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Inventory management with advance demand information and flexible shipment consolidation

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Abstract

In this paper, we study a stochastic single-item, single-stage inventory system, in which orders from several production facilities are placed at one warehouse. An (R, Q) policy is applied to control the inventory at the warehouse, and orders arrive according to a Poisson process and include a due date such that some information about future demand is available. This advance demand information (ADI) can be used to adapt a time-based shipment consolidation policy applied to replenish stock at the production facilities. We develop a model to incorporate flexible deliveries, indicating that orders can be shipped before their due date if sufficient reserved transportation capacity is available. We derive analytical, approximate expressions for the expected inventory and shipment costs and therefore enable the evaluation of different inventory and shipment policies including outbound transportation capacities. We additionally show how to compute the optimal policy parameters and conduct a detailed numerical study. Our computational experiments indicate that our approximation works extremely well, with an average total cost deviation of 0.20%, and finds optimal policy parameters in more than 90% of our instances. In line with existing research, we can show that ADI leads to large cost reductions. However, the main cause of the cost reduction is the flexible delivery option. To be able to completely utilize this option, even larger safety stocks are obtained compared to systems without ADI, but savings due to a more efficient transportation policy far exceed the cost increase due to higher safety stocks.

Keywords Inventory system · Advance demand information · Time-based shipment consolidation · Continuous review · Outbound transportation capacity

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1 Introduction

It is well known that demand and supply uncertainties influence the stock levels in a supply chain because safety stock is needed to hedge against delivery delays and demand peaks and to prevent stock-outs. However, the large safety stocks required for high service levels are related to high inventory costs. Therefore, measures to reduce the inventory levels while maintaining the same service level are needed. One way to lower safety stocks without a reduction in high service levels is to improve forecasts or gather advance order (advance demand) information (ADI) (Ahmadi et al. 2019a). ADI is enabled by technological innovations and developments in recent years, allowing for the transmission and evaluation of an enormous amount of data rapidly and inexpensively.

In this paper, ADI is obtained through a preorder strategy, in which customers are motivated to order in advance, while financial incentives are provided depending on their willingness to wait Chen (2001). In the literature, these preorder strategies are often modeled by a so-called *demand lead time*, which represents the time between order placement and due date (Hariharan and Zipkin 1995). For example, the brand Star Labs sells Linux laptops in a variety of configurations. By offering a discount of 5% on the retail price, the company induces the customer to order in advance, even if he or she must accept a lead time of several weeks. Due to this preorder strategy, information about future demand can also be accessible to the supplier of components such that he or she can reduce his or her inventory in the warehouse without decreasing the service level.

Additionally, ADI can also be used to reduce transportation costs because demand uncertainties require third-party logistics (3PL) service providers to charge high prices when warehouses cannot make reliable commitments to the 3PL regarding the quantities to be dispatched. Commonly, companies enter into contracts for a fixed shipment volume at fixed times, but doing so can lead to low utilization of the reserved volume or many emergency deliveries if the period demand exceeds the reserved capacity. To react to this issue, we allow for satisfaction of facility orders before they are required, which is known as *flexible delivery*. Wang and Toktay (2008) were the first to study the option of flexible deliveries for a single-stage inventory system when ADI is available. However, they investigated flexible deliveries in a business-to-consumer (B2C) environment, whereas we focus on business-to-business (B2B). While consumers in a B2C setting expect their orders to be satisfied immediately, orders can be consolidated over time, as well as over several production facilities in the B2B setting, which is investigated in this article. Since each shipment causes fixed transportation costs, e.g., for salary, insurance cost, and fuel, it is reasonable to apply a time-based shipment consolidation (SCL) program in which small loads can be accumulated into one large load over a fixed interval. Including an aggregation of several production facilities has the advantage that individual production facilities can be supplied with smaller shipments more frequently, while the utilization of the transportation mean remains high.

The research presented in this paper can be viewed as an extension of the model studied in Wang and Toktay (2008). Similar to their research, we investigate the influences of ADI and flexible deliveries in a single-stage inventory system. In contrast to their work, in which demands are satisfied directly after each period, we investigate a flexible time-based SCL policy. The warehouse receives production facility orders with due dates, which are consolidated and shipped to the facilities at regular points in time. As soon as the due date arrives, the orders are prioritized for shipment. If the reserved capacity is not fully utilized with prioritized orders, the remaining capacity can be filled with nonprioritized orders, indicating that the facilities can obtain early deliveries and stock them prior to their actual demand. This SCL policy results in orders being dispatched after the due date and orders being dispatched before the due date is reached due to the flexible shipment policy. Delayed deliveries reduce the available stock at the production facilities, requiring a higher safety stock level to maintain the service level. Since orders have to wait until the next shipment date, waiting costs are incurred to compensate the facilities for additional stock-keeping. If an order is shipped before the due date, more stock is kept at the production facilities, which is reflected by early-delivery costs.

An outside supplier replenishes the inventory at the warehouse with a deterministic supply lead time according to an (R, Q) replenishment policy. Therefore, stock-keeping costs are incurred, which consist of the costs for cycle stock and safety stock. Finally, costs for each reserved capacity unit at the 3PL are incurred, as well as additional costs if the available capacity is not sufficient to ship all goods and emergency shipments must be organized.

In contrast to the existing literature, we model the trade-off between inventory and transportation costs for a problem in which ADI is available, and transportation capacity limits the total consolidated load (TCL). We are the first to consider ADI in a B2B setting, in which it is reasonable to apply time-based SCL. Additionally, we answer the following research questions. First, what is the added value of a preorder strategy combined with a flexible time-based SCL policy? Second, what are the impacts of flexible deliveries, enabled due to ADI, on the expected total cost and the stock levels in a single-stage inventory system? And finally, how do flexible deliveries influence the optimal outbound capacity reservation under time-based SCL? We present a mathematical model, derive expressions for evaluating different policies including transportation capacities and show how the optimal policy parameters can be computed. A numerical study is conducted to derive managerial insights and to answer the above-formulated research questions. We also show that, to fully exploit the benefit of ADI, it is necessary to adapt the shipment policy, and it might also be necessary to increase the safety stock at the warehouse to enable early shipments.

The organization of the remainder of the paper is as follows. First, we provide an overview of the related literature regarding ADI and SCL in section 2. Based on a detailed problem description in section 3, we develop the mathematical expressions for the expected total cost function in section 4. In section 5, we show how to optimize the reorder level, the length of the SCL cycle and the outbound transportation capacity, helping us to derive managerial insights in section 6. In the final section, we outline the main results and present directions for future research.

2 Related literature

Our paper is related to two large streams of literature: articles related to ADI for inventory management and articles about the coordination of SCL and inventory management.

We start our discussion with contributions, in which ADI is perfect and represented by a demand lead time in inventory systems. If perfect ADI is assumed, an order cannot be changed or canceled after order placement. Hariharan and Zipkin (1995) were the first to investigate a single-echelon continuous review inventory system with Poisson demand and a demand lead time. They concluded that ADI in the form of a demand lead time reduces the effective supply lead time (supply lead time - demand lead time) and, consequently, the safety stock. Thus, if the demand lead time exceeds the supply lead time, a make-to-order situation is reflected. Ahmadi et al. (2019a) extended this model by a cost component for commitment. These commitment costs represent a bonus for companies which accept a preorder strategy. Under linear commitment cost, they found the optimal commitment lead time is either zero or equal to the replenishment lead time. Further, Ahmadi et al. (2019b) and Ahmadi et al. (2020) studied commitment costs for assemble-to-order systems and for inventory systems with time-based service constraints, respectively, and received similar results. Other continuous review models with a positive demand lead time were presented in Lu et al. (2003), who investigated an assemble-to-order system with stochastic lead time and ADI. A divergent inventory system with nonidentical demand lead times for different customers was studied by Marklund (2006), who compared different reservation policies. He showed that the last-minute reservation policy (reserving at the moment when the due date is reached) avoids potential wrong prioritization of customers in cases of nonidentical demand lead times. Du and Larsen (2017) investigated a single-stage model with ADI and different customer classes and applied different reservation policies.

In addition to continuous-time models, discrete-time models are discussed in the literature. Gallego and Özer (2001) were the first to study a single-echelon model with periodic review and heterogeneous ADI using a modified inventory position, indicating that the inventory position is reduced by the known future demand. The conclusion is that using the modified inventory position outperforms the regular inventory position for state-dependent (s, S) ordering policies. Dellaert and Melo (2003) investigated a similar situation and modeled it using a Markov decision process.

Based on this research, there have been several extensions; for example, Gallego and Özer (2003) and Özer (2003) considered a serial inventory system and divergent inventory system, respectively. There have also been other extensions, such as Özer and Wei (2004), Angelus and Özer (2016) and Wang and Toktay (2008). The latter investigated flexible deliveries (i.e., orders can be fulfilled before the due date is reached) for a single-echelon inventory system without SCL with the result that the expected cost could be reduced with flexible delivery and ADI. Additionally, Bourland et al. (1996) showed the benefits of ADI in a periodic two-stage inventory system, and DeCroix and Mookerjee (1997) showed the benefits in a setting in which actual demand information can be purchased.

The influence of ADI has also been considered in the context of production-inventory systems with limited capacity under a make-to-stock policy. Buzacott and Shanthikumar (1994) investigated the relationship between safety stock and safety lead time for an $M|M|1$ queue, whereas Karaesmen et al. (2002) studied the discrete-time version. Karaesmen et al. (2004) investigated, among other things, early deliveries for $M|M|1$ queues and determined that manufacturers can observe significant cost savings if early deliveries are allowed. Further, Liberopoulos (2008) showed the relationship between inventory and demand lead time for different settings when considering a make-to-stock queue.

There also exists literature regarding imperfect ADI, indicating that the due date or the order quantity is not certain, e.g., Bourland et al. (1996); Tan et al. (2007, 2009) and Topan et al. (2018). In Thonemann (2002), customers are divided into two classes that provide different information to the manufacturer. Other ways to model ADI can be found in Heath and Jackson (1994) and Graves et al. (1998); Güllü (1996, 1997); Toktay and Wein (2001); Schoenmeyr and Graves (2009); Bernstein and DeCroix (2015) and Papier (2016), in which ADI is developed by dynamic forecast updates.

All of the contributions discussed above have in common that each order is related to a single shipment or a single production order, and no consolidation occurs. In contrast, in our paper, we investigate how ADI can be used to adapt a time-based shipment policy to replenish inventories.

This approach leads us to the discussion of the second stream of literature relevant to our research: contributions devoted to SCL. Higginson and Bookbinder (1995) considered different SCL programs in stochastic single-stage inventory systems and analyzed them using simulation. Çetinkaya and Lee (2000) studied an (s, S) ordering policy for a single-stage system with time-based SCL to a group of retailers under private carriage and derived expressions for the approximate long-run average cost to obtain optimal inventory and shipment policy parameters, whereas Axsäter (2001) presented an exact optimization approach for the same model. Çetinkaya et al. (2006) focused on the same setting with quantity-based SCL and found out that a quantity-based dispatch policy outperforms the time-based dispatch policy in terms of expected total cost. However, with the time-based consolidation the customers have an upper bound on the maximum waiting time. In contrast, Chen et al. (2005) focused on the (R, Q) policy for the single-stage problem and evolved an optimization method with bounded enumeration to find optimal policy parameters again for both dispatch policies under private carriage. Çetinkaya and Bookbinder (2003) investigated both quantity-based and time-based SCL for private and common carriage and derived formulas for a near-optimal dispatch quantity and SCL cycle length similar to the EOQ formula. Ülkü (2012) integrated carbon emissions in a time-based dispatch model with private carriage and showed that an increase of utilization of the transportation capacity decreases the environmental damage. A price- and time-sensitive demand under time-based SCL is investigated in Ülkü and Bookbinder (2012a) and Ülkü and Bookbinder (2012b) when private and common carriage is applied, respectively.

