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Electricity Market Design: Clearing and Pricing in Non-Convex Markets

Johannes Knörr

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1. Prof. Dr. Martin Bichler
2. Prof. Dr. Martin Weibelzahl
3. Assistant Prof. Anthony Papavasiliou

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Abstract

This dissertation explores allocation and pricing in non-convex and coupled electricity markets, addressing the absence of Walrasian equilibrium prices and proposing alternative pricing rules grounded in convex optimization and duality theory. The work is divided into three main parts, each contributing novel insights and solutions to the challenges posed by the evolving landscape of electricity markets.

Electricity markets play a critical role in modern economies, serving as the mechanism through which electricity, a fundamental commodity, is traded between producers and consumers. Traditionally, electricity markets were subject to central planning, where a single entity, often a monopoly, controlled generation, transmission, and distribution. However, with advancements in technology, policy shifts, and the desire for increased competition and efficiency, many electricity markets have transitioned to competitive structures, where multiple participants, including generators, consumers, and transmission operators interact through market-based mechanisms.

One of the foundational concepts in economic theory relevant to electricity markets is the Walrasian equilibrium. In a Walrasian equilibrium, prices and allocation balance supply and demand and ensure that no bidder has an incentive to deviate from their assigned bundles of items. In markets with convex preference functions, this equilibrium is characterized by an efficient resource allocation, maximizing overall welfare.

However, in non-convex and coupled electricity markets, achieving a Walrasian equilibrium becomes challenging due to various factors such as non-convex bid languages, transmission constraints, and the integration of renewable energy sources with intermittent generation patterns. These complexities lead to deviations from the idealized conditions assumed in traditional economic models, necessitating the development of alternative pricing rules and allocation mechanisms.

In the first part, this dissertation addresses the impact of increasing demand response on pricing in U.S. electricity markets, where established pricing rules violate envy-freeness and budget balance. A new pricing rule that runs in polynomial time is proposed, reducing make-whole payments while maintaining efficiency and individual rationality. This pricing rule introduces penalties on paradoxically rejected bids to ensure stability in regulated electricity markets.

The second part extends the analysis to a multi-objective optimization framework, identifying conflicting design goals of established pricing rules and proposing the *Join* pricing rule. This rule optimizes multiple classes of lost opportunity costs simultaneously, achieving lower make-whole payments and improved congestion signals than existing pricing rules. The Join pricing rule leverages techniques from convex optimization and duality theory to design prices that jointly minimize incentives to deviate locally and incentives to exit the market.

The third part focuses on the impact of different power flow models on allocation and pric-

ing in nodal electricity markets. The study emphasizes the ability of tighter relaxations of the Alternating Current Optimal Power Flow (ACOPF) problem to enhance the quality of price signals and improve economic outcomes. By investigating second-order conic and semidefinite programming relaxations for the ACOPF, this dissertation demonstrates the substantial impact of power flow models on allocation and redispatch in coupled electricity markets where power flows adhere to physical laws.

Overall, this dissertation contributes valuable insights and methodologies to the ongoing discourse on allocation and pricing in non-convex and coupled electricity markets, offering pragmatic solutions to achieve a balance among efficiency, individual rationality, and budget balance while navigating the complexities of modern energy systems. The proposed pricing rules and approaches provide a framework for designing market mechanisms that address the challenges posed by non-convexities, demand response, and structural transformations in electricity markets, ultimately aiming for more efficient, equitable, and sustainable energy allocation.

Zusammenfassung

Diese Dissertation befasst sich mit der Allokation und Preisbildung in nicht-konvexen und gekoppelten Strommärkten. Dabei wird das Fehlen von Walrasianischen Gleichgewichtspreisen thematisiert und es werden alternative Preisbildungsregeln vorgeschlagen, die auf konvexer Optimierung und Dualitätstheorie basieren. Die Arbeit ist in drei Hauptteile gegliedert, die jeweils neue Einsichten und Lösungen für die Herausforderungen auf den sich stets weiterentwickelnden Strommärkten bieten.

Strommärkte spielen in modernen Volkswirtschaften eine entscheidende Rolle, da sie als Mechanismus dienen, über den Elektrizität zwischen Erzeugern und Verbrauchern gehandelt wird. Traditionell funktionierten Strommärkte nach zentraler Steuerung, bei denen eine einzige Organisation, oft ein Monopolist, die Erzeugung, Übertragung und Verteilung kontrollierte. Im Zuge des technologischen Fortschritts, politischer Veränderungen und des Wunsches nach mehr Wettbewerb und Effizienz gingen viele Strommärkte jedoch zu Wettbewerbsstrukturen über, bei denen mehrere Teilnehmer, darunter Erzeuger, Verbraucher und Übertragungsnetzbetreiber, durch marktbasierte Mechanismen zusammenwirken.

Eines der grundlegenden Konzepte der Wirtschaftstheorie, das für die Strommärkte von Bedeutung ist, ist das Walrasianische Gleichgewicht. In einem Walrasianischen Gleichgewicht sorgen Preise und Allokation für ein Gleichgewicht zwischen Angebot und Nachfrage und gewährleisten, dass kein Bieter einen Anreiz hat, von den ihm/ihr zugeteilten Bündel an Gütern abzuweichen. Auf Märkten mit konvexen Präferenzfunktionen ist dieses Gleichgewicht durch eine effiziente Ressourcenzuweisung charakterisiert, die die Gesamtwohlfahrt maximiert.

Auf nicht-konvexen und gekoppelten Strommärkten wird das Erreichen eines Walrasianischen Gleichgewichts jedoch durch verschiedene Faktoren wie nicht-konvexe Gebotssprachen, Übertragungsbeschränkungen und die Integration erneuerbarer Energiequellen mit intermittierenden Erzeugungsmustern erschwert. Diese Komplexität führt zu Abweichungen von den idealisierten Bedingungen, die in traditionellen ökonomischen Modellen angenommen werden, und macht die Entwicklung alternativer Preisbildungsregeln und Allokationsmechanismen erforderlich.

Der erste Teil dieser Dissertation befasst sich mit den Auswirkungen der zunehmenden Nachfrageflexibilität auf die Preisbildung in US-amerikanischen Strommärkten, wo die etablierten Preisbildungsregeln keine Neidfreiheit und kein Budgetgleichgewicht erzielen. Es wird eine neue Preisregel mit polynomialer Zeitkomplexität vorgeschlagen, die Ausgleichszahlungen reduziert und gleichzeitig Effizienz und individuelle Rationalität beibehält. Diese Preisregel führt Sanktionen für paradoxerweise abgelehnte Gebote ein, um die Stabilität auf Strommärkten zu gewährleisten.

Im zweiten Teil wird die Analyse auf einen multizentrischen Optimierungsrahmen ausgeweitet, indem widersprüchliche Ziele etablierter Preisregeln identifiziert und die *Join*-Preisregel vorgeschlagen werden. Diese Regel optimiert mehrere Klassen von Opportunitätskosten gleichzeitig und erzielt niedrigere Ausgleichszahlungen und bessere Netzsi-

gnale als bestehende Preisregeln. Die Join-Preisregel nutzt Techniken der konvexen Optimierung und der Dualitätstheorie, um Preise zu berechnen, die gleichzeitig Anreize für lokale Abweichungen und Anreize zur Nicht-Teilnahme am Markt minimieren.

Der dritte Teil befasst sich mit den Auswirkungen verschiedener Stromflussmodelle auf Allokation und Preisbildung in nodalen Strommärkten. Die vorgestellte Studie unterstreicht die Fähigkeit besserer Relaxierungen des Alternating Current Optimal Power Flow (ACOPF) Problems, die Qualität der Preissignale und der Allokation zu verbessern. Durch die Untersuchung konischer und semidefiniter Relaxationen für das ACOPF-Problem zeigt diese Dissertation den bedeutenden Einfluss von Stromflussmodellen auf Allokation und Redispatch in gekoppelten Strommärkten auf, in denen Stromflüsse physikalischen Gesetzen folgen.

Insgesamt leistet diese Dissertation einen wertvollen Beitrag zum laufenden Diskurs über die Allokation und Preisbildung in nicht-konvexen und gekoppelten Strommärkten. Sie bietet pragmatische Lösungen, um einen Ausgleich zwischen Effizienz, individueller Rationalität und Budgetgleichgewicht zu finden, bei gleichzeitiger Bewältigung der Komplexität moderner Energiesysteme. Die vorgeschlagenen Preisbildungsregeln und -ansätze bieten einen Rahmen für die Gestaltung von Marktmechanismen, die die Herausforderungen von Nichtkonvexität, Nachfrageflexibilität und strukturellem Wandel in Strommärkten angehen und letztlich auf eine effizientere, gerechtere und nachhaltigere Energieallokation abzielen.

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Contents

1. Introduction

In classical microeconomic theory, *markets* serve as mechanisms to efficiently allocate goods or items from supply to demand, employing prices as central means of coordination. Based on this premise, the field of *market design* studies the design of mechanisms and rules to achieve favorable market outcomes. Leveraging tools from mathematical optimization, mechanism design, and game theory, the domain of market design has produced pioneering results across various markets (e.g., Gale and Shapley, 1962; Myerson, 1981; Roth, 1984; Milgrom, 2000), advancing our understanding of how to allocate goods, resources, or labor in a financially efficient, equitable, and incentive-compatible manner. The significance of market design for the modern economy has been acknowledged by the awarding of Nobel Memorial Prizes in Economic Sciences in 2007, 2012, and 2020 (The Nobel Prize, 2024).[1]

1.1. Motivation

A fundamental concept in market design is that of *competitive* or *Walrasian equilibria*, named after French economist León Walras (Walras, 1874). This equilibrium denotes an allocation and prices where supply equals demand and every bidder maximizes their payoff. Essentially, a Walrasian equilibrium implies a stable market, in the sense that no bidder has an incentive to deviate from the outcome. Prices are linear (i.e., the price of a bundle equals the sum of the prices of its items) and anonymous (i.e., prices apply equally to all bidders) and provide an efficient signal for market entry or exit.

In their seminal paper, Arrow and Debreu (1954) demonstrate that a set of competitive equilibrium prices always exists under the assumptions of convex preferences, perfect competition, and demand independence. The Arrow-Debreu model and its extensions (McKenzie, 1959; Gale, 1963) form the basis of the celebrated *welfare theorems* (Mas-Colell et al., 1995), establishing a link between competitive equilibria and allocative efficiency, i.e., an allocation that maximizes the social welfare of all market participants.

However, the assumptions of general equilibrium theory and the Arrow-Debreu model often do not align with real-world markets. In practical scenarios, items are rarely perfectly

[1] 2007: L. Hurwicz, E. Maskin, and R. Myerson ”for having laid the foundations of mechanism design theory”.
2012: A. Roth and L. Shapley ”for the theory of stable allocations and the practice of market design”.
2020: P. Milgrom and R. Wilson ”for improvements to auction theory and inventions of new auction formats”.

divisible and bidders do not typically exhibit convex preferences. Thus, a large stream of literature investigates necessary conditions for the existence of Walrasian equilibria in markets with non-convexities, employing the concept of *quasilinear* utility functions, where market participants seek to maximize their payoff as the difference between (possibly non-convex) valuation and price. The existence of Walrasian equilibria necessitates strong assumptions on bidders' preference functions (Baldwin and Klemperer, 2019), such as gross-substitutes (Kelso and Crawford, 1982) and their generalizations (Gul et al., 2000; Milgrom and Strulovici, 2009; Leme, 2017). In general, Walrasian equilibria exist if and only if the linear relaxation of the *winner determination problem* has an integer solution (Bikhchandani and Mamer, 1997). For arbitrary valuations, this may require prices to be non-linear and personalized (Bikhchandani and Ostroy, 2002), or such prices may not exist at all (Bichler and Waldherr, 2017).

Prominent examples of markets featuring indivisibilities and non-convex preferences include transportation markets (Caplice and Sheffi, 2003), industrial procurement (Bichler et al., 2005), or spectrum auctions (Bichler et al., 2014). This dissertation focuses on *electricity markets*, recognizing them as among the most pivotal commodity markets globally.

In recent decades, electricity markets across many countries have transitioned from centralized monopolies to competitive wholesale markets. In most industrialized countries, the majority of electricity is now allocated through market-based mechanisms. For example, as of 2022, the European day-ahead market has successfully cleared an annual volume of 1683 TWh (NEMO Committee, 2023). Consequently, the design of power markets and systems has become a fundamental pillar of modern society.

While various market designs have been implemented, they all converge on the central role of the spot market price, serving as the foundational element from which all other contract prices are derived. As described above, under the assumption of convexity, Walrasian equilibrium prices provide perfect short- and long-run equilibria in electricity markets (Perez-Arriaga and Meseguer, 1997). However, electricity is unlike any other commodity, and the necessary conditions for Walrasian equilibrium prices are no longer met.

Firstly, the value and cost functions of bidders introduce *non-convexities* to the allocation problem (Liberopoulos and Andrianesis, 2016). For instance, conventional electricity generators have minimum load requirements, ramping constraints, or minimum running time obligations that are typically modeled with binary decision variables in the central market clearing problem.

Secondly, electricity markets are *coupled*, meaning electricity transmission from generators to consumers traverses expansive power networks. Power flows adhere to physical laws, necessitating a representation of the transmission network in the market clearing problem. A central challenge arises from the fact that an accurate network representation, the so-called *Alternating Current Optimal Power Flow* (ACOPF) problem (Molzahn and Hiskens, 2019), introduces non-linear and non-convex constraints that render the allocation problem intractable and Walrasian equilibria infeasible.

Thirdly, in the wake of climate change, many nations aspire to achieve net zero carbon emissions and increased electrification of the economy (United Nations, 2015; IEA, 2021). These endeavors coincide with a fundamental transformation of the power sector, moving from conventional to variable renewable energy sources and incorporating demand response and storage into the market. Such changes necessitate market clearing and pricing rules that are not only economical but also scalable, sustainable, and capable of accommodating the structural shifts occurring in the power sector.

To summarize, computing an allocation and prices in non-convex and coupled electricity markets remains an open and fundamental problem. The challenges posed by non-convex preferences, transmission constraints, and structural transformations are a topic of active discussion.

In Europe, prices are required to be uniform, i.e., to avoid so-called *paradoxically accepted bids* (NEMO Committee, 2019b), which would require individual side-payments to compensate losses. As Walrasian equilibrium prices are unattainable, this requirement can only be satisfied at the expense of welfare losses (Meeus et al., 2009). Moreover, the representation of the European transmission network is much simplified by aggregating nodes into larger price zones. Given that the resulting economic allocation may not align with physical power flows, *redispatch* is necessary to attain a physically feasible outcome. The rise of renewable energy sources has heightened the need for redispatch, leading to increasingly higher costs. As a consequence of welfare losses, redispatch costs, and scalability problems, there is an ongoing discourse about transitioning to non-uniform or nodal pricing (Ashour Novirdoust et al., 2021; Eicke and Schittekatte, 2022; NEMO Committee, 2023).

In contrast, in liberalized United States (U.S.) electricity markets, a linearized version of the ACOPF, known as the *Direct Current OPF* (DCOPF) problem (Molzahn and Hiskens, 2019), is utilized to compute nodal prices. Employing a non-uniform pricing rule, system operators implement the welfare-maximizing allocation but need to make side-payments to bidders that are unable to recover their costs or have an incentive to deviate. With these out-of-market payments on the rise in recent years, established pricing rules have come under scrutiny (FERC, 2014). Furthermore, the anticipated increase in non-convexities due to demand response and storage resources raises new questions regarding price formation in electricity markets.

1.2. Contributions

This dissertation seeks to contribute to the discussions on clearing and pricing in non-convex and coupled electricity markets. It builds on the dilemma that spot market prices are considered crucial for short- and long-term efficiency, yet the theoretical ideal of Walrasian equilibrium prices does not exist. The primary objective of this thesis is to propose alternative pricing rules that – founded on principles of convex optimization and duality theory – yield favorable economic outcomes.

1.2.1. Part I: Pricing in the Presence of Demand Response

The first publication of this dissertation (Bichler et al., 2023a, see Chapter 3) addresses the absence of Walrasian equilibrium prices in the context of increasing levels of price-elastic demand and the rise of side-payments in U.S. markets.

The welfare theorems imply that Walrasian equilibrium prices ensure efficiency of the allocation (i.e., maximizing social welfare), individual rationality (i.e., ensuring non-negative payoffs for each bidder), and envy-freeness (i.e., maximizing individual payoffs), along with overall budget balance. In non-convex markets, these prices cease to exist. Established pricing rules in U.S. electricity markets fulfill efficiency and individual rationality but compromise on envy-freeness and budget balance. Specifically, budget balance is disrupted by so-called *make-whole payments*, i.e., individual side-payments compensating bidders that are unable to recover bid costs. Increasing make-whole payments have been identified as a concern, undermining price signals and creating flawed incentives (FERC, 2020).

We establish that, assuming price-inelastic demand, one can always obtain an outcome that satisfies efficiency, individual rationality, and budget balance. We propose a corresponding pricing rule and suggest that, in regulated wholesale electricity markets, penalties on remaining lost opportunity costs may ensure stability.

However, this conclusion need not hold in the presence of demand response and price-elastic demand. We demonstrate that there can be no pricing rule that simultaneously satisfies efficiency, individual rationality, and budget balance with linear and anonymous prices. To address concerns about increasing make-whole payments, we propose another pricing rule that maintains efficiency and individual rationality with minimum budget balance violation and polynomial time complexity. Numerical experiments indicate that, on average, this pricing rule does not yield higher prices compared to established rules but significantly reduces make-whole payments, addressing a crucial policy concern.

Bibliographical information:

Bichler, M., **Knörr, J.**, Maldonado, F. (2023). "Pricing in Nonconvex Markets: How to Price Electricity in the Presence of Demand Response." *Information Systems Research 34(2):652-675*. https://doi.org/10.1287/isre.2022.1139.

1.2.2. Part II: Pricing as a Multi-Objective Optimization Problem

The second publication (Ahunbay et al., 2023b, see Chapter 4) of this dissertation extends the approach of Bichler et al. (2023a) and adopts a more holistic perspective of pricing in non-convex and coupled electricity markets.

Due to the non-existence of Walrasian equilibrium prices, seeking linear and anonymous prices for the efficient allocation in non-convex markets inevitably results in the violation of either envy-freeness or budget balance. In other words, bidders do not maximize individual profits and – unless compensated – have an incentive to deviate from the efficient outcome. These incentives are quantified by the *lost opportunity costs* (LOCs) of a bidder (Schiro et al., 2016).

Our work is based on the observation that established pricing rules each minimize a specific *class* of LOCs. Leveraging techniques from convex optimization and duality theory, we establish a connection between each pricing rule, its dual problem, and the corresponding class of LOCs. Specifically, *Convex Hull pricing* (Hogan and Ring, 2003; Gribik et al., 2007) corresponds to minimizing *global* LOCs and as such is considered a favorable yet intractable pricing rule for non-convex markets. The most common implementation in practical markets, Integer Programming (IP) pricing (O'Neill et al., 2005), corresponds to minimizing *local* LOCs and – as we show – thereby maintains that prices signal the marginal value of transmission capacity. Finally, make-whole payments, as minimized in Bichler et al. (2023a), represent a third class of LOCs.

Through this formal analysis, we conclude that the design goals of established pricing rules are conflicting, and we thus frame pricing in non-convex markets as a multi-objective optimization problem. As main contribution, we utilize our primal-dual framework to design novel pricing rules that jointly optimize multiple classes of LOCs. Specifically, we design the *Join pricing* rule, which – in polynomial time – yields prices that jointly minimize incentives to deviate locally and incentives to exit the market, thus striking a balance between local LOCs and make-whole payments. We prove that Join pricing always achieves lower make-whole payments than IP pricing and satisfies a weak form of Pareto optimality. Extensive numerical experiments demonstrate that Join pricing provides promising outcomes, with prices effectively signaling the marginal value of transmission capacity and requiring minimal make-whole payments.

Bibliographical information:

Ahunbay, M.S., Bichler, M., **Knörr, J.** (2023). "Pricing Optimal Outcomes in Coupled and Non-Convex Electricity Markets." *Proceedings of the 24th ACM Conference on Economics and Computation (EC '23)*. https://doi.org/10.1145/3580507.3597732, https://arxiv.org/abs/2209.07386. Currently in minor revision for Operations Research.

1.2.3. Part III: Pricing and Optimal Power Flow Problems

Finally, the third publication of this dissertation (Bichler and Knörr, 2023, see Chapter 5) shifts the focus to the coupled nature of electricity markets and examines the impact of different power flow models on allocation and prices.

The representation of the transmission network is crucial for deriving price signals that accurately reflect the locational scarcity of electricity. Given the intractability of the underlying ACOPF problem, current market models resort to simplified versions. In Europe, the transmission network is aggregated into zones, and only cross-zonal flows and critical network elements are considered for the allocation problem. U.S. markets utilize the linearized DCOPF for market clearing, generating nodal price signals. Regulators in both jurisdictions acknowledge that these simplifications adversely impact the economic outcome of the allocation (Cain and O'Neill, 2012; ACER, 2021).

Recent years have witnessed significant advances in convex optimization, prompting a surge in literature seeking tighter approximations for the ACOPF problem (Molzahn and Hiskens, 2019). This literature primarily focuses on the optimality of solutions and the computational costs involved. In our paper, we establish a link to the the literature on electricity market pricing. Specifically, we investigate second-order conic and semidefinite programming relaxations for the ACOPF in terms of the allocation and prices they produce, considering non-convexities in bidders' preferences at the same time.

Our findings carry considerable policy implications. Different power flow models result in substantial deviations in allocation and redispatch. A key observation is that prices derived from the DCOPF approximation can significantly differ from those of tighter convex relaxations. Specifically, the DCOPF allocation suggests congestion on certain transmission network lines, leading to considerable price differences across nodes. However, the underlying physical power flows exhibit no such congestion, rendering the observed high price peaks under the DCOPF economically unjustified. In contrast, tighter convex relaxations provide a more accurate depiction of physical power flows, and markedly enhance the quality of price signals.

Bibliographical information:

Bichler, M., **Knörr, J.** (2023). "Getting Prices Right on Electricity Spot Markets: On the Economic Impact of Advanced Power Flow Models." *Energy Economics 126:106968.* https://doi.org/10.1016/j.eneco.2023.106968.

1.2.4. Other Publications

Besides the referenced papers, the author of this dissertation was involved in multiple other publications that study the intersection of electricity markets, economic theory, and market design. Even though these papers are not included in this dissertation, short summaries are provided below:

Bichler, M., Buhl, H., **Knörr, J.**, Maldonado, F., Schott, P., Waldherr, S., Weibelzahl, M. (2022). "Electricity Markets in a Time of Change: A Call to Arms for Business Research." *Schmalenbach Journal of Business Research 74:77-102.* https://doi.org/10.1007/s41471-021-00126-4.

In this work, we acknowledge that the energy transition presents challenges not confined to engineering and natural sciences but extends to the business research discipline as well.

Based on an extensive literature review, we offer an up-to-date overview of fundamental questions in electricity market design and outline research challenges spanning various business research disciplines.

Ahunbay, M.S., Bichler, M., Dobos, T., **Knörr, J.** (2023). ”Solving Large-Scale Electricity Market Pricing Problems in Polynomial Time”. https://arxiv.org/abs/2312.07071. Currently in revision for European Journal of Operational Research.

In this study, we address scalability issues identified in non-convex allocation problems within electricity markets. We leverage a recent proposal for approximating competitive equilibria in polynomial time, promising approximate efficiency, no budget deficit, and computational tractability (Milgrom and Watt, 2021). We present experimental results for this mechanism in the context of electricity markets and observe that it strikes a promising balance between computational scalability and favorable economic outcomes.

Dobos, T., Bichler, M., **Knörr, J.** (2024). ”Finding Stable Price Zones in European Electricity Markets: Aiming to Square the Circle?”. *Working Paper*.

The European electricity market is split into large bidding zones. As a result of structural congestion and increasing redispatch costs, the European Commission launched the Bidding Zone Review process to assess alternative zone configurations. In this study, we employ data from the Bidding Zone Review to construct zones through node clustering. Notably, we find that the composition of clusters varies significantly based on the selected clustering attributes, algorithms, and time frames. Given the ongoing and considerable transformations in power systems and markets, we posit that establishing large, stable bidding zones may prove unattainable.

Knörr, J., Bichler, M., Dobos, T. (2024). ”Zonal vs. Nodal Pricing: An Analysis of Different Pricing Rules in Germany”. http://arxiv.org/abs/2403.09265. *Working Paper*.

In this work, we utilize a recent data publication in the context of the European Bidding Zone Review to evaluate various market clearing and pricing rules for the German electricity market. Based on real-world data, our findings confirm that a nodal market clearing leads to the lowest system costs by mitigating costly redispatch. The average price increase from zonal to nodal pricing is moderate. Especially non-uniform pricing rules enhance computational scalability, provide clearer location-based signals, and entail minimal additional side-payments.

