A multimodal network interaction model for the macroscopic fundamental diagram (MFD)

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1. The problem

Urban transportation is complex due to interactions of many road users with many different transportation modes, e.g. cars, bicycles, buses, taxis, pedestrians etc. In general, the mere existence of any road user in the network imposes negative externalities to all other users leading to delayed journeys. Thus, which allocation of time and space resources to transportation modes is optimal for a city? This classic question has frequently been asked and answered in many ways, but, so far, no comprehensive methodology exists that quantifies the multimodal interactions at the network level. However, the macroscopic fundamental diagram (MFD) offers a novel approach for understanding network-wide traffic (Daganzo, 2007; Daganzo and Geroliminis, 2008). Some studies have already investigated interactions between cars and buses (Geroliminis et al., 2014; Loder et al., 2017; Castrillon and Laval, 2018), and cars and pedestrians (Daganzo and Knoop, 2016).

In this paper, we propose a novel and general methodology to describe analytically the effects of local and microscopic disturbances in multimodal traffic, e.g. cars, buses, bicycles, on the overall performance of urban networks. We use a recently formulated functional form for the MFD (Ambühl et al., 2018) in conjunction with the two-fluid theory of town traffic by Herman and Prigogine (1979) to link additional delays for cars generated by such disturbances to the MFD shape.

We organize this paper as follows. Section 2 introduces to the mechanism to link the delays caused by the interaction between modes to the MFD. Thereafter, we introduce to our idea of formulating the analytical delay functions in Section 3. In the full paper, we provide the full analytical delay functions and illustrate how optimal mode shares given network topology and demand can be derived with this proposed model.

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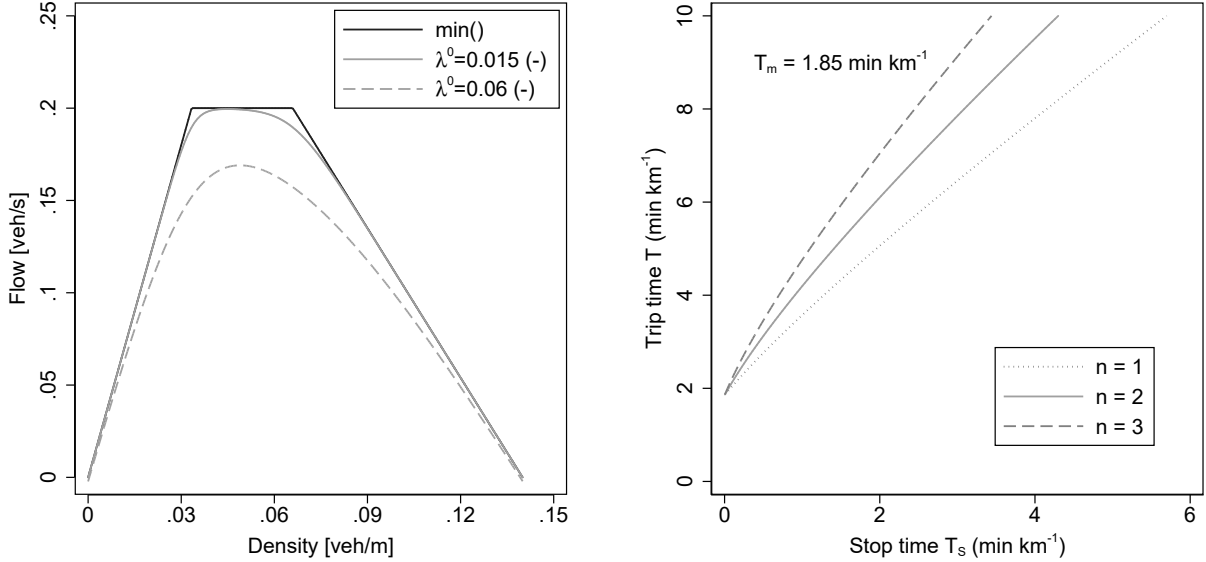
(a) Illustration of the λ^0 parameter.

(b) Illustration of the two-fluid parameter.

Figure 1: Fundamental relationships of urban-scale traffic.