Additionally, several papers have faced divergent two-echelon inventory systems with time-based shipment policies. Marklund (2011) considered one warehouse with an (R, Q) policy and N retailers with base-stock policies and a time-based shipment policy. He developed a fast recursive procedure to obtain the expected total cost of the system and presented a method to find an optimal reorder level, as well as optimal base-stock levels and optimal length of the SCL cycles under Poisson demand. This basic model has been extended in several directions: Howard and Marklund (2011) studied different reservation policies according to a myopic allocation policy with a single retailer group, whereas Stenius et al. (2018) included a limited transportation capacity, where a reserved transportation capacity can be extended by expensive and spontaneous deliveries (primary and alternative transportation option). Stenius et al. (2016) extended the research and allowed compound Poisson processes to model customer demand. They presented an exact approach that requires large computation times; therefore, Johansson et al. (2020) derived approximations for the same model. In contrast to these papers, Kiesmüller and de Kok (2005) investigated a quantity-based shipment policy in a divergent system with (R, Q) policies at all locations and compound renewal demand.

Based on our literature study, we can conclude that ADI and SCL programs have already been discussed separately in detail. However, how ADI can be integrated into a SCL program to increase the utilization of transportation capacity has not been the subject of earlier studies. In this paper, we combine both topics and study an inventory system in which ADI enables flexible deliveries in the context of a SCL program. We show how to compute the optimal length of the SCL cycle, the optimal reserved transportation capacity and the optimal reorder points in this setting.

3 Problem formulation

We consider a single-item continuous review inventory system composed of one warehouse and N production facilities, which belong to the same company. The warehouse supplies these production facilities, which order according to Poisson processes with rate λ_n. Consequently, the order process at the warehouse is also Poisson with rate $\lambda = \sum_{n=1}^{N} \lambda_n$. The inventory at the warehouse is replenished from an outside supplier with sufficient capacity and constant supply lead time L_s according to an (R, Q) policy, indicating that a replenishment order of Q units is placed if the modified inventory position (inventory level - backorders + outstanding replenishment orders - observed orders) reaches the reorder level R. The decision maker has to determine the value of R, and we assume that Q is fixed due to a contract with the outside supplier. As an approximation, Q can also be predetermined applying the EOQ formula, which has been shown to be close to optimal in previous research (Axsäter 2015). Therefore, Q is not a decision variable in our model.

The company uses a preorder strategy, and therefore the production facilities are forced to place each order L_d time units before the actual demand occurs, indicating that each order is combined with a due date. The time between order placement and due date is known as the demand lead time L_d. For clarity, we call orders that have not reached the due date *orders* and orders that have reached the due date *demands*.

At the moment when the due date is reached, the warehouse is obligated to satisfy this demand with the next delivery if sufficient stock on hand is available. Companies accept such a preorder strategy when they receive a bonus, e.g., in the form of lower unit costs. Since the warehouse and the facilities belong to the same company, we do not include the bonus in our model. This also makes it possible to determine the pure value of inserting ADI. We further assume constant and identical demand lead times for all production facilities, such that it is not necessary to consider the issue of stock allocation. Moreover, we focus on perfect order information, meaning that the order quantity and due date are certain and cannot be changed after order placement. Order cancelations may occur, but due to the associated high cancelation costs, we assume that this happens very seldom, and we exclude imperfect ADI for the sake of simplicity. In this paper, we investigate the case in which $L_d \leq L_s$ (make-to-stock) because otherwise there is no need to have stock at the warehouse (make-to-order) (Fig. 1).

In contrast to other studies that consider ADI, the warehouse does not satisfy all demands immediately but applies a time-based SCL policy. Therefore, the N production facilities are aggregated into a group to consolidate demands not only over time but also across several production facilities. After a fixed time period T, called SCL cycle length, a load with all accumulated demands of the considered group is dispatched from the warehouse to the production facilities. We assume a central control of the system where the decision maker has to determine the length of the consolidation cycle as well as the amount of safety stock needed at the warehouse. He or she can further determine in advance how much transportation capacity to reserve for a low price. In the case of stock-outs, demand is backordered on the shipment day. When sufficient inventory is available again, backorders are satisfied with the next scheduled shipment. Time-based shipping strategies are popular in industry because scheduling, administration and coordination of processes at production facilities are easy to manage.

Due to stochastic order processes, it is evident that the number of accumulated demands during a SCL cycle is also random. The warehouse does not have its own fleet of trucks and therefore engages a 3PL for the transportation from the warehouse to the group of production facilities (including local deliveries). Although 3PL can react quickly to requests, this flexible strategy is quite costly. To reduce costs, companies negotiate contracts in which a fixed transportation capacity is reserved for a lower

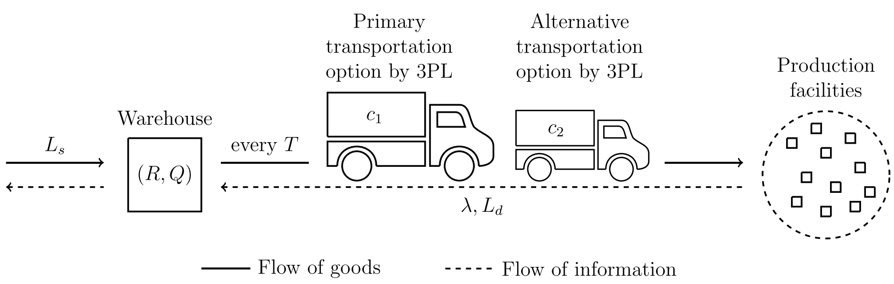

Fig. 1 Flow of information and goods in the considered inventory system

price. Thus, we consider a primary option and an alternative transportation option. The primary transportation option reflects the capacity reserved at the 3PL for periodic shipments from the warehouse to the production facilities, which are limited to a self-chosen capacity reservation *Cap* and can be extended by the alternative option, which has ample capacity but is more expensive. The alternative option can only be used on scheduled shipment days and if the reserved capacity is exhausted. For the primary option, fixed costs $\alpha(Cap)$ per shipment occur, which depend on the chosen reserved transportation capacity. The fixed transportation costs represent nothing more than variable transportation costs that have to be paid for each reserved transportation capacity unit ($c_1 \cdot Cap$). However, these costs are charged independently of the realized TCL. If the realized TCL exceeds the reserved capacity of the primary transportation option, the warehouse has to pay variable costs c_2 for each unit shipped by the alternative option. Note that $c_2 > c_1$ to reflect that the alternative transportation is more expensive than a capacity reservation per unit. Dispatching, sorting and consolidating costs are included in the before mentioned transportation costs paid to the 3PL. As we focus on the inventory and transportation decisions at the warehouse, and all demand has to be satisfied, the transportation lead time from the warehouse to the production facilities is not relevant for the decision and therefore is not included in the model.

Considering that production facilities order in advance, the question arises regarding how to include information about future demand in the transportation schedule. It is allowed that orders can be satisfied before their due date is reached, known as flexible delivery. In our approach, all demands must be shipped to the production facilities on the upcoming shipment day, if necessary, by the alternative option. In contrast, orders can only be dispatched if there is remaining capacity at the primary transportation option. However, it is only allowed to ship an order one shipment date ahead because the production facilities cannot keep a large amount of additional stock. The application of this approach results in increased utilization of the reserved capacity. At the same time, the usage of the expensive emergency option can be reduced if the reserved capacity is not fully exhausted on average. At the warehouse, orders and demands are allocated according to the first-come-first-served principle.

In addition to the shipment costs, three other types of costs occur and influence decision making. First, stock-keeping costs h are charged for each unit on stock per time unit. Second, waiting costs w arise for each unit that waits due to the SCL policy per time unit. These costs are related to a discount for the production facilities since they must hold more safety stock to reach the same service level. Backorder costs at the warehouse are included in the waiting costs because backorders lead to a longer waiting time. Third, costs for early deliveries are considered, compensating the facilities that require more space to store units shipped prior to the due date. Early-delivery costs e are charged for each early shipped unit per time unit. In the following, we use the term inventory cost for the sum of these three types of costs, which are assumed to be linear.

Thus, the system's expected total cost per time unit $TC(R, T, Cap)$ is composed of the expected inventory cost $TIC(R, T, Cap)$ and expected shipment costs $TSC(R, T, Cap)$:

$$TC(R, T, Cap) = TIC(R, T, Cap) + TSC(R, T, Cap). \tag{1}$$

A low stock level at the warehouse leads to high waiting and shipment costs because orders can hardly ever be shipped in advance; thus, the flexible delivery option cannot be used at all. However, a high stock level would cause high stock-keeping and early-delivery costs at the warehouse since there is always sufficient stock to dispatch orders before the due date. Conversely, a small SCL cycle length results in low waiting and early-delivery costs but leads to high transportation costs. The right balance between these types of costs must be found; therefore, the following optimization problem is formulated:

$$\min TC(R, T, Cap) \qquad R \in \mathbb{Z}, T \in \mathbb{N}, Cap \in \mathbb{N}_0. \tag{2}$$

To determine the optimal policy parameters (R^*, T^*, Cap^*), it is necessary to be able to evaluate a policy; therefore, mathematical expressions for the expected total cost must be derived, which are the focus of the next section. The notation used is summarized in Table 1.

4 Analysis

In this section, we derive expressions for the expected shipment costs $TSC(R, T, Cap)$ per time unit and the expected inventory cost $TIC(R, T, Cap)$ per time unit. Our analysis of the latter relies on the unit tracking methodology introduced by Axsäter

Table 1 Notation

Decision variables	
R	Reorder level
T	Length of the SCL cycle
Cap	Reserved capacity at the primary transportation option
Input data	
λ	Order arrival rate at the warehouse
h	Stock-keeping costs per unit and per time unit at warehouse
w	Waiting costs per unit and per time unit at warehouse
e	Early-delivery costs per unit and per time unit at the warehouse
L_d	Demand lead time
L_s	Supply lead time from supplier to warehouse
Q	Replenishment quantity
$\alpha(Cap)$	Fixed costs for each scheduled shipment by the primary transportation option
c_1	Variable costs per unit reserved at the primary transportation option
c_2	Variable costs per unit shipped by the alternative transportation option
General notations	
x^+	$\max(0, x)$ and analogously, $x^- = \max(0, -x)$
$\lfloor x \rceil$	$\lfloor x + 0.5 \rfloor$
$\underset{R,Q}{\mathrm{mod}}(x)$	$x + kQ$ where $k \in \mathbb{N}_0$ such that $R < x + kQ \leq R + Q$

(1990), where each unit going through the system is observed separately, which enables us to calculate the related expected costs for each unit. However, we must adapt this methodology for a situation with ADI, limited transportation capacity and flexible deliveries.