1.3. Outline

The remaining sections of this dissertation are organized as follows: Chapter 2 provides a foundation of electricity market design and offers essential definitions and notation related to auctions and our market model. It discusses bidding formats that introduce non-convexities and explores various representations of the transmission network. Chapter 3 presents the first project of this dissertation, wherein a pricing rule is formulated to minimize make-whole payments. Chapter 4 encompasses the second publication, introducing novel pricing rules from a multi-objective optimization perspective. Lastly, Chapter 5 includes the third paper of this dissertation, establishing a link between optimal power flow problems and pricing in electricity markets. The concluding Chapter 6 wraps up the dissertation and outlines potential avenues for future research.

2. Theoretical Background

This chapter serves as an introduction to the subsequent publications presented in the remainder of this dissertation. It begins with an exploration of key insights into electricity market design, followed by a review of fundamental notions and concepts from competitive equilibrium theory. Building upon this foundation, two central complexities of electricity market design are considered as the primary focus of this dissertation: bid languages and transmission constraints. The section concludes by outlining the current practices in market clearing and pricing observed in U.S. and European markets. The topics touched upon in this chapter, such as electricity market design, mechanism design, and general equilibrium theory, have a rich and expansive history. As a result, this chapter addresses selected aspects from these fields that are most relevant for the contributions of this dissertation.

2.1. Overview of Electricity Market Design

Electricity is a crucial commodity with unique characteristics. It must be transmitted from generators to consumers through designated networks, adhering to the laws of physics by following the path of least resistance. Supply and demand in the power system must be balanced at every moment. Moreover, even though electricity can be generated from multiple fuel sources, it is inherently a homogeneous commodity without differences in quality.

Power grids are considered the most extensive and interconnected system ever built by humanity (Glover et al., 2023), necessitating intricate management to ensure secure and affordable electricity. Historically, the generation and distribution of electricity started as local monopolies. Over the years, regulators gradually introduced market structures to power systems, aiming to enhance both short-run and long-run efficiency. For example, in U.S. markets, power pools emerged to link local monopoly markets, enabling electricity trade across regions (Cramton, 2017). In Europe, several European Union (EU) legislative acts facilitated the establishment of network codes and guidelines to foster an internal energy market (Meeus, 2020). Ultimately, competitive wholesale electricity markets were introduced in both jurisdictions to facilitate efficient trading and pricing of electricity.

Because of the crucial relevance and technical properties of electricity, power markets demand a careful design to ensure reliability of supply, maximize social welfare, control carbon emissions, foster efficient investment incentives, and achieve various other objectives. The California electricity crisis of 2001 serves as a stark reminder that flawed market designs have the potential to incur billions of dollars in costs for consumers (Borenstein, 2002). While differing in details, practical market designs typically revolve around integrated markets with nodal pricing, as seen in U.S. markets, or exchange-based markets with zonal pricing, as observed in Europe.

Electricity can be traded in various time horizons. Futures and forward markets allow to hedge risk exposures up to years ahead of delivery. Balancing markets are required to react at short notice to ensure continuous balance of supply and demand and stable grid frequencies. This dissertation focuses primarily on *spot markets*, which determine the allocation of electricity and provide key price signals for market participants. Spot markets comprise *day-ahead* markets, where electricity is cleared the day before delivery, and *intraday* or *real-time* markets, which operate until shortly before physical delivery.

In the United States, not all regions have shifted to liberalized wholesale markets. However, in those that have (CAISO, ERCOT, ISO-NE, MISO, NYISO, PJM, SPP), a central entity known as an Independent System Operator (ISO) manages both the power market and the power system. Figure 2.1 illustrates the geographical boundaries of the ISO-managed areas.[1] On day-ahead markets, market participants submit complex multi-part bids reflecting their operating characteristics (e.g., variable and fixed costs) and technical constraints (e.g., minimum and maximum output). The ISO then solves a comprehensive mixed-integer problem, known as the *Security-Constrained Unit Commitment and Economic Dispatch* (SC-UCED) problem, to optimize electricity and reserve allocation, incorporating a linearized version of power flow equations (DCOPF) to ensure approximate physical feasibility. Additionally, the ISO computes electricity prices for each node in the transmission network. The provision of such locational price signals is also referred to as *nodal pricing*. Real-time markets involve solving a linear economic dispatch problem every five minutes to determine the final dispatch and prices.

In European markets, system operation and market operation are decoupled. Transmission System Operators (TSOs) manage the grid's stability and balance, while Nominated Electricity Market Operators (NEMOs) collect bids and determine allocation and prices. European spot markets are jointly cleared through the *Single Day-Ahead Coupling* (SDAC) and *Single Intraday Coupling* (SIDC) mechanisms. Figure 2.2 depicts the current geographical scope of SDAC.[2] On both the SDAC and SIDC market, bidders do not submit technical constraints for a central dispatch to NEMOs; instead, they utilize more economical orders (e.g., hourly and block bids), effectively shifting some responsibility for optimizing generation resource operations onto market participants (Herrero et al., 2020). A single Pan-European clearing algorithm (EUPHEMIA) determines allocation and prices on the day-ahead market (NEMO Committee, 2019b).

[1]The shapefiles were obtained from https://www.census.gov/geographies/mapping-files/time-series/geo/carto-boundary-file.html and https://github.com/electricitymaps.

[2]The shapefile was obtained from https://ec.europa.eu/eurostat/web/gisco/geodata/reference-data/administrative-units-statistical-units/nuts.

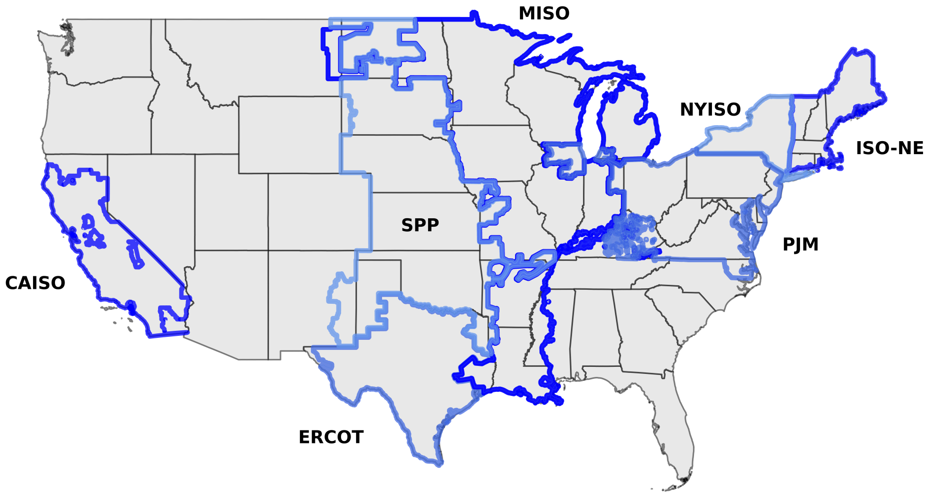

Figure 2.1.: U.S. ISO Markets

However, only a small subset of transmission constraints – those referring to cross-zonal lines and critical network elements – are considered by the algorithm. An elaborate capacity calculation precedes the market clearing to determine line capacities according to the available transfer capacity (ATC) and flow-based market coupling (FBMC) methodology (NEMO Committee, 2019b).[3] The simplification of transmission constraints has two main consequences. On the one hand, a single price is computed for every large price zone, often coinciding with national borders, referred to as *zonal pricing*. On the other hand, the resulting allocation is far from being physically feasible, and so-called *redispatch* conducted by the TSOs is necessary to achieve feasible power flows. In intraday markets, continuous trading occurs after an initial opening auction, enabling market participants to adjust their positions in real time through various types of orders (NEMO Committee, 2019a).

The remainder of this dissertation focuses primarily on *day-ahead markets* due to their significance in clearing volume and price signaling. Day-ahead markets are conceptualized as auctions, and this dissertation approaches them from this perspective. In this chapter, after introducing notation (Section 2.2) and important concepts from auction and equilibrium theory (2.3), attention shifts to the complexities of day-ahead electricity auctions: non-convex bid languages (2.4) and non-linear non-convex transmission constraints (2.5). The chapter concludes by introducing U.S. and European day-ahead markets more formally (2.6). Building on this foundation, the three contributions detailed in Chapters 3-5 introduce in-

[3] See also https://www.entsoe.eu/network_codes/ccr-regions/.

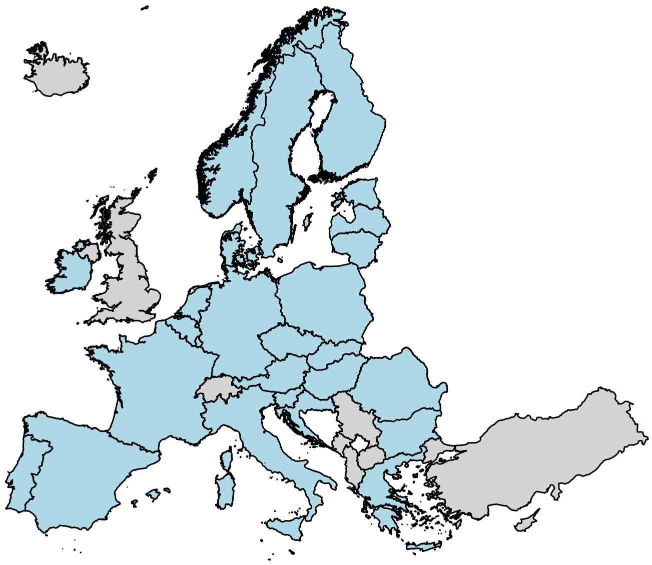

Figure 2.2.: SDAC Market

novative market clearing and pricing rules to tackle the challenges posed by non-convex bid languages and transmission constraints within real-world electricity markets.

2.2. Preliminaries

Day-ahead markets can be described in the framework of general combinatorial auctions, i.e., a set of buyers and sellers that compete for acquiring or selling electricity. Formally, an auction consists of a set of *buyers* $i \in \mathcal{I}$ and a set of *sellers* $j \in \mathcal{J}$, with $\mathcal{L} = \mathcal{I} \cup \mathcal{J}$ as the set of *bidders* and $\mathcal{I} \cap \mathcal{J} = \emptyset$. Together, they transact a set of indivisible *items* $k \in \mathcal{K}$, where an item k corresponds to electricity at a specific location and time period. Multiple homogeneous *units* of each item are available (e.g., multiple kWh of electricity at a specific location and period). However, as shown by Bikhchandani and Mamer (1997), the key results for single-unit markets generalize to multi-unit markets by considering each unit as a separate item. Therefore, the single-unit case will be considered in the following, unless stated otherwise.

In full generality, bidders can submit bids for *bundles* of items, defined as the power set $2^{\mathcal{K}}$ of all possible combinations of the items, with the aim to be allocated exactly one bundle. This so-called *exclusive-or (XOR)* bid language allows bidders to express any possible valuation (Nisan, 2000). An auctioneer – possibly subject to allocative restrictions such as transmission constraints – collects all bids and determines an *allocation* as well as *payments* for every bidder. Formally, each bidder $\ell \in \mathcal{L}$ is assigned a bundle $S_\ell \subseteq \mathcal{K}$ and a corresponding (positive or negative) payment $p_\ell \in \mathbb{R}$. The *outcome* $o = (x, p)$ of an auction is defined as the tuple of allocation $x = (S_\ell)_{\ell \in \mathcal{L}}$ and payments $p = (p_\ell)_{\ell \in \mathcal{L}}$.

To quantify the preferences for bundles, each bidder possesses a *valuation function* v_ℓ, assigning a value to each possible bundle.

$$v_\ell : 2^{\mathcal{K}} \to \mathbb{R}$$

For a buyer $i \in \mathcal{I}$, the valuation function v_i is usually non-negative and represents the value for a bundle $S \subseteq \mathcal{K}$, while for a seller $j \in \mathcal{J}$, the valuation function v_j is usually non-positive and represents the costs of supplying a bundle $S \subseteq \mathcal{K}$.[4] Similarly, the payment p_i of a buyer i is usually non-negative, while the payment p_j of a seller j is usually non-positive, indicating a revenue for selling items.

It is assumed that standard assumptions regarding valuation functions are satisfied:

- *Monotonicity*: The value of the empty bundle is always 0, i.e., $v_\ell(0) = 0 \; \forall \ell \in \mathcal{L}$. The valuation function of a buyer $i \in \mathcal{I}$ is weakly increasing, i.e., $v_i(S') \leq v_i(S)$ for $S' \subseteq S$. The valuation function of a seller $j \in \mathcal{J}$ is weakly decreasing, i.e., $v_j(S') \geq v_j(S)$ for $S' \subseteq S$.
- *Independent Private Values*: The valuation of a bidder remains private information and is contingent only on the received bundle. Specifically, it remains unaffected even when information regarding other bidders' valuations or allocations becomes known.
- *Quasilinearity*: A bidder's *direct utility* u_ℓ for a bundle $S_\ell \subseteq \mathcal{K}$ in outcome o is defined as the difference between valuation and payments, i.e., $u_\ell(o) = v_\ell(S_\ell) - p_\ell(S_\ell)$ where $p_\ell : \; 2^{\mathcal{K}} \to \mathbb{R}$ describes the payment function of bidder ℓ.

Bidders may decide to not report their true valuations in order to achieve a more favorable outcome for themselves individually. In particular, a bidder ℓ places bids according to a bidding function $b_\ell : 2^{\mathcal{K}} \to \mathbb{R}$ where b_ℓ may not necessarily correspond to v_ℓ. If $b_\ell(S) = v_\ell(S) \; \forall \; S \subseteq \mathcal{K}$, bidder ℓ bids *truthfully*. Unless specified otherwise, this dissertation assumes truthful bidding.

Given a payment rule p_ℓ under outcome o, a bidder ℓ seeks a bundle that maximizes their direct utility. The collection of these bundles is known as the *demand set* $D_\ell(p_\ell)$:

$$D_\ell(p_\ell) = \arg\max_{S \subseteq \mathcal{K}} v_\ell(S) - p_\ell(S)$$

The utility associated with any bundle in the demand set under outcome o is quantified by the *indirect utility* $\hat{u}_\ell(o)$:

$$\hat{u}_\ell(o) = \max_{S \subseteq \mathcal{K}} v_\ell(S) - p_\ell(S)$$

[4]In electricity markets, it has been observed that sellers may bid with negative costs, being willing to pay for providing electricity. This would result in a positive valuation v_j.

An auctioneer collects bids and determines an allocation of items between sellers and buyers. A common objective in auctions is to maximize social welfare, quantified as the sum of buyers' values for the items or bundles they receive minus the sum of sellers' costs for items or bundles sold. Finding such an allocation is equivalent to solving a combinatorial optimization problem referred to as *Winner Determination Problem* (WDP) (Blumrosen and Nisan, 2011).

$$
\begin{aligned}
\text{maximize} \quad & \sum_{\ell \in \mathcal{L}} \sum_{S \subseteq \mathcal{K}} v_\ell(S) x_\ell(S) && \text{(WDP)} \\
\text{subject to} \quad & \sum_{i \in \mathcal{I}} \sum_{S \subseteq \mathcal{K}: k \in S} x_i(S) \leq 1 && \forall\, k \in \mathcal{K} \\
& \sum_{j \in \mathcal{J}} \sum_{S \subseteq \mathcal{K}: k \in S} x_j(S) \leq 1 && \forall\, k \in \mathcal{K} \\
& \sum_{i \in \mathcal{I}} \sum_{S \subseteq \mathcal{K}: k \in S} x_i(S) - \sum_{j \in \mathcal{J}} \sum_{S \subseteq \mathcal{K}: k \in S} x_j(S) = 0 && \forall\, k \in \mathcal{K} \\
& \sum_{S \subseteq \mathcal{K}} x_\ell(S) \leq 1 && \forall\, \ell \in \mathcal{L} \\
& x_\ell(S) \in \{0, 1\} && \forall\, \ell \in \mathcal{L},\ S \subseteq \mathcal{K}
\end{aligned}
$$

Note that the WDP formulation above assumes truthful bidding and a single-unit market. The binary variable $x_\ell(S)$ takes the value 1 if bundle S is allocated to bidder ℓ, and 0 otherwise. The first and second constraints guarantee that each item is bought or sold by at most one buyer or seller, respectively. The third constraint ensures that supply and demand are balanced for each item. The fourth constraint ensures that each bidder obtains at most one bundle. The final constraint asserts the indivisibility of each item.

The market clearing problem in day-ahead electricity markets is inherently linked to the WDP, i.e., allocating electricity from generators to consumers in a manner that maximizes welfare. However, applying the standard WDP directly to real-world markets is impractical. Specifically, it is well known that the WDP is NP-complete, i.e., no algorithm is known to solve the WDP efficiently (in polynomial time) for every problem instance (Lehmann et al., 2006). Additionally, designing an ideal payment rule $p_\ell(S)$ that guarantees a stable and balanced outcome poses a non-trivial challenge. These foundational problems have been central to two major research domains in economics: *mechanism design* and *general equilibrium theory*. The next section will discuss important but selected aspects from these areas as they relate to electricity market design.

2.3. Economic Design Goals

The field of *mechanism design* is concerned with the design of effective mechanisms in environments where self-interested agents engage in strategic interactions while holding private information about their preferences. It is viewed as a counterpart to traditional game theory. Auctions represent a specific category of mechanisms where a collection of bids is translated into an outcome.

Day-ahead electricity markets serve as a prime example of these auction mechanisms. This section outlines essential concepts and design criteria important for market design, while subsequent sections will elaborate on how these concepts are pertinent to the context of electricity markets.

Formally, let $\mathcal{M}$ denote a mechanism that maps bids to an auction outcome $o = (x, p)$. In particular, a mechanism is said to implement a social choice function $f : \mathcal{B}^{\mathcal{L}} \to O$ with $\mathcal{B}^{\mathcal{L}}$ as the set of all bid functions $b_\ell : 2^{\mathcal{K}} \to \mathbb{R}$ over all bidders $\ell \in \mathcal{L}$ and O as the set of auction outcomes. A traditional social choice function is the *utilitarian welfare function*, wherein social welfare is defined as the sum of bidders' valuations. Assuming truthful bidding, the WDP describes a mechanism to achieve such an outcome with an *efficient* allocation. More generally, an auction mechanism can be assessed by its *allocative efficiency*.

Definition 2.1 (ALLOCATIVE EFFICIENCY)**.** *Let $x^* = (S^*_\ell)_{\ell \in \mathcal{L}}$ describe an allocation that maximizes social welfare. Let $x = (S_\ell)_{\ell \in \mathcal{L}}$ describe the allocation obtained by an auction mechanism $\mathcal{M}$. The allocative efficiency of $\mathcal{M}$ is defined as:*

$$\frac{\sum_{\ell \in \mathcal{L}} v_\ell(S_\ell)}{\sum_{\ell \in \mathcal{L}} v_\ell(S^*_\ell)}.$$

This definition assumes that the maximum social welfare $\sum_{\ell \in \mathcal{L}} v_\ell(S^*_\ell)$ is positive, indicating that allocating bundles from sellers to buyers can create value.

Another notion of efficiency is *Pareto optimality* (or *Pareto efficiency*) which characterizes an outcome where it is impossible to improve the direct utility of one bidder without simultaneously decreasing the direct utility of another.

Definition 2.2 (PARETO OPTIMALITY)**.** *Given a mechanism $\mathcal{M}$ that enacts a social choice function f, an outcome o is Pareto optimal if and only if there does not exist any alternative outcome o' that weakly improves the utility of all bidders $\ell \in \mathcal{L}$ and strongly improves the utility of at least one bidder $\ell' \in \mathcal{L}$,*

$$\nexists\, o' \text{ such that } \forall\, \ell \in \mathcal{L},\ u_\ell(o') \geq u_\ell(o) \text{ and } \exists\, \ell' \in \mathcal{L} \text{ where } u_{\ell'}(o') > u_{\ell'}(o).$$

Pareto optimality is a broader concept that can be seen as a necessary condition for allocative efficiency. If an allocation is allocatively efficient, it must be Pareto optimal. However, not every Pareto optimal outcome is allocatively efficient.

In an attempt to lower their own payments or harm competitors, bidders may decide to strategically misreport their valuations or enact in collusive behavior. Therefore, a mechanism should make sure that agents cannot obtain higher profits from bidding untruthfully. An auction mechanism that ensures that truthful bidding is a dominant strategy is called *strategyproof*.

Definition 2.3 (STRATEGYPROOFNESS). *Let $b_{-\ell}$ denote the bid functions of all other bidders $\mathcal{L} \setminus \{\ell\}$ and $u_\ell(o)$ be the direct utility of bidder ℓ under outcome o. A mechanism $\mathcal{M}$ that implements a social choice function f is strategyproof or dominant-strategy incentive-compatible (DSIC) if for all bidders $\ell \in \mathcal{L}$ and bid functions $(b_\ell)_{\ell\in\mathcal{L}} \in \mathcal{B}^{\mathcal{L}}$,*

$$u_\ell(f(v_\ell, b_{-\ell})) \geq u_\ell(f(b_\ell, b_{-\ell})).$$

Various other notions of incentive compatibility have been proposed. For example, regarding an auction as a Bayesian game, a mechanism is *Bayes-Nash incentive-compatible (BNIC)* if truthful bidding constitutes a Bayes-Nash equilibrium. Specifically, given their beliefs about other players, a bidder maximizes their expected utility by bidding truthfully. A strategyproof mechanism is necessarily BNIC, but the reverse need not be the case. Strategyproofness can also be extended to groups of bidders, ensuring that no coalition can profit from jointly misreporting valuations. This stronger notion of strategyproofness is referred to as *group-strategyproofness*.

Bidders participate in an auction mechanism with the goal to maximize their utility. If bidders prefer an outcome that is not in their demand set, given a specific payment rule, the outcome might not be stable. In particular, bidders might *envy* competing bidders.

Definition 2.4 (ENVY-FREENESS). *A mechanism $\mathcal{M}$ is envy-free if every bidder $\ell \in \mathcal{L}$ receives a bundle S_ℓ in their demand set,*

$$S_\ell \in D_\ell(p_\ell).$$

Even if bidders do not maximize their utility and envy-freeness is violated, a common requirement for auction outcomes – such as those in day-ahead electricity markets – is that bidders do not incur losses. If this property of *individual rationality* is not guaranteed, a bidder might not voluntarily participate in an auction mechanism.

Definition 2.5 (INDIVIDUAL RATIONALITY). *A mechanism $\mathcal{M}$ is individually rational if for all bid functions $(b_\ell)_{\ell\in\mathcal{L}} \in \mathcal{B}^{\mathcal{L}}$, no bidder $\ell \in \mathcal{L}$ pays more than their bid for their allocated bundle S_ℓ,*

$$b_\ell(S_\ell) \geq p_\ell(S_\ell).$$

Assuming truthful bidding, where $b_\ell(S_\ell) = v_\ell(S_\ell)$, individual rationality thereby ensures that every bidder ℓ has a non-negative direct utility $u_\ell(o)$ from the auction outcome o.

Furthermore, an ideal mechanism should achieve all these properties without monetary transfers into or out of the auction. *Budget balance* requires that no external subsidies are required, and that no money is withdrawn from the market.

Definition 2.6 (BUDGET BALANCE). *A mechanism $\mathcal{M}$ is budget balanced if the sum of payments of buyers equals the sum of payments to sellers,*

$$\sum_{i\in\mathcal{I}} p_i(S_i) + \sum_{j\in\mathcal{J}} p_j(S_j) = 0.$$

Lastly, while payments p_ℓ can, in principle, be *non-linear* and *personalized*, many markets – day-ahead electricity markets being a prime example – require a unique price signal for each item, e.g., as a reference for hedging instruments or to provide unbiased investment signals. Usually, this corresponds to the requirement of *linear* and *anonymous* prices.

Definition 2.7 (LINEAR AND ANONYMOUS PRICES). *A mechanism $\mathcal{M}$ implements linear and anonymous prices if the payment of any bidder $\ell \in \mathcal{L}$ for any bundle S_ℓ depends only on item-level prices p_k for each item $k \in \mathcal{K}$,*

$$p_i(S_i) = \sum_{k \in S_i} p_k \text{ and } p_j(S_j) = -\sum_{k \in S_j} p_k.$$

In other words, two bidders are charged the same price for the same bundle (anonymity), and the price of a bundle simply equals the sum of the prices of its items (linearity).

Unfortunately, it has been long established that the aforementioned properties cannot be jointly satisfied. In a social choice context, Gibbard (1973) and Satterthwaite (1975) famously show that any non-dictatorial voting mechanism with more than two options cannot be strategyproof. Generally, Hurwicz (1972) demonstrates that under very mild assumptions, there can be no strategyproof, individually rational, and Pareto optimal mechanism. It was later shown that the well-known *VCG mechanism* (Vickrey, 1961; Clarke, 1971; Groves, 1973) is the unique mechanism to achieve efficiency and strategyproofness (Green and Laffont, 1979). However, it violates group-strategyproofness and requires non-linear and personalized prices. Myerson and Satterthwaite (1983) prove that no mechanism can simultaneously satisfy efficiency, Bayes-Nash incentive compatibility, individual rationality, and budget balance.