2. Methodology

The presented multimodal network interaction model uses the following notation. The model generally applies to the interaction of m modes, but here we focus on cars (subscript c), buses (subscript b), and bicycles (subscript v, for velo - french bicycle). We define that $\mathbf{k}$ is the three-dimensional state vector of the system with elements car density, k_a, bus density, k_b, and bicycle density, k_v. Each mode has a well-defined corridor macroscopic fundamental diagram (MFD). Generally, the MFD is a lower envelope to all possible states in the relationships between network's average flow, q, and density, k (Daganzo, 2007; Daganzo and Geroliminis, 2008). Originally defined for car traffic only, we simply transfer the idea to buses and bicycles as the three modes basically only differ in propulsion and some operational characteristics, e.g. speeds, passenger occupancy, vehicle size. We denote this lower envelope as the upper MFD (uMFD) that is known a priori (Ambühl et al., 2018; Daganzo et al., 2017). All observed traffic states will always be located *below* this uMFD due to traffic heterogeneity (e.g. Mazloumian et al., 2010; Geroliminis and Sun, 2011; Gayah and Daganzo, 2011; Daganzo et al., 2011) and network dynamics (e.g. Mariotte et al., 2017). Here, we use a functional form for the MFD proposed by Ambühl et al. (2018) that captures in particular the gap between the a priori known uMFD and the observed MFD with just a single parameter, λ^0. This parameter can be seen as a quantification of network homogeneity or the between-vehicle interactions. Eqn. 1 shows this functional form for a trapezoidal uMFD. Such uMFD has been used, for example, by Daganzo et al. (2017). For the reader's convenience we omit the subscript m for the mode as it appears at every item.

$$q\left(k\right) = -\lambda^0 \ln\left(\exp\left(-\frac{v^f k}{\lambda^0}\right) + \exp\left(-\frac{Q}{\lambda^0}\right) + \exp\left(-\frac{(\kappa - k)\,w}{\lambda^0}\right)\right) \tag{1}$$

Here, v^f is the free flow speed in the network, Q is the network's capacity as defined by the most constraining intersection (Daganzo and Geroliminis, 2008), κ is the jam spacing in the network, and w the backward wave speed. Arguably, each mode m has its characteristic values. In Figure 1a we illustrate the behavior of this MFD function for different values of λ^0 in comparison with the uMFD as defined by the minimum operator

with the trapezoidal shape (Eqn. 1). With λ^0 approaching zero, the resulting curve approaches the uMFD, in this case the trapezoidal shape. When λ^0 increases, the curve moves further down but still describes the familiar MFD shape.

We also define the total travel time per kilometer T_m of mode m. It consists of two parts as given by Eqn. 2. The first term, $T_m^0\left(k_m\right)$, denotes the homogeneous travel time of mode m given its current accumulation levels, k_m, without any interactions with other modes. The second term, $\Gamma_m\left(\mathbf{k}\right)$, describes the additional delays caused by the interactions of modes among each other on mode m. In other words, the homogeneous travel time of m is given by each modes' own MFD, while the delays Γ_m jointly depend on all modes' accumulations.

$$T_m\left(\mathbf{k}\right) = T_m^0\left(k_m\right) + \Gamma_m\left(\mathbf{k}\right) \tag{2}$$

To then obtain each modes' MFD capturing the interaction effects, we must update with the travel times from Eqn. 2 each modes' MFD in Eqn. 1. In other words, we want to find $\mu\left(\mathbf{k}\right)$ as defined in Eqn. 3, that augments λ^0 due to the interactions between modes to $\check{\lambda}$ at a given state vector $\mathbf{k}$. Here, $\mu\left(\mathbf{k}\right)$ is a function with $\mathbb{R}^m \to \mathbb{R}$. The model proposed in this paper allows to calculate $\mu\left(\mathbf{k}\right)$, allowing to calculate either $\check{\lambda}$ or λ^0 if one of the latter two values is measured.

