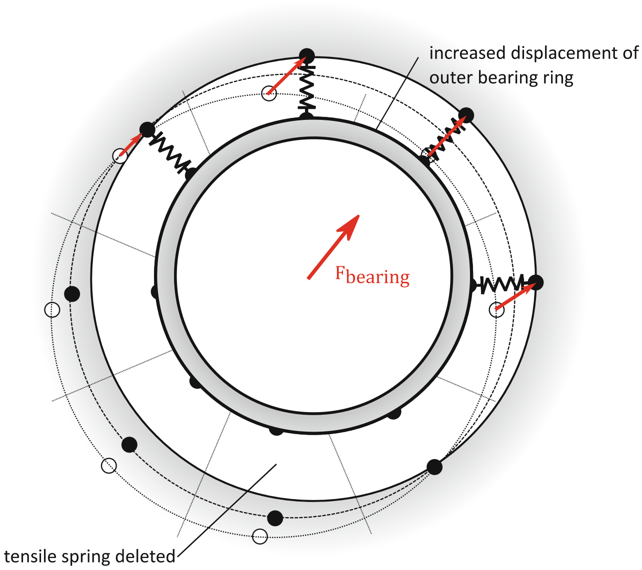

Fig. 7 Deformation behaviour of outer bearing ring with deactivated tensile springs

The penetration of the coupling points l_d is the result of the comparison of the lengths $|a_{\|,s}|$ and $|b_{\|,s}|$

$$l_d = |\boldsymbol{a}_{\|,s}| - |\boldsymbol{b}_{\|,s}| \tag{4}$$

If $l_d > 0$, the considered spring transmits pressure forces, while tensile forces act for $l_d < 0$. In this case, the coupling springs between the outer bearing ring and the bearing seat segment are deleted and the tensile forces are deactivated. As long as tensile forces can still be found on any bearing-seat segment, the described process will be iteratively repeated [10].

Fig. 7 shows the deformation behaviour of the outer bearing ring and the housing discretisation points with deactivated tensile springs. The bearing force acts on fewer housing discretisation points which is why the displacement of the outer bearing ring increases. This approach represents a more detailed modelling of the contact between the outer bearing ring and the bearing seats of the housing.

4 Influence of housing modeling method

This section describes the influence of different modelling methods for the housing in the simulation program RIKOR on the bearing outer ring and the shaft deflections of a hypoid gearbox system. The hypoid gearbox consists of two shafts, with two bearing on each shaft. Fig. 8 shows the labelling of the shafts and the bearing, which is used in further diagrams and explanations. Shaft One (S1) is a ring gear shaft and runs on two angular contact ball bearings S1B1 and S1B2. Bearing Two (S2B2) on Shaft Two (S2) is a cylindrical roller bearing, while Bearing One (S2B1) is also an angular contact ball bearing. The Figure also shows the coordinate systems of the shafts for the calculation of the bending lines.

The influence of the modelling method is investigated by the shaft bending lines and the outer bearing-ring deflections. Besides the advanced method (label: *discr*), the method with a central reduction point (label: centre) and calculations with rigid housings (label: rigid) are performed to show the effect of the housing modelling method.

Fig. 9 and 10 show the shaft deformation and the outer bearing deflection in the w- and v-direction with the three different housing modelling methods. In these casees, the maximum input torque of 500 Nm is acting in coast operation. The diagrams indicate a high influence of the housing modelling method. Especially S2B2 and S1B1 show a growing deflection in the w-direction with an increasing level of detail. Higher deflections at the bearing also result in a higher shaft deformation at the tooth contact at $u = 0mm$ on Shaft 2. This is equally true with the shaft and bearing deflections in v-direction in Fig. 10.