Furthermore, additional complexities occur in real-world markets. For example, electricity markets have restricted and non-convex bid languages (Section 2.4) and the auctioneer has to consider non-linear transmission constraints (Section 2.5). This renders market clearing and pricing fundamentally difficult, and market designers have to develop mechanisms that reasonably balance between the aforementioned properties.

In the absence of an ideal market clearing mechanism, an often desired outcome is that of a *competitive equilibrium*. From the properties listed above, a competitive equilibrium satisfies envy-freeness and budget balance, consequently leading to a stable outcome where each bidder receives a bundle from their demand set, and supply and demand are in equilibrium. Note that envy-freeness implies that individual rationality holds as well.

Definition 2.8 (COMPETITIVE EQUILIBRIUM). *An allocation $x = (S_\ell)_{\ell \in \mathcal{L}}$ and payments $p = (p_\ell)_{\ell \in \mathcal{L}}$ form a competitive equilibrium if for each bidder ℓ, $S_\ell \in D_\ell(p_\ell)$, $\cup_{i \in \mathcal{I}} S_i = \cup_{j \in \mathcal{J}} S_j$, and $\sum_{i \in \mathcal{I}} p_i + \sum_{j \in \mathcal{J}} p_j = 0$.*

A competitive equilibrium has been considered desirable due to positive existence results. Arrow and Debreu (1954) and McKenzie (1959) prove that a competitive equilibrium always exists in a market with perfect competition if bidders have convex preferences and if demand is independent of the price or availability of items. This notion formalizes the equilibrium concept first noted by Walras (1874). In his honor, a competitive equilibrium with linear and anonymous prices is called a *Walrasian equilibrium*.

Definition 2.9 (WALRASIAN EQUILIBRIUM)**.** *An allocation $x = (S_\ell)_{\ell \in \mathcal{L}}$ and item prices $p \in \mathbb{R}^{|\mathcal{K}|}$ form a Walrasian equilibrium if for each bidder ℓ, $S_\ell \in D_\ell(p)$ and $\cup_{i \in \mathcal{I}} S_i = \cup_{j \in \mathcal{J}} S_j$.*

The model by Arrow and Debreu (1954) and the notion of Walrasian equilibria led to the celebrated *fundamental theorems of welfare economics*, revealing the inherent connection between competitive equilibria and allocative efficiency. The welfare theorems are often considered a theoretical justification for a market-based allocation of resources.

Theorem 2.1 (FIRST WELFARE THEOREM)**.** *Let allocation x and prices p constitute a Walrasian equilibrium. Then the allocation x maximizes social welfare.*

Theorem 2.2 (SECOND WELFARE THEOREM)**.** *Let allocation x be Pareto optimal. Then there exist linear and anonymous prices p such that (x, p) constitutes a Walrasian equilibrium.*

While the welfare theorems were initially introduced for fully divisible items and fractional allocations, Blumrosen and Nisan (2011) demonstrate that they also hold for the case of indivisible items. Equivalently, a Walrasian equilibrium exists if and only if the solution to the WDP has zero integrality gap. Moreover, if the linear relaxation of the WDP has an integral solution, a Walrasian equilibrium always exists (Bikhchandani and Mamer, 1997).

The welfare theorems laid the foundation for general equilibrium theory, i.e., characterizing the existence and properties of Walrasian equilibria. While they imply a stable market outcome, mechanisms that implement Walrasian equilibria are generally not strategyproof. However, it has been demonstrated that with increasing market size, Walrasian equilibria achieve approximate strategyproofness (Azevedo and Budish, 2018; Jackson and Manelli, 1997).

Due to the welfare theorems, Walrasian equilibria are considered a desirable outcome of auctions, including day-ahead electricity markets. However, the welfare theorems and existence of Walrasian equilibria generally only hold under the model by Arrow and Debreu (1954). Unfortunately, the underlying assumptions are fairly strong and restrict the application of the welfare theorems to real-world markets. In particular, when bidders have *non-convex preferences*, a Walrasian equilibrium might cease to exist. In many markets – including day-ahead electricity markets – such non-convexities are an inherent part of the market design, since bidders are permitted to submit non-convex bids by design of the *bid language*.

2.4. Bid Languages

Bid languages establish the structure for bids that bidders can use to communicate their valuation or costs. Designing a bid language necessitates ensuring computational scalability for the allocation problem, providing completeness to accurately express valuations, and accommodating market and bidder-specific idiosyncrasies.

The most general bid language is the XOR language discussed in Section 2.2, as it allows bidders to express a value for each of the $2^{|\mathcal{K}|} - 1$ possible non-empty bundles. However, employing such a bid language faces practical challenges for several reasons. Due to the large number of bundles, bidders may find it impractical to assign a value to each one, and the *communication complexity* of preference elicitation by the auctioneer is too high (Nisan and Segal, 2006). If valuations are provided only for a subset of bids, the *missing bids problem* can cause substantial welfare losses (Kroemer et al., 2016; Bichler et al., 2014). Lastly, the WDP grows exponentially in the number of items, and the resulting *computational complexity* of the allocation problem prevents practical application.

As a result, market designers frequently leverage their understanding of bidders' valuation and cost structures to craft domain-specific, concise, and scalable bid languages. Notable examples include spectrum auctions (Bichler et al., 2023b), procurement auctions (Bichler et al., 2011), and day-ahead electricity markets (Herrero et al., 2020; Conejo and Sioshansi, 2018). The remainder of this section will focus on U.S. ISO and European day-ahead markets as representative examples.

In U.S. ISO markets, the day-ahead market operates through a unit commitment and economic dispatch system. The ISO, acting as both a central market and system operator, aims to dispatch resources in a viable and economically efficient manner. To that end, they provide bid languages that enable bidders to express the operating restrictions and cost structures of their assets. Examples of these bid languages can be accessed through the operations manuals of the California ISO (CAISO, 2023) and Midcontinent ISO (MISO, 2023b).

While bid languages can vary between ISOs, they often share common features. Generators typically submit *multi-part bids* to convey their operating costs, consisting of a (piecewise constant or piecewise linear) variable cost function, no-load costs representing fixed costs to operate a generation unit, and start-up costs representing fixed costs to start a unit. Generators can also specify various technical parameters of their assets, such as minimum and maximum output levels, ramping rates, minimum and maximum running times after start-ups, or minimum cooling times after shut-downs. Although these technical constraints have been tailored primarily to thermal units, some ISOs have introduced additional bid formats for multi-stage generators (e.g., combined cycle gas turbines), pumped-hydro units, or storage resources (Herrero et al., 2020). Given the increased volatility in supply resulting from renewable energy sources, bid languages that allow to express flexibility are a topic of discussion (Mays, 2021). Moreover, since ISOs co-optimize the actual dispatch and the provision of ancillary services, generators can submit bids for various reserve products. On the demand side, buyers can express fixed, price-inelastic demand as well as (piecewise constant or piecewise linear) bid curves for price-elastic demand.

In Europe, market operation and system operation is institutionally separated. Consequently, generation assets are not centrally dispatched; instead, the NEMOs provide bid languages that facilitate a more decentralized decision-making by bidders (Herrero et al., 2020). For example, a generation firm owning multiple assets might submit portfolio bids that aggregate the characteristics of their generation units.

The available market orders are outlined in the public documentation of the Pan-European Hybrid Electricity Market Integration Algorithm (EUPHEMIA, NEMO Committee (2019b)). All order types, except for seller complex orders, are available to both sellers and buyers. The standard bid format are *hourly orders* as single-part offers to sell or buy electricity at specific prices. An hourly order consists of price-quantity pairs that translate into aggregate (piecewise constant, piecewise linear, or hybrid) supply and demand curves.[5]

Block orders describe collections of hourly orders whose acceptance is contingent on specific conditions. *Regular block orders* specify a number of periods, a desired volume for each period, a price limit, and a minimum acceptance ratio.[6] The clearing algorithm can either accept the volumes in each period (at the minimum acceptance ratio or higher), or reject all volumes entirely. Several derivations of block orders exist to express more complex valuation functions. *Linked block orders* are composed of a parent block order and a child block order which may only be accepted if the parent order is cleared. *Loop block orders* extend this logic such that two blocks can only be accepted or rejected together. Finally, *exclusive block orders* represent a collection of block orders for which the sum of the accepted ratios must not exceed one.

Only hourly orders and regular block orders are accessible to bidders across all NEMOs. Besides, other order types have been introduced in specific NEMOs to accommodate local idiosyncrasies and regulations. For example, *complex orders*, available in Ireland, Portugal, and Spain, allow to specify a minimum income condition (MIC), ensuring that the revenue collected from a set of hourly orders covers the specified costs, consisting of a variable and fixed cost term. MIC orders can be combined with a scheduled stop to avoid abrupt deactivations of underlying assets. Complex orders with ramping constraints, limiting the difference in cleared energy between two hours, are termed load gradient orders. *Scalable complex orders* enhance computational scalability by directly including the fixed cost term in the objective function, unlike MIC orders. In Italy, *PUN orders* (Prezzo Unico Nazionale) must be cleared at a nationwide price rather than the price of the respective bidding zone. *Merit orders* describe hourly orders that will be accepted in the order of a specified ranking. Lastly, *flexible hourly orders* in the Nordic market describe hourly orders that can be shifted to any hour of the day.

[5] A transition from 60-minute to 15-minute market time unit is expected to go live in 2025 (NEMO Committee, 2023), which will supplement hourly orders with quarter-hourly orders.
[6] Block orders with a minimum acceptance ratio of less than one are also known as *curtailable block orders*.

Although bid languages in U.S. and European markets exhibit considerable differences, they both possess a common characteristic: *non-convexities*. Specifically, both the unit commitment in U.S. markets and the block orders in European markets require a modeling via binary variables, rendering the allocation problem a non-convex mixed-integer optimization problem. Consequently, the convexity assumption underlying the model by Arrow and Debreu (1954) is violated, and Walrasian equilibrium prices generally do not exist, as the following example demonstrates.

Example 2.1 (NON-EXISTENCE OF WALRASIAN EQUILIBRIUM)**.** *Assume a simplified European market setting with a single zone and a single hour, involving one seller and two buyers. Seller S1 submits a regular block order for 2 MW, with a price limit of EUR 30 and a minimum acceptance ratio of 1. Buyers B1 and B2 each submit a regular block order for 1 MW and a minimum acceptance ratio of 1. B1 has a price limit of EUR 10 and B2 a price limit of EUR 40.*

The unique welfare-maximizing allocation is to accept all block orders, yielding a welfare of EUR $10 + 40 - 30 = 20$*. At any linear price* p*, equivalent to an hourly electricity price, the market is balanced, both in budget and in supply and demand.*

However, as illustrated in Figure 2.3, achieving envy-freeness is impossible with any linear price p*. For* $p \leq 10$ *EUR/MWh (Case A), the revenue of S1 would be at most EUR 20, failing to cover their costs and violating individual rationality and envy-freeness. In Figure 2.3, the loss incurred by S1 is depicted in blue. Formally, at* $p \leq 10$ *EUR/MWh, the demand set of S1 would consist of the empty bundle, and the assigned bundle (selling 2 MW) is not in the demand set.*

Similarly, at $p > 10$ *EUR/MWh (Case B), buyer B1 would pay more than their bid, again violating individual rationality and envy-freeness. As a result, a Walrasian equilibrium cannot be achieved.*[7]

Mathematically, the non-existence of Walrasian equilibria follows from the non-convexities implied by the block orders. Consequently, the allocation problem becomes a mixed-integer problem, with its solution exhibiting a positive duality gap. This duality gap implies *lost opportunity costs* incurred by market participants, i.e., $\sum_{\ell \in \mathcal{L}} \hat{u}_\ell(o) - u_\ell(o) > 0$ for any feasible outcome o with linear and anonymous prices. The lost opportunity costs quantify the violation of envy-freeness, and a Walrasian equilibrium thus becomes unattainable in markets with non-convexities.

In contrast, a Walrasian equilibrium exists if the linear relaxation of the allocation problem yields an integral solution (Bikhchandani and Mamer, 1997), coinciding with a zero duality gap by strong duality. Additionally, extensive research efforts have focused on identifying conditions ensuring the existence of Walrasian equilibria.

[7] A Walrasian equilibrium is also unattainable for the second feasible allocation, which involves rejecting all block orders. Under this allocation, if $p < 40$ EUR/MWh, envy-freeness would be violated for B2. Conversely, $p \geq 40$ EUR/MWh would lead to a violation of envy-freeness for S1.

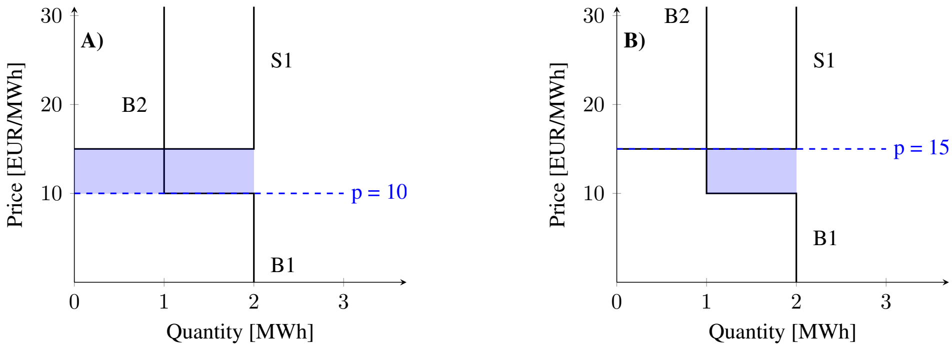

Figure 2.3.: Exemplary Market Clearing at p =10 EUR/MWh and p =15 EUR/MWh

Kelso and Crawford (1982) introduce the *gross substitutes* condition for single-unit markets. Essentially, this condition posits that increasing the price of one or more items does not result in a decrease in demand for any other items, with equivalent definitions provided by Gul and Stacchetti (1999). When all bidders' valuation functions adhere to the gross substitutes condition, a Walrasian equilibrium is guaranteed to exist, e.g., for unit-demand or additive valuation functions. Given gross substitutes, a Walrasian equilibrium can be implemented as an ascending auction (Gul et al., 2000). The concept of gross substitutes has also been extended to multi-unit markets (so-called *strong substitutes*, Milgrom and Strulovici (2009)) and valuations with complementarities (so-called *gross substitutes and complements*, Sun and Yang (2006)).

Baldwin and Klemperer (2019) provide generalized *demand types* for which a Walrasian equilibrium exists. For personalized and non-linear prices, it has been demonstrated that a competitive equilibrium always exists when there is only a single seller in the market (Bikhchandani and Ostroy, 2002). Lastly, leveraging the Shapley-Folkman Lemma (Starr, 1969), an *approximate equilibrium* can be attained even without convex preferences. Starr (1969) and Heller (1972) bound the distance from an approximate equilibrium to a *pseudoequilibrium* obtained from a convexified market.

However, achieving a Walrasian equilibrium is typically unfeasible in electricity markets, as none of the aforementioned conditions are met. Consequently, allocation and pricing rules must find a trade-off between allocative efficiency, budget balance, and envy-freeness. The first (Bichler et al., 2023a, see Chapter 3) and second (Ahunbay et al., 2023b, see Chapter 4) publication of this dissertation expand on this topic, proposing pricing rules that aim to balance different dimensions of envy-freeness and budget balance.

2.5. Transmission Constraints

Electricity markets are characterized by another pivotal challenge that sets them apart from other markets: *transmission constraints*. The power system infrastructure is widely regarded as one of humanity's most extensive engineering achievements. Market clearing and pricing must acknowledge that electricity transmission operates under the laws of physics rather than economics. Consequently, transmission constraints impose limitations on the auctioneer, who must determine a dispatch that adheres to feasible network flows.

Power flow models have received considerable attention in academic research (Cain and O'Neill, 2012; Frank et al., 2012; Molzahn and Hiskens, 2019; Bienstock et al., 2020). The power flow equations describe the relationship between injections and withdrawals of power at nodes (also known as buses) and the resulting transmission line flows. In a basic alternating current (AC) power circuit, transmission lines exhibit *resistance* R [Ω] and *reactance* X [Ω], depending on factors such as length and conductor material. Resistance describes the property of a material to oppose the flow of current I [A], and power dissipated by resistors is known as *real* or *active* power P [W], described by Ohm's Law: $P = RI^2$. Real power represents the usable power to perform work.

Reactance, on the other hand, represents the opposition to the change in current caused by inductors and capacitors in AC circuits[8], and the respective power component is known as *reactive* power Q [VAR]. Reactive power does not perform any useful work but instead oscillates in the circuit. It is essential for maintaining the voltage profile of the system and supporting the transfer of active power in AC transmission networks.

Real and reactive power together are represented by the complex *apparent* power $S = P + \boldsymbol{j}Q$ [VA] with $\boldsymbol{j}$ as the imaginary unit.[9] Apparent power represents the total power flowing in an AC circuit, including both real and reactive components.

Each node in an AC circuit has a voltage V and each line a current I, often represented as sinusoidal phasors, where the magnitude represents the amplitude of the voltage or current, and the angle represents the phase shift. The phase shift θ [rad] between voltage and current determines the distribution of real and reactive power. When voltage and current are in phase ($\theta = 0$), all power is real, and no reactive power is exchanged. When there is a phase difference ($\theta \neq 0$), a portion of the power becomes reactive and oscillates in the circuit.

In a large, meshed transmission network, phase angle differences between nodes determine the flow of real power. Changes in voltage magnitude ($|V|$) primarily influence the flow of reactive power. This interplay between phase angles and voltage magnitudes regulates the distribution of both real and reactive power throughout the network.

[8] Capacitors cause the current phasor to lead the voltage phasor, generating reactive power. Inductors cause the voltage phasor to lead the current phasor, absorbing reactive power.

[9] The bold notation $\boldsymbol{j}$ is used to distinguish the imaginary unit from a seller j.

Each line in an AC circuit has a resistance and reactance, which together form the *impedance* $Z = R + \boldsymbol{j}X$ [Ω]. The inverse of the impedance is the *admittance* Y [$S = \Omega^{-1}$], consisting of a real *conductance* G and an imaginary *susceptance* B, i.e., $Y = G + \boldsymbol{j}B$.

With this notation, the AC power flow equations describe the flow of electricity in a power grid.

Definition 2.10 (AC Power Flow Equations). *Let $\mathcal{N}$ be the set of nodes and pairs of nodes (n, m) encode transmission lines. Let $x \in \mathbb{C}^{|\mathcal{L}|\times|N|}$ be the bundles allocated to buyers and sellers at each node. With $V_n = V_{dn} + \boldsymbol{j}V_{qn}$ as complex voltage at node n and V_n^{min},V_n^{max} as minimum/maximum voltages, the AC power flow equations are defined as:*

$$\begin{aligned}
\sum_{j\in\mathcal{J}} x_{jn}^{Re} - \sum_{i\in\mathcal{I}} x_{in}^{Re} &= \sum_{m\in\mathcal{N}} V_{dn}(G_{nm}V_{dm} - B_{nm}V_{qm}) + V_{qn}(B_{nm}V_{dm} + G_{nm}V_{qm}) && \forall n \in \mathcal{N} \\
\sum_{j\in\mathcal{J}} x_{jn}^{Im} - \sum_{i\in\mathcal{I}} x_{in}^{Im} &= \sum_{m\in\mathcal{N}} V_{dn}(-B_{nm}V_{dm} - G_{nm}V_{qm}) + V_{qn}(G_{nm}V_{dm} - B_{nm}V_{qm}) && \forall n \in \mathcal{N} \\
V_{dn}^2 + V_{qn}^2 &= |V_n|^2 && \forall n \in \mathcal{N} \\
(V_n^{min})^2 \leq |V_n|^2 &\leq (V_n^{max})^2 && \forall n \in \mathcal{N} \\
V_n &\in \mathbb{C} && \forall n \in \mathcal{N}
\end{aligned}$$

x^{Re} denotes real power, while x^{Im} represents reactive power. If line (n, m) does not exist, it is assumed that $B_{nm} = G_{nm} = 0$. Several alternative formulations of the AC power flow equations exist (Molzahn and Hiskens, 2019).

To find a feasible allocation, the market clearing problem in electricity markets must account for AC power flows, meaning the assigned bundles have to satisfy above equations. This gives rise to the *AC Optimal Power Flow* (ACOPF) problem. However, solving the ACOPF is NP-hard and computationally intractable, as the feasible space defined by the power flow equations is highly non-convex (Hiskens and Davy, 2001). Currently, there are no scalable algorithms capable of finding the global optimum of the ACOPF. To address the computational complexities associated with solving such optimization problems, various power flow relaxations and approximations have been devised (Molzahn and Hiskens, 2019). These approaches often leverage techniques from *convex optimization*.

The most established convex optimization technique is *linear programming* (Dantzig, 1963; Bertsimas and Tsitsiklis, 1997). A canonical linear program is defined as follows:

$$\min_{x\in\mathbb{R}^d} \{c^T x : Ax \geq b\}$$

Solver techniques for linear programming have seen significant advancements over the years. The simplex method, for example, has a worst-case exponential time complexity in the number of variables, but in practice, it often performs well and converges in a reasonable number of iterations, particularly for sparse LP instances. Interior-point methods typically have polynomial time complexity and are known for their efficiency on dense and large-scale LP problems. Modern solvers can handle problems with thousands or even millions of variables and constraints. Due to this scalability, linear programming approximations of the ACOPF have been chosen for implementation in real-world electricity markets.

In U.S. markets with nodal pricing, the linear *Direct Current Optimal Power Flow* (DCOPF) problem has been used (Stott et al., 2009; Eldridge et al., 2018; Molzahn and Hiskens, 2019). The DCOPF simplifies the power flow equations by making three key assumptions: (i) it disregards line resistances and reactive power, (ii) it fixes voltage magnitudes $|V_n|$ at each node to 1, and (iii) it assumes that voltage angle differences between nodes are sufficiently small such that $\sin(\theta_n - \theta_m) \approx \theta_n - \theta_m$ and $\cos(\theta_n - \theta_m) \approx 1$. As a consequence, the power flow equations become linear constraints.

Definition 2.11 (DC Power Flow Equations)**.** *Let $x \in \mathbb{C}^{|\mathcal{L}|\times|N|}$ be the electricity allocated to buyers and sellers at each node. With θ_n as voltage phase angle at node n, the DC power flow equations are defined as:*

$$\sum_{j\in\mathcal{J}} x_{jn}^{Re} - \sum_{i\in\mathcal{I}} x_{in}^{Re} = \sum_{m\in\mathcal{N}} -B_{nm}(\theta_n - \theta_m) \qquad \forall n \in \mathcal{N}$$

$$\theta_n \in \mathbb{R} \qquad \forall n \in \mathcal{N}$$

In U.S. ISO markets, more refined versions of the DCOPF are employed for market clearing, while maintaining the computational scalability of the DCOPF as a linear program (Li et al., 2022).

In European zonal markets, sets of nodes are aggregated into large bidding zones $\mathcal{Z}$, with only cross-zonal lines and critical network elements considered as linear constraints in the allocation problem. This subset of lines is denoted by $\mathcal{H}$. The EUPHEMIA algorithm accommodates two types of linear constraints to model $\mathcal{H}$.

Firstly, under the available transfer capacity (ATC) methodology (ENTSO-E, 2000), TSOs compute available capacities for each line $h \in \mathcal{H}$ that can be utilized for spot market trading. ATCs are derived as the total transfer capacity minus reliability margins and notified transmission flows. Simple linear constraints in the market clearing problem ensure that cross-zonal flows (potentially with linearized losses) do not exceed the ATC of each line.

Secondly, under the flow-based market coupling (FBMC) methodology (NEMO Committee, 2021; Schönheit et al., 2021), the impact of all trades on flows on cross-zonal lines and critical network elements is directly accounted for. Based on a linearized network representation, TSOs obtain *power transfer distribution factors* (PTDFs) for each node in the grid. For a specific node $n \in \mathcal{N}$, its PTDFs specify the line flows on each line given an injection at n. TSOs further estimate the contribution of an injection at node n to a change of the net position of each zone $z \in \mathcal{Z}$, termed as *generation shift keys* (GSKs). The nodal PTDFs and GSKs are then utilized to construct zonal PTDFs that specify how changes in the zone net positions affect the flows on each line $h \in \mathcal{H}$ (Schönheit et al., 2021).

$$PTDF_{hz}^{Zonal} = \sum_{n \in \mathcal{N}} GSK_{nz} PTDF_{hn}^{Nodal}$$

Moreover, TSOs compute *remaining available margins* (RAMs) for each line $h \in \mathcal{H}$, representing the maximum transmission capacity that can be auctioned in the day-ahead market. With net_z denoting the net position of each zone z, the linear FBMC constraints are formulated as a matrix-vector product (NEMO Committee, 2021).

$$PTDF^{Zonal} net \leq RAM$$

The DCOPF, ATC, and FBMC approximations are linear, but may not yield solutions that are physically feasible with respect to the AC power flow equations. Consequently, ISOs or TSOs must conduct *redispatch*, modifying the computed dispatch to achieve an ACOPF-feasible outcome. Redispatch costs are notably high in European markets, where the majority of transmission constraints are not represented. These costs have risen significantly in recent years (Bundesnetzagentur, 2022), prompting concerns and leading to an ongoing Bidding Zone Review (BZR) process to reassess the configuration of European bidding zones.