4.1 Expected shipment costs

First, we focus on the expected shipment costs per time unit, where fixed costs $\alpha(Cap)$ occur for each shipment to reflect the reservation costs at the 3PL. If the capacity of the primary transportation option is exceeded on the shipment day, variable costs c_2 arise for $(m - Cap)^+$ demanded units shipped by the alternative option, where m denotes the realized TCL. Note that $m = 0$ also causes fixed costs (reservation at the logistic provider); thus, we assume that fixed costs for the primary option are charged for $m \in \mathbb{N}_0$, yielding the following expression for the expected shipment costs per time unit:

$$\begin{aligned} TSC(R, T, Cap) &= \frac{1}{T} \sum_{m=0}^{\infty} P(M = m)(\alpha(Cap) + c_2(m - Cap)^+) \\ &= \frac{1}{T} \sum_{m=0}^{\infty} P(M = m)(c_1 \cdot Cap + c_2(m - Cap)^+), \end{aligned} \tag{3}$$

where M is defined as a random variable representing the TCL from the warehouse to the production facilities on the day of shipment. Hence, $P(M = m)$ represents the probability mass function (pmf) of the TCL on a shipment day.

Obviously, for the computation of $TSC(R, T, Cap)$, the pmf of M is needed first. For the case without a preorder strategy ($L_d = 0$), we refer to Stenius et al. (2018), whereas $L_d > 0$ and the extension with early deliveries is studied in this paper. As already mentioned, units that have reached their due date before the shipment date (which are demanded) must be shipped, and units that have not reached the due date (which are ordered) can be shipped in case of available capacity on the primary transportation option, provided that the warehouse has sufficient stock. To describe this shipment policy mathematically, we introduce the following variables:

$M(t)$	TCL from the warehouse to the production facilities by time t
$K(t)$	remaining units that cannot be shipped by time t due to a lack of stock at the warehouse or limited transportation capacity
$D(t_1, t_2)$	orders at the warehouse during the time interval $(t_1, t_2]$, $t_1 < t_2$
$IL(t)$	inventory level by time t
$IP(t)$	inventory position by time t.

In the following, we show how the pmf of the TCL can be approximately computed. For this analysis, we distinguish between different cases, which can be separated depending on the length of L_d, L_s and T, and they are shown in Table 2. A detailed explanation of all different situations will be provided during the analysis.

Table 2 Ranges for different calculation of remaining units

Case	Range of L_d	Range of L_s
1	$L_d \leq T$	$L_s \leq T$
2	$L_d \leq T$	$T < L_s \leq T + L_d$
3	$L_d \leq T$	$T + L_d < L_s \leq 2T$
4	$L_d \leq T$	$L_s > 2T$
5	$T < L_d \leq 2T$	$T < L_s \leq 2T$
6	$T < L_d \leq 2T$	$2T < L_s \leq T + L_d$
7	$T < L_d \leq 2T$	$L_s > T + L_d$
8	$2T < L_d \leq 3T$	$2T < L_s \leq T + L_d$
9	$2T < L_d \leq 3T$	$L_s > T + L_d$
10	$L_d > 3T$	$3T < L_s \leq T + L_d$
11	$L_d > 3T$	$L_s > T + L_d$

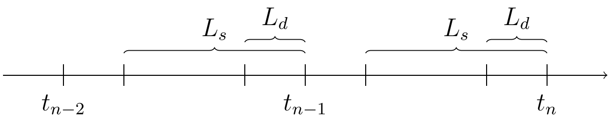

Fig. 2 Shipment cycle when $L_d < T$ and $L_s \leq T$

4.1.1 The cases $L_d \leq T$

We start our discussion with the first case in which $L_d \leq T$ and $L_s \leq T$, also illustrated in Fig. 2, where t_n ($t_n = nT, n \in \mathbb{N}$) represents the nth shipment day.

The TCL $M(t_n)$ is composed of the following parts. First, the remaining units $K(t_{n-1})$ of the previous shipment day are included in the TCL. These units were backordered due to a lack of stock at the warehouse or ordered units when the due date was not reached, and there was not sufficient capacity available to ship them earlier. Since $L_s \leq T$, all backordered units at time t_{n-1} can be shipped at t_n, and all ordered units before t_{n-1} must at the latest be shipped by t_n. Second, all orders and demands during the interval $(t_{n-1}, t_n]$ can be shipped at t_n if there is sufficient stock and shipment capacity available. Otherwise, backordered or ordered units will be left behind, indicating that we must subtract $K(t_n)$. These considerations lead to the following expression for the TCL at t_n

$$M(t_n) = D(t_{n-1}, t_n) + K(t_{n-1}) - K(t_n). \tag{4}$$

This expression reveals that we must characterize the stochastic process describing the remaining units at each shipment point $K(t_n)$. As mentioned above, there are two reasons why units cannot be shipped. First, if there is a lack of stock at the warehouse, then even demanded units cannot be shipped, and $IL(t_n)^-$ units are backordered. Second, all units that are ordered but not demanded must wait until the subsequent shipment dispatches if there is not enough reserved capacity at t_n available. The total number of units, which exceeds the capacity of the primary transportation option, is given as $(Cap - D(t_{n-1}, t_n) - K(t_{n-1}))^-$, but since all demanded units must be shipped by the alternative transportation option, at maximum, all orders $D(t_n - L_d, t_n)$ remain at the warehouse and must wait until the next shipment. Doing so yields

$$K(t_n) = \max\left(IL(t_n)^-, \min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n) - K(t_{n-1}))^-)\right). \quad (5)$$

It is easy to see that the following limit holds,

$$K^\infty(t_n) := \lim_{Cap\to\infty} K(t_n) = IL(t_n)^- \quad (6)$$

which shows that, in the case of high capacity, the shipment policy strives to ship all units earlier because only in the case of a lack of stock at the warehouse backorders must wait until the next dispatch time. In contrast, no early deliveries will occur in the case of small available capacity, as seen in (7).

$$K^0(t_n) := \lim_{Cap\to 0} K(t_n) = \max(IL(t_n)^-, D(t_n - L_d, t_n)) \quad (7)$$

For the remaining analysis, we split the time interval $(t_{n-1}, t_n]$ into several subintervals, and we express the number of backorders at time t_n based on the inventory position at time $t_n - L_s$ because $t_n - L_s$ is the last time when a replenishment order can be placed that arrives before or at t_n. Reformulating (5) leads to

$$\begin{aligned} K(t_n) = \max\Big(&(IP(t_n - L_s) - D(t_n - L_s, t_n - L_d) - D(t_n - L_d, t_n))^-,\\ &\min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n - L_s) - D(t_n - L_s, t_n - L_d)\\ &- D(t_n - L_d, t_n) - K(t_{n-1}))^-)\Big). \end{aligned} \quad (8)$$

Although the distributions of the demand and the inventory position are known and independent, the conditional distribution $P(K(t_n) = j \mid K(t_{n-1}) = i)$ has to be considered; therefore, the pmf of $K(t_n)$ cannot be computed directly by applying (8). When computing $K(t_{n-1})$, the number of backorders is obtained based on the inventory position at $t_{n-1} - L_s$, which is why we must also include information about the demand and the inventory position before time t_{n-1}, and we replace $IP(t_n - L_s)$ with

$$IP(t_n - L_s) = \underset{R,Q}{\mathrm{mod}}(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s)). \quad (9)$$

We obtain

$$\begin{aligned} K(t_n) = \max\Big(\Big(&\underset{R,Q}{\mathrm{mod}}\,(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d)\\ &- D(t_{n-1} - L_d, t_{n-1}) - D(t_{n-1}, t_n - L_s)) - D(t_n - L_s, t_n - L_d)\\ &- D(t_n - L_d, t_n)\Big)^-, \min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n - L_s)\\ &- D(t_n - L_s, t_n - L_d) - D(t_n - L_d, t_n) - K(t_{n-1}))^-)\Big). \end{aligned} \quad (10)$$

To obtain $K(t_n)$, we first have to compute $K(t_{n-1})$, which can be specified as

$$\begin{aligned} K(t_{n-1}) = \max\Big(& (IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ & - D(t_{n-1} - L_d, t_{n-1}))^-, \min(D(t_{n-1} - L_d, t_{n-1}), (Cap \\ & - D(t_{n-2}, t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ & - D(t_{n-1} - L_d, t_{n-1}) - K(t_{n-2}))^-)\Big). \end{aligned} \tag{11}$$

This expression means that the number of remaining units $K(t_n)$ depends on the number of remaining units of all previous SCL cycles. We assume that the impact of these quantities decreases the further we look back into the past; therefore, we only include the information of the previous two SCL cycles to determine $K(t_n)$. However, we have to deal with $K(t_{n-2})$. We suggest replacing it with a constant value and providing more information about this approximation at the end of this section.

Hence, the TCL given in (4) can be modified, and we replace in (12) the expressions for $K(t_n)$ and $K(t_{n-1})$ with (10) and (11) to compute the probability distribution for the TCL:

$$\begin{aligned} M(t_n) = & D(t_{n-1}, t_n - L_s) + D(t_n - L_s, t_n - L_d) \\ & + D(t_n - L_d, t_n) + K(t_{n-1}) - K(t_n). \end{aligned} \tag{12}$$

As the demand follows a Poisson process, the demand during a specific time interval is Poisson distributed. Further, $IP(t_{n-1} - L_s)$ is uniformly distributed between $R + 1$ and $R + Q$ (Axsäter 2015). Since all random variables are independently distributed, convolutions can be used to calculate the pmf of M. For the determination of $K(t_{n-2})$, we rely on an approximation presented at the end of this subsection, and we perform convolutions to obtain the probability mass function of the TCL $M(t_n)$. Due to our simplifying assumptions, the obtained distribution is an approximation, the performance of which is tested in a numerical study in section 6.

We obtain three additional cases when $L_d \leq T$ and $L_s > T$ based on the length of L_s, as shown in Table 2. In the second case, the supply lead time is in the range of $T < L_s \leq T + L_d$, which is why the time $t_{n-1} - L_d$ is before $t_n - L_s$, as illustrated in Fig. 3. The sequence of the latter time points changes if $T + L_d < L_s \leq 2T$, which legitimates the third case. The fourth case occurs if $L_s > 2T$ because $t_n - L_s$ is before t_{n-2}. The main difference from the first case is that the time $t_n - L_s$ of the last warehouse replenishment order, which will arrive at the latest by t_n, is before the previous shipment day at t_{n-1}, which is why we cannot assure that all backorders at an arbitrary shipment date can be satisfied by the following shipment day. This change in sequences leads to different time intervals during $t_{n-1} - L_s$ and t_n and therefore to different formulas of $K(t_n)$ and $K(t_{n-1})$, which are shown in Appendix.