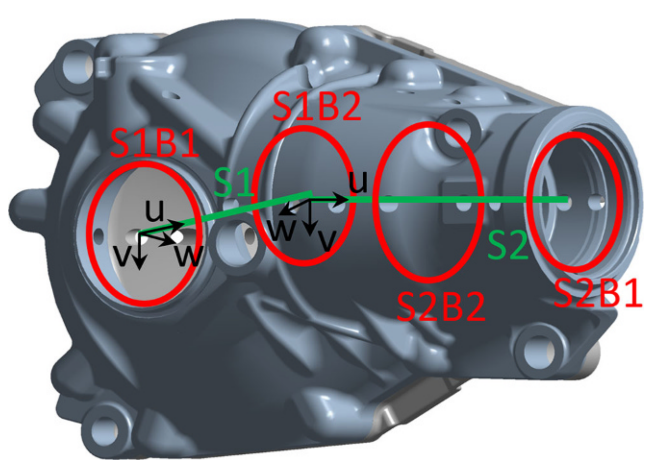

Fig. 8 Labelling of the hypoid gear shaft (S1), the pinion shaft (S2), the bearings and the shaft coordinate systems

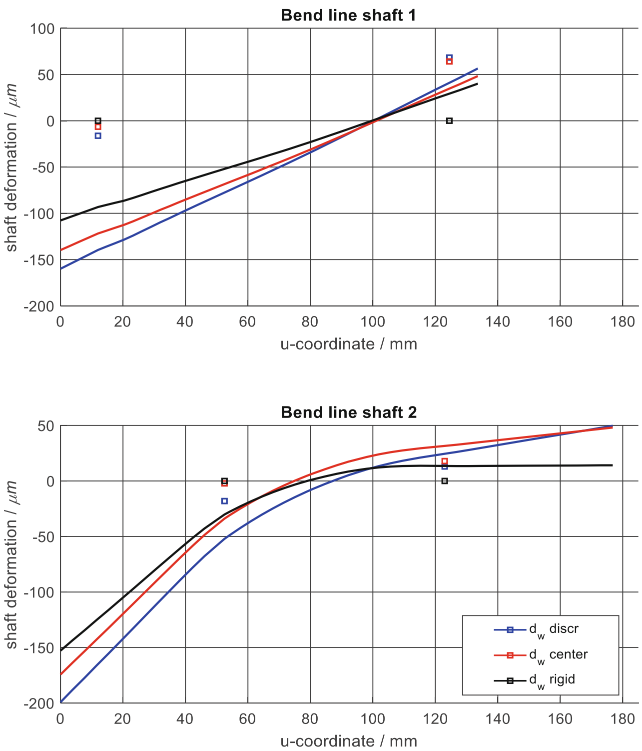

Fig. 9 Shaft deformations and outer bearing ring deflections in w-direction for the different modelling methods and maximum input torque in coast mode (–500 Nm)

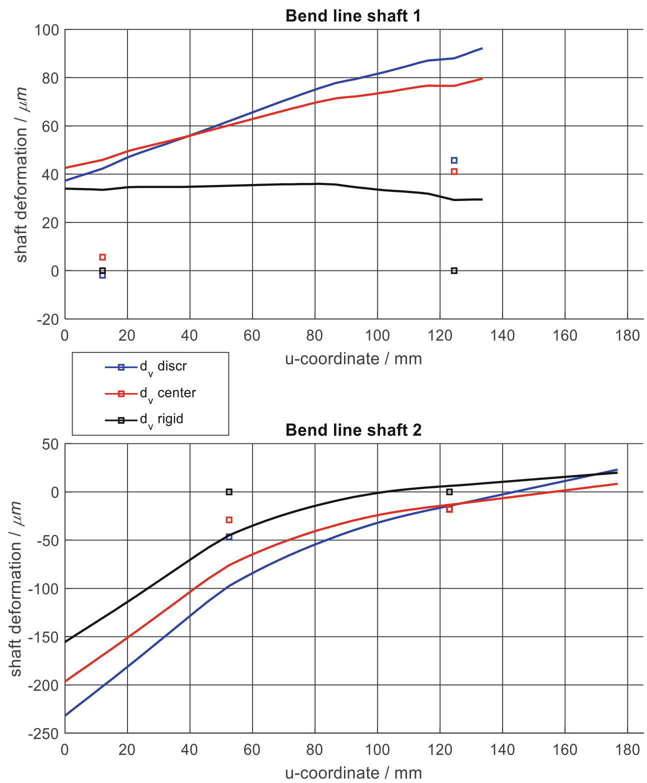

Fig. 10 Shaft and bearing displacement in v-direction at maximum input torque in coast operation (–500 Nm)

5 Results of validation

This section describes the validation of the advanced housing modelling approach by comparisons with measurement results of a static test rig. For this purpose, Daffner [3] measured the relative displacement of the pinion and gear shaft in the tooth contact under load.