Another approach to better approximate the AC power flow equations is to employ tighter convex relaxations than simple linear approximations. In particular, the canonical linear program can be rewritten as $\min_{x \in \mathbb{R}^d} \{c^T x : Ax - b \in \mathbb{R}_+^m\}$. The idea of *conic programming* is to replace the non-negative orthant $\mathbb{R}_+^m$ with other non-linear, convex, and well-behaved cones $\mathcal{C}$:

$$\min_{x \in \mathbb{R}^d} \{c^T x : Ax - b \in \mathcal{C}\}$$

The most developed form of conic programs are *second-order cone* programs (SOCPs) (Boyd and Vandenberghe, 2004):

$$\min_{x \in \mathbb{R}^d} \{c^T x : \|A_i x - b_i\|_2 \leq g_i^T x - e_i, \forall i = 1, .., m\}$$

The notation $\|\cdot\|_2$ denotes the Euclidean norm. Particularly, the objective function of an SOCP is linear, and the constraints are defined by convex quadratic inequalities over second-order cones. Modern SOCP solvers utilize interior-point methods that often have polynomial-time complexity (Alizadeh and Goldfarb, 2003; Andersen et al., 2003). SOCPs generalize linear programming and can address a wider range of optimization problems. It has also been leveraged in optimal power flow applications. The *second-order conic* (SOC) relaxation (Jabr, 2006) and *quadratic convex* (QC) relaxation (Hijazi et al., 2017; Coffrin et al., 2016) are two SOCP formulations that closely approximate the ACOPF. Sufficient conditions for the exactness of these SOCP relaxations, particularly in radial networks, have been established, and they have demonstrated good empirical results (Gan et al., 2012; Low, 2014). Unlike the DCOPF, SOC and QC represent *relaxations* rather than *approximations*, implying that the feasible space of ACOPF is a subset of the feasible sets of SOC and QC.

Another variant of conic programming involves the positive semidefinite cone $\mathcal{S}_+^m$, i.e., the cone of all positive semidefinite matrices. A *semidefinite* program (SDP) is defined as follows:

$$\min_{x \in \mathbb{R}^d} \{c^T x : \mathcal{A}x - B \succeq 0\} = \min_{x \in \mathbb{R}^d} \{c^T x : \sum_i x_i A_i - B \in \mathcal{S}_+^m\}$$

An SDP is characterized by a linear objective function and linear matrix inequality constraints. Similar to SOCPs, SDP solvers leverage interior-point methods and typically achieve polynomial-time complexity (Vandenberghe and Boyd, 1996). The SDP relaxation of the ACOPF (Shor, 1987) is proven to be tighter than the SOC relaxation and solves numerous ACOPF instances exactly (Lavaei and Low, 2012). However, the relaxation may not always yield exact solutions, and efforts to enhance its tightness often encounter computational challenges. Leveraging the sparsity of the network has been recognized as a strategy to mitigate computational costs (Grone et al., 1984).

Figure 2.4 illustrates the hierarchy of convex power flow models that seek to approximate the non-convex feasible space of the ACOPF, ranging from the tight SDP relaxation to the fully linearized DCOPF approximation. The ATC and FBMC approximations were excluded from the diagram, as they refer specifically to zonal market clearing.

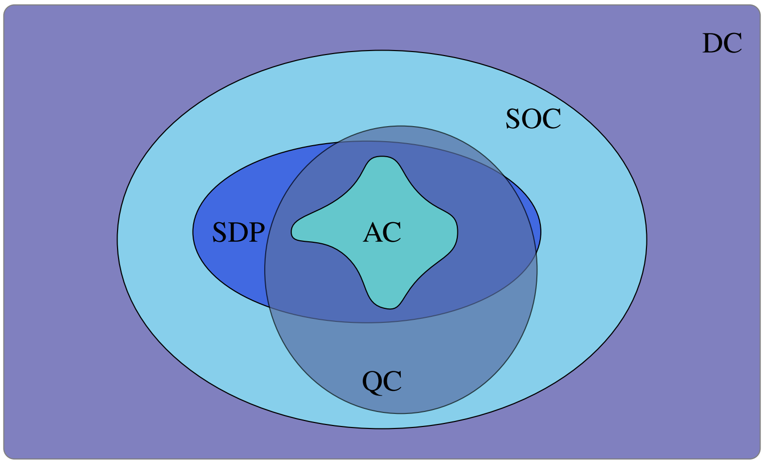

Figure 2.4.: Hierarchy of Power Flow Models

The non-negative orthant, the second-order cone, and the positive semidefinite cone exhibit a property known as *self-duality*. This implies that the dual cone $\mathcal{C}_*$ coincides with the cone

$\mathcal{C}$ itself. Consequently, both *weak* and *strong conic duality* hold without requiring additional conditions. Specifically, for any primal feasible x and any dual feasible y, weak duality dictates that $c^T x \geq b^T y$, and under the condition of strict feasibility (Slater's condition), strong duality ensures that both primal and dual problems are solvable with identical optimal objective value. Therefore, results such as the welfare theorems remain applicable even when the allocation problem is formulated as an SOCP or SDP.

Solvers for mixed-integer versions of second-order cone and semidefinite programs are less mature (Molzahn and Hiskens, 2019). In the third publication of this dissertation (Bichler and Knörr, 2023, see Chapter 5), the impact of various power flow relaxations and approximations on allocation and pricing in non-convex markets is explored. This publication extends the discussion on SOCP and SDP formulations and demonstrates that the choice of power flow model can have considerable impact on the auction outcome.

2.6. Clearing and Pricing in Electricity Markets

As discussed in the previous sections, non-convex bid languages and transmission constraints pose significant challenges to day-ahead electricity markets, leading to two main consequences: the market clearing problem becomes NP-hard, and Walrasian equilibria cease to exist. Slightly modifying the notation, the general formulation of the electricity market clearing problem is as follows:

$$\begin{aligned} \max_x \sum_{\ell \in \mathcal{L}} v_\ell(x_\ell) \\ \text{s.t. } x \in \Psi \end{aligned}$$

In this notation, x_ℓ represents the bundles assigned to buyers and sellers, indicating the amount of electricity sold or purchased in a specific time period and location. Buyers express their valuation function and sellers their cost function using a given bid language $v_\ell(\cdot)$, which may, for example, include multi-part offers or block orders. The set Ψ encompasses all allocations complying with feasible power flows according to a representation of the transmission network, such as ACOPF, DCOPF, or ATC/FBMC. The market operator aims to find an allocation that maximizes welfare while ensuring feasibility with respect to power flows.

This notation readily identifies the two main complexities addressed in this dissertation: non-convex bid languages encoded by $v_\ell(\cdot)$, and the presence of non-convex physical grid constraints within Ψ.

The discussion on the design of bid languages and transmission network representations in U.S. and European markets has been previously outlined. Building upon this foundation, this section further elaborates on the mechanisms for clearing and pricing in these representative markets. Figure 2.5 illustrates the schematic clearing and pricing processes in U.S.

ISO and European markets, the details of which will be explained in the subsequent paragraphs.

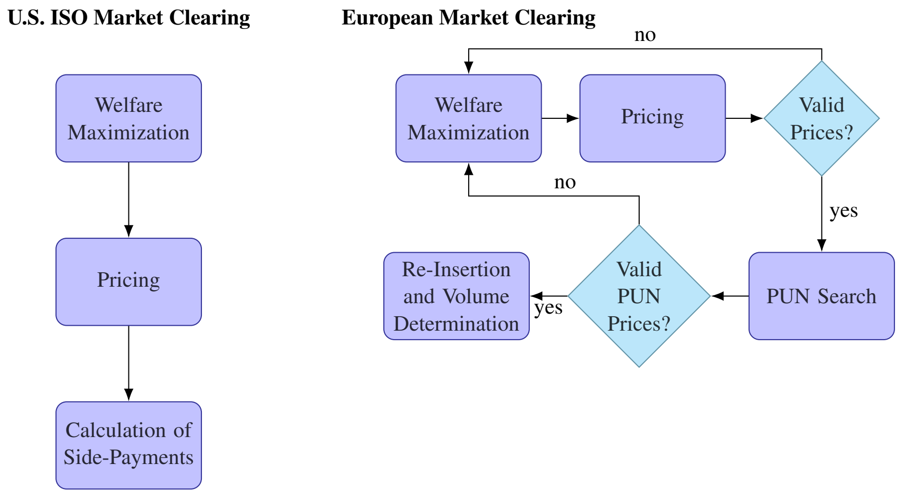

Figure 2.5.: Schematic Clearing and Pricing Processes

U.S. ISO Market Clearing

In U.S. ISO markets, the day-ahead market clearing is based on multi-part bids and a DCOPF representation of the transmission network. The resulting mixed-integer optimization problem is solved to optimality to obtain the welfare-maximizing allocation x^*. In subsequent and separate steps, linear and anonymous market prices p together with non-uniform side-payments are determined based on some linearized version of the allocation problem.

Different ISOs employ diverse methodologies to calculate prices, often with subtle variations distinguishing them. One of the most commonly employed approaches is Integer Programming (IP) pricing, as introduced by O'Neill et al. (2005). Its widespread adoption stems from its notion of "marginal" pricing, although this terminology may lack precision within non-convex markets. IP pricing eliminates integer constraints by fixing integer variables at their optimal values, derived from the welfare-maximizing allocation. This effectively assumes predetermined commitments for thermal generators to operate or remain idle during specific periods. Consequently, the problem transforms into a convex one, and hourly and nodal electricity prices are obtained as dual variables of the supply-demand balance constraints.

These prices constitute Walrasian equilibrium prices for the *convexified* market, albeit not for the original non-convex market. In particular, generators may have incentives to deviate from their committed status, or may be unable to recover costs associated with their integer variables (e.g., start-up costs) from the market payment. Therefore, O'Neill et al. (2005) propose a payment rule that incorporates personalized side-payments to bidders, set at the dual variables of the constraints fixing the integer variables. This ensures that every bidder attains zero profits; thus, unprofitable suppliers receive positive side-payments, while profitable ones are assigned negative side-payments. The optimal allocation, linear and anonymous market prices, and personalized side-payments together satisfy efficiency, supply-demand balance, and envy-freeness. However, the zero-profit condition for bidders may be overly restrictive. In practice, side-payments are often limited to positive payments, compensating only for losses incurred by bidders under the market payments. Such payments, known as *make-whole payments* (MWPs), are defined as follows.

$$MWP_\ell = \max(-u_\ell(o^*), 0)$$

In this context, $o^* = (x^*, p)$ indicates the outcome defined by the welfare-maximizing allocation x^* and the determined market prices p. By providing MWPs to bidders, the outcome maintains individual rationality, but envy-freeness and budget balance may still be violated. As a result, the outcome of IP pricing does not constitute a Walrasian equilibrium. Although IP prices can exhibit high volatility (Bjørndal and Jörnsten, 2008), IP prices signal the marginal value of additional transmission capacity, ensuring efficient signaling of congestion within the transmission network.

As IP pricing can require large and discriminatory MWPs that can lead to misaligned incentives (Herrero et al., 2015), alternative pricing mechanisms are sought to minimize the need for such side-payments. In this regard, Convex Hull (CH) pricing, proposed by Hogan and Ring (2003) and Gribik et al. (2007), exhibits promising theoretical properties. Like IP pricing, CH pricing first determines the optimal allocation and then computes prices based on a separate convexified pricing problem. Specifically, CH pricing substitutes non-convex cost functions $-v_j(x_j)$ with their convex envelopes $\overline{conv}(-v_j)(x_j) = \max\{g(x_j) | g \leq -v_j \text{ is closed and convex}\}$. This transformation yields a convex problem, with prices derived from the dual variables of supply-demand balance constraints.

CH prices represent Walrasian equilibrium prices only if the duality gap of the allocation problem is zero. In cases where a positive duality gap exists, bidders incur *lost opportunity costs* (LOCs), leading to violations of envy-freeness. LOCs quantify the foregone payoffs of a bidder at the market price p, given as follows.

$$LOC_\ell = \hat{u}_\ell(o^*) - u_\ell(o^*)$$

The appeal of CH pricing stems from its ability to minimize the aggregate LOCs of bidders (Schiro et al., 2016). Zero LOCs indicate that all bidders receive a bundle from their demand set, thereby satisfying envy-freeness. Note that as envy-freeness implies individual rationality, MWPs are a subset of LOCs. However, determining convex envelopes of non-convex functions is generally intractable. Efforts have been made to identify conditions under which CH prices can be computed using simple linear programs, such as the linear relaxation of the allocation problem. This pricing rule, known as Extended Locational Marginal Pricing (ELMP, MISO (2023a)), has been utilized to price fast-start units in various U.S. ISO markets. For restricted bid languages, it was demonstrated that ELMP is equivalent to CH pricing (Hua and Baldick, 2017). While the minimization of LOCs is desirable, CH prices may not effectively signal congestion, and offline units may end up setting the price (Schiro et al., 2016).

In practice, pricing methodologies in U.S. ISOs are tailored to the specific characteristics of the market area, such as the treatment of block-loaded and fast-start units or the selection of integer constraints relaxed in the pricing problem. Given the limitations associated with IP and CH prices, numerous alternative pricing rules have been proposed (Liberopoulos and Andrianesis, 2016). Broadly speaking, these pricing rules fall into several categories: they may involve different types of external side-payments, permit internal zero-sum transfers among bidders, or depart from the allocation that maximizes welfare. Since the welfare theorems do not hold, it becomes necessary to compromise at least one of efficiency, envy-freeness, budget balance, or the attainment of linear and anonymous prices. This dissertation seeks to contribute to this discussion by developing novel pricing rules that offer desirable trade-offs.

European Market Clearing

In European markets, the allocation x and prices p are jointly computed by the EUPHEMIA algorithm (NEMO Committee, 2019b). From an algorithmic perspective, EUPHEMIA solves a mixed-integer quadratic program subject to complementarity constraints. The *master problem* seeks a welfare-maximizing allocation by accepting and rejecting the submitted market orders, subject to flow constraints according to the ATC and FBMC methodologies. Additionally, the objective function integrates a penalty term to ensure equitable distribution of curtailment among bidding zones if necessary. The mixed-integer problem is solved to optimality, yielding a candidate allocation.

Subsequently, the candidate allocation is input into the *price determination sub-problem*. Specifically, EUPHEMIA aims to determine hourly zonal prices that are coherent with the candidate allocation. This entails that there must be no *paradoxically accepted bids* (PABs), which violate individual rationality and would necessitate a make-whole payment. Furthermore, no in-the-money hourly order must be rejected, corresponding to *paradoxically rejected bids* (PRBs). In-the-money block orders, however, may be paradoxically rejected. Additionally, complex orders must meet their minimum income requirement, and power trades must not oscillate along a line connecting two zones with negative prices. Optionally, EUPHEMIA may prevent adverse flows from more expensive to cheaper zones.

In cases where such prices cannot be determined, EUPHEMIA introduces a cut to the master problem, discarding the current candidate allocation. The algorithm then revisits the master problem and initiates the next iteration to search for a new candidate allocation.

Upon successfully completing the price determination, EUPHEMIA advances to the *PUN search sub-problem*. This step in the algorithm addresses Italian PUN orders, which require clearance at a single national price, unlike regular orders cleared at the zonal price of one of Italy's four bidding zones. Determining PUN prices is not trivial, as cleared orders must avoid paradoxical acceptance, and the PUN price can only deviate from the weighted average zonal price by a specified tolerance. If valid PUN prices do not exist, EUPHEMIA reverts to the master problem and introduces an additional cut.

When the PUN search yields a viable outcome, EUPHEMIA proceeds with a *re-insertion* step aimed at rectifying false paradoxically rejected MIC orders and other PRBs. This involves iteratively attempting to accept each PRB and identifying valid market clearing prices in a feasible solution where social welfare does not decrease. The re-insertion process continues until either no paradoxically rejected bids remain or the time limit is reached. Lastly, EUPHEMIA proceeds with the *volume indeterminacy sub-problem* to resolve indeterminacies arising from multiple feasible solutions, aiming at minimizing curtailment or maximizing accepted volumes.

Since EUPHEMIA introduces cuts to the original welfare maximization problem, it will not lead to an outcome with allocative efficiency. Due to PRBs that violate envy-freeness, EUPHEMIA prices also do not constitute Walrasian equilibrium prices. However, unlike in U.S. ISO markets, EUPHEMIA eliminates the need for side-payments by avoiding PABs, thus ensuring individual rationality and budget balance at the expense of reduced welfare.

Another significant concern in European markets pertains to the runtime of EUPHEMIA. With a requirement to provide a solution within 17 minutes, the iterative nature of the algorithm implies scalability issues. The impending introduction of 15-minute products in day-ahead markets is anticipated to exacerbate this issue (NEMO Committee, 2023). Lastly, European market clearing computes a zonal allocation, leading to increasing amounts of costly redispatch to modify the allocation to satisfy physical power flow constraints (Bundesnetzagentur, 2022). This dissertation aims to facilitate an understanding of the impact of different power flow models on allocation and prices.

3. Part I: Pricing in the Presence of Demand Response

Peer-Reviewed Journal Paper

Title: Pricing in Nonconvex Markets: How to Price Electricity in the Presence of Demand Response.

Authors: Martin Bichler, Johannes Knörr, Felipe Maldonado.

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Abstract: A Walrasian competitive equilibrium defines a set of linear and anonymous prices where no coalition of market participants wants to deviate. Walrasian prices do not exist in nonconvex markets in general, with electricity markets as an important real-world example. However, the availability of linear and anonymous prices is important for derivatives markets and as a signal for scarcity. Prior literature on electricity markets assumed price-inelastic demand and introduced numerous heuristics to compute linear and anonymous prices on electricity markets. At these prices, market participants often make a loss. As a result, market operators provide out-of-market side-payments (so-called make-whole payments) to cover these losses. Make-whole payments dilute public price signals and are a significant concern in electricity markets. Moreover, demand-side flexibility becomes increasingly important with growing levels of renewable energy sources. Demand response implies that different flexibility options come at different prices, and the proportion of price-sensitive demand that actively bids on power exchanges will further increase. We show that with price-inelastic demand there are simple pricing schemes that are individually rational (participants do not make a loss), clear the market, support an efficient solution, and do not require make-whole payments. With the advent of demand-side bids, budget balanced prices (no subsidies are necessary) cannot exist anymore, and we propose a pricing rule that minimizes make-whole payments. We describe design desiderata that different pricing schemes satisfy and report results of experiments that evaluate the level of subsidies required for linear and anonymous prices on electricity spot markets with price-sensitive demand.

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Pricing in Non-Convex Markets: How to Price Electricity in the Presence of Demand Response

Martin Bichler[1], Johannes Knörr[1], Felipe Maldonado[2]
[1]Department of Computer Science, Technical University of Munich
[2] Department of Mathematical Sciences, University of Essex
{bichler, knoerr,}@in.tum.de; felipe.maldonado@essex.ac.uk

A Walrasian competitive equilibrium defines a set of linear and anonymous prices where no coalition of market participants wants to deviate. Walrasian prices do not exist in non-convex markets in general, with electricity markets as an important real-world example. However, the availability of linear and anonymous prices is important for derivatives markets and as a signal for scarcity. Prior literature on electricity markets assumed price-inelastic demand and introduced numerous heuristics to compute linear and anonymous prices on electricity markets. At these prices market participants often make a loss. As a result, market operators provide out-of-market side-payments (so-called make-whole payments) to cover these losses. Make-whole payments dilute public price signals and are a significant concern in electricity markets. Besides, demand-side flexibility becomes increasingly important with growing levels of renewable energy sources. Demand response implies that different flexibility options come at different prices, and the proportion of price-sensitive demand that actively bids on power exchanges will further increase. We show that with price-inelastic demand there are simple pricing schemes that are individually rational (participants do not make a loss), clear the market, support an efficient solution and do not require make-whole payments. With the advent of demand-side bids, budget balanced prices (no subsidies are necessary) cannot exist anymore, and we propose a pricing rule that minimizes make-whole payments. We describe design desiderata that different pricing schemes satisfy and report results of experiments that evaluate the level of subsidies required for linear and anonymous prices on electricity spot markets with price-sensitive demand.

Key words: Electricity market design, demand flexibility, non-convexities, pricing

1. Introduction

In many parts of the world, electricity markets have developed from monopolies to competitive wholesale markets. For example, European countries and large parts of the United States liberalized their electricity markets in the 1990s. Short-term electricity procurement is now carried out via power exchanges in these jurisdictions. These power exchanges determine central price signals for over-the-counter trades and futures markets (Shah and Chatterjee 2020). Typically, on day-ahead markets hourly products for the next day are traded. After the day-ahead markets, the market operators use real-time markets in the United States (or intraday markets in Europe) to deal with changes in supply and demand that are closer to the actual dispatch. We will distinguish these types of *electricity spot markets* from *futures markets* where participants can hedge against longer-term price risks.

Spot markets are significant in size. In 2020, European coupled day-ahead markets alone cleared 1,530 TWh in 27 countries with average prices between 30 and 40 EUR/MWh (NEMO Committee 2021). Similarly, the cost of serving load amounted to $8.9 billion in the Californian market, covering 26,000 circuit miles, roughly 1,000 power plants, a population of 30 million, and about 9,700 pricing nodes (California ISO 2018, 2021).

With climate change and a transition to renewable energy sources (RES) such as wind and solar power, we move to an economy with many thousands of small generators and a more price-sensitive demand side that actively bids in electricity markets and offers flexibility to cope with variability in the supply (IRENA 2019, Hytowitz et al. 2020). Changes on electricity markets are not only relevant for market operators, but they also impact generators, industrial and retail consumers alike. These changes in the market have led to renewed interest in the design of electricity markets. While many aspects of electricity market design are similar to other markets, a few features stand out. First, demand and supply need to be balanced at all times to guarantee a stable electricity grid. Second, electricity markets are characterized by non-convex preferences. For example, electricity suppliers often incur fixed costs for starting up and running their generators. On the demand side, industrial customers typically need a certain volume of electricity over consecutive hours to finish production or maintain energy-intensive services. Such consumption profiles can sometimes be shifted over time, but the profiles themselves must not be altered. These non-convexities in the preferences typically lead to non-convex optimization problems that need to be solved in order to determine the efficient or welfare-maximizing dispatch and prices. We will use the term *non-convex markets* in what follows.

Deciding over a particular pricing rule in non-convex market is a complex problem. In this paper we will describe a series of desirable properties for such pricing rules, outlining when those properties are feasible in real-life settings. We will start describing the theoretical ideal called *Walrasian Equilibrium*, where prices are linear (per MWh) and anonymous (independent of the participant), they are also envy-free such that nobody would want to deviate from the optimal allocation or dispatch at these prices, and they are budget-balanced, i.e. no subsidies are required. Unfortunately, it is a well-known fact that in non-convex markets, Walrasian equilibrium prices do not exist in general. Therefore both academics and practitioners have proposed alternatives where necessarily some of the properties are violated. For instance, electricity markets in the United States compute linear and anonymous market prices but they violate budget-balance. Pricing rules as they are used by market operators stipulate out-of-market side-payments, so-called *make-whole payments*, to market participants who would make a loss at the market prices. However, make-whole payments are currently under scrutiny by regulators, which demands new approaches as we will discuss below.

1.1. Contributions

First, we prove that make-whole payments cannot be avoided in markets with a price-sensitive buy-side. While the literature suggest that demand response and price-sensitive demand will play an increasing role in the future, standard models in electricity market design mostly assume price-inelastic demand.

We introduce a pricing rule that minimizes make-whole payments and compare it to existing payment rules used in practice and to other academic proposals. This is in contrast to other literature on electricity market pricing, which is largely based on heuristics to minimize *lost opportunity costs*, i.e. incentives to deviate from the optimal allocation. We argue that lost opportunity costs are less of a concern, since market operators on electricity markets typically enforce stability via penalties. Our experimental results show that high make-whole payments on electricity markets as they are challenged by regulators can either be avoided or reduced substantially with the new pricing rule. This is not at the expense of higher market prices, and the changes in the overall payments of market participants are very small. The new pricing rule can be solved efficiently with standard linear programming techniques and it is easy to understand. The approach is generic and also of interest to non-convex markets beyond those for electricity only.

1.2. Organization of the Paper

The rest of the paper is structured as follows: Section 2 discusses related literature, while Section 3 provides a short introduction to electricity market design. In Section 4, we discuss competitive equilibrium theory and show when anonymous and linear prices are possible on budget-balanced electricity markets. Section 5 introduces optimization models to compute prices in environments with price-inelastic and price-sensitive demand. We briefly characterize existing proposals for electricity prices before we provide results of experiments in Section 6. Section 7 provides a summary and conclusions.

2. Related Literature

The literature on competitive equilibrium has a long history. In this section, we summarize central theoretical findings before we discuss the related literature on electricity market design. We briefly survey the literature on demand response that leads to price-sensitive demand and the challenges arising from this change for market design. Finally, we discuss the connections between the different literature streams.