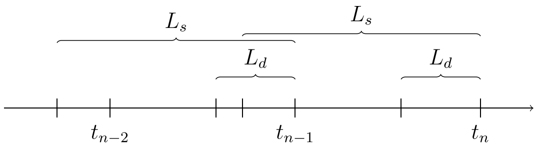

Fig. 3 Shipment cycle when $L_d \leq T, T < L_s \leq T + L_d$

4.1.2 The cases $L_d > T$

Now, the focus is on all seven cases in which $L_d > T$. To express the differences between $L_d \leq T$ and $L_d > T$, we investigate the fifth case with $T < L_d \leq 2T, T < L_s \leq 2T$, as illustrated in Fig. 4 in more detail. The other situations can be handled similarly. The shipment policy only allows for shipping an ordered unit one shipment date earlier to the production facilities. As long as $L_d \leq T$, this assumption is fulfilled automatically. However, it is not true for $L_d > T$. Orders during $(t_{n+1} - L_d, t_n)$ cannot be shipped at t_n because their official shipment date is at t_{n+2}. The earliest shipment date for these orders is t_{n+1}. Therefore, the demand during $t_{n+1} - L_d$ and t_n is directly added to the number of remaining units at t_n, which can be obtained by

$$\begin{aligned}K(t_n) = D(t_{n+1} - L_d, t_n) + \max\Bigg(\Bigg(& \operatorname*{mod}_{R,Q} (IP(t_{n-1} - L_s) \\ & - D(t_{n-1} - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-2}) - D(t_{n-2}, t_n - L_s)) \\ & - D(t_n - L_s, t_n - L_d) - D(t_n - L_d, t_{n-1}) - D(t_{n-1}, t_{n+1} - L_d)\Bigg)^{-}, \\ & \min(D(t_n - L_d, t_{n-1}) + D(t_{n-1}, t_{n+1} - L_d), (Cap - D(t_{n-1}, t_{n+1} - L_d) \\ & - K(t_{n-1}))^{-})\Bigg).\end{aligned} \tag{13}$$

This property can also be applied for determining $K(t_{n-1})$, where the orders $D(t_n - L_d, t_{n-1})$ will be considered on the shipment day at t_n. Doing so yields

$$\begin{aligned}K(t_{n-1}) = D(t_n - L_d, t_{n-1}) + \max\Big(& (IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ & - D(t_{n-1} - L_d, t_{n-2}) - D(t_{n-2}, t_n - L_s) - D(t_n - L_s, t_n - L_d))^{-}, \\ & \min(D(t_{n-1} - L_d, t_{n-2}) + D(t_{n-2}, t_n - L_s) + D(t_n - L_s, t_n - L_d), \\ & (Cap - K(t_{n-2}))^{-}\Big).\end{aligned} \tag{14}$$

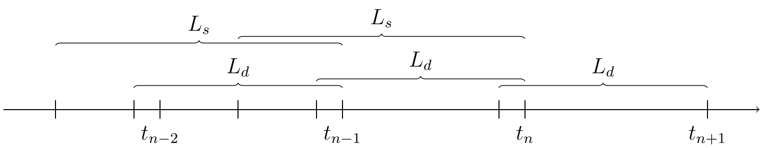

Fig. 4 Shipment cycle when $T < L_d \leq 2T, T < L_s \leq 2T$

The TCL is the sum of demands during the considered SCL cycle plus $K(t_{n-1})$ minus $K(t_n)$, as given in (4).

$$
\begin{aligned}
M(t_n) = D(t_{n-1}, t_{n+1} - L_d) + D(t_{n+1} - L_d, t_n) \\
+ D(t_n - L_d, t_n) + K(t_{n-1}) - K(t_n)
\end{aligned}
\tag{15}
$$

When we still consider the range $T < L_d \leq 2T$ for the demand lead time, the sequence of time points changes depending on the length of L_s, which is why the sixth and seventh cases arise. The time $t_n - L_s$ is between $t_{n-1} - L_d$ and t_{n-2} if $2T < L_s \leq T + L_d$ and accordingly between $t_{n-1} - L_s$ and $t_{n-1} - L_d$ if $T + L_d < L_s$. Similarly, all ranges for the remaining cases can be obtained. All formulas for $K(t_n)$ and $K(t_{n-1})$ are shown in Appendix.

4.1.3 Iterative procedure to determine $K(t_{n-2})$

While we have derived expressions for $K(t_n)$ and $K(t_{n-1})$, it remains to determine $K(t_{n-2})$ to be able to compute the pmf of the TCL $M(t_n)$. As mentioned before, we neglect the remaining units of prior shipment days before t_{n-2} and use a fixed value $\overline{K}$ for $K(t_{n-2})$, which is updated in an iterative procedure. The idea is to replace the number of remaining units with its expectation. Therefore, we start with a given value of $\overline{K}$, determine the probability distribution of $K(t_n)$ and use it to compute the expectation of $K(t_n)$. We also consider a factor that reflects the capacity utilization of the vehicle when we update the value for $\overline{K}$. In general, the obtained value for $\overline{K}$ is not an integer, rendering the computation of the convolutions impossible. As a solution, we determine the pmf for the TCL and the remaining units at t_n two times. First, we use the rounded-down value $\lfloor \overline{K} \rfloor$ and the second time the rounded-up value $\lceil \overline{K} \rceil$. Both results are merged as follows:

$$
\begin{aligned}
P(M(t_n) = k) = (\lceil \overline{K} \rceil - \overline{K})) \cdot P(M(t_n) = k \mid \lfloor \overline{K} \rfloor) \\
+ (\overline{K} - \lfloor \overline{K} \rfloor) \cdot P(M(t_n) = k \mid \lceil \overline{K} \rceil).
\end{aligned}
\tag{16}
$$

As a starting value for the iterative procedure, we have chosen $(Cap - \lambda T - \frac{1}{2}\lambda L_d)^-$, which is obtained by replacing the random demand in the expression with the total number of units, which exceeds the capacity of the primary transportation option, with its expectation. Furthermore, the remaining units $K(t_{n-1})$ are replaced with 50% of the expected number of ordered units with a due date after the shipment time. In summary, we provide a sketch of the algorithm to compute the expected shipment costs.

Step 1: Start with $\overline{K} = (Cap - \lambda T - \frac{1}{2}\lambda L_d)^-$.
Step 2: Compute for $\lceil \overline{K} \rceil$ the pmf of $K(t_n)$ from (11) and (10) for case 1, or use the corresponding formulas for the other cases.
Step 3: Compute for $\lfloor \overline{K} \rfloor$ the pmf of $K(t_n)$ from (11) and (10) for case 1, or use the corresponding formulas for the other cases.
Step 4: Compute the pmf of $K(t_n)$, similar to (16).

Step 5: Compute the expected number of remaining units at t_n by $\bar{K}(t_n) = \sum_{j=0}^{\infty} jP(K(t_n) = j)$.
Step 6: If $|\overline{K} - \bar{K}(t_n)\frac{Cap}{\lambda T}| < 0.1$, go to step 7; otherwise, set $\overline{K} = \left\lfloor \bar{K}(t_n)\frac{Cap}{\lambda T} \cdot 10 \right\rceil : 10$ and go to step 2.
Step 7: Compute the pmf of the TCL according to (16).
Step 8: Compute $TSC(R, T, Cap)$ with current pmf of the TCL by (3).

4.2 Expected inventory cost at the warehouse

For the analysis of the expected inventory cost per time unit, we adapt the methodology introduced in Marklund (2011), who studied an inventory system with time-based SCL without ADI and without flexible deliveries, reflecting the case of $L_d = 0$. Therefore, we only discuss the case of $L_d > 0$ with flexible deliveries in this paper. We observe each unit going through the system separately, calculate the cost for each unit and then consider the expectation of this cost. In a first step, we show how the (R, Q) policy is connected to a base-stock policy with base-stock level S because we use this relationship in our further analysis.

Let us denote with t_r the time where the warehouse just placed a replenishment order of size Q at the outside supplier, which arrives after a supply lead time L_s. The units of this order are then consumed in the defined order $1, 2, \dots, Q$. The first unit of this batch is needed for the $(R + 1)$ th order at the warehouse after t_r because, at t_r, there are still R units on stock that are used first. The observation of the first unit represents the situation in which the warehouse uses an $(S - 1, S)$ policy with base-stock level $S = R + 1$. Similarly, we can relate a base-stock policy to each unit of the batch, for example, using a base-stock level of $S = R + 2$ for the second unit and finally $S = R + Q$ for the final unit of the batch. Thus, one possibility for obtaining the system's expected total inventory cost is to replace the (R, Q) policy by Q base-stock policies with base-stock levels $S = R + 1, R + 2, \dots, R + Q$. Therefore, the expected inventory cost for the system for an (R, Q) policy can be calculated as shown in (17), where $TIC(S, T, Cap)$ represents the expected inventory cost for the system per time unit when using a base-stock policy with base-stock level S and a SCL cycle length T with a capacity reservation of Cap units.

$$TIC(R, T, Cap) = \frac{1}{Q} \sum_{S=R+1}^{R+Q} TIC(S, T, Cap) \tag{17}$$

In the case of $S > 0$, the warehouse orders the considered unit at the outside supplier before a facility orders it, whereas $S \leq 0$ implies that the warehouse orders the considered unit at the outside supplier after or at the same time at which the facility orders it at the warehouse. These two situations have to be discussed separately, and we have to introduce some additional notation for further analysis:

$\Omega(x)$ length of the time interval between the replenishment moment at the warehouse and the moment when the xth order from the production facilities arrives at the warehouse, random variable,

V shipment delay, defined as the length of the time interval between the time when a unit is demanded and available for a shipment, and the subsequent shipment day, random variable,
$G^x(t)$ cumulative distribution function of an Erlang (x, λ) distribution,
$g^x(t)$ density of an Erlang (x, λ) distribution,
$U(t)$ cumulative distribution function of a uniform distributed random variable on the interval $[0, T]$,
$u(t)$ density function of a uniform distributed random variable on the interval $[0, T]$.

We call a unit *prequalified* if it is available and ordered because it can be shipped before it is actually demanded since we allow for early deliveries. A unit is denoted as *qualified* if it is available and demanded and thus has to be shipped with the primary or alternative transportation option on the subsequent shipment day.

4.2.1 The case of $S > 0$

Observing a specific unit on its way through the system, we recognize four essential points in time. First, a unit is available for satisfying a demand at the warehouse exactly L_s time units after the replenishment order was placed, thus at time $t_a = t_r + L_s$. The following important event is the time when the facility order for this unit arrives at the warehouse, which is given as $t_o = t_r + \Omega(S)$, where $\Omega(S)$ represents the time until the Sth facility order occurs. The third interesting time is when the status of the unit changes from an order to a demand, which occurs at $t_d = t_r + \Omega(S) + L_d$. Finally, there is the time t_s when the unit is shipped to the production facility, depending on the available reserved transportation capacity.

Which types of costs occur and to what extent depend on the sequence of the events. For the derivation of the formulas, we have to distinguish all possible situations, which are denoted with $i \in \{A, B, C, D, E, F, G\}$. Situation A is illustrated in Fig. 5, and we explain all of the cases in the following in more detail.

The first three situations (A,B,C) are related to a situation in which the unit is available when the order occurs, which means that $t_a < t_o$. Otherwise, the unit is ordered while there is no available stock for the considered unit at the warehouse (D,E,F,G). We further differentiate between the situations in which prequalification and qualification occur at different points in time (D,E,F) and at the same time (G), whereby the latter case only occurs if the unit is available after it is demanded. If the unit is available before demand occurs, then the unit is prequalified at time $\max(t_a, t_o)$ and qualified at time t_d. In the following, $t_n \in \mathbb{N}$ denotes an arbitrary shipment day.

A $t_a < t_o, t_{n-1} < t_o < t_d < t_n, t_s = t_n$ Since prequalification and qualification occur in the same SCL cycle, the unit must be shipped on the next shipment day ($t_s = t_n$),

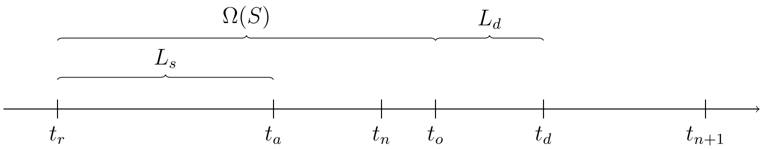

Fig. 5 Important moments in time in situation A

indicating there are no early-delivery costs, and the unit must wait V time units. Stock-keeping costs are charged for a time interval with length $\Omega(S) - L_s + L_d + V$.