$$\check{\lambda} = \lambda^0 + \mu\left(\mathbf{k}\right) \tag{3}$$

We establish the link between delays Γ and λ with the well-established *two-fluid theory* of urban traffic (Herman and Prigogine, 1979). The two-fluid theory of urban traffic is analogous to the Bose-Einstein condensation of particles at low temperatures. In this theory, traffic is considered as consisting of running vehicles (subscript r) and stopped vehicles (subscript s), where the running speed of vehicles v_r is related to the fraction of running vehicles f_r by Eqn. 4 and by definition Eqn. 5. Here, n is a network-wide constant and assumed to result from driving behavior, network topology and signal settings, and v^f is the free-flow speed. As in case of using the MFD idea for all modes, we use also apply this theory to all modes m.

$$v_r = v^f\left(f_r\right)^n \tag{4}$$

$$v = v_r f_r \tag{5}$$

Further, by definition the fraction of vehicles stopped f_s and the fraction of running vehicles f_r always add up to one: $f_s + f_r \equiv 1$. Then, the space-mean speed in the network v results from Eqn. 6.

$$v = v^f\left(1 - f_s\right)^n f_r = v^f\left(1 - f_s\right)^{n+1} \tag{6}$$

Importantly, Herman and Prigogine (1979) point out that f_s is proportional to a power law with exponent p of the density to jam density ratio. In an empirical study, Lu et al. (2018) report that $p \approx 1$ making computation easier, but we carry the p further along in this section as it can be context specific.

$$f_s = \left(\frac{T_s}{T}\right) \propto \left(\frac{k}{\kappa}\right)^p \tag{7}$$

The fundamental equation of the two-fluid theory results from Eqns. 4 and 6 and is given by Eqn. 8. This equation establishes a relationship between the total travel time per kilometer T, the running time

per kilometer T_r, the free flow speed v^f as well as the network parameter n. We illustrate the functional behavior of this equation in Figure 1b for different values of n. For higher values of n, the fraction of stop time out of the total trip time decreases.

$$\log T_r = \frac{n}{n+1} \log T + \frac{1}{n+1} \log \left(\frac{1}{v^f} \right) \tag{8}$$

With the MFD expressed by Eqn. 1, we can algebraically derive formulae for T, T_r, T_s. The total trip time per kilometer or pace, T, is simply obtained by the inverse of the space-mean speed in the MFD as shown by Eqn. 9.

$$T(k) = \frac{k}{-\lambda^0 \ln \left(\exp \left(-\frac{v^f k}{\lambda^0} \right) + \exp \left(-\frac{Q}{\lambda^0} \right) + \exp \left(-\frac{(\kappa - k)w}{\lambda^0} \right) \right)} \tag{9}$$

Then, we obtain the running time per kilometer, T_r, by using Eqns. 5, 7 and $T(k)$ from Eqn. 9.

$$T_r(k) = \left(1 - \left(\frac{k}{\kappa} \right)^p \right) T(k) \tag{10}$$

Last, we obtain the stopping time per kilometer, T_s, by subtracting the running time from the total trip time as given by Eqn. 11.

$$T_s(k) = T(k) - T_r(k) \tag{11}$$

In their empirical work, Herman and Prigogine (1979), Ardekani and Herman (1985) and Ardekani et al. (1992) estimate n econometrically from measurements of T_r and T. As we obtained formulae for T and T_r, we can derive n analytically. Thus, after some algebra, we can solve Eqn. 8 for n, resulting in Eqn. 12.

$$n = \frac{\log \left(v^f \right)^{-1} - \log T_r}{\log T_r - \log T} \tag{12}$$

In the following, we introduce the following notation. We define λ^0, n^0, T^0, T_r^0 and T_s^0 with superscript 0 to denote the case of a homogeneous and steady-state car traffic stream without any disturbances. Let us then consider that the interactions with other transport modes create additional delays Γ. We assume that these delays either affect only the stopping time with $\Gamma_s(\mathbf{k})$ or the running time $\Gamma_r(\mathbf{k})$. We discuss these functions in detail later in this section. Here, Γ is a scalar function with $\mathbb{R}^m \to \mathbb{R}$. Accordingly, the two-fluid travel time variables can be rewritten for the case with additional delays with Eqns. 13-15.