2.1. Equilibrium Theory

Early in the research on markets, general equilibrium theory was developed to study demand, supply, and prices for multiple goods or objects on markets. The Arrow-Debreu model shows that under convex preferences, perfect competition, and demand independence, there must be a set of competitive equilibrium prices (Arrow and Debreu 1954, McKenzie 1959, Gale 1963, Kaneko 1976). The results derived from the Arrow-Debreu model led to the well-known welfare theorems, representing important arguments for markets to be used as efficient or welfare-maximizing means to allocate scarce resources such as electricity. The first theorem states that any Walrasian equilibrium leads to a Pareto efficient allocation of resources. The second theorem states that any efficient allocation can be attained by a Walrasian equilibrium under the Arrow-Debreu model assumptions (Mas-Colell et al. 1995). Walrasian equilibrium prices are such that there is a single price for each product (i.e., prices for a package are *linear*) and this is the same price for all participants (i.e., *anonymous* prices with no price differentiation).

However, standard general equilibrium theory assumes divisible goods and convex preferences. Most real-world markets such as those for electricity, transportation, radio frequency spectrum, or environmental access rights are traded as indivisible goods and participants have non-convex preferences and complex constraints. Such markets have led to substantial interest in the question when Walrasian equilibria exist. Unfortunately, in markets with indivisible goods, it is well known that only very restricted types of valuations (e.g., substitutes valuations) allow for convex allocation problems and Walrasian equilibria (Kim 1986, Bikhchandani and Mamer 1997, Leme 2017, Baldwin and Klemperer 2019).

This raises the question how prices can be computed in the presence of non-convex preferences for indivisible goods and which properties we can hope to achieve compared to Walrasian equilibria. Established market design desiderata are *efficiency* (i.e., maximization of welfare or gains from trade), *individual rationality* (i.e., participants should not make a loss), *budget balance* (i.e., the market operator should not make a loss or a gain), and *envy-freeness* (i.e., participants would not want a different allocation at the prices). These axioms are not only central to economic theory (Mas-Colell et al. 1995), but are widely adopted and natural design desiderata for practical market design. If the allocation problem is convex, duality theory and dual prices in convex optimization provides a principled way to determine competitive equilibrium prices that satisfy these desiderata (Bichler et al. 2020). In non-convex markets, it is well known that competitive equilibrium prices might need to be non-linear and personalized and such prices might not even exist (Bichler and Waldherr 2017). Thus, in a combinatorial auction or a combinatorial exchange that allows for supply and demand bids on packages of items, each bidder might need to have a different price for the same package (personalized prices), and each package price could differ from the sum of the item prices in this package (non-linear prices). As a simple example, consider a single supplier with an (indivisible) sell bid of 2 MWh for \$30, while there is one buyer asking for 1 MWh for at most \$10, and another buyer asking for 1 MWh for \$28. Linear and anonymous market prices could not be higher than \$10/MWh and as such there would be no trade and no gains from trade. With price differentiation, trade would be possible. However, non-linear and personalized prices would convey little information other than that a bidder lost or won. Besides, if prices should serve as a baseline for derivatives as is the case for options or futures, this is hardly possible with non-linear prices that differ among participants. In other words, anonymity and linearity are important requirements for prices on electricity markets but also in other domains (Bichler et al. 2018).

2.2. Pricing on Electricity Spot Markets

Electricity spot markets are composed of varying levels of “demand” (load) and matching levels of “supply” (generation). Market participants submit supply and demand bids according to a certain *bid language* that determines the form of the allocation problem (which yields the efficient dispatch) and the pricing rule. For example, in markets in the United States (U.S.), generators can express start-up or no-load costs, economies of scale (by means of piecewise-linear cost functions), or minimum-generation requirements. These and other elements of the bid language then translate into non-convex allocation problems (Herrero et al. 2020). In 2005, the Pennsylvania, Jersey, Maryland Power Pool (PJM) introduced mixed integer programming (MIP) in order to address these non-convexities and to determine the efficient allocation or dispatch (O’Neill et al. 2020). Since 2018, all Independent System Operators (ISOs) in the U.S use MIPs to compute the efficient dispatch instead of the Lagrangian relaxation that was used before. Dual prices as they are accessible for convex optimization problems are not available in such markets, which led to a fundamental question: How can market prices per hour be computed in such non-convex markets?

One approach followed by European day-ahead markets is to sacrifice efficiency. The EUPHEMIA algorithm that is used to clear European day-ahead markets first solves a welfare maximization allocation problem as a mixed-integer program and then iteratively tries to find linear and anonymous prices that clear the market. If such prices cannot be found, additional constraints are added to the welfare maximization problem (Committee 2020). However, it is unclear how much of the gains from trade are sacrificed this way. Furthermore, this approach inevitably leads to paradoxically rejected bids (Meeus et al. 2009). In particular, there are generators with an ask price that is less than the market price, yet they will not be dispatched. Such prices are also not *envy-free* and hence not a Walrasian equilibrium. We will not further discuss this approach in our paper and focus on market designs as in the United States that implement the efficient outcome.

Over the years, several pricing rules have been suggested aiming to mimic competitive equilibrium prices on such MIP-based electricity markets (Liberopoulos and Andrianesis 2016). Locational marginal pricing (LMP) rules of many ISOs are based on IP pricing (aka. Integer Programming pricing), where the allocation problem is solved to optimality, the integer variables are fixed, and the prices are then derived from the dual variables of the demand-supply constraint of the resulting (convex) linear program (O’Neill et al. 2005). IP pricing computes anonymous and linear prices, but these prices do not constitute competitive equilibrium prices. Some generators might not maximize their individual profits and want to deviate, i.e., switch to a different dispatch at those prices, and IP prices are thus not *envy-free*. The latter is central to the definition of a competitive equilibrium and it leads to stability of the outcome. Importantly, besides a lack of stability, the generators often make a loss at the IP prices, i.e., prices are not even *individually rational*. Pricing in

U.S. ISO markets has changed in an attempt to reduce the weight of uplift and to internalize all operational costs into market prices as far as possible (Herrero et al. 2020). Some ISOs switched from IP pricing to Extended LMPs (ELMPs) in the recent years, which are based on the dual variables from the demand-supply constraint in the LP relaxation of the underlying MIP. However, similar issues arise. As a consequence, U.S. ISOs continue to search for improvements via new formulations for ELMP.[1]

ISOs use personalized side-payments to address the fact that the public market prices from IP pricing or ELMP are neither envy-free nor individually rational. This effectively differentiates the linear and anonymous market prices from the payments of the market participants, which are then non-linear and personalized. Typically, two kinds of external side-payments are considered in the literature: *lost opportunity costs* and *make-whole payments* (Schiro et al. 2016). *Lost opportunity costs* describe payments that are so high that no generator would want to change its dispatch and envy-freeness is achieved. Such payments may be very large if the market contains non-convexities, and these payments could even go to generators that were not scheduled (Eldridge et al. 2019). However, electricity markets are highly regulated markets and as such there are alternative means to enforce stability other than high lost opportunity cost payments. Actually, most ISOs only pay *make-whole payments* to ensure individual rationality of all generators and stipulate penalties that a generator has to pay if it indeed deviates from the optimal dispatch. In other words, they relax envy-freeness to only individual rationality requirements. We refer to such outcomes as having *penalty-based stability*.

However, even the make-whole payments are a significant concern (Hytowitz et al. 2020). The U.S. Federal Energy Regulatory Commission (FERC) regulates the U.S. wholesale power markets to promote just competition. In 2018, the FERC found that the practices of several ISOs were unjust and ordered them to change their pricing because prices did not accurately reflect the cost of serving load (O'Neill et al. 2019). Make-whole payments are not reflected in the public price signals, and they lead to biased investment signals. This also constitutes a problem for futures markets, where spot market prices serve as the key reference. In addition, the FERC has released several orders and notices about pricing, which argue that "the use of side-payments can undermine the market's ability to send actionable price signals."[2] Similarly, O'Neill et al. (2019) state that "the make-whole payments are not transparent to other market participants and are allocated too broadly to provide correct price incentives for market participants to make efficient entry and exit decisions as well as efficient investments in facilities and equipment." In summary, a challenge in U.S. ISO markets is to reduce side-payments, which are a clear sign of inefficient pricing, while still ensuring individual rationality of all market participants.

[1] https://www.misoenergy.org/stakeholder-engagement/stakeholder-feedback/msc-elmp-iii-whitepaper-20190117/

[2] https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation

In our first contribution, we introduce an optimization model which always computes prices that are individually rational and budget balanced and clear the market at the efficient dispatch without make-whole payments under the assumptions of price-inelastic demand and strict demand-supply equality. These assumptions are standard in the electricity market literature (Liberopoulos and Andrianesis 2016).

2.3. Price-Sensitive vs. Price-Inelastic Demand

While the academic literature on electricity market pricing almost exclusively relies on the assumption of price-inelastic demand, this assumption is unlikely to hold in the future (Herrero et al. 2020). Power systems are changing profoundly due to the introduction of large volumes of RES. The largest proportion of RES capacity are Variable Energy Resources (VER) such as solar and wind power. The characteristic variability and uncertainty of these VER require an integration of demand flexibility (Reihani et al. 2016). Demand response is the most immediately available way of increasing demand flexibility and the cheaper option compared to storage technologies (EU 2016). For example, industrial processes for the production of pulp and paper are able to provide demand response with a duration of up to three hours without any notice time (EU 2016). Still, this flexibility comes at a cost and bidders want lower prices if they provide more flexibility. As indicated, it is expected that in the future we will see a much increased price-responsive demand (Hytowitz et al. 2020). The recent FERC order 2222 from 2020 also aims at an active demand side to bid in wholesale markets. However, such price-sensitive bids for flexible demand make market design more challenging. First, new bid formats lead to additional non-convexities which even increase the make-whole payments needed with currently used pricing rules. Second, prices that are individually rational and clear the market at the efficient dispatch cannot always be budget balanced anymore, as we will show.

In our second contribution, we introduce alternative pricing rules that minimize make-whole payments while they still clear the market at the efficient dispatch with price-sensitive demand. The pricing rules introduced in this paper are based on a mathematical program which differs significantly from IP pricing, ELMP, or other proposals in the literature. Similar to existing pricing rules on ISO markets, it treats efficiency and individual rationality as first-order goals (i.e., enforces these directly in the model), while budget balance and envy-freeness are treated as second-order goals. However, in contrast to existing literature, we prioritize budget balance over envy-freeness in a lexicographical way. This lexicographical ordering is motivated by the fact that the violation of budget balance and the resulting side-payments have led to concerns by regulators and market participants, as we discussed earlier. Envy-freeness should lead to stability of the outcome in markets as participants do not have an incentive to deviate. In highly regulated and transparent electricity markets, stability can be achieved by imposing penalties, which is already common on ISO markets today. Participants cannot easily deviate from the efficient dispatch determined by the market operator, and the level of penalties (that generators would only have to pay if they deviated from the efficient dispatch) is much less of a concern than high personalized side-payments by the market operator that are not reflected in the market prices.

The new pricing rule (PE-A) that we propose can be computed in polynomial time and scales to large problem sizes. Importantly, we show that average locational hourly prices do not increase compared to other established pricing rules in our experiments and the impact on the payments of individual market participants are small. However, PE-A avoids large make-whole payments as they occur with IP pricing, even with price-sensitive demand. In Section 6.2 we analyze our proposed pricing schemes based on a widely-used benchmark data set: the IEEE RTS benchmark market consisting of 24 nodes, 24 hours, 32 generators (with non-convex cost functions), and 17 consumers. The average make-whole payments for PE-A pricing only amount to 0 - 0.15% of the total costs in all treatments. In contrast, for IP pricing or ELMP the make-whole payments were 4 - 5% on average for all generators. Actually, for some generators the make-whole payments could be more than 10% of their payment with IP pricing. Such high make-whole payments can be avoided with PE-A and we achieve almost budget-balanced outcomes in all experiments. We also compare PE-A with a simple implementation of Average Incremental Cost (AIC) pricing, a pricing rule that was recently proposed to address high side-payments on electricity spot markets. PE-A is not restricted to specifics of the allocation problem on electricity markets and can also be applied to other types of non-convex and two-sided markets.

2.4. Positioning in the Literature

This paper draws on different streams in the literature. The fundamental problem of pricing in multi-object markets is central to micro-economic theory and the management sciences. The fact that non-convex preferences lead to problems in equilibrium theory has been known for a long time (see for example Farrell (1959)). Several contributions such as the well-known Shapley-Folkman-Starr lemma (Starr 1969) suggest that nearly competitive equilibria are possible if the market grows large. A number of more recent articles suggest that Walrasian prices can be approximated in (very) large markets and that such markets are approximately incentive-compatible (Azevedo et al. 2013, Azevedo and Budish 2019).

Electricity markets are already very large with hundreds of participants, but their non-convex nature adds an extra complexity to the pricing problem. The question how actual pricing rules for such non-convex electricity markets should be designed, has led to a number of heuristics such as IP pricing and ELMP in the operations research and power engineering literature (O'Neill et al. 2005, 2016, Liberopoulos and Andrianesis 2016, Eldridge et al. 2019, O'Neill et al. 2019). We will revisit this literature in Section 5.4. These heuristics typically aim to approximate a competitive equilibrium and relax budget balance and envy-freeness.

The Information Systems literature has made numerous contributions to market design and pricing in non-convex markets. Some of the work deals with pricing in combinatorial auctions (Xia et al. 2004, Adomavicius and Gupta 2005, Adomavicius et al. 2012, Guo et al. 2012, Petrakis et al. 2013, Bichler et al. 2013, Adomavicius et al. 2020), while other articles deal with combinatorial exchanges (Guo et al. 2012, Bichler et al. 2018). The design of energy markets has also received attention in Information Systems more recently (Ketter et al. 2016, Valogianni and Ketter 2016, Koolen et al. 2018). This paper combines these two strands suggesting a new approach to pricing in electricity markets that substantially reduces or even eliminates the need for side-payments. Our approach can well be relevant to other non-convex markets such as those used in transportation (Caplice and Sheffi 2003, Garrido 2007) or for the trading fishery access rights (Bichler et al. 2019).

3. Bid Languages and Demand-Side Flexibility

Let us provide a brief overview of electricity market design and the role of demand response for future market designs. The pricing rules that can be employed on a market depend on the underlying allocation problem, which again depends on the types of bids or the bid language available on a market. The bid languages on electricity markets today are specific and aim at reflecting the underlying cost functions of generators and – in part – valuation functions of the demand side. They allow the participants to communicate their valuations or cost structures effectively. The market operator then solves the allocation problem and determines a schedule of generation and prices (Cramton 2017). Day-ahead markets are complemented by intraday (Europe) or real-time (U.S.) markets. These markets modify the day-ahead schedule to determine the actual physical dispatch. Especially in European countries, the day-ahead market is considered to be the main reference market, while in the United States it mostly possesses the notion of a forward market for the real-time market that determines the dispatch (Antonopoulos et al. 2020).

Bid languages allow for the expression of the underlying costs in order to enable efficient outcomes (Cramton 2003). For instance, generators typically incur certain fixed costs for starting up and running a generator, as well as variable electricity production costs. Moreover, the operation is often subject to technical conditions, e.g., referring to minimum runtimes or ramping constraints. On the demand side, market participants might want to express certain flexibility options, and this will become much more of an issue in the future with increasing levels of RES. Let us briefly summarize the state-of-the-practice.

In European markets, aside from regular bids for individual hours of the day, the bid language allows for *block bids*. The latter represent a set of individual bids that can be executed only in total or not at all (Committee 2020). Cost structures are communicated as single-part offers, requiring market participants to aggregate various cost components into a single parameter. The communication of multiple cost components is explicitly avoided in order to promote decentralized decision-making on the part of the market participants (Herrero et al. 2020). Most European markets allow for price-sensitive bids on the demand side, although between 2010 and 2015 an estimated 82 - 89% of the bids were not price-sensitive (EU 2016). In Europe, the market is cleared with (zonal) linear and anonymous prices without any side-payments which leads to efficiency losses in the dispatch (Meeus et al. 2009). Overall, the bid language permits a less detailed expression of cost functions than, for instance, bid languages used in the United States.

Market participants in the United States are generally permitted to indicate their costs in a more granular way than in European markets (Madani et al. 2018). Cost structures can be communicated with *multi-part bids*, usually consisting of start-up costs, no-load costs as well as an offer curve. Furthermore, generators can express technical constraints such as minimum up and down times, minimum and maximum output levels, ramp rates, or start-up times. This allows generators to express their cost characteristics very effectively (Cramton 2017). So-called self-schedules are pure quantity bids specifying an amount of energy that needs to be dispatched regardless of price levels or cost structures. Demand-side bids comprise price-inelastic self-scheduling as well as price-sensitive bid curves (Cramton 2017).

As an example of an ISO bid language, PJM allows for fixed-demand bids and price-sensitive bids on the demand side. A fixed-demand bid or self-schedule is price-inelastic and defines a level of energy to be purchased at any price over a particular hour at a location or node. In contrast, price-sensitive bids specify a defined level of energy, a location and a price, above which the demand bid is zero. More than 90% of the bids in the PJM market were fixed-demand bids in 2019, and only a very small proportion is price-sensitive at this point (Monitoring Analytics 2019). This explains why most proposals for pricing rules in the literature assume only price-inelastic demand. However, this is expected to change with increasing levels of demand response, which specifies flexible bids to be executed but only up to a certain price.

U.S. ISO markets aim to find a welfare-maximizing dispatch based on bids, yet in contrast to European markets, they first determine the efficient dispatch before they compute prices. While European markets compute prices for large price zones, the prices on U.S. electricity markets are computed per node in the electricity grid. The nodal system aims to consider physical grid constraints in the optimization. In U.S. nodal markets, bids and offers, resource constraints, network constraints, transmission losses and certain ancillary service requirements are all optimized simultaneously[3] (Cramton 2017). As a result, the electricity price reflects the marginal cost of supplying electricity at a specific node in the network (assuming the underlying problem was convex). Locational marginal prices have also been suggested for European markets (Purchala 2018, Ashour Novirdoust et al. 2021). For the remainder of this paper, we discuss markets as they are operated by ISOs in the United States, in Australia, in South American markets, and many other parts of the world.

As indicated, demand-side bidding is central to accommodate the volatile nature of VER in the future. ISOs in the United States have already taken steps to accommodate demand-side flexibility

[3] This co-optimization of energy and ancillary services differs from European markets, where reserves are cleared in separate markets.

and price-sensitive bids. For example, the Midcontinent ISO (MISO) is undergoing reforms[4] to better incorporate Demand Responsive Resources (DRRs) into the price formation (in both day-ahead and real-time markets). The above mentioned FERC Order 2222 promotes participation of the demand side, in particular distributed energy resources, and storage in wholesale electricity markets. There is significant potential for industrial demand flexibility, but industry will invest in flexibility options only if it comes with lower electricity prices (EU 2016). Therefore, the increase of price-sensitive bids in wholesale electricity markets is to be expected in the future.

A number of proposals have been made for the demand side to express flexibility (Liu et al. 2015, Ottesen and Tomasgard 2015, Ottesen et al. 2016). Flexibility extensions of a bid language on the demand side can include shiftable volumes (asking to meet a certain volume within a certain time frame), shiftable profiles (allowing to shift a pre-determined demand profile over time), or adjustable demand (involving extensible or curtailable demand). Such flexibility options in the bid language would be a powerful way to address the intermittent nature of VER, but they lead to substantial non-convexities due to additional integer variables in the allocation problem. For example, thermal power plants have ramping constraints that make the production available in one period dependent on the production in the preceding and following periods. The introduction of renewable energy sources leads to an increased use of the of thermal units, and ramping constraints are expected to be binding more frequently (Herrero et al. 2020). Ignoring such constraints in the day-ahead schedule can significantly degrade the efficiency of the dispatch. Thus, one cannot expect the non-convexities on electricity markets to vanish, especially in a future with large proportions of renewable energy sources. Such demand flexibility and price-sensitive demand have ample consequences on the properties of prices that we can compute, as we will show.

4. Competitive Equilibrium

In this section, we introduce necessary notation, summarize existing theory on pricing in non-convex markets, and discuss design desiderata for electricity markets.

4.1. Notation and Economic Environment

In the auction market, there are K types of items (goods; hours and locations in a day-ahead market), denoted by $k \in \mathcal{K} = \{1, \ldots, K\}$, buyers $i \in \mathcal{I} = \{1, \ldots, I\}$ and sellers $j \in \mathcal{J} = \{1, \ldots, J\}$. In the multi-unit case, we have multiple *homogeneous* units (e.g., the minimum bid increment) for each of the *heterogeneous* K items $k \in \mathcal{K}$. A bundle of interest to buyer i (seller j) is described by a vector $x_i \in \mathcal{X}$ ($y_j \in \mathcal{Y}$) where $\mathcal{X}$ ($\mathcal{Y}$) is a compact subset of $\mathbb{Z}^K_{\geq 0}$. Each buyer i (seller j) has a monotonously increasing (decreasing) value function $v_i\colon \mathcal{X} \to \mathbb{R}_{\geq 0}$ ($v_j\colon \mathcal{Y} \to \mathbb{R}_{\geq 0}$) over bundles of items or objects x_i (y_j).

[4] https://www.misoenergy.org/stakeholder-engagement/issue-tracking/update-to-demand-response-deployment-tools/

An auctioneer wants to find an allocation of items to bidders. The auctioneer aims for *allocative efficiency*. This means the auctioneer wants to maximize *social welfare*, which is the gains from trade for all participants (the buyers and sellers). The goal of the auctioneer is to find an efficient allocation $(\mathbf{x}, \mathbf{y}) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ and linear and anonymous market clearing prices $\lambda = \{\lambda(k)\}_{k \in \mathcal{K}} \in \mathbb{R}^K_{\geq 0}$. The *linearity of prices* refers to the property that individual prices are set for each item $k \in \mathcal{K}$; the price for a bundle x_i is then simply the sum of the prices of its components, i.e., it is given by the dot product $\lambda' x_i$. *Anonymity* means that the resulting prices λ are the same for all bidders and there is no price differentiation. Competitive equilibrium prices might also be non-linear and personalized, but linearity and anonymity are crucial on electricity and other real-world markets as we discussed earlier. We assume, buyer i's (direct) utility from bundle x_i is given by $\pi_i(x_i, \lambda) = v_i(x_i) - \lambda' x_i$, and seller j's utility from bundle y_j is given by $\pi_j(y_j, \lambda) = \lambda' y_j - v_j(y_j)$. Such utility functions are linear in price and referred to as quasilinear utility functions. All market participants are assumed to be price-takers, meaning that they cannot influence the market prices on their own. Social welfare can now be defined as $\sum_{i \in \mathcal{I}} v_i(x_i) - \sum_{j \in \mathcal{J}} v_j(y_j)$, as prices cancel when the utilities of market participants are added.

With linear and anonymous prices $\lambda = (\lambda(1), \ldots, \lambda(k), \ldots, \lambda(K))$, the *indirect utility function* is defined as

$$u_i(\lambda) = \max_{x \in \mathcal{X}} \{v_i(x) - \lambda' x\} \quad \text{and} \quad u_j(\lambda) = \max_{y \in \mathcal{Y}} \{\lambda' y - v_j(y)\}.$$

The indirect utility function is widely used in economics and returns the maximal utility that bidder i can obtain at prices λ. The *demand correspondence* $D_i(\lambda)$ and $D_j(\lambda)$, resp. describe the set of bundles that maximize the indirect utility function at prices λ, i.e.,

$$D_i(\lambda) = \arg\max_{x \in \mathcal{X}} \{v_i(x) - \lambda' x\} \quad \text{and} \quad D_j(\lambda) = \arg\max_{y \in \mathcal{Y}} \{\lambda' y - v_j(y)\}.$$

4.2. Competitive Equilibrium

If in an outcome (consisting of an allocation and prices) all bidders are allocated a bundle from their demand correspondence, then the outcome is *envy-free* (EV). No bidder would want to get another bundle, as a bidder cannot increase her utility at these prices. If we have EV and the market is *budget-balanced* (BB), we have a *competitive equilibrium* (CE). If competitive equilibrium prices are linear and anonymous (LA), we also refer to this as a Walrasian equilibrium.

Definition 1 (Walrasian (competitive) equilibrium, (WE)). A price vector λ^* and a feasible allocation $(\mathbf{x}, \mathbf{y})$ form a *Walrasian equilibrium* if $\sum_{i \in \mathcal{I}} x_i = \sum_{j \in \mathcal{J}} y_j$, $x_i \in D_i(\lambda^*)$ for every buyer $i \in \mathcal{I}$, $y_j \in D_j(\lambda^*)$ for every seller $j \in \mathcal{J}$, and budget is balanced with $\sum_{i \in \mathcal{I}} \lambda^{*\prime} x_i = \sum_{j \in \mathcal{J}} \lambda^{*\prime} y_j$.

The BB condition implies that an unallocated item has a price of zero. Note that getting a bundle from the demand correspondence implies individual rationality (IR), because if bidders would make a loss with a bundle it would never be in their demand correspondence. However, EV is a much stronger condition than

IR. In summary, a Walrasian equilibrium (WE) has the properties $BB \wedge EV \wedge LA$. Later we will distinguish between linear and anonymous (LA) prices and linear and anonymous payments (LAP). For now, we assume that prices coincide with the payments.