B $t_a < t_o, t_o < t_{n-1} < t_d < t_n, t_s = t_{n-1}$ This situation can only occur if there is sufficient reserved transportation capacity available at t_{n-1} to initiate an early delivery, which is then associated with early-delivery costs for $T - V$ time units. The unit is kept on stock for $\Omega(S) - T - L_s + L_d + V$ time units.

C $t_a < t_o, t_o < t_{n-1} < t_d < t_n, t_s = t_n$ There is not sufficient transportation capacity available to allow for early delivery of the considered unit in this situation. Then, waiting costs for V time units are due, and stock-keeping costs are incurred for $\Omega(S) - L_s + L_d + V$ time units.

D $t_o \leq t_a, t_{n-1} < t_a < t_d < t_n, t_s = t_n$ Similar to situation A, there are no early-delivery costs, and waiting costs are incurred for V time units. The unit is kept on stock for $\Omega(S) - L_s + L_d + V$ time units, which is shorter than A because the unit is available after t_{n-1}.

E $t_o \leq t_a, t_a < t_{n-1} < t_d < t_n, t_s = t_{n-1}$ This situation can only occur if there is sufficient reserved transportation capacity available at t_{n-1} to initiate an early delivery, which is then associated with early-delivery costs for $T - V$ time units. The unit is kept on stock for $\Omega(S) - T - L_s + L_d + V$ time units.

F $t_o \leq t_a, t_a < t_{n-1} < t_d < t_n, t_s = t_n$ This situation is comparable to C such that waiting costs for V time units are, due and stock-keeping costs are incurred for $\Omega(S) - L_s + L_d + V$ time units.

G $t_o \leq t_a, t_o < t_d < t_a < t_n, t_s = t_n$ Since prequalification and qualification occur at the same moment in time, there are no early-delivery costs. Stock-keeping costs only occur for V time units, and waiting costs (including the backorder costs) are charged for $V + L_s - L_d - \Omega(S)$ time units.

In summary, we provide all of the cost expressions for situations $i \in \{A, B, C, D, E, F, G\}$ in Table 3.

Obviously, for a given S, T and Cap, the inventory cost depends on the random variables $\Omega(S)$ and V and the available reserved transportation capacity at t_n. However, we only know the pmf of the TCL at t_n after all demands and orders of the considered SCL cycle occurred, which again is why we rely on an approximation. We denote the inventory cost for case $S > 0$ in the following analysis with $C(\Omega(S), V, M(t_n))$. Denoting the joint density function of $\Omega(S)$, V and $M(t_n)$ with $f(x, y, z)$, we can obtain the expected inventory cost by

Table 3 Different cost expressions for the different situations

Situation i	Stock-keeping costs	Waiting costs	Early-delivery costs
A	$h(\Omega(S) - L_s + L_d + V)$	wV	0
B	$h(\Omega(S) - T - L_s + L_d + V)$	0	$e(T - V)$
C	$h(\Omega(S) - L_s + L_d + V)$	wV	0
D	$h(\Omega(S) - L_s + L_d + V)$	wV	0
E	$h(\Omega(S) - T - L_s + L_d + V)$	0	$e(T - V)$
F	$h(\Omega(S) - L_s + L_d + V)$	wV	0
G	hV	$w(V + L_s - L_d - \Omega(S))$	0

$$E[C(\Omega(S), V, M(t_n))] = \int_0^\infty \int_0^T \int_0^\infty C(x,y,z) f(x,y,z) dz\, dy\, dx = \sum_{\substack{i \in \{A,B,C,\\ D,E,F,G\}}} \int_0^\infty \int_0^T \int_0^\infty C_i(x,y,z) f_i(x,y,z)\, dz\, dy\, dx, \tag{18}$$

which means that we can split the derivation into several parts, where each part corresponds to one of the aforementioned situations. For the following analysis, we neglect the dependency between $M(t_n)$ and the other random variables such that we obtain

$$E[C(\Omega(S), V, M(t_n))] = \sum_{\substack{i \in \{A,B,C,\\ D,E,F,G\}}} \int_0^\infty \int_0^T \int_0^\infty C_i(x,y,z) f_i(x,y) f(z) dz\, dy\, dx. \tag{19}$$

Therefore, we can reformulate (19) as

$$\begin{aligned} E[C(\Omega(S), V, M(t_n))] &= \sum_{\substack{i \in \{A,B,C,\\ D,E,F,G\}}} \int_0^\infty \int_0^T \int_0^\infty C_i(x,y) f_i(x,y) f(z) dz\, dy\, dx \\ &= \sum_{i \in \{A,D,G\}} \int_0^\infty \int_0^T C_i(x,y) f_i(x,y)\, dy\, dx \\ &\quad + P(M(t_n) < Cap) \sum_{i \in \{B,E\}} \int_0^\infty \int_0^T C_i(x,y) f_i(x,y)\, dy\, dx \\ &\quad + P(M(t_n) \geq Cap) \sum_{i \in \{C,F\}} \int_0^\infty \int_0^T C_i(x,y) f_i(x,y)\, dy\, dx, \end{aligned} \tag{20}$$

where $C_i(x, y)$ and $f_i(x, y)$ with $i \in \{A, B, C, D, E, F, G\}$ represent the inventory cost depending on $\Omega(S)$ and V and the joint distribution function of $\Omega(S)$ and V, respectively.

Since the cost expressions can easily be derived from Table 3, it remains to determine the functions $f_i(x, y), i \in \{A, B, C, D, E, F, G\}$. It can be shown that the functions $f_i(x, y)$ are positive on different domains (Table 4). Further, in the range where they are unequal to zero, the functions have the form $g^S(x)u(y)$ for all situations i, $i \in \{A, B, C, D, E, F, G\}$. In the following, we show the derivation for situation A, whereas the remaining derivations are given in Appendix.

$$\begin{aligned} F_A(x, y) &= P(\Omega(S) \le x, V \le y, t_a < t_o, t_{n-1} < t_o < t_d < t_n, t_s = t_n) \\ &= P(\Omega(S) \le x, V \le y, t_r + L_s < t_r + \Omega(S), \\ &\quad t_{n-1} < t_r + \Omega(S) < t_r + \Omega(S) + L_d < t_n) \\ &= P(\Omega(S) \le x, V \le y, L_s < \Omega(S), \\ &\quad 0 < t_r + \Omega(S) - t_{n-1} < t_r + \Omega(S) + L_d - t_{n-1} < t_n - t_{n-1}) \\ &= P(\Omega(S) \le x, V \le y, L_s < \Omega(S), \\ &\quad 0 < t_r + \Omega(S) - t_{n-1} < t_r + \Omega(S) + L_d - t_{n-1} < T) \end{aligned} \tag{21}$$

Since $T = V + L_d + t_r + \Omega(S) - t_{n-1}$ (Fig. 5), we obtain for $x > L_s$ and $y \le (T - L_d)^+$

$$\begin{aligned} F_A(x, y) &= P(L_s < \Omega(S) \le x, V \le y, \\ &\quad 0 < t_r + \Omega(S) - t_{n-1} < t_r + \Omega(S) + L_d - t_{n-1} < T) \\ &= P(L_s < \Omega(S) \le x, V \le y, 0 < T - V - L_d < T - V < T) \\ &= P(L_s < \Omega(S) \le x, V \le y, V < (T - L_d)^+) \\ &= P(L_s < \Omega(S) \le x, V \le y) \\ &= \left(G^S(x) - G^S(L_s)\right)U(y). \end{aligned} \tag{22}$$

Since V is uniformly distributed between [0, T] (Tijms 2003), the upper bound of y cannot be smaller than 0. Thus, $f_A(x, y)$ is given as the partial derivative with respect to x and y

$$f_A(x, y) = \begin{cases} g^S(x)u(y) & \text{if } L_s < x < \infty, 0 \le y \le (T - L_d)^+ \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Table 4 Range for x and y dependent on situation i, $i \in \{A, B, C, D, E, F, G\}$

Situation i	x	y
A	$L_s < x < \infty$	$0 \le y \le (T - L_d)^+$
B	$L_s < x < \infty$	$(T - L_d)^+ < y \le T$
C	$L_s < x < \infty$	$(T - L_d)^+ < y \le T$
D	$L_s - L_d < x \le L_s$	$0 \le y \le (T - L_d + L_s - x)^+$
E	$L_s - L_d < x \le L_s$	$(T - L_d + L_s - x)^+ < y \le T$
F	$L_s - L_d < x \le L_s$	$(T - L_d + L_s - x)^+ < y \le T$
G	$0 \le x \le L_s - L_d$	$0 \le y \le T$

Since demand follows a Poisson process, $\Omega(S)$ is Erlang distributed with the parameters S and λ and is independent of the shipment delay V (Tijms 2003). Therefore, the expectation of the inventory cost for each situation can be computed, and the results for

$$E[C_i(\Omega(S), V)] = \int_0^{\infty} \int_0^{T} C_i(x, y) f_i(x, y)\, dy\, dx \quad \forall i \in \{A, B, C, D, E, F, G\} \tag{24}$$

are given in Tables 5 and 6. Since the range for y depends on T and L_d, we must consider the cases $L_d \leq T$ and $L_d > T$ separately. The derivation is again reported in Appendix.

4.2.2 The case of $S \leq 0$

If $S = 0$, then the considered unit is ordered by the production facility at the same time that the warehouse orders it at the outside supplier. Thus, no safety stock is kept at the warehouse. Focusing on $S < 0$, the warehouse orders the considered unit at the outside supplier after the next $|S|$th facility orders occur. Due to $L_d \leq L_s$, both cases imply that the considered unit is always demanded before it is available for shipment. Therefore, warehouse stock-keeping costs arise for the time interval V, whereas waiting costs occur for the time interval $V + L_s - L_d + \Omega(|S|)$. As $t_a \geq t_d$, the flexible delivery option cannot be used. Therefore, inventory cost $\tilde{C}(\Omega(S), V)$ for case $S \leq 0$ is independent of $M(t_n)$, and the expectation of this cost can be obtained by

$$E[\tilde{C}(\Omega(S), V)] = h\frac{T}{2} + w\left(\frac{|S|}{\lambda} + L_s - L_d + \frac{T}{2}\right). \tag{25}$$

In summary, the expected inventory cost of the system is given by

$$TIC(S, T, Cap) = \begin{cases} \lambda E\left[C(\Omega(S), V, M(t_n))\right], & \text{for } S > 0 \\ \lambda E\left[\tilde{C}(\Omega(S), V)\right], & \text{for } S \leq 0. \end{cases} \tag{26}$$

5 Approximation method

This section shows how to minimize the expected total cost $TC(R, T, Cap)$ per time unit when $L_d \geq 0$ by determining the near-optimal reorder level R^*, the near-optimal length of the SCL cycle T^* as well as the near-optimal capacity reservation Cap^*.