$$T_r(\mathbf{k}) = T_r^0(k) + \Gamma_r(\mathbf{k}) \tag{13}$$

$$T_s(\mathbf{k}) = T_s^0(k) + \Gamma_s(\mathbf{k}) \tag{14}$$

$$T(\mathbf{k}) = T_r^0(k) + \Gamma_r(\mathbf{k}) + T_s^0(k) + \Gamma_s(\mathbf{k}) \tag{15}$$

With the additional delays on travel times quantified, we can use Eqn. 12 to calculate $\check{n}$ of the system that describes the network performance in the presence of interactions across modes according to Eqn. 16.

$$\check{n}\left(\mathbf{k}\right) = \frac{\log\left(v^f\right)^{-1} - \log\left(T_r^0\left(k\right) + \Gamma_r\left(\mathbf{k}\right)\right)}{\log\left(T_r^0\left(k\right) + \Gamma_r\left(\mathbf{k}\right)\right) - \log\left(T_r^0\left(k\right) + \Gamma_r\left(\mathbf{k}\right) + T_s^0\left(k\right) + \Gamma_s\left(\mathbf{k}\right)\right)} \tag{16}$$

At this stage, with given λ^0 and $\mathbf{k}$ we have calculated $\check{n}$. However, recall that we are interested in the effects on $\check{\lambda}$ from Eqn. 3, and ultimately in the MFD capacity of each mode. For this, we equate in Eqn. 17 the space-mean speed of the λ trapezoidal function from Eqn. 1 and the speed of the two-fluid theory from Eqn. 6. Note that the information of $\mathbf{k}$ is now carried along with $\check{n}$ and that the right-hand side of Eqn. 17 is similar to the inverse of Eqn. 9, but where λ^0 is replaced by $\check{n}$ to calculate the interaction effects.

$$v^f\left(1 - \left(\frac{k}{\kappa}\right)^p\right)^{\check{n}(\mathbf{k})+1} = -\check{\lambda}\frac{\ln\left(\exp\left(-\frac{v^f k}{\check{\lambda}}\right) + \exp\left(-\frac{Q}{\check{\lambda}}\right) + \exp\left(-\frac{(\kappa-k)w}{\check{\lambda}}\right)\right)}{k} \tag{17}$$

Eqn. 17 can simply then be solved as a root problem in mathematical software when transformed into Eqn. 18. The only unknown is $\check{\lambda}$.

$$0 = -\check{\lambda}\frac{\ln\left(\exp\left(-\frac{v^f k}{\check{\lambda}}\right) + \exp\left(-\frac{Q}{\check{\lambda}}\right) + \exp\left(-\frac{(\kappa-k)w}{\check{\lambda}}\right)\right)}{k} - v^f\left(1 - \left(\frac{k}{\kappa}\right)^p\right)^{\check{n}(\mathbf{k})+1} \tag{18}$$

The problem formulated in Eqn. 18 must be solved for each mode m separately and because of the high non-linearity of model, we propose to solve Eqn. 18 for each demand situation separately, i.e. for all possible values of $\mathbf{k}$, instead of assuming constant n or λ values over all densities.

3. Delay functions

In the following, we focus on identifying delay functions, i.e. $\Gamma\left(\mathbf{k}\right)$ for an urban corridor with given MFDs for each mode. Here, we consider that the interactions between modes are continuously distributed along the corridor. The methodology presented above is generic and allows to use any formulation of $\Gamma_s\left(\mathbf{k}\right)$ and $\Gamma_r\left(\mathbf{k}\right)$ functions. Here, we use the following notation: $\Gamma_s^{c\to b}$ describes the additional stopping delays caused by cars on buses. We use the $\to$ operator to indicate which mode affects which other mode. Where we do not provide the $\to$ operator, Γ corresponds to the total additional delay caused by all other modes. Intuitively, the delay functions are a function of the network topology, i.e. in case all modes run on dedicated infrastructure the interaction delays are zero, while they are non-zero when their infrastructure is (partially) overlapping.