The question is now under which conditions Walrasian equilibria exist and whether they support efficient (welfare-maximizing) outcomes (EF). To study these questions in a market with quasilinear utilities and independent private values, we use the following mathematical optimization problem describing a (combinatorial) exchange, which allows for arbitrary package bids. This bid language does not impose any restrictions on the types of valuations or cost functions and can be seen as the most general form of non-convex markets. As a matter of fact, the most prominent element of bid languages used in European day-ahead markets are block bids, i.e., package bids on adjacent time slots, and they can be easily captured in the following optimization problem. Electricity markets in the USA stipulate different bid languages to reduce the number of bids that participants need to submit, but they can be seen as a specific type of combinatorial exchange.

Let $\mathcal{X}_i \subseteq \mathbb{Z}^K_{\geq 0}$ denote all bundles for which buyer i submitted a bid, and $\mathcal{Y}_j \subseteq \mathbb{Z}^K_{\geq 0}$ denote all bundles for which seller j submitted an ask. For simplicity, we make the natural assumption that every bidder submits a bid with value 0 for the empty bundle. Let $z_i(x) \in \{0, 1\}$ be a binary decision variable denoting whether buyer i wins bundle $x \in \mathcal{X}_i$, and $z_j(y) \in \{0, 1\}$ be a binary decision variable denoting whether seller j wins bundle $y \in \mathcal{Y}_j$. The parameters $x(k)$ and $y(k)$ describe how many units a buyer wants or a seller provides of item k in a bundle. The allocation or *winner determination problem* (WDP) can then be written as an integer program as follows:

$$
\begin{aligned}
\max \quad & \sum_{i \in \mathcal{I}} \sum_{x \in \mathcal{X}_i} v_i(x) z_i(x) - \sum_{j \in \mathcal{J}} \sum_{y \in \mathcal{Y}_j} v_j(y) z_j(y) && && \text{(WDP)} \\
\text{s.t.} \quad & && && \\
& \sum_{x \in \mathcal{X}_i} z_i(x) \leq 1 && \forall i \in \mathcal{I} && (\pi_i) \\
& \sum_{y \in \mathcal{Y}_j} z_j(y) \leq 1 && \forall j \in \mathcal{J} && (\pi_j) \\
& \sum_{i \in \mathcal{I}} \sum_{x \in \mathcal{X}_i} x(k)\, z_i(x) \leq \sum_{j \in \mathcal{J}} \sum_{y \in \mathcal{Y}_j} y(k) z_j(y) && \forall k \in \mathcal{K} && (\lambda(k)) \\
& z_i(x) \in \{0, 1\} && \forall i \in \mathcal{I}, \forall x \in \mathcal{X}_i && \\
& z_j(y) \in \{0, 1\} && \forall j \in \mathcal{J}, \forall y \in \mathcal{Y}_j &&
\end{aligned}
$$

The WDP determines an allocation of bundles maximizing gains from trade, i.e., an efficient outcome. It assumes that participants specify a package bid for each possible package of interest, but they can win at most one. This is also referred to as an XOR bid language. While such a bid language is fully expressive, it requires exponentially many bids and is impractical for most applications. This is why electricity markets specify compact bid languages assuming some knowledge of the cost functions of generators. Bikhchandani and Mamer (1997) describe a multi-item, single-unit market. Their central theorem shows that there exist clearing prices for the indivisible single-unit problem if and only if the LP relaxation of WDP has an integer solution. In this case, the dual variables $\lambda(k)$ constitute Walrasian equilibrium (WE) prices, and the dual variables π_i and π_j determine the surplus of buyer i and seller j, respectively. The result can be proven via the strong duality theorem and the complementary slackness conditions in linear programming. As was already noted by Bikhchandani and Mamer (1997), the result for multi-item, *multi*-unit markets also directly follows from their result, by considering each of the multiple units as separate items. As a result, the welfare theorems hold in the quasilinear model:

Theorem 1 **(First and second welfare theorem)**. *Let* $(\mathbf{x}, \mathbf{y})$ *be an equilibrium allocation induced by a Walrasian equilibrium price vector* λ*, then* $(\mathbf{x}, \mathbf{y})$ *yields the optimal social welfare. Conversely, if* $(\mathbf{x}, \mathbf{y})$ *is a Pareto efficient allocation, then it can be supported by a Walrasian price vector* λ *so that* $(\lambda, \mathbf{x}, \mathbf{y})$ *forms a Walrasian equilibrium.*

Unfortunately, the LP relaxation of WDP does not yield integer solutions in general, and thus we cannot expect WE to exist in general. In fact, it is well known that WE only exist for restricted types of valuations for which the LP relaxation actually yields a feasible integer solution. For example, if all bidders' valuations are strong substitutes, this is a sufficient condition for WE to exist (Bikhchandani and Mamer 1997, Leme 2017, Baldwin and Klemperer 2019, Bichler et al. 2020). In practice, these conditions are rarely satisfied. In particular, non-convex cost functions on electricity markets lead to non-convex allocation problems that do not satisfy conditions for WE.

Competitive equilibrium prices do not need to be linear and anonymous. Bikhchandani and Ostroy (2002) show that, for combinatorial auctions with arbitrary valuations, competitive equilibrium prices need to be personalized and non-linear. Such prices convey little information other than that a particular package was winning or losing. In fact, for combinatorial exchanges with multiple buyers and sellers, there can even be situations where no competitive equilibrium exists (Bichler and Waldherr 2017). As discussed earlier, linear and anonymous prices on day-ahead electricity markets are an important baseline for forward markets and they serve as investment signals. Therefore, we need to relax some of the design desiderata of Walrasian equilibria.

4.3. Penalty-Based Stability

We discussed that prices should be linear and anonymous (LA), they should support the efficient allocation (EF), and neither the participants (IR) nor the auctioneer (BB) should make a loss. If envy-freeness (EV) was additionally satisfied, prices would support a Walrasian equilibrium. The welfare theorems (Theorem 1) suggest that all of these axioms are satisfied in convex markets. With general preferences in non-convex markets, however, this is impossible to achieve (Bikhchandani and Ostroy 2002, Bichler and Waldherr 2017). Market operators might not want to relax EF and IR as welfare should be maximized and no participants should incur losses from submitting bids. Current pricing schemes such as IP pricing and ELMP sacrifice BB and EV, but the side-payments that arise from the violation of BB have led to controversy as outlined in the introduction.

EV describes stability at prices from which no participant would want to deviate. In highly regulated and transparent markets such as electricity markets, stability can also be enforced without prices. As a matter of fact, U.S. ISOs such as ERCOT, MISO, NYISO, or CAISO enforce stability of the outcome via penalties in case a generator deviates from the efficient dispatch (O'Neill et al. 2020). As compared to WE, they relax the EV condition and only ask for IR. In what follows, we will show that with price-inelastic demand and a strict demand-supply equivalence, we can always find prices that satisfy $IR \wedge BB \wedge LA \wedge EF$. While we focus on an electricity market example, the insights are relevant to all types of non-convex markets. Let us introduce a simplified example of a single hour traded on an electricity market to illustrate which properties we can hope to achieve with linear and anonymous prices. From now on, we require strict demand-supply equivalence.

Example 1. Suppose we have three generators G1, G2, and G3 (the sellers on electricity markets). G1 produces 10 MWh and asks for \$500 (\$50/MWh). G2 produces 20 MWh and asks for \$300 (\$15/MWh). Finally, G3 produces 30 MWh and asks for \$700 (\$23.3/MWh). A buyer needs exactly 30 MWh in this hour and can either purchase from G1 and G2 or from G3, where buying from G3 is the efficient dispatch. Bids are indivisible. There are several options for the ISO:

1. The ISO could select the efficient dispatch, but set the price just below \$15/MWh, the ask of G2. The efficient dispatch with G3 is selected, but G3 makes a loss. In order to achieve IR, the market maker can pay G3 $\$700 - 30\text{MWh} * \$15/\text{MWh} = \$250$ as a *make whole payment*. These side-payments are commonly used in U.S. electricity markets, but they violate BB.[5]
2. The ISO could select the efficient dispatch and set the price at the ask of G3, i.e., \$23.3/MWh. At this price it would be attractive for G2 to produce, and her ask is "paradoxically rejected." It is common on U.S. electricity markets to define a penalty for G2 in case she does. This penalty would be at the difference of her ask and the market price. In our example, this penalty for G2 would be $20\text{MWh} * \$23.3/\text{MWh} - \$300 = \$166.67$. The market satisfies EF, IR, and BB, but not EV, as G2 does not maximize her payoff at the prices. As such, it is efficient but not a WE.

[5] Much of the literature on pricing in electricity markets and their current implementations suggests that the results are a CE. As introduced earlier, a CE requires envy-freeness and budget balance, but budget balance is not satisfied here.

3. The ISO could pick the inefficient dispatch with generators G1 and G2 and set the price at \$50/MWh. No side-payments by an ISO are needed, but there is a welfare loss of \$100. This alternative is implemented on European day-ahead markets. G3 is paradoxically rejected.

If we use penalties to enforce stability of the outcome, we can define new design desiderata for pricing on non-convex markets. Recall that π_k is the direct utility for market participant k.

Definition 2 (Penalty-based stable, budget-balanced, and efficient outcome (PBE)). A linear and anonymous price vector λ^* and an efficient allocation $(\mathbf{x}, \mathbf{y})$ form a penalty-based stable and efficient outcome if $\pi_i(x_i, \lambda^*) \geq 0$, $\pi_j(y_j, \lambda^*) \geq 0$ for every buyer $i \in \mathcal{I}$ and every seller $j \in \mathcal{J}$, if the market is budget balanced with $\sum_{i \in \mathcal{I}} \lambda^{*\prime} x_i = \sum_{j \in \mathcal{J}} \lambda^{*\prime} y_j$.

Note that budget balance and linear and anonymous prices in non-convex markets imply a strict demand-supply equivalence. If buyers and sellers have the very same anonymous and linear price vector (λ) and buyers buy less than what the sellers sell, then make-whole payments are required and BB is violated. To see this, assume that a seller sells a package of 2 MWh and a buyer is interested in only 1 MWh. We have a non-convexity arising from the indivisible package bid of the seller which does not allow us to price one of the two MWh in the seller's package at zero. Even if the buyer has a higher value for 1 MWh than what the seller asks for the package, we cannot achieve budget balance with a single price λ. Thus, the auctioneer needs to compensate the seller for the second MWh. But even if we have strict demand-supply equivalence, a PBE might not be possible as the following example shows.

Example 2. Suppose there are generators G1 and G2 both asking for \$30 for 3 MWh. Buyer B1 wants to buy 4 MWh for \$20 in total, and buyer B2 is price-inelastic with a demand of 2 MWh. With an ask price of \$10/MWh, the two generators ask for \$60 in total. However, as the market price cannot be higher than \$5/MWh, which is what B1 is willing to pay, the buyers will pay only \$30 for the 6 MWh in total. The ISO would need to pay a total of \$30 of make-whole payments to the two generators to facilitate the efficient trade at a price of \$5/MWh. The ISO could also set a different market price, but at any price it is inevitable to compensate the losses of some of the market participants.

The efficient trade would be possible only if the bids of the demand side are all higher than the average ask price or all buyers are price-inelastic. As indicated, the latter is the standard assumption in the literature on electricity market design.

Definition 3. Buyer $i \in \mathcal{I}$ is *price-inelastic* if for any bundle $x \in \mathcal{X}$, $v_i(x) - \lambda' x \geq 0$ for all $\lambda \in \mathbb{R}^K_{\geq 0}$. Such a condition implies that, for any price vector λ, $\pi_i(x_i, \lambda) \geq 0$.

Proposition 1. *A combinatorial exchange can implement a PBE, if the demand is price-inelastic and demand equals supply.*

Proof: We assume that we can solve the WDP to optimality, providing an efficient allocation (EF), $(\mathbf{x}, \mathbf{y}) = ((x_i)_{i\in\mathcal{I}}, (y_j)_{j\in\mathcal{J}})$ such that $\sum_{i\in\mathcal{I}} x_i = \sum_{j\in\mathcal{J}} y_j$. Furthermore, if we assume that all the buyers are price-inelastic, we can choose a linear and anonymous price vector $\lambda^* = (\lambda^*(1), \ldots, \lambda^*(K))$ large enough such that $\pi_i(x_i, \lambda^*) \geq 0$ for all $j \in \mathcal{J}$. For example, one can set $\lambda^*(k)$ as the highest average cost for item k, such that IR is satisfied for all generators. Finally the condition $\sum_{i\in\mathcal{I}} \lambda^{*\prime} x_i = \sum_{j\in\mathcal{J}} \lambda^{*\prime} y_j$ gives us budget balance (BB). As a result, this combinatorial exchange can implement a PBE. Q.E.D.

With price-inelastic demand and strict demand-supply equivalence, we can increase the linear and anonymous price until we obtain IR for the generators. The same would hold true if some buyers are price-sensitive but all their bids are higher than the average cost of the sellers. Since these conditions are rarely met on electricity markets, it is common to deviate from budget balance (BB) by providing make-whole payments that ensure individual rationality of the generators. We want these make-whole payments to be minimal, because such personalized payments are not reflected in the public market prices. Let us now define a penalty-based stable and efficient outcome (PE):

Definition 4 (Penalty-based stable and efficient outcome (PE)). A linear and anonymous market price vector λ^*, personalized make-whole payments δ_i, δ_j, and an efficient allocation $(\mathbf{x}, \mathbf{y})$ form a penalty-based stable and efficient outcome if $\pi_i(x_i, \lambda^*) + \delta_i \geq 0$, $\pi_j(y_j, \lambda^*) + \delta_j \geq 0$ for every buyer $i \in \mathcal{I}$ and every seller $j \in \mathcal{J}$. For PE prices, we demand the total of the personalized make-whole payments to be minimal.

Here the make-whole payments compensate (aggregate) losses that result from the allocated bundle. As we will see, there can be different notions of make-whole payments on electricity markets, such as compensating item-level losses. Next, we will introduce optimization problems to compute PBE whenever it exists or PE otherwise.

5. Pricing Rules

For price computation, we want prices to be linear and anonymous and we enforce efficiency, while other design goals can be relaxed. We treat BB as first-order design goal and price-based stability as second-order goal. This means that we first aim for linear and anonymous prices, eliminating or minimizing the make-whole payments that the ISO needs to pay. Such prices better reflect the value of electricity, as compared to pricing schemes where the price signal is significantly distorted due to large private and personalized make-whole payments.

PBE (and also PE) prices are not unique. Therefore, we select those prices that minimize incentives to deviate. Unfortunately, these are computationally intractable problems if we want to compute them exactly, as we will show. In lieu thereof, we choose the price vector that is closest to the dual variables of the LP relaxation of the allocation problem. If the allocation problem were a convex optimization problem, such dual prices would constitute a competitive equilibrium, i.e., a stable solution that satisfies EV.

Before we get to price computation, let us introduce an abstract version of the central allocation problem on electricity markets. Then, we introduce optimization models to compute prices on markets with price-inelastic and with price-sensitive demand to compute PBE or PE prices, respectively. Finally, we compare these pricing rules with other approaches in the literature.

5.1. Allocation Problem

In the last section, we have discussed combinatorial exchanges with package bidding as they do not restrict the types of preferences that a participant might have. Combinatorial exchanges with package bids are impractical for electricity markets because they would require bidders to submit an exponential set of bids. Rather, electricity markets use compact bid languages (Goetzendorff et al. 2015) that only require generators to specify a small number of parameters describing their underlying cost functions and technical constraints as well as buyers to specify their bid curves.

Unit commitment (UC) problems represent our starting point. Operational constraints on thermal generation units such as ramping limits and minimum up/down times require those units to be committed in advance of when they are needed, typically via day-ahead unit commitment. Unit commitment models determine the optimal scheduling of a given set of power suppliers in order to meet electricity demand. Such models minimize total system costs subject to market clearing conditions (supply meets demand) and technical power plant constraints (Stott et al. 2009). Unit commitment models are generation scheduling models, determining the output of each generator. We use the term security constrained unit commitment model (SCUC) if it additionally includes network characteristics and constraints (van den Bergh et al. 2014).

The SCUC problem can be formulated as a mixed-integer non-linear problem. The non-linearity comes from the fact that transmission lines are typically high-voltage alternating current (AC). An AC optimal power-flow model (ACOPF) provides a non-linear system which describes the energy flow through each transmission line accurately, and is theoretically the best approach to solve the SCUC (Carpentier 1985). The ACOPF is non-linear, non-convex and an NP-hard mixed-integer optimization problem (Zohrizadeh et al. 2020). Although there are various approaches to global optimization, an exact solution to the ACOPF can be considered intractable for realistic networks (Watson et al. 2015). This has led to significant research into convex relaxations of the problem (Zohrizadeh et al. 2020). The linear relaxation is also referred to as the direct current (DC) optimal power-flow model (DCOPF), and versions of this are widely used among U.S. ISOs to compute the efficient dispatch and prices (Eldridge et al. 2017).

In our paper, we focus on pricing and thus assume a generic DCOPF model that is given by a mixed-integer linear program (MIP). Appendix A provides an overview of the notation. In the abstract formulation, buyers and generators / sellers are again denoted by the sets $\mathcal{I}$ and $\mathcal{J}$, respectively. The set of traded goods $\mathcal{K}$ can now be described as the Cartesian product $\mathcal{N} \times \mathcal{T}$, where $\mathcal{N}$ represents a set of network nodes and $\mathcal{T}$ a set of time periods. Nodes are connected through a set of transmission lines $\mathcal{L}$. The objective of DCOPF aims at maximizing welfare, taking into account buyers' valuations (v) and generators' variable and fixed costs (c and h, respectively). Both buyers (2) and generators (1) can specify constraint matrices A, G and Q, R, respectively, in order to communicate their preferences and feasible bundles. DC power flows (f) are determined in (3) (with P as inverse matrix of the power transfer distribution factors and W and Z as mappings of buyers and sellers to their respective nodes), with a requirement of aggregate balance (4) and a consideration of line flow limits (5). The decision variables include buying (x) and selling (y) quantities, the associated binary variables (d and u), as well as power flows (f). As indicated by the integer multipliers r and s, the binary variables d and u can account for several categories such as start-up and commitment variables for generators.

$$\max_{x,y,u,d,f} \quad v'x - c'y - h'u \qquad \text{(DCOPF)}$$
$$\text{s.t.}$$
$$Ay + Gu \geq b \qquad (1)$$
$$Qx + Rd \leq e \qquad (2)$$
$$Pf = Wy - Zx \qquad (3)$$
$$\overline{W}y - \overline{Z}x = 0 \qquad (4)$$
$$\underline{F} \leq f \leq \overline{F} \qquad (5)$$
$$x \geq 0 \qquad (6)$$
$$y \geq 0 \qquad (7)$$
$$u \in \{0,1\}^{sJT} \qquad (8)$$
$$d \in \{0,1\}^{rIT} \qquad (9)$$
$$f \in \mathbb{R}^{LT} \qquad (10)$$

For convenience, we define vector x_i to include only the buying quantities of buyer $i \in \mathcal{I}$ as non-zero components, i.e., $\sum_{i\in\mathcal{I}} x_i = x$. Similarly, we define the vectors d_i, y_j, and u_j for buyers $i \in \mathcal{I}$ and generators $j \in \mathcal{J}$, respectively. The utility of buyer i is then defined as $\pi_i(x_i, \lambda) = v'x_i - \lambda' Z x_i$ with λ being the market price vector. The utility of generator j is $\pi_j(y_j, \lambda) = \lambda' W y_j - c' y_j - h' u_j$. Similarly, x_t and y_t are the vectors containing the buying and selling quantities of all buyers and generators resp. in period $t \in \mathcal{T}$ as non-zero components. The vectors x_{it} and y_{jt} consequently only include one non-zero component, namely the particular quantity of buyer i and generator j resp. in period t.

The DCOPF model does not allow for non-linear costs or non-linear AC power flows. We also abstract from transmission network elements such as transformers, shunts, or auxiliary services. However, the DCOPF formulation provides the overall structure of a MIP used for unit commitment problems, that allows us to perform a meaningful analysis of different pricing rules in our experiments in Section 6.

5.2. PBE Pricing with Price-Inelastic Demand

We first focus on the case of price-inelastic demand. This complies with the traditional notion of electricity as a basic and indispensable necessity. If demand x has no attached valuations, $v'x$ can be removed from the objective function of DCOPF, and the generators' cost will be minimized. Demand flexibility can be taken into account (by constraint 2), as long as buyers are price-inelastic. Let x^*, u^*, y^*, d^*, and f^* denote the optimal solution to this modified problem, which is efficient with demand-supply equivalence. As the buyers are price-inelastic, there will always be a price profile $\lambda^{PBE} \in \mathbb{R}^{NT}_{\geq 0}$ over locations and time periods such that no generator incurs losses (see Proposition 1).

The following bilevel integer program PBE-P computes prices such that at the efficient dispatch no generator makes a loss at any time (first constraint) and that there are no negative congestion revenues (second constraint). The latter prevents that nodal prices are set low at demand-intensive nodes and high at supply-intensive nodes, implying missing money only due to nodal price discrepancies. In the first constraint, individual rationality is based on *hourly* losses incurred by the respective market participant. Even if a loss in a certain hour is offset by a higher gain in the subsequent hour, the loss is compensated by a make-whole payment. The third constraint makes sure that incentives for generators to deviate from the efficient solution are minimal. π_j describes the payoff that a generator $j \in \mathcal{J}$ would have at the prices λ, if she could choose her dispatch such that it maximizes her payoff. The latter is computed in the lower level optimization (fourth constraint). This model would lead to prices that are IR and BB and it would minimize the gains by deviating from the efficient solution by an individual. In a Walrasian equilibrium of a convex economy, also coalitions of market participants cannot deviate. We do not consider such blocking coalitions in this model.

$$
\begin{aligned}
\min_{\lambda,\pi,\gamma} \quad & \sum_{j\in\mathcal{J}} \gamma_j && && \text{(PBE-P)} \\
\text{s.t.} \quad & && && \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 && && \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \geq 0 && && \\
& \pi_j - (\lambda' W y^*_j - c' y^*_j - h' u^*_j) \leq \gamma_j && && \forall j \in \mathcal{J} \\
& \pi_j = \max_{y,u} (\lambda' W y_j - c' y_j - h' u_j) \quad \text{s.t. } (1), (7), (8) && && \forall j \in \mathcal{J} \\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \pi \in \mathbb{R}^{J}_{\geq 0}, \gamma \in \mathbb{R}^{J} && &&
\end{aligned}
$$

Solving bilevel mixed integer programming problems is Σ_2^p-hard (Jeroslow 1985), a complexity class that is clearly intractable. While this is no proof that the specific problem PBE-P is in this complexity class, realistic problem sizes of PBE-P are very large and the problems need to be solved in due time. For example, the time to compute allocation and pricing in European day-ahead markets is only 17 minutes (NEMO Committee 2021).[6]

[6] Prior to 08/07/2021, the allowed computation time was only 12 minutes.

Given the associated practical complexity of solving PBE-P in the required time frames, we suggest an alternative based on Extended Locational Marginal Pricing (ELMP), which will be described in Section 5.4.2 in greater detail. In essence, ELMP is a tractable heuristic where binary variables of the DCOPF (constraints 8 and 9) are relaxed to continuous variables and prices are retrieved from the duals of the nodal demand-supply constraints. ELMP aims at minimizing lost opportunity costs, and if a market is convex it actually does. Therefore, instead of PBE-P we solve PBE-A. The latter omits the lower-level optimization and instead minimizes the difference between the price vector that satisfies the individual rationality constraints (λ) and the ELMP prices (λ^{ELMP}). Note that ELMP are computed by a linear program, and therefore λ^{ELMP} can be computed effectively with state-of-the-art linear programming solvers (see numerical results in Section 6.2).

$$
\begin{aligned}
\min_{\lambda} \quad & \|\lambda - \lambda^{ELMP}\|_1 && \text{(PBE-A)} \\
\text{s.t.} \quad & \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} \geq 0 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \geq 0 \\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}
\end{aligned}
$$

Here we use $\|\lambda - \lambda^{ELMP}\|_1$ in an attempt to minimize incentives to deviate from the efficient dispatch. With an L_1 norm in the objective, PBE-A can also be modeled as a linear program which can be solved in polynomial time. One can also minimize the squared Euclidean norm, which makes this a quadratic problem which might lead to less variation in the components of the price vector. Wolfe's combinatorial algorithm is widely used to solve such problems. Even though this algorithm does not run in polynomial time in the worst case (De Loera et al. 2020), it is very effective in practice and can serve as an alternative.