Examples reveal that $TC(R, T, Cap)$ is not jointly convex in R, T and Cap. However, during all of our numerical experiments, we could not find any example in which $TC(R, T, Cap)$ was not convex in T for a fixed R and Cap (not convex in Cap for a fixed R and T). Using this property, we perform a bounded enumeration. In our numerical study, we focus on the optimization of R^* and T^* for a fixed Cap

Table 5 Expected inventory cost conditioned on the different situations for the case $L_d \leq T$

i	$E[C_i(\Omega(S), V)]$
A	$h\frac{T-L_d}{T}\left(\left(\frac{T+L_d}{2}-L_s\right)\left(1-G^S(L_s)\right)+\frac{S}{\lambda}\left(1-G^{S+1}(L_s)\right)\right)+w\frac{(T-L_d)^2}{2T}\left(1-G^S(L_s)\right)$
B	$h\frac{L_d}{T}\left(\frac{L_d}{2}-L_s\right)\left(1-G^S(L_s)\right)\frac{S}{\lambda}\left(1-G^{S+1}(L_s)\right)\Big)+e\frac{L_d^2}{2T}\left(1-G^S(L_s)\right)$
C	$h\frac{L_d}{T}\left(\frac{L_d}{2}-L_s+T)\left(1-G^S(L_s)\right)+\frac{S}{\lambda}\left(1-G^{S+1}(L_s)\right)\right)+w\frac{2TL_d-L_d^2}{2T}\left(1-G^S(L_s)\right)$
D	$h\Big(\frac{T^2-(L_s-L_d)^2}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)+\frac{(L_s-L_d)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right)-\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
	$+w\Big(\frac{(T-L_d+L_s)^2}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)-\frac{(T-L_d+L_s)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right)+\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
E	$h\Big(\frac{(L_d-L_s)^2}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)+\frac{(L_d-L_s)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right))+\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
	$+e\Big(\frac{(L_d-L_s)^2}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)+\frac{(L_d-L_s)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right)+\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
F	$h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)+\frac{(L_d-L_s+T)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right)+\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
	$+w\Big(\frac{2T(L_d-L_s)-(L_d-L_s)^2}{2T}\left(G^S(L_s)-G^S(L_s-L_d)\right)+\frac{(T-L_d+L_s)S}{T\lambda}\left(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\right)-\frac{S(S+1)}{2T\lambda^2}\left(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\right)\Big)$
G	$h\frac{T}{2}G^S(L_s-L_d)+w\Big((L_s-L_d+\frac{T}{2})G^S(L_s-L_d)-\frac{S}{\lambda}G^{S+1}(L_s-L_d)\Big)$

and on the optimization of R^* and Cap^* for a fixed T, respectively. T and Cap are highly dependent on each other since a high utilization of the reserved transportation capacity mainly depends on the expected number of orders during a consolidation cycle, thus on λT. In general, a modified given Cap changes the optimal T^*, and inversely, a modified given T changes the optimal Cap^*. To limit the computational time, we assume that either Cap or T is given. This is also sufficient to answer the research questions.

First, we define a lower and an upper bound on the decision variable R, denoted by R^L and R^U, respectively. A proven lower bound is $R^L = -Q$ (Axsäter 1998). When increasing R, there is a point at which another increase in R does not influence the waiting, early-delivery and shipment costs because the stock on hand remains sufficient to always satisfy all orders and demands; thus, the flexible delivery option is already used to some extent. The only effect then is an increase in stock-keeping costs. The reorder level is high enough when the demand during the supply lead time never exceeds R, indicating that backorders do not occur. Therefore, $R^U = \min(R : P(D(0, L_s) > R) < \epsilon)$, where ϵ is a small number close to zero. Note that the bounds do not depend on T or on Cap.

5.1 Determination of R^* and T^* for a given Cap

Since the length of the SCL cycle can only take natural numbers, we define the lower bound for T as $T^L = 1$. The upper bound $T^U(R, Cap)$ can be found for each R and a given Cap individually, where we use the convexity property. For a given $R \in \{R^L, \dots, R^U\}$ and Cap, we increase T by 1 until $TC(R, T-1, Cap) < TC(R, T, Cap)$.

Step 1: Determine $R^U = \min(R : P(D(0, L_s) > R) < \epsilon)$ and $R^L = -Q$, and fix Cap.
Step 2: For all given $R = R^L, R^L + 1, \dots, R^U$, compute $TC(R, T, Cap)$ with $T = T^L = 1$.
Step 3: Increase T by 1 and compute $TC(R, T, Cap)$ for all relevant R.
Step 4: If $TC(R, T-1, Cap) < TC(R, T, Cap)$ for all $R = R^L, R^L + 1, \dots, R^U$, go to step 5; else continue with step 3.
Step 5: Find R^* and T^*, which minimize the expected total cost $TC(R, T, Cap)$ for a fixed Cap.

5.2 Determination of R^* and Cap^* for a given T

Obviously, the transportation capacity reserved cannot be negative and has to be integer. Therefore, the lower bound for Cap is $Cap^L = 0$, where only the alternative transportation option can be used why early deliveries are not allowed at all. The upper bound $Cap^U(R, T)$ can be found for each R and a given T individually, where we use the convexity property. For a given $R \in \{R^L, \dots, R^U\}$ and T, we increase Cap by 1 until $TC(R, T, Cap - 1) < TC(R, T, Cap)$.

Table 6 Expected inventory cost conditioned on the different situations for the case $L_d > T$

i	$E[C_i(\Omega(S), V)]$
A	0
B	$h\Big((L_d - L_s - \frac{T}{2})\Big(1 - G^S(L_s)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big) + e\frac{T}{2}\Big(1 - G^S(L_s)\Big)$
C	$h\Big((L_d - L_s + \frac{T}{2})\Big(1 - G^S(x)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big)\Big) + w\frac{T}{2}\Big(1 - G^S(x)\Big)$
D	$h\Big(\frac{T^2-(L_s-L_d)^2}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) + \frac{(L_s-L_d)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$-\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big)\Big)$
	$+w\Big(\frac{(T-L_d+L_s)^2}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) - \frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big)\Big)$
E	$h\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) + \frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big) + (L_d - L_s - \frac{T}{2})\Big(G^S(L_s) - G^S(L_s - L_d + T)\Big) + \frac{S}{\lambda}\Big(G^{S+1}(L_s) - G^{S+1}(L_s - L_d + T)\Big)\Big)$
	$+e\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) + \frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big) + \frac{T}{2}\Big(G^S(L_s) - G^S(L_s - L_d + T))\Big)\Big)$
F	$h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) + \frac{(L_d-L_s+T)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big) + (L_d - L_s + \frac{T}{2})\Big(G^S(L_s) - G^S(L_s - L_d + T)\Big) + \frac{S}{\lambda}\Big(G^{S+1}(L_s) - G^{S+1}(L_s - L_d + T)\Big)\Big)$
	$+w\Big(\frac{2T(L_d-L_s)-(L_d-L_s)^2}{2T}\Big(G^S(L_s - L_d + T) - G^S(L_s - L_d)\Big) + \frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s - L_d + T) - G^{S+1}(L_s - L_d)\Big)$
	$-\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s - L_d + T) - G^{S+2}(L_s - L_d)\Big) + \frac{T}{2}\Big(G^S(L_s) - G^S(L_s - L_d + T)\Big)\Big)$
G	$h\frac{T}{2}G^S(L_s - L_d) + w\Big((L_s - L_d + \frac{T}{2})G^S(L_s - L_d) - \frac{S}{\lambda}G^{S+1}(L_s - L_d)\Big)$

Step 1: Determine $R^U = \min(R : P(D(0, L_s) > R) < \epsilon)$ and $R^L = -Q$, and fix T.
Step 2: For all given $R = R^L, R^L + 1, \dots, R^U$, compute $TC(R, T, Cap)$ with $Cap = Cap^L = 0$.
Step 3: Increase Cap by 1 and compute $TC(R, T, Cap)$ for all relevant R.
Step 4: If $TC(R, T, Cap - 1) < TC(R, T, Cap)$ for all $R = R^L, R^L + 1, \dots, R^U$, go to step 5; else continue with step 3.
Step 5: Find R^* and Cap^*, which minimize the expected total cost $TC(R, T, Cap)$ for a fixed T.

6 Numerical study

In this section, we first present the results of a numerical study to investigate the performance of the applied approximation for the expected total cost $TC(R, T, Cap)$. Second, we investigate the influences of ADI and the flexible SCL program on the expected total cost, as well as on the variables to be optimized.

In the numerical study, we focus on bulky and expensive items to show how a company can apply the presented model. We start with a definition of a base case, where the parameters related to the inventory system are in a similar range as in Marklund (2011) and other references. The order rate of the item at the warehouse is given as $\lambda = 2$. The stock-keeping costs parameter h at warehouse equals 1 per unit and time unit, whereas the waiting and early-delivery cost parameters at the warehouse are fixed to $w = 2$ and $e = 2$ per unit and time unit, respectively. Due to a time-based SCL program, production facilities must hold more safety stock, wherefore we consider waiting costs. Early-delivery costs represent stock-keeping costs at production facilities. Both reasons justify cost parameters w and e close to h. We rely on similar ranges for waiting costs as, e.g., in Çetinkaya et al. (2008). The replenishment quantity Q equals 10 to limit computational time, and the transportation capacity is fixed at $Cap = 10$, which is reasonable for a bulky product. Shipment costs depend on the fixed cost parameter $\alpha(Cap)$ for reserved transportation capacity and the variable parameter c_2 in case the reserved capacity is exceeded. In our base case, we fix the variable reservation cost to $c_1 = 10$ and obtain in the base case for $Cap = 10$ a value for the fixed transportation costs $\alpha(Cap) = 100$. Additionally, variable shipment costs depend on c_1 according to $c_2 = 2c_1$. When the primary transportation capacity is fully utilized, a cost of c_1 arises per unit shipped for the primary transportation option. We double the unit shipment costs for the alternative transportation option for the base case. A reasonable supply lead time from the outside supplier to the warehouse is $L_s = 2$, whereas $L_d = 1$ time units.

6.1 Performance of the approximation

Before deriving managerial insights, we validate our approximation method with a simulation study. For this study, we focus on the optimization of R and T for a given Cap. Therefore, we define a mixed-level fractional factorial design, which relates to the base case. For parameters λ, Cap, $\frac{w}{h}$ and $\frac{e}{h}$, the base case defines the medium level, which is extended by low and high levels. Since h, w and e represent inventory holding cost at the warehouse and indirectly at the production facilities, we only investigate a changed relation between h and w (between h and e). For parameters c_2, L_d and L_s, we define low and high levels as follows: $c_2 = xc_1$ with $x \in \{1.5, 2\}$, $L_d \in \{1, 2\}$ and $L_s \in \{2, 4\}$. We do not vary c_1 since the relation of cost for reservations and spontaneous shipments changes by modifying c_2. Additionally, we fix the replenishment quantity Q as mentioned in the base case, because the replenishment costs do not have an influence on the optimal decisions. This yields $3^4 \cdot 2^3 = 648$ instances.

For these 648 instances, we determine the parameters R^*_{app} and T^*_{app}, solving the optimization problem (2), relying on the results of sections 4 and 5. We use a simulation to evaluate the system's expected cost of a policy and call this cost the exact expected cost to validate these results. The length of each simulation run is 52000 days, while the last 50000 days are used for the cost computation. We use sequential sampling and stop if the half-width of the 95% confidence interval of the average total cost is smaller than 0.5% of the average total cost of the considered instance.