There are several conventions to describe the relative displacements of the pinion and the gear shaft. Below, the convention according to Klingenberg [9], visualised in Fig. 11, is used.

The following parameters are the result of a combination of separately measured displacements of the pinion and the shaft:

ΔΣ = Displacement of the shaft angle (°)
ΔV = Displacement of the centre distance (*μm*)
ΔH = Displacement along the pinion axis (*μm*)
ΔJ = Displacement along the gear axis (μm)

Fig. 12 shows the relative displacements of the measurement and RIKOR calculations with a rigid housing. The axial force of the gear mesh leads to an additional bending displacement of the pinion shaft, which is why the relative displacements in this type of operation are greater. The direction of the calculated and measured displacements ΔV, ΔH and ΔJ are in conformity. The largest deviations between calculation and measurement are shown by the curves ΔV, ΔH and ΔJ in coast operation. The modelling of the geometrically complex hypoid gear in rikor does not take local deformation of the mating gear into account [3]. That is the reason for the large deviations of the calculated parameters ΔΣ from the measured angles. The calculated relative displacements in Fig. 13 for the modeling with the centre stiffness matrix show a better accordance with the measurements. In particular, ΔV and ΔJ in coast operation show a high influence of the housing modelling method. The

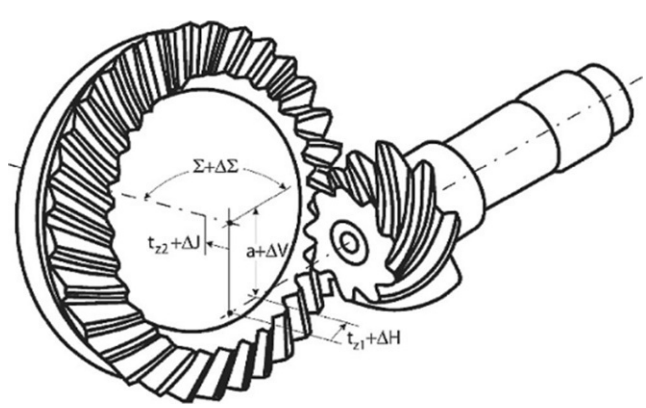

Fig. 11 Relative displacement in the tooth contact of hypoid gears in accordance with Klingenberg [9]

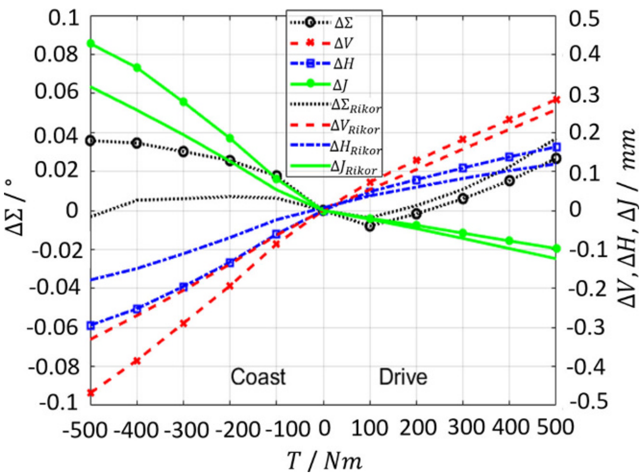

Fig. 12 Relative displacement parameters of the measurement and RIKOR calculation with rigid housing

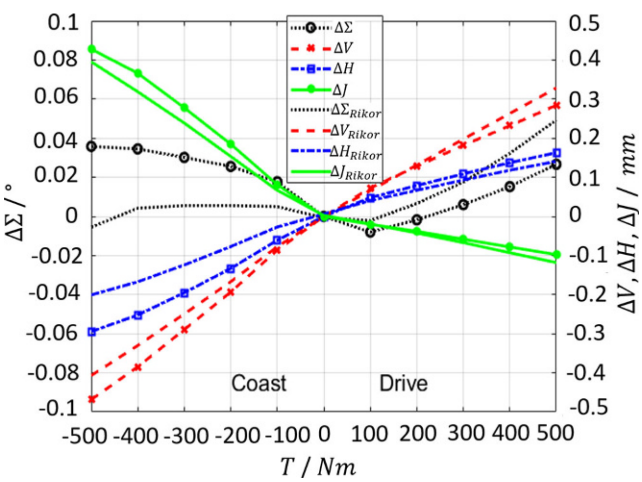

Fig. 13 Relative displacements parameters of measurement and RIKOR calculation with the centre housing stiffness matrix