5.3. PE Pricing with Price-Sensitive Demand

We now assume price-sensitive demand, i.e., some or all of the buyers submit valuations v, as represented by the DCOPF. As we have shown, a PBE does not always exist for DCOPF. We can sacrifice budget balance (BB) but still ensure EF and IR. As a result, market prices are still linear and anonymous (LA), but individual payments are not (no LAP). Let $x^*, y^*, u^*, d^* f^*$ be the optimal solution to DCOPF. We define the following problem to compute the minimal make-whole payments associated to a price vector λ.

$$
\begin{aligned}
\min_{\lambda, \delta^{\mathcal{I}}, \delta^{\mathcal{J}}} \quad & \|\delta^{\mathcal{I}}\|_1 + \|\delta^{\mathcal{J}}\|_1 && \text{(PE-}\alpha\text{)} \\
\text{s.t.} \quad & \\
& v' x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \geq 0 \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \\
& \lambda' W y^*_{jt} - c' y^*_{jt} - h' u^*_{jt} + \delta^{\mathcal{J}}_{jt} \geq 0 \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \\
& \lambda' Z x^* - \lambda' W y^* \geq 0
\end{aligned}
$$

$$\lambda \in \mathbb{R}^{NT}_{\geq 0}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\geq 0}$$

The variables $\delta^{\mathcal{I}}$ and $\delta^{\mathcal{J}}$ represent the required make-whole payments to buyers $\mathcal{I}$ and generators $\mathcal{J}$. Note that we again consider *hourly* losses for the calculation of make-whole payments which slightly extends the requirements for make-whole payments compared to Definition 4 (which only asks for no aggregate losses over all hours). The optimal make-whole payments from PE-α are given as $\delta^{\mathcal{I}*}$ and $\delta^{\mathcal{J}*}$. Again, the resulting price vectors are not unique, and we could formulate a bilevel integer program aiming to satisfy individual rationality and to minimize incentives to deviate to another dispatch at these prices.

$$\begin{aligned}
\min_{\lambda,\pi,\gamma,\delta^{\mathcal{I}},\delta^{\mathcal{J}}} \quad & \sum_{i\in\mathcal{I}} \gamma_i + \sum_{j\in\mathcal{J}} \gamma_j && \text{(PE-P)}\\
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \geq 0 && \forall i \in \mathcal{I}, t \in \mathcal{T}\\
& \lambda' W y^*_{jt} - c'y^*_{jt} - h'u^*_{jt} + \delta^{\mathcal{J}}_{jt} \geq 0 && \forall j \in \mathcal{J}, t \in \mathcal{T}\\
& \lambda' Z x^* - \lambda' W y^* \geq 0 &&\\
& \delta^{\mathcal{I}} = \delta^{\mathcal{I}*} &&\\
& \delta^{\mathcal{J}} = \delta^{\mathcal{J}*} &&\\
& \pi_i - (v'x^*_i - \lambda' Z x^*_i) \leq \gamma_i && \forall i \in \mathcal{I}\\
& \pi_i = \max_{x,d}(v'x_i - \lambda' Z x_i) \quad \text{s.t. (2), (6), (9)} && \forall i \in \mathcal{I}\\
& \pi_j - (\lambda' W y^*_j - c'y^*_j - h'u^*_j) \leq \gamma_j && \forall j \in \mathcal{J}\\
& \pi_j = \max_{y,u}(\lambda' W y_j - c'y_j - h'u_j) \quad \text{s.t. (1), (7), (8)} && \forall j \in \mathcal{J}\\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \pi \in \mathbb{R}^{I+J}_{\geq 0}, \gamma \in \mathbb{R}^{I+J}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\geq 0} &&
\end{aligned}$$

Similar to our discussion on the case with price-inelastic demand, we replace the bilevel integer program by a tractable linear program that minimizes the distance to ELMP prices, subject to having the minimal make-whole payments given by PE-α.

$$\begin{aligned}
\min_{\lambda,\delta^{\mathcal{I}},\delta^{\mathcal{J}}} \quad & \|\lambda - \lambda^{ELMP}\|_1 && \text{(PE-A)}\\
\text{s.t.} \quad & \\
& v'x^*_{it} - \lambda' Z x^*_{it} + \delta^{\mathcal{I}}_{it} \geq 0 && \forall i \in \mathcal{I}, t \in \mathcal{T}\\
& \lambda' W y^*_{jt} - c'y^*_{jt} - h'u^*_{jt} \geq 0 && \forall j \in \mathcal{J}, t \in \mathcal{T}\\
& \lambda' Z x^* - \lambda' W y^* \geq 0 &&\\
& \delta^{\mathcal{I}} = \delta^{\mathcal{I}*} &&\\
& \delta^{\mathcal{J}} = \delta^{\mathcal{J}*} &&\\
& \lambda \in \mathbb{R}^{NT}_{\geq 0}, \delta^{\mathcal{I}} \in \mathbb{R}^{IT}_{\geq 0}, \delta^{\mathcal{J}} \in \mathbb{R}^{JT}_{\geq 0} &&
\end{aligned}$$

This basic formulation of PE-A (as well as PBE-A) might lead to prices that differ among nodes even though there is no congestion in the transmission network. With a few additional constraints, one can make sure that two adjacent nodes have the same price if there is no congestion. For this purpose, one only has to add price equivalence constraints for edges adjacent to those nodes where there is no congestion. In our experimental evaluation, we observed that such constraints modify the price profile and increase the necessary make-whole payments, yet the make-whole payments required by PE-A are still significantly lower compared to alternative pricing rules.

5.4. Comparison to Existing Pricing Rules

There is a significant literature on pricing rules for electricity spot markets, and a detailed discussion of all proposals is beyond the scope of this paper. An excellent and up-to-date overview of pricing in electricity markets is provided by Liberopoulos and Andrianesis (2016). Note that the literature in their paper is entirely based on the assumption of price-inelastic demand.

In our discussion, we focus on Integer Programming (IP) pricing and Extended Locational Marginal Pricing (ELMP) since they are used by U.S. ISOs in practice. Furthermore, we consider Average Incremental Cost (AIC) pricing, a recent proposal that also addresses the problem of significant make-whole payments. As introduced earlier, relevant criteria are efficiency (EF), individual rationality (IR), budget balance (BB), linear and anonymous prices (LA), and linear and anonymous payments (LAP). Note that IP pricing, ELMP and AIC satisfy EF and IR, which are widely considered essential on electricity markets. With price inelastic demand, PBE-A provides a straightforward way to *guarantee BB and LAP*. In case of price-sensitive demand, PE-A is the only pricing rule that *minimizes make-whole payments* under linear and anonymous prices. Let us now provide a brief description of IP, ELMP, and AIC pricing.

5.4.1. IP Pricing IP pricing (O'Neill et al. 2005) was an early and widely adopted proposal for pricing on electricity markets. First, the efficient dispatch is computed via DCOPF. Then the integer variables are fixed to their optimal values, resulting in a linear program. The duals of the nodal balance constraints provide linear and anonymous market prices, while the duals associated to constraints with integer variables determine individual uplift payments. O'Neill et al. (2005) originally describe a problem with price-inelastic demand. IP pricing can, however, also be adapted to settings with price-sensitive demand (Madani et al. 2018). It was also extended to multi-period, multi-nodal markets in many U.S. ISOs, including CAISO, PJM, or SPP. In practice, the uplift payments are restricted to be non-negative make-whole payments. Thus, market participants can retain their profits, and only individual losses are compensated by make-whole payments to ensure individual rationality. Budget balance is violated due to the make-whole payments, and the prices do not constitute a competitive equilibrium.

5.4.2. ELMP Pricing ELMP relaxes binary variables of the DCOPF to continuous variables and takes the duals of the relaxed problem as market prices. MISO introduced ELMP in 2011, but similar approaches were implemented by ISO-NE (O'Neill et al. 2019). Similar to IP pricing, there are individual make-whole payments, and the stability of the solution is enforced via penalties. Lost opportunity costs (LOCs) describe the forgone profit from the most profitable alternative level of electricity production at the prices. In total, the make-whole payments and the required penalties yield the LOCs of a generator. ELMP pricing represents an approximation of Convex Hull Pricing (CHP), as introduced by Gribik et al. (2007). CHP computes prices that indeed minimize LOCs, but it is computationally expensive and thus it has not been implemented in the field (Schiro et al. 2016). However, for simple problem formulations ELMP and CHP prices are equivalent (Hua and Baldick 2017). Evidence by MISO suggests that lost opportunity costs can be reduced by ELMP pricing compared to IP pricing, yet the general economic properties of ELMP remain unclear (Schiro et al. 2016).

5.4.3. AIC Pricing In a series of essays, O'Neill et al. (2019) challenge established pricing rules on electricity markets and criticize that the resulting make-whole payments lead to biased market prices. They suggest AIC pricing, which implements IP pricing as a first stage. In a second step, the AIC price computation relaxes the integer variables of generators that make a loss for the actual AIC pricing run and adjusts their objective function coefficients to reflect the average costs, i.e., it distributes the fixed costs of a generator over the quantity allocated to the generator. In a stylized market with only a single period this would eliminate the make-whole payments of the generators. In a market with multiple periods, O'Neill et al. (2020) suggests an iterative process comprising several pricing runs to achieve budget balance. The approach does not consider make-whole payments for the demand side, but proposes price differentiation among buyers via Ramsey-Boiteux-like pricing.

AIC pricing provides an innovative new approach to electricity market pricing. Similar to PBE-A or PE-A, the goal is to eliminate or minimize make-whole payments. But there are also differences. First, PE-A minimizes make-whole payments for both sides of the market in a single optimization. Second, unlike AIC pricing, PE-A does not involve price differentiation on the demand side, but sticks to linear and anonymous prices. Price-differentiation among buyers can be a very useful tool to deal with the non-convexities in a market. However, it is also challenging. First, personalized prices lead to some level of intransparency in the market compared to an anonymous linear price for all market participants. Again, not all information is contained in the public price signal. Second, there is a difference between differential and anonymous prices in terms of manipulability. Uniform multi-unit auctions and the Walrasian mechanism are known to be strategy-proof in the large (Azevedo and Budish 2019). This means, with many participants truth-telling is approximately optimal and the impact of a single participant on the price becomes negligible with many participants. This is no longer the case if the payments of a participant are personalized. A pay-as-bid pricing scheme is manipulable and bidders will not reveal their true preferences. The only exception is the Vickrey-Clarke-Groves payment rule which is the unique payment rule that is dominant-strategy incentive-compatible (Green and Laffont 1979).

In our experiments we show that the make-whole payments necessary to achieve linear and anonymous prices are negligible even with price-sensitive demand. We argue that, if make-whole payments are so low, there is no need to restrict to discriminatory prices for each buyer or many anonymous but non-linear prices (say for different volumes of electricity demanded), because the market price includes "almost" all information about supply and demand.

For our experiments in Section 6, we will ignore price differentiation in AIC, but instead compute make-whole payments to allow for a comparison to other pricing rules. Besides, we consider only a single AIC pricing run, and not multiple iterations.

5.4.4. Alternative Proposals Various other pricing rules have been suggested in the past two decades. Some, such as Direct Minimum Uplift (DMU) pricing, refrain from linear and anonymous prices and are thus beyond the focus of this paper. Others, such as the Equilibrium-Constrained (EC) pricing framework by Azizan et al. (2020) are restricted to price-inelastic demand. Moreover, many rules have been investigated only under very specific assumption (e.g., Generalized Uplift pricing, Semi-Lagrangean pricing).

Toczyłowski and Zoltowska (2009) introduce the DMU approach, which postulates a bid-ask-spread between the market prices for buyers and generators. DMU pricing aims at finding a spread that allows for minimal side-payments and that compensates lost opportunity costs. The side-payments are designed as uniform per-unit payments for buyers and sellers. However, the ISO has to give up a single linear price vector. DMU pricing has been proposed for multi-period power flow problems with price-sensitive demand.

More recently, Azizan et al. (2020) proposed the EC pricing scheme that is applicable to general non-convex settings with price-inelastic demand. Dispatch and payments are determined simultaneously to achieve EF and IR, as well as to ensure no incentives to deviate, rendering penalties unnecessary. Consequently, the price and payment functions must be general enough and hence allow for non-linear and personalized components. One upside is the broad applicability of their pricing framework to established price and payment functions. Moreover, the authors provide a polynomial-time approximation algorithm for general non-convex cost functions. The authors do not account for price-sensitive demand. Therefore, their settings are restricted to those where a PBE is feasible. In contrast to PBE-A, equilibrium-constrained pricing gives up budget balance and linear and anonymous payment functions to ensure stability without further penalties. Similar to O'Neill et al. (2019), we instead argue for maintaining budget balance with linear and anonymous payments and treat lost opportunity costs as secondary objective. In regulated electricity markets penalties are an accepted means to achieve stability.

Generalized Uplift pricing, introduced by Motto and Galiana (2002) and Galiana et al. (2003), has been proposed for a single-period problem with price-inelastic demand and seeks to find minimum zero-sum uplift payments that ensure stability. Minimum Zero-Sum Uplift Pricing by Liberopoulos and Andrianesis (2016) seeks the minimum prices that ensure a PBE. In contrast to PBE-A, Minimum Zero-Sum Uplift pricing allows for uplift charges for profitable generators. Starting at marginal cost, it increases prices and redistributes the additional gains of profitable generators to the loss-making generators. It terminates as soon as individual rationality is ensured for every generator. The Semi-Lagrangean pricing scheme by Araoz and Jörnsten (2011) also achieves a PBE, but their formulation is restricted to price-inelastic demand. The Primal-Dual pricing rule by Ruiz et al. (2012) aims at uniform IR prices with price-inelastic demand by relaxing efficiency. Finally, O'Neill et al. (2016) introduce the Dual Pricing algorithm which starts with the dual of the IP pricing problem and adds restrictions to ensure individual rationality and budget balance. By employing Ramsey-Boiteux pricing, it results in personalized prices for buyers. In Section 6 we will focus only on IP pricing, ELMP and AIC pricing for the reasons mentioned above.

6. Numerical Experiments

In what follows, we compare the different pricing rules experimentally. We start with small illustrative examples before we report aggregate results for the IEEE RTS System, which is frequently used as a benchmark.

6.1. Illustrative Examples

In our illustrative examples we go from simple to more complex environments. We start with a simple convex setting, consisting of two generators G1 and G2 and two buyers B1 and B2 at a single node and over three time periods (i.e., hours). We will gradually extend this example to reflect non-convexities, as well as price-sensitive and flexible demand-side bids. We will benchmark IP and ELMP pricing as established rules used by ISOs and further include AIC pricing as a promising rule that has not yet been employed in practice. These existing rules are compared to PBE-A and PE-A, respectively.

6.1.1. Base Case: Convex Supply, Price-Inelastic Demand G1 offers up to 15 MW for $5/MWh, and G2 offers up to 20 MW for $3/MWh. B1 and B2 schedule price-inelastic demand according to the following table:

[MWh]	B1	B2
t=1	4	3
t=2	6	6
t=3	10	12

Table 1 **Base Case: Price-inelastic Demand**

	G1	G2
Max Load [MW]	15	20
Offer Price [$/MWh]	5	3

Table 2 **Base Case: Convex Supply**

The optimal solution is obviously to let G2 satisfy the entire demand in the first two periods, while G1 satisfies only the residual demand of 2 MWh in excess of the maximum load of G2 in the third period. IP, ELMP, AIC, and PBE-A prices are identically set and constitute a WE, PBE, and PE.

[$/MWh]	IP	ELMP	AIC	PBE-A
t=1	3.00	3.00	3.00	3.00
t=2	3.00	3.00	3.00	3.00
t=3	5.00	5.00	5.00	5.00
MWP	0.00	0.00	0.00	0.00

Table 3 Base Case: Prices

[MWh]	G1	G2	B1	B2
t=1	0	7	4	3
t=2	0	12	6	6
t=3	2	20	10	12

Table 4 Base Case: Dispatch

6.1.2. Non-Convexities Next, we introduce non-convexities for the generators, i.e., G1 has a minimum load of 2 MW per period as well as no-load costs of $8 that occur as fixed costs when G1 is committed. G2 has a minimum load of 10 MW and no-load costs of $10. Therefore, G2 can no longer satisfy the demand in the first period and is replaced by G1. It is also assumed that G1 requires a minimum runtime of three periods. That is, if G1 is committed, it must sell at least its minimum load in every period. Consequently, the optimal dispatch now involves G1 satisfying the entire demand in $t = 1$ and running at a minimum load in the remaining periods, while G2 satisfies the residual demand.

[$/MWh]	IP	ELMP	AIC	PBE-A
t=1	5.00	3.50	6.14	6.14
t=2	3.00	3.50	3.00	9.00
t=3	5.00	6.10	9.00	9.00
MWP	38.00	40.29	22.00	0.00

Table 5 Non-convexities: Prices

[MWh]	G1	G2	B1	B2
t=1	7	0	4	3
t=2	2	10	6	6
t=3	2	20	10	12

Table 6 Non-convexities: Dispatch

Neither pricing rule yields a WE. IP, ELMP, and AIC result in individual losses, at least for some of the hours, and thus fail to produce a PBE. PBE-A avoids any make-whole payments and yields a PBE. Even if the aggregate profits were considered, AIC pricing cannot ensure individual rationality, at least after a single pricing run.

6.1.3. Price-Sensitive Demand We now introduce price-sensitive demand. We assume that half of the price-inelastic demand is retained as price-inelastic. For the remaining half, B1 bids $10/MWh and B2 bids $2/MWh in each period, respectively. The dispatch thus changes as it is not welfare-optimal to satisfy the entire demand. Due to the price-sensitive demand, we now use PE-A instead of PBE-A.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	6.45	6.45
t=2	2.00	3.50	4.00	4.17
t=3	3.00	3.50	3.71	8.58
MWP	64.00	50.75	26.57	17.00

Table 7 Price-Sensitive Demand: Prices

[MWh]	G1	G2	B1	B2
t=1	5.5	0	4	1.5
t=2	2	10	6	6
t=3	2	14	10	6

Table 8 Price-Sensitive Demand: Dispatch

Under the welfare-optimal dispatch, no PBE is possible. In order to satisfy the price-inelastic fraction of demand, both generators need to produce at least at their minimum loads, and make-whole payments thus become inevitable. PE-A achieves the lowest aggregate make-whole payments ($10.50 to G1, $6.50 to B2), which are as close to budget balance as possible.

6.1.4. Flexible Demand We now additionally convert some of the inflexible demand into flexible demand. B1 has converted 2 MWh from $t = 1$ and 1 MWh from $t = 2$ into a shiftable volume of 3 MWh that can be satisfied in an arbitrary pattern over the considered time frame.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	7.29	7.29
t=2	2.00	3.50	4.00	9.00
t=3	3.00	3.50	3.71	9.00
MWP	64.00	44.75	22.57	7.00

Table 9 Flexible Demand I: Prices

[MWh]	G1	G2	B1	B2
t=1	3.5	0	2	1.5
t=2	2	10	8	4
t=3	2	14	10	6

Table 10 Flexible Demand I: Dispatch

The shiftable volume of B1 is completely served in $t = 2$ and replaces some of the price-sensitive demand of B2. This allows for a significant reduction in make-whole payments for PE-A. Assume now that B1 adds an additional 1 MWh from $t = 2$ to the shiftable volume. Making use of this flexibility allows for PBE prices.

[$/MWh]	IP	ELMP	AIC	PE-A
t=1	5.00	3.50	7.29	7.29
t=2	5.00	3.50	6.14	6.14
t=3	3.00	3.50	3.56	9.00
MWP	38.00	43.75	10.89	0.00

Table 11 Flexible Demand II: Prices

[MWh]	G1	G2	B1	B2
t=1	3.5	0	2	1.5
t=2	7	0	4	3
t=3	2	18	14	6

Table 12 Flexible Demand II: Dispatch

This example also illustrates the advantages of demand-side bidding and bid languages that permit the expression of flexibility dimensions.

6.2. Experiments Based on the IEEE RTS System

Finally, we report results of numerical experiments based on the IEEE RTS System introduced by Grigg et al. (1999), in order to better understand prices in a larger and realistic test system. This system has been used in a variety of studies on electricity markets (Garcia-Bertrand et al. 2006, Morales et al. 2009, Zoltowska 2016, Hytowitz et al. 2020, Zocca and Zwart 2021) and includes non-convexities (no-load costs, minimum loads, minimum runtimes), price-sensitive demand, as well as several nodes and time periods. Therefore, it is well suited to study prices and make-whole payments under different pricing schemes.

Grigg et al. (1999) provide a stylized system topology, transmission network parameters, hourly (nodal) demand data as well as characteristics of generating units. In accordance with Zoltowska (2016), we select the single area, 24-node topology by Grigg et al. (1999) for a representative 24-hour winter day with 32 generators (total capacity: 6.81 GW) and 17 consumers (average hourly demand: 2.60 GWh). For data on (non-convex) generation costs or demand valuations we rely on the bid and offer curves provided by the cases studies of Garcia-Bertrand et al. (2006) and Zoltowska (2016) on this system. The experiments were conducted on an Intel(R) Core(TM) i7-8565U CPU with 16 GB RAM.

Our base setting includes 32 generators with minimum and maximum loads, minimum runtimes, as well as no-load costs and an offer curve representing variable costs. The demand of the 17 consumers is assumed to be price-inelastic at first and later extended to price-sensitive and flexible demand. Generators and consumers are embedded in a DC power flow model with 24 nodes. Appendix C provides heatmaps of the hourly nodal prices and Table 13 reports statistics on prices, make-whole payments (MWP) as well as the magnitude of penalties necessary to avoid generators to deviate from the efficient dispatch. Note that instead of penalties an ISO could also just prohibit deviations from the efficient dispatch. In all scenarios we will see that the make-whole payments for PBE-A (in case of price-inelastic demand) or PE-A (in case of price-sensitive demand) are zero or very low compared to other pricing rules. Also, the make-whole payments for AIC prices are reduced compared to IP pricing, but remain significant after a single pricing run and with hourly loss compensation. Make-whole payments per generator can be found in Appendix B.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	22.32	8.81	35,749.28	0.00	0.00	0.00	4.63%	1.39s
ELMP	22.62	6.40	6,193.37	0.00	2,460.20	0.00	0.80%	1.39s
AIC	29.62	16.92	26,114.72	0.00	46,095.51	0.00	3.38%	2.68s
PBE-A	23.35	7.53	0.00	0.00	17,966.49	0.00	0.00%	1.46s

Table 13 IEEE RTS Statistics with Price-Inelastic Demand

Under price-inelastic demand, a PBE (Definition 2) is achieved only by PBE-A. All other pricing rules require make-whole payments to ensure individual rationality. Classical IP pricing requires high make-whole payments to the generators, resulting in a violation of budget balance for the market operator. AIC prices are high on average, especially in the peak periods $t = 18$ and $t = 19$ (see Figure 5 in Appendix C), contributing to a large standard deviation of the prices at the same time. The price peaks allow for overall profitability for the generators, but as discussed before, individual periodic losses are still compensated, resulting in make-whole payments during low-price periods. In contrast, ELMP produces a smooth price profile with little volatility and low lost opportunity costs (as reflected by the sum of make-whole payments and penalties). PBE-A adjusts this price profile only slightly in order to ensure a PBE, mainly by increasing prices at the nodes 101 and 115, where most of the otherwise unprofitable generators are situated. As a consequence, the price average and standard deviation are slightly increased, but no make-whole payments are required outside the market price. Penalties are necessary but are still lower than the lost opportunity costs required under IP or AIC pricing.

Next, we consider price-sensitive demand, taking into account the bid curves as described by Garcia-Bertrand et al. (2006). In particular, each buyer submits some minimum price-inelastic demand and a piecewise-constant demand curve on top of that. Accounting for buyer valuations naturally decreases prices compared to the price-inelastic case, which is also evident from Table 14 and Figure 6 in Appendix C.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.63	5.03	14,272.57	0.33	0.00	0.42	2.52%	1.57s
ELMP	21.02	5.35	490.39	781.43	257.43	803.04	0.22%	1.55s
AIC	21.27	6.36	11,048.85	945.38	0.00	0.00	2.12%	2.96s
PE-A	20.85	5.23	0.00	112.43	929.52	804.66	0.02%	1.66s

Table 14 IEEE RTS Statistics with Price-Sensitive Demand

It is not possible to achieve a PBE in this environment. IP prices produce the lowest average price and standard deviation. Similar to the price-inelastic case, it diverges most from budget balance, with make-whole payments amounting to 2.5% of the total incurred generation costs. ELMP prices are higher on average, resulting in less make-whole payments for the generators. However, these prices do not minimize total make-whole payments. PE-A requires make-whole payments of only \$112.41. Only IP prices are on average lower than PE-A prices, and the total lost opportunity costs of PE-A (as reflected by the sum of make-whole payments and penalties) are minimal among the pricing rules under consideration. PE-A prices are minimal in make-whole payments, closest to stability, and imply low and smooth price profiles.

Finally, we introduce demand flexibility. The following Tables 15 and 16 reflect prices where 20% of the previously price-inelastic demand is converted to either shiftable profiles or shiftable volumes. Here, each shiftable demand is a randomly sampled 5-hour interval of inelastic demand that can either be shifted as a profile by 4 hours (shiftable profile) or the aggregate volume can be satisfied within the original 5 hours in an arbitrary fashion (shiftable volume).