To determine the optimal policy parameters (R^*, T^*), we combine the simulation with a neighborhood search and use R^*_{app} and T^*_{app} as initial values. The neighborhood includes all points $(R^*_{app} + g, T^*_{app} + G)$ with $g, G \in \{-1, 0, 1\}$. If the neighborhood offers a better average total cost value than the initials, the neighborhood search is repeated for the best value in the neighborhood. This iterative procedure can be stopped if no better average total cost value can be found. The obtained policy parameters are locally optimal and define the optimal decision (R^*, T^*).

We are interested in the relative average total cost increase caused by not making the optimal decision with our approach. Therefore, we calculate the relative total cost difference between the average total cost of the optimal policy $TC_{sim}(R^*, T^*)$ and the average total cost of the policy determined by our approach $TC_{sim}(R^*_{app}, T^*_{app})$ for all instances. $TC_{sim}(R, T)$ represents the average total cost computed by simulation, which we assume to be the correct average total cost values. We define the relative cost difference as

$$\delta TC_{sim} = \frac{TC_{sim}(R^*_{app}, T^*_{app}) - TC_{sim}(R^*, T^*)}{TC_{sim}(R^*, T^*)} \cdot 100 \tag{27}$$

and also compute aggregate values. In Table 7, the aggregated results of all 648 examples are provided to investigate the impact of the input parameters on the performance. The average (maximum) total cost deviation is 0.20% (10.54%). The worst case is observed in a situation in which the reserved transportation capacity is small compared to the average demand. In these situations, $K(t_{n-2})$ has a more significant influence on the TCL at t_n, explaining the decreasing performance. However, from an economic point of view, a small reserved capacity is only acceptable if the demand is comparatively low.

For 598 of 648 instances, we found the optimal policy parameters using our approximate approach. Only in 7.72% of the instances could we not find the optimal values; however, for 31 of these 50 examples, the optimal values were located in the direct neighborhood ($R^*_{app} + g, T^*_{app} + G$) with $g, G \in \{-1, 0, 1\}$. For only 19 examples, larger deviations in the optimal policy parameters were observed. The maximum deviation between R^*_{app} and R^* for all 648 examples is four, whereas the deviation between T^*_{app} and T^* is one at maximum.

Focusing only on the 50 examples in which we do not derive the optimal policy parameter, we observe an average (a maximum) total cost increase of 2.58% (10.54%). A more detailed look at the results also reveals that a nonoptimal SCL cycle length has a larger effect on the expected cost than a nonoptimal reorder level. Choosing a SCL cycle length that is one day shorter (longer) than the optimal cycle length means that more demands must be shipped by the alternative transportation option (the transportation capacity is less utilized on average), explaining this observation.

In summary, we conclude that our approximation has an excellent performance for the most relevant cases, and even in the other situations, it is acceptable. Therefore, we can use our model to generate managerial insights.

6.2 Managerial insights

In this section, we quantify the added value of ADI under flexible SCL and investigate the impact of the length of the demand lead time on the optimal SCL cycle length and on the optimal capacity of the primary transportation option. Therefore, we use a different experimental design to reduce computational time without losing insights and reduce the level for the order rate ($\lambda \in \{1, 2\}$), while we increase the levels for the demand lead time $L_d \in \{0, 2, 4, 6, 8\}$ and fix L_s to 10. The other parameters are the same as in the base case and the former study.

6.2.1 Cost improvements by ADI and flexible deliveries

First, we focus on a given capacity and on the influence of an increasing demand lead time on the expected total cost. We compute the average marginal relative decrease in the expected total cost when the demand lead time L_d is stepwise increased by two time units, while the other parameters are fixed. The results are presented in Table 8 and indicate that large cost reductions can be achieved if customers are willing to place orders in advance. In general, it can be seen that the longer the demand lead time, the greater the total cost reductions. The maximal marginal relative cost reduction when L_d is increased from 0 to 2 is 20.29%, whereas we can achieve a maximum decrease of the expected cost when L_d is increased from 0 to 8 of 35.57%. Further, it can be observed that an increase in the demand lead time of two time units can have a different effect depending on the starting point. For example, coming from the situation where $L_d = 0$ to a situation where $L_d = 2$, on average, the expected total cost per time unit can be decreased from 62.01 to 55.54 (10.43%), whereas the expected total cost per time unit can be reduced from 55.54 to 52.91 (4.74%) when increasing the demand lead time from 2 to 4. Thus, the marginal value of the ADI decreases with an increasing amount of information.

There are two possible sources for the cost reduction. First, as already observed in (Hariharan and Zipkin 1995), a longer demand lead time reduces the effective lead time ($L_s - L_d$) and therefore the safety stock. Second, due to the flexible shipment policy, a longer demand lead time results in more orders available for earlier shipments. Thus, better utilization of the transportation capacity can be attained, and fewer emergency deliveries are necessary. Since early deliveries are only allowed when enough free capacity is available, it is clear that the reserved capacity must have an impact on the benefit of ADI in our setting. It can be observed that the marginal relative cost reduction decreases drastically with increasing L_d for the situation $Cap = 5$ (Table 8). The mentioned effect is not so large for $Cap = 20$. Although the marginal relative cost reduction also decreases as L_d increases, it decreases much less, and even when the demand lead time is increased from 2 to 4, 6.61% of the expected total cost can still be saved. In situations with a small reserved transportation capacity, the capacity is already fully utilized on many shipping days when $L_d = 2$, so further information will not result in additional early deliveries in many cases. With a higher reserved capacity, there is a higher probability of unused capacity for orders, even if information is already available. Since the reserved capacity seems to be an important variable, we will later also determine the optimal reserved capacity.

To investigate the proportion of the average total cost decrease caused by reducing the safety stock at the warehouse and by early shipments, respectively,

Table 7 Average and maximum of the relative cost deviation for all examples

Parameter	Value	Average relative cost deviation	Maximum relative cost deviation
w	1	0.1150	10.2782
	2	0.1035	5.3411
	5	0.3779	10.5391
e	1	0.2380	10.5391
	2	0.2239	10.2782
	5	0.1346	7.6342
c_2	$1.5c_1$	0.1775	10.5391
	$2c_1$	0.2201	10.2782
λ	1	0.0043	0.4215
	2	0.0345	1.3202
	4	0.5576	10.5391
L_d	1	0.0000	0.0003
	2	0.3976	10.5391
L_s	2	0.2715	10.5391
	4	0.1261	10.3451
Cap	5	0.5802	10.5391
	10	0.0119	0.9755
	20	0.0043	0.3361
Total		0.1988	10.5391

we computed the expected total cost for the same 540 examples when flexible deliveries are not allowed at all but ADI is still available.

Additionally, we are interested in the question of whether cost can be reduced even more if more early shipments are allowed. An evident and simple policy to investigate is to dispatch all orders regardless of the remaining transportation capacity. Thus, the primary transportation option may be already exhausted, but additional orders will also be shipped to the facilities by using the alternative option. Our analysis can be easily adapted to obtain formulas for the computation of the expected total cost because remaining units will only occur when the warehouse is running out of stock.

In Fig. 6, we depict the average total cost for all three policies as a function of L_d. It can be seen that a shipment policy without flexible deliveries performs worst. For such a policy, we observe an almost linear cost decrease with increasing demand lead time of approximately 0.7% due to a reduction in the effective supply lead time, resulting in less stock at the warehouse.

Significant cost savings can be obtained with the introduction of flexible deliveries. However, shipping all orders one shipment day ahead and neglecting the available capacity can further be improved by our shipment policy where the reserved capacity is taken into account when deciding about the TCL. Then, an increase in the demand lead time from 0 to 2 results in a decrease of the average total cost by more than 10%, indicating that a reduction of approximately 9.5 percentage points is caused by adapting the time-based SCL policy and allowing for flexible deliveries. We can conclude that the more significant part of the cost reduction is induced by the flexible delivery option and not by reducing the safety stock.

With increasing L_d, this effect is reduced until we reach a point at which additional ADI will only decrease the stock because the flexible delivery option is used to its extent. This is shown in Fig. 7, where we illustrate the marginal relative cost reduction of all three policies.

We can also observe that with increasing demand lead, the cost difference of the two flexible delivery concepts is increasing. The marginal relative total cost reduction for the simple policy is even lower than for the shipment policy without flexible deliveries, because for large demand lead times the alternative transportation option has to be used too often, which prevents further cost reductions.

6.2.2 Optimal length of the SCL cycle

Moreover, to enable early deliveries, even more safety stock is kept at the warehouse compared to a situation without flexible deliveries. This situation is illustrated in Table 9, where the optimal policy parameters R^* and T^* are presented for different values of the reserved transportation capacity, demand lead time and order rate. The remaining parameters are fixed at $h = 1, w = 2, e = 2, c_1 = 20, c_2 = 2\frac{\alpha}{Cap}$, and $L_s = 10$. While the demand lead time L_d has a significant impact on the numerical value of the reorder level, the influence on the optimal length of the SCL cycle is much less. This condition holds for both situations, with and without flexible deliveries. The optimal SCL cycle length is influenced by the shipment costs, as well as

Table 8 Marginal relative cost reduction enabled by ADI and flexible deliveries

Parameter	Value	Marginal relative cost difference $L_d = 0 \rightarrow L_d = 2$	Marginal relative cost difference $L_d = 2 \rightarrow L_d = 4$	Marginal relative cost difference $L_d = 4 \rightarrow L_d = 6$	Marginal relative cost difference $L_d = 6 \rightarrow L_d = 8$	Relative cost difference $L_d = 0 \rightarrow L_d = 8$
w	1	6.1140	2.2723	0.7054	0.3227	9.1885
	2	8.4178	3.8752	1.5760	0.6367	13.9058
	5	15.0111	7.3582	3.5730	1.6296	25.3152
e	1	11.1651	6.0846	3.2606	1.6605	20.6308
	2	10.7273	5.1953	2.3103	0.9814	18.1320
	5	9.3987	2.9626	0.6694	0.1328	12.7873
c_2	$1.5c_1$	9.9334	4.7338	2.0145	0.9201	16.6990
	$2c_1$	10.9141	4.7391	2.0976	0.8890	17.6548
λ	1	9.5306	5.8501	3.1113	1.5726	18.7710
	2	11.0043	4.0146	1.3856	0.4869	16.1709
Cap	5	11.3573	2.6547	0.9616	0.3394	14.8303
	10	10.9027	4.3526	1.2452	0.4499	16.2206
	20	9.3313	6.6097	3.5824	1.7425	19.7802
Total		10.4304	4.7365	2.0564	0.9044	17.1834

by the relationship between the average demand and the capacity. An increasing order rate raises the reorder level and reduces the SCL cycle length to utilize the reserved transportation capacity and to avoid expensive additional shipments.

6.2.3 Optimal transportation capacity

We are further interested in the influence of flexible deliveries on the optimal capacity of the primary transportation option for a given length of the SCL cycle. In Table 10, it can be seen that flexible deliveries lead to equal or larger reorder levels for a fixed and given SCL cycle length. This means that more safety stock is required to enable flexible deliveries. Additionally, more reserved transportation capacity Cap^* is needed in situations with large values of T to exploit the benefit of flexible deliveries fully. However, the influence of the flexible deliveries on the optimal capacity to be reserved is negligible.