curves in Fig. 14 show the measurement and the RIKOR calculation with the advanced housing modelling method and the discretised stiffness matrix. In comparison with the less detailed calculations, the values in coast operation show a further raised accuracy. The graph ΔΣ still shows the highest discrepancy in coast and drive operation as the hypoid gear shaft modelling was not changed and therefore, the local deformations of the mating gear is not included in the rikor calculation. Table 1 lists the displacement values for maximum input torque (–500 Nm/500 Nm) for coast and drive operation and the total deviation of the three modelling methods from the measurement.

The advanced housing modelling method offers the highest accuracy in each direction, except ΔV_{drive}. Just as in the calculation with the centre stiffness matrix, the shaft deflection exhibits overly high values. One reason for this could be the low load on S2B1, which is relieved in drive mode.

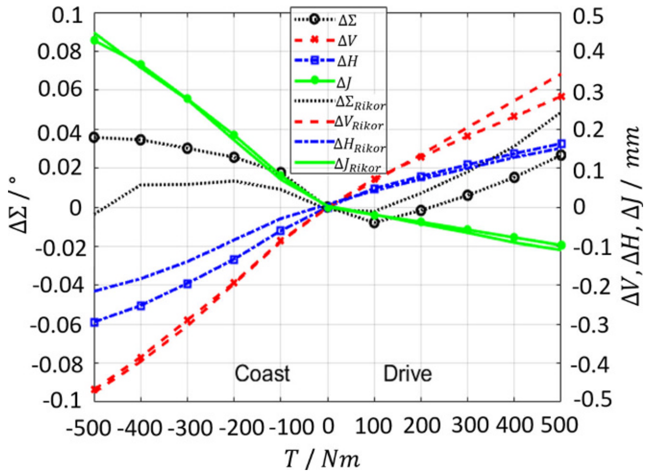

Fig. 14 Relative displacement of the measurement and RIKOR calculation with the discretised stiffness matrix

Table 1 Displacement values for maximum input torque and total relative deviation from the measurement

	Meas	Rigid	Centre	Discr
ΔV_{coast} (μm)	–468	–331	–405	–475
ΔJ_{coast} (μm)	427	316	394	447
ΔH_{coast} (μm)	–294	–177	–199	–216
ΔV_{drive} (μm)	284	257	330	339
ΔJ_{drive} (μm)	–98	–125	–120	–109
ΔH_{drive} (μm)	163	118	139	152
Deviation (%)	–	26.6	17.8	11.7

For lightly loaded bearings, there is an increased error probability in both the calculation and the measurement.

The overall deviation of the calculated displacements from the measurements drops from 26.6% in the case of a rigid housing to 17.8% for the centre stiffness matrix and to 11.7% when using the more detailed advanced method.

6 Summary and outlook

A high accuracy in the deformation computation of shaft-bearing-housing systems is a main requirement for gear-calculation programs, as the deformations can cause deviations in the tooth contact. The RIKOR program offers a method to include reduced stiffness matrices of housings into the computation of the shaft-bearing system. Especially in the case of hypoid gearboxes with aluminium housings, housing deformations and crossover influences between the bearing seats become more important. Reduced stiffness matrices can be computed by using the finite element method (FEM). The established method uses stiffness matrices with one reduction point in the centre of each bearing.

This study presents an advanced method of including stiffness matrices with multiple reduction points at each bearing, which are distributed equally over the circumference of the drillings. An iterative solving method in RIKOR deletes tensile forces between the outer bearing ring and the bearing seat, as they do not occur in reality. This advanced method offers a more detailed modelling of housing geometries in the RIKOR calculation program.

The approach was validated by comparing measured and calculated values for the relative displacements of the pinion and crown wheel shaft. The results of the validation show an increasing accuracy of the calculated displacements with the advanced modelling method in comparison with the measurement.

In addition, the advanced housing modelling method offers the opportunity to model clearance and slip fits of the outer bearing ring in the housing. Further investigations will be carried out on this topic, to integrate the modelling of fits of different types into the housing inclusion in RIKOR.

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