As aforementioned, $\Gamma\left(\mathbf{k}\right)$ has *two* mechanism: stopping delays $\Gamma_s\left(\mathbf{k}\right)$ and running delays $\Gamma_r\left(\mathbf{k}\right)$. We assume additivity of delays within each mechanism as formulated in Eqn. 19 for the additional stopping delays for cars $\Gamma_s^a\left(\mathbf{k}\right)$, i.e. we calculate additional delays pairwise, which sum is then the total additional delay. In other words, this assumes no combined or second order effects, e.g. from bicycles and buses on cars.

$$\Gamma_s^c\left(\mathbf{k}\right) = \Gamma_s^{b\to c}\left(k_c, k_b\right) + \Gamma_s^{v\to c}\left(k_c, k_v\right) \tag{19}$$

The full paper then provides the mathematical formulations of the delay functions for each mode.

4. Conclusions

In this paper, we presented a novel multimodal interaction model for the MFD. The model is generic and flexible to accommodate interactions between all sorts of transportation modes, but in this analysis we restricted ourselves to cars, buses and bicycles. This multimodal interaction model for the MFD is a substantial contribution, not only in modeling urban traffic flow, but also in policy making as it allows to discuss the optimal mode share given demand urban network topology.

Acknowledgement

References

Ambühl, L., Loder, A., Bliemer, M. C., Menendez, M., Axhausen, K. W., 2018. A functional form for the macroscopic fundamental diagram with a physical meaning. Transportation Research Part B (in press), 12–19.

Ardekani, S. A., Herman, R., 1985. A comparison of the quality of traffic service in downtown networks of various cities around the world. Traffic Engineering and Control, 574–581.

Ardekani, S. A., Williams, J. C., Bhat, S., 1992. Influence of urban network features on quality of traffic service. Transportation Research Record: Journal of the Transportation Research Board 1358, 6–12.

Castrillon, F., Laval, J., 2018. Impact of buses on the macroscopic fundamental diagram of homogeneous arterial corridors. Transportmetrica B: Transport Dynamics 6 (4), 286–301.

Daganzo, C. F., 2007. Urban gridlock: Macroscopic modeling and mitigation approaches. Transportation Research Part B: Methodological 41, 49–62.

Daganzo, C. F., Gayah, V. V., Gonzales, E. J., 2011. Macroscopic relations of urban traffic variables: Bifurcations, multivaluedness and instability. Transportation Research Part B: Methodological 45 (1), 278–288.

Daganzo, C. F., Geroliminis, N., 2008. An analytical approximation for the macroscopic fundamental diagram of urban traffic. Transportation Research Part B: Methodological 42, 771–781.

Daganzo, C. F., Knoop, V. L., 2016. Traffic flow on pedestrianized streets. Transportation Research Part B: Methodological 86, 211–222.

Daganzo, C. F., Lehe, L. J., Argote-Cabanero, J., 2017. Adaptive offsets for signalized streets. Transportation Research Part B: Methodological, 1–9.

Gayah, V. V., Daganzo, C. F., 2011. Clockwise hysteresis loops in the macroscopic fundamental diagram: An effect of network instability. Transportation Research Part B: Methodological 45, 643–655.

Geroliminis, N., Sun, J., 2011. Properties of a well-defined macroscopic fundamental diagram for urban traffic. Transportation Research Part B: Methodological 45 (3), 605–617.

Geroliminis, N., Zheng, N., Ampountolas, K., 2014. A three-dimensional macroscopic fundamental diagram for mixed bi-modal urban networks. Transportation Research Part C: Emerging Technologies 42, 168–181.

Herman, R., Prigogine, I., 1979. A two-fluid approach to town traffic. Science 204, 148–151.

Loder, A., Ambühl, L., Menendez, M., Axhausen, K. W., 2017. Empirics of multi-modal traffic networks – Using the 3D macroscopic fundamental diagram. Transportation Research Part C: Emerging Technologies 82, 88–101.

Lu, S., Knoop, V. L., Keyvan-Ekbatani, M., 2018. Using taxi GPS data for macroscopic traffic monitoring in large scale urban networks: calibration and MFD derivation. Transportation Research Procedia.

Mariotte, G., Leclercq, L., Laval, J. A., 2017. Macroscopic urban dynamics: Analytical and numerical comparisons of existing models. Transportation Research Part B: Methodological 101, 245–267.

Mazloumian, A., Geroliminis, N., Helbing, D., 2010. The spatial variability of vehicle densities as determinant of urban network capacity. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 368 (1928), 4627.