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.63	5.03	14,275.82	9,116.75	0.00	0.00	4.14%	1.76s
ELMP	20.74	5.14	1,034.49	21,420.72	257.43	0.00	3.97%	1.71s
AIC	20.51	5.76	11,882.69	20,773.64	0.00	0.00	5.78%	3.27s
PE-A	20.42	4.92	188.54	322.96	929.52	538.11	0.09%	1.84s

Table 15 IEEE RTS Statistics with 20% Shiftable Profiles

In both cases, welfare gains can be realized by using the demand-side flexibility in a welfare-maximizing fashion. The increase in make-whole payments for the buyers is a result of the modeling decision to assign the highest valuation in the bid curve to the – formerly price-inelastic and now price-sensitive and flexible –

demand. As the flexible demand needs to be satisfied within the boundaries set by the flexibility parameters, this can create a loss on the part of the buyer if her highest valuation is still below the generation cost. This results in the significantly higher make-whole payments for buyers. Again PE-A has by far the lowest make-whole payments and little price volatility, etc.

	Price Mean	Price Std. Dev.	MWP Sell	MWP Buy	Penalty Sell	Penalty Buy	MWP / Total Cost	Computation Time
IP	19.98	5.29	12,725.92	10,491.12	0.00	0.00	4.18%	1.83s
ELMP	21.04	5.36	436.60	22,686.77	257.43	0.00	4.16%	1.74s
AIC	21.66	6.65	10,888.60	35,141.68	0.00	0.00	8.29%	3.34s
PE-A	20.74	5.17	564.35	198.47	741.80	687.45	0.14%	1.84s

Table 16 IEEE RTS Statistics with 20% Shiftable Volumes

The numerical tests indicate that PBE-A and PE-A can substantially reduce or even eliminate make-whole payments compared to conventional pricing schemes. As a result, there are no or only very low side-payments that are not reflected in the public market price anymore. Approaching budget balance comes at the expense of higher penalties to ensure a stable market outcome. As discussed in the previous sections, we argue that penalties are less of a concern, since they are already established and enforced in highly regulated electricity markets (O'Neill et al. 2020).

7. Conclusions

Electricity markets have seen significant change among U.S. ISOs recently. While all ISOs moved to mixed-integer programming in order to determine the efficient dispatch, there is still a significant discussion about out-of-market make-whole payments paid by the ISOs to some of the generators. These payments can be significant and they distort the market price signals as has been pointed out by the U.S. FERC and domain experts. We show that, with the standard assumption of price-inelastic demand and demand-supply equivalence, no make-whole payments are necessary.

With the advent of variable energy sources, demand response becomes increasingly important. To adequately reflect flexibility on the demand side, ISOs need new bid formats that likely lead to additional non-convexities and price-sensitive demand. We prove that in such markets zero make-whole payments are impossible in general. Based on this insight, we introduce the PE-A pricing rule that minimizes make-whole payments and compare it to existing payment rules used by ISOs and the AIC pricing rule. Rather than trying to mimic competitive equilibrium prices based on linear relaxations of the underlying non-convex allocation problem, we treat envy-freeness as second-order design goal and optimize these objectives directly. The results show that high side-payments on electricity markets as they are challenged by regulators can either be avoided or reduced substantially.

The experiments provide evidence that prices under PE-A do not increase on average compared to established pricing rules, and the changes in the overall payments of market participants are very small. Moreover, make-whole payments are avoided or they are negligible in all experiments that we ran. The new pricing rules are based on optimization problems that can be solved in polynomial time and whose principles are easy to understand and communicate. The new pricing rule is also general without dependencies on the specifics of the underlying allocation problem and can be applied to other non-convex markets as well.

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Appendix A: Notation for DCOPF

Sets

- $\mathcal{I} = \{1, ..., I\}$: Buyers (index i)
- $\mathcal{J} = \{1, ..., J\}$: Generators (index j)
- $\mathcal{T} = \{1, ..., T\}$: Time periods (index t)
- $\mathcal{N} = \{1, ..., N\}$: Nodes (index n)
- $\mathcal{L} = \{1, ..., L\}$: Lines (index l)

Parameters

- $v \in \mathbb{R}^{IT}$: Buyer valuations
- $c \in \mathbb{R}^{JT}$: Generator variable cost
- $h \in \mathbb{R}^{sJT}$: Generator fixed costs
- $A \in \mathbb{R}^{m\times JT}$: Generator constraint matrix I
- $G \in \mathbb{R}^{m\times sJT}$: Generator constraint matrix II
- $b \in \mathbb{R}^{m}$: Generator constraint right-hand side
- $Q \in \mathbb{R}^{k\times IT}$: Buyer constraint matrix I
- $R \in \mathbb{R}^{k\times rIT}$: Buyer constraint matrix II
- $e \in \mathbb{R}^{k}$: Buyer right-hand side
- $P \in \mathbb{R}^{NT\times LT}$: Inverse PTDF matrix (calculated from susceptance and network incidence matrix; also includes reference node)
- $W \in \mathbb{R}^{NT\times JT}$: Generator to node and period mapping matrix
- $Z \in \mathbb{R}^{NT\times IT}$: Buyer to node and period mapping matrix
- $\overline{W} \in \mathbb{R}^{T\times JT}$: Generator to period mapping matrix
- $\overline{Z} \in \mathbb{R}^{T\times IT}$: Buyer to period mapping matrix
- $\overline{F} \in \mathbb{R}^{LT}$: Upper flow limits
- $\underline{F} \in \mathbb{R}^{LT}$: Lower flow limits

Decision Variables

- $x \in \mathbb{R}^{IT}$: Buying quantities
- $d \in \{0, 1\}^{rIT}$: Buy-side binary variables (r as integer multiplier to account for several binaries [different dimensions of flexibility, etc.])
- $y \in \mathbb{R}^{JT}$: Selling quantities
- $u \in \{0, 1\}^{sJT}$: Generator commitment and other binaries (s as integer multiplier to account for several binaries [commitment, start-up, etc.])
- $f \in \mathbb{R}^{LT}$: Line flows

Appendix B: Make-Whole Payments

The following figures (see online version for better visualization) provide histograms of make-whole payments per generator (in $) in the different environments with price-inelastic, price-sensitive, and flexible demand (shiftable profiles and shiftable volumes) for the IEEE RTS system. With IP pricing, ELMP, and AIC some generators receive very high make-whole payments that are not reflected in the public prices. With PBE-A / PE-A such make-whole payments are negligible.

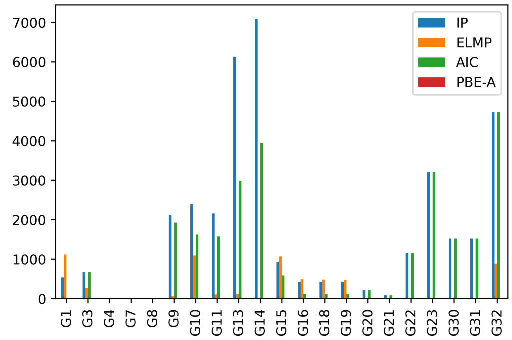

Figure 1 **Make-Whole Payments with Price-Inelastic Demand**

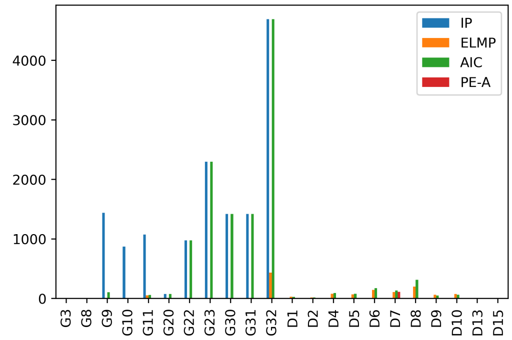

Figure 2 **Make-Whole Payments with Price-Sensitive Demand**

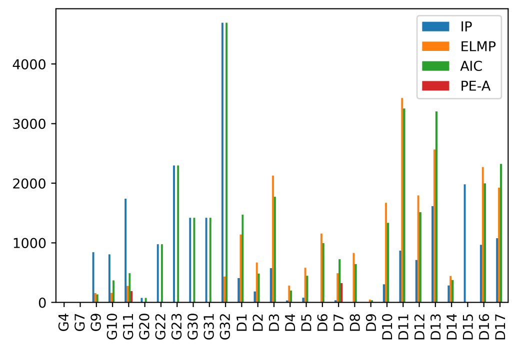

Figure 3 **Make-Whole Payments with 20% Shiftable Profiles**

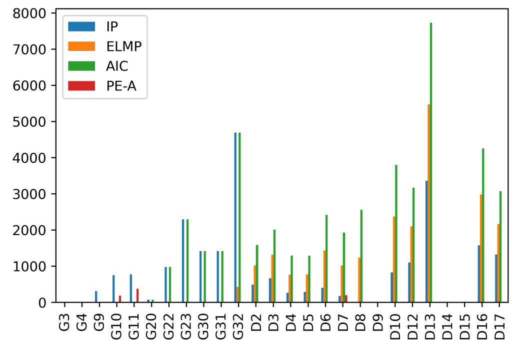

Figure 4 **Make-Whole Payments with 20% Shiftable Volumes**

Appendix C: Heatmaps of Prices

The following heatmaps describe hourly nodal prices (in $/MWh) for different nodes across the day for the IEEE RTS system. Darker colors describe higher prices. Each panel describes the outcome of one pricing rule (IP, ELMP, AIC, and PBE-A or PE-A) for the environments with price-inelastic demand, price-sensitive demand, 20% shiftable profiles or 20% shiftable volumes. In general, the prices with price-sensitive and flexible demand tend to be higher.

Interestingly, the prices of PBE-A / PE-A and ELMP tend to be similar in spite of significantly lower make-whole payments with PBE-A / PE-A.

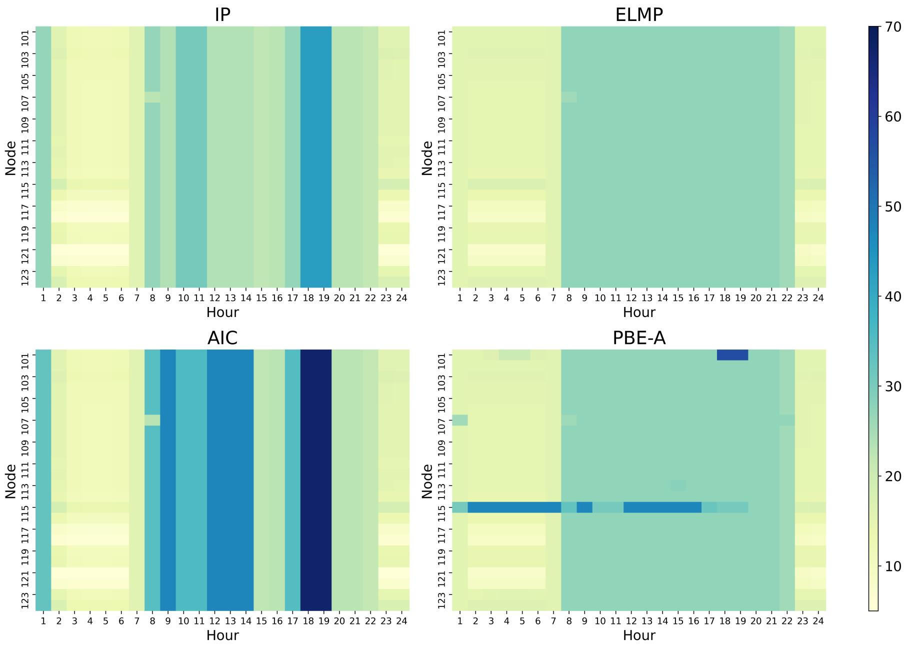

Figure 5 **IEEE RTS Prices with Price-Inelastic Demand**

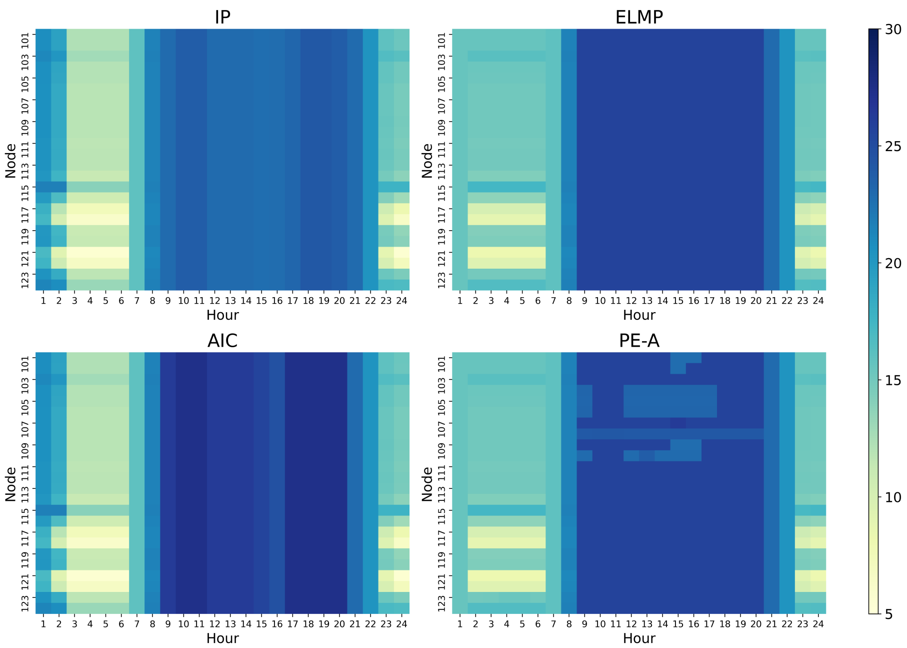

Figure 6 **IEEE RTS Prices with Price-Sensitive Demand**

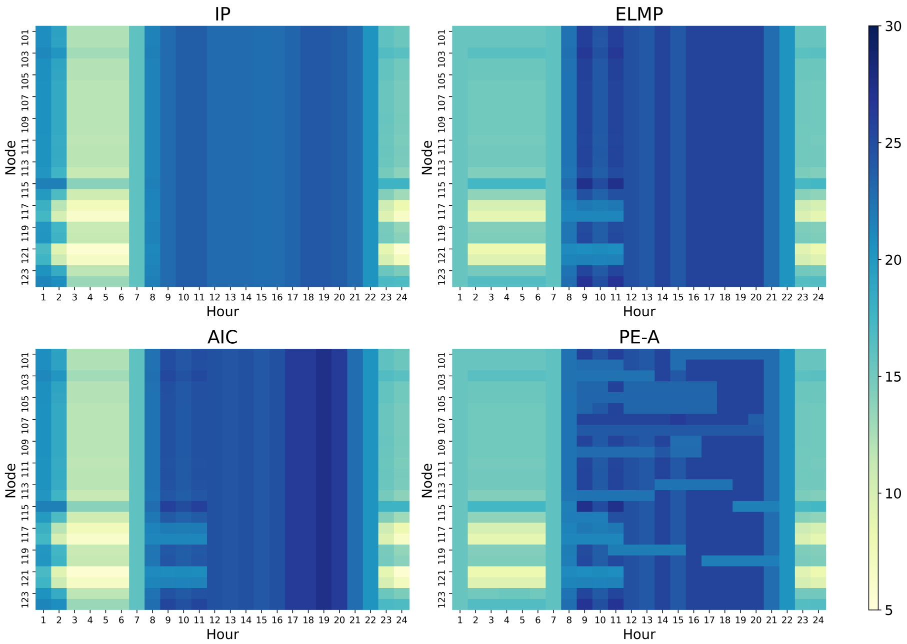

Figure 7 **IEEE RTS Prices with 20% Shiftable Profiles**

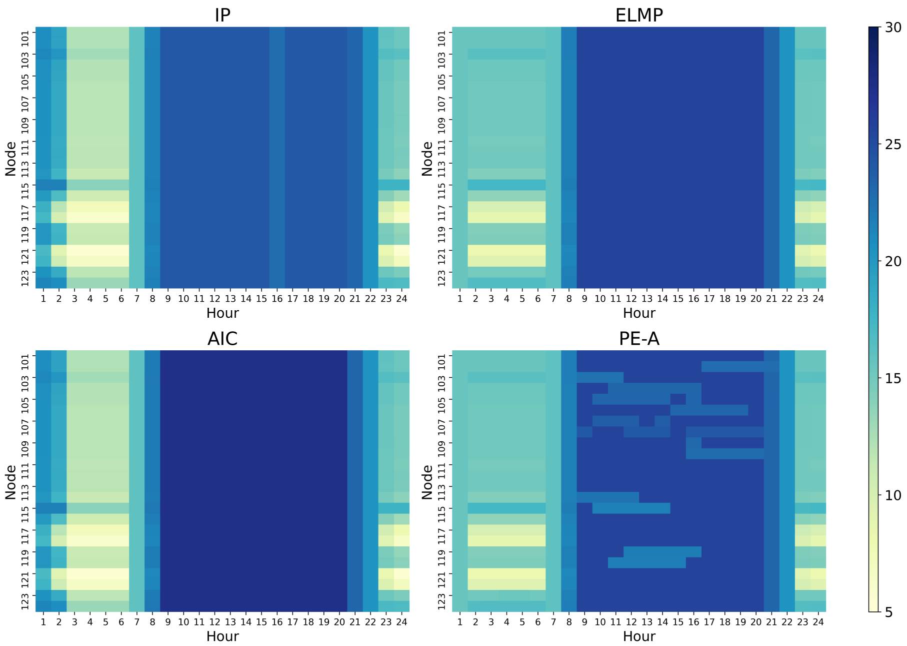

Figure 8 **IEEE RTS Prices with 20% Shiftable Volumes**

Additional experiments can be found in an online supplement.

4. Part II: Pricing as a Multi-Objective Optimization Problem

Peer-Reviewed Conference Paper

Title: Pricing Optimal Outcomes in Coupled and Non-Convex Electricity Markets.

Authors: Mete Şeref Ahunbay, Martin Bichler, Johannes Knörr.

In: Proceedings of the 24th ACM Conference on Economics and Computation (EC '23).

Abstract: According to the fundamental theorems of welfare economics, any competitive equilibrium is Pareto efficient. Unfortunately, competitive equilibrium prices only exist under strong assumptions such as perfectly divisible goods and convex preferences. In many real-world markets, participants have non-convex preferences and the allocation problem needs to consider complex constraints. Electricity markets are a prime example, but similar problems appear in many real-world markets, which has led to a growing literature in market design. To rectify the non-existence of Walrasian equilibria, in U.S. electricity markets, the welfare-optimal allocation is currently supported by linear prices based on the dual of a relaxed allocation problem. These relaxations consider convexified valuation functions for market participants, such as full convex envelopes (Convex Hull [CH] pricing) or restricted convexifications (e.g. Integer Programming [IP] pricing or Minimum Make-Whole Payment [Min-MWP] pricing). However, with increasing levels of renewable energy sources, these pricing rules have come under scrutiny. In particular, they lead to high out-of-market side-payments to some participants and/or to incorrect pricing across (un)congested transmission lines, resulting in revenue shortfalls for transmission operators. We show that existing pricing heuristics optimize specific design goals that can be conflicting. More precisely, standard pricing rules each optimize a specific class of lost opportunity costs that correspond to desirable economic properties, such as minimum violation of obedience constraints (CH pricing), pricing at marginal cost/valuation (IP pricing), or minimum violation of individual rationality (Min-MWP pricing). However, the resulting trade-offs can be substantial, and we establish that the design of pricing rules is fundamentally a multi-objective optimization problem addressing different incentives. We consider two approaches to balance these trade-offs. First, we consider traditional multi-objective optimization techniques using weighing of individual objectives. Second, we introduce a novel parameter-free pricing rule that minimizes incentives for market participants to deviate locally. We perform extensive numerical experiments on stylized and real-world data. Our theoretical and experimental findings show how the new pricing rule capitalizes on the upsides of existing pricing rules under scrutiny today. It leads to prices that incur low make-whole payments while providing

adequate congestion signals and low lost opportunity costs. Our suggested pricing rule does not require weighing of objectives, it is computationally scalable, and balances trade-offs in a principled manner, addressing an important policy issue in electricity markets. A full version of this paper can be found at https://arxiv.org/abs/2209.07386.

Contribution of dissertation author: Methodology, software, experimental design, investigation, visualization, joint paper management.

Citation: Ahunbay et al. (2023b).

Comment: The long version of this article is currently in minor revision for the journal *Operations Research*.

Pricing Optimal Outcomes in Coupled and Non-Convex Electricity Markets

METE ŞEREF AHUNBAY, MARTIN BICHLER, and JOHANNES KNÖRR, Technical University of Munich, Germany

According to the fundamental theorems of welfare economics, any competitive equilibrium is Pareto efficient. Unfortunately, competitive equilibrium prices only exist under strong assumptions such as perfectly divisible goods and convex preferences. In many real-world markets, participants have non-convex preferences and the allocation problem needs to consider complex constraints. Electricity markets are a prime example, but similar problems appear in many real-world markets, which has led to a growing literature in market design.

To rectify the non-existence of Walrasian equilibria, in U.S. electricity markets, the welfare-optimal allocation is currently supported by linear prices based on the dual of a relaxed allocation problem. These relaxations consider convexified valuation functions for market participants, such as full convex envelopes (Convex Hull [CH] pricing) or restricted convexifications (e.g. Integer Programming [IP] pricing or Minimum Make-Whole Payment [Min-MWP] pricing). However, with increasing levels of renewable energy sources, these pricing rules have come under scrutiny. In particular, they lead to high out-of-market side-payments to some participants and/or to incorrect pricing across (un)congested transmission lines, resulting in revenue shortfalls for transmission operators. We show that existing pricing heuristics optimize specific design goals that can be conflicting. More precisely, standard pricing rules each optimize a specific class of lost opportunity costs that correspond to desirable economic properties, such as minimum violation of obedience constraints (CH pricing), pricing at marginal cost/valuation (IP pricing), or minimum violation of individual rationality (Min-MWP pricing). However, the resulting trade-offs can be substantial, and we establish that the design of pricing rules is fundamentally a multi-objective optimization problem addressing different incentives.

We consider two approaches to balance these trade-offs. First, we consider traditional multi-objective optimization techniques using weighing of individual objectives. Second, we introduce a novel parameter-free pricing rule that minimizes incentives for market participants to deviate locally. We perform extensive numerical experiments on stylized and real-world data. Our theoretical and experimental findings show how the new pricing rule capitalizes on the upsides of existing pricing rules under scrutiny today. It leads to prices that incur low make-whole payments while providing adequate congestion signals and low lost opportunity costs. Our suggested pricing rule does not require weighing of objectives, it is computationally scalable, and balances trade-offs in a principled manner, addressing an important policy issue in electricity markets.

A full version of this paper can be found at https://arxiv.org/abs/2209.07386.

CCS Concepts: • **Applied computing → Multi-criterion optimization and decision-making**; • **Theory of computation** → *Computational pricing and auctions.*

Additional Key Words and Phrases: electricity markets, non-convex markets, multi-objective optimization

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1 INTRODUCTION

Low transaction costs on electronic markets have led to an increased use of market mechanisms to allocate scarce resources. The promise of markets rests on fundamental theoretical results such as the welfare theorems [Arrow and Debreu, 1954]. These theorems state that in markets with convex preferences, a Walrasian equilibrium will maximize social welfare and that every welfare-maximizing allocation can be supported by Walrasian equilibrium prices. At those prices no market participant would want to deviate from what they are assigned, and the outcome is envy-free and budget-balanced. The welfare theorems provide the theoretical rationale for using markets in a wide variety of applications.

Unfortunately, more recent literature on competitive equilibrium theory for markets with multiple indivisible goods shows that Walrasian equilibrium prices only exist under restrictive assumptions on the valuation functions [Baldwin and Klemperer, 2019, Bikhchandani and Mamer, 1997, Bikhchandani and Ostroy, 2002, Gul and Stacchetti, 1999]. Most real-world markets have non-convex value and cost functions leading to non-convex welfare maximization problems, where Walrasian equilibrium prices do not exist in general. Electricity markets are a prime example of such non-convex markets, but similar characteristics can be found in industrial procurement, spectrum auctions, and transportation markets [Cramton et al., 2006].

While the fundamental problems that arise in pricing goods in non-convex markets are independent of the domain, electricity market design is particularly challenging due to the specific bid languages used and the fact that supply and demand need to be balanced at all times. More importantly, electricity markets consist of coupled markets where trade happens on interlinked nodes in the electricity grid. As a result, congestion on network links needs to be reflected in the prices as well. These specifics lead to one of the most challenging market design problems, one that is receiving renewed attention due to the ongoing energy transition.

In a Walrasian equilibrium, one price aligns all incentives such that no participant wants to deviate from the welfare-maximizing outcome, but such equilibria generally only exist in convex markets [Bikhchandani and Mamer, 1997]. European day-ahead electricity markets therefore accept welfare losses while ensuring linear prices, whereas wholesale electricity markets in the U.S. implement optimal outcomes and determine linear and anonymous market prices that are complemented by personalized side-payments. However, in both jurisdictions, the current pricing rules have come under scrutiny. [1]

1.1 Multiple Pricing Objectives

This paper is based