6.3 Impact of the policy assumption

When we defined the flexible shipment consolidation policy, we only allowed orders to be shipped one shipment day ahead. They have to be shipped if enough stock and remaining reserved transportation capacity is available. This assumption could be limiting in situations where L_d is much larger than T because orders could potentially be shipped several shipment days in advance. We want to investigate the impact of

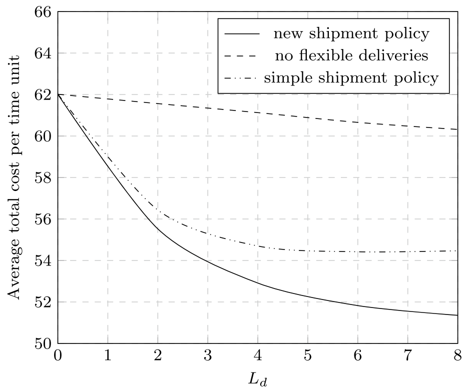

Fig. 6 Average total cost per time unit

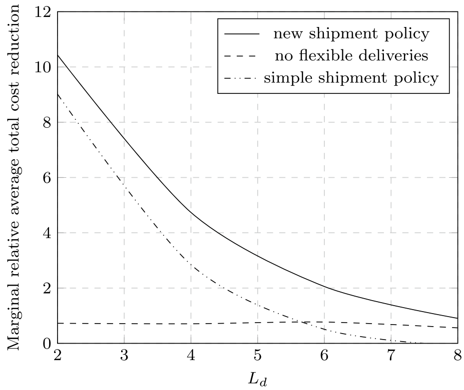

Fig. 7 Marginal relative total cost reduction

this assumption on the optimal decision and expected total cost. Therefore, we compare our policy with a policy where early shipments are always allowed if stock and capacity are available. That means orders are shipped as early as possible. For a fair comparison, we have to determine the optimal reorder level and the optimal length of the SCL cycle, which is done by a simulation-based approach.

We concentrate on a large demand lead time ($L_d = 8$) and a large demand rate ($\lambda = 4$), because in these situations it is more likely that the optimal SCL cycle length is smaller than L_d. L_s is fixed to 10 to reflect make-to-stock situations and different reserved capacities are considered.

We expect a larger impact if waiting and early-delivery costs are large and therefore select the following cost parameters to test our conjecture: $w \in \{2, 10, 100\}$ and $e \in \{2, 10, 100\}$. The remaining parameters correspond to the base case.

Table 11 columns two and three show the optimal decisions with and without the shipment assumption, while the last column presents the relative total cost deviation according to

$$\delta TC_{asm} = \frac{TC_{sim}(R^*_{asm}, T^*_{asm}) - TC_{sim}(R^*, T^*)}{TC_{sim}(R^*, T^*)} \cdot 100, \tag{28}$$

where $TC_{sim}(R^*, T^*)$ represents the minimal total cost without the shipment assumption and $TC_{sim}(R^*_{asm}, T^*_{asm})$ the minimal total cost with shipment assumption both determined by simulation.

Our numerical experiments reveal that the shipment assumption impacts the optimal reorder level. If early shipments are allowed only one shipment day ahead, then more safety stock is needed at the warehouse to enable flexible deliveries in many situations. On the other hand, if early shipments are not restricted, then the early-delivery costs are controlled by a reduction of the optimal reorder level, making early deliveries less likely, as stock-outs occur more frequently.

A contrary effect can be observed if the waiting costs parameter is very high ($w = 100$) and the early-delivery costs parameter is very low ($e = 2$). In such a situation, early deliveries are preferred to avoid waiting times, which results in larger safety stocks under an “as early as possible" shipment policy.

Our numerical results also show that the optimal length of the SCL cycle is relatively robust against the shipment assumption. Thus, total cost differences are induced by the different safety stock quantities at the warehouse. Although the reorder level can control the number of early deliveries, the opportunities are limited. This explains the lower minimal total cost if early deliveries are only allowed one shipment day ahead. Only in situations where early deliveries are much cheaper than the waiting costs it is more beneficial to allow shipments as early as possible. However, it is improbable that such large differences occur in reality since early-delivery and waiting costs are related to stock-keeping costs and, therefore, pretty much the same.

Based on our numerical study, we can conclude that, in line with the existing literature, ADI can lead to large cost reductions in inventory management. However, to fully exploit the benefit of ADI, the shipment policy should also be adapted, and flexible deliveries should be integrated into SCL programs. Doing so could entail larger inventories, but the savings due to a more efficient transportation policy far exceed the cost increases due to larger safety stocks.

Table 9 Optimal reorder level and optimal SCL cycle length

Cap	λ	(R^*, T^*) shipment policy					(R^*, T^*) without flexible deliveries				
		$L_d = 0$	$L_d = 2$	$L_d = 4$	$L_d = 6$	$L_d = 8$	$L_d = 0$	$L_d = 2$	$L_d = 4$	$L_d = 6$	$L_d = 8$
5	1	(8,5)	(7,5)	(5,5)	(4,5)	(2,5)	(8,5)	(6,5)	(4,5)	(2,5)	(0,5)
	2	(20,3)	(16,3)	(12,3)	(7,3)	(3,3)	(20,3)	(15,3)	(11,3)	(6,3)	(2,3)
	4	(41,2)	(37,1)	(29,1)	(20,1)	(11,1)	(41,2)	(32,2)	(23,2)	(14,2)	(8,1)
10	1	(6,9)	(6,8)	(5,8)	(3,9)	(2,9)	(6,9)	(4,8)	(2,8)	(0,8)	(-2,8)
	2	(16,5)	(14,5)	(11,5)	(8,5)	(4,5)	(16,5)	(12,5)	(8,5)	(4,5)	(0,5)
	4	(37,3)	(30,3)	(22,3)	(14,3)	(6,3)	(37,3)	(29,3)	(20,3)	(12,3)	(4,3)
20	1	(6,15)	(5,15)	(5,15)	(4,15)	(3,15)	(6,15)	(4,15)	(2,15)	(0,15)	(-2,15)
	2	(17,9)	(15,9)	(14,9)	(11,9)	(7,9)	(17,9)	(12,9)	(8,9)	(4,9)	(0,9)
	4	(37,5)	(34,5)	(26,5)	(17,5)	(12,4)	(37,5)	(29,5)	(20,5)	(12,5)	(4,5)

Table 10 Optimal reorder level and optimal capacity of the primary transportation option

T	λ	(R^*, Cap^*) with flexible deliveries					(R^*, Cap^*) without flexible deliveries				
		$L_d = 0$	$L_d = 2$	$L_d = 4$	$L_d = 6$	$L_d = 8$	$L_d = 0$	$L_d = 2$	$L_d = 4$	$L_d = 6$	$L_d = 8$
3	1	(9,3)	(8,3)	(7,3)	(4,3)	(2,3)	(9,3)	(7,3)	(5,3)	(2,3)	(0,3)
	2	(20,6)	(19,6)	(15,6)	(11,6)	(7,6)	(20,6)	(15,6)	(11,6)	(6,6)	(2,6)
	4	(38,11)	(38,12)	(25,12)	(20,12)	(10,12)	(38,11)	(30,11)	(22,12)	(14,12)	(6,12)
5	1	(8,5)	(7,5)	(6,5)	(4,5)	(2,5)	(8,5)	(6,5)	(4,5)	(2,5)	(0,5)
	2	(16,10)	(17,10)	(15,10)	(11,10)	(6,10)	(16,10)	(12,10)	(8,10)	(4,10)	(0,10)
	4	(37,20)	(37,21)	(23,20)	(20,20)	(21,21)	(37,20)	(29,20)	(20,20)	(12,20)	(4,20)
10	1	(6,10)	(6,10)	(6,11)	(5,11)	(4,11)	(6,10)	(4,10)	(2,10)	(0,10)	(-2,10)
	2	(16,20)	(17,21)	(16,22)	(16,22)	(14,22)	(16,20)	(12,20)	(8,20)	(4,20)	(0,20)
	4	(37,40)	(36,42)	(36,43)	(37,43)	(40,42)	(37,40)	(29,40)	(20,40)	(12,40)	(4,40)

7 Summary and outlook

In this paper, we have investigated a one-stage inventory model with ADI and a flexible time-based SCL program with a reserved transportation capacity. We derive approximate mathematical expressions to compute shipment and inventory costs at the warehouse and thus are able to determine the warehouse reorder level and the SCL cycle length for the given situation. We have shown in a simulation study that our approximations have excellent performance and can be used to determine near-optimal policy parameters because the optimal decisions are found in more than 90% of our instances, and the average total cost deviation is 0.1988%.

The main finding is that companies can benefit greatly from ADI in the context of inventory management. However, they will miss opportunities if they only focus on the reduction of safety stocks and on a single logistic process. Cost reductions can be increased even more if connected logistic processes are adapted, such as the SCL policy with flexible deliveries. In the investigated setting, the largest part of the cost reduction is induced by the flexible delivery option. Thus, the full potential of ADI can only be exploited if whole logistic processes are adapted.

Although we have not studied it in detail in this paper, we believe that our integrated logistic approach does not only reduce cost, but also has environmental benefits by increasing the utilization of the reserved transportation capacity. Further research can elaborate on these environmental aspects in more detail. It would be very interesting to understand how the optimal policy parameters and the optimal reserved transportation capacity behave if, besides the minimization of cost, also the minimization of carbon emissions is an aim.

We have assumed perfect ADI and homogeneous demand lead times in our model. A logical next step is to replace the limiting assumptions and allow imperfect ADI as well as heterogeneous demand lead times. This will increase the

Table 11 Influence of shipment assumption on optimal reorder level and optimal SCL cycle length

		(R^*, T^*) with shipment assumption			(R^*, T^*) without shipment assumption			Relative total cost deviation		
Cap	*w*	$e = 2$	$e = 10$	$e = 100$	$e = 2$	$e = 10$	$e = 100$	$e = 2$	$e = 10$	$e = 100$
5	2	(10,2)	(6,3)	(4,3)	(9,2)	(6,3)	(4,3)	– 0.5647	– 0.0191	0.0107
	10	(15,2)	(10,2)	(8,3)	(14,2)	(9,2)	(8,3)	– 1.8850	– 2.8433	– 0.4990
	100	(18,2)	(18,2)	(11,1)	(20,2)	(16,2)	(9,2)	4.6285	– 15.7399	– 14.5374
10	2	(8,2)	(5,3)	(2,3)	(7,2)	(5,3)	(2,3)	– 0.7968	– 0.0074	– 0.1349
	10	(13,2)	(8,2)	(6,3)	(12,2)	(7,2)	(6,3)	– 2.1444	– 4.5346	– 0.6919
	100	(18,2)	(18,2)	(10,1)	(19,2)	(14,2)	(7,2)	4.6304	– 18.5438	– 21.7269
20	2	(8,2)	(4,3)	(2,3)	(5,3)	(4,3)	(2,3)	– 1.7276	– 0.0583	– 0.2639
	10	(11,2)	(8,2)	(5,3)	(10,2)	(7,2)	(4,3)	– 2.1704	– 6.6513	– 0.5613
	100	(17,2)	(16,2)	(9,1)	(18,2)	(13,2)	(5,2)	4.3675	– 19.8509	– 23.2847

complexity of the model and requires complete new analysis and therefore, it was not possible to investigate these aspects within the scope of this work.

Another direction for future research is an extension of the inventory system. Instead of only studying one warehouse, a divergent inventory system can be the object of future research. Another extension can be a more general demand model such as a compound Poisson process.

However, we are convinced that these extensions will not change the main finding of our paper, that ADI should not only be used to reduce stock levels but also to adapt related logistic processes. We expect that all extensions come with more complexity and will require a heuristic solution approach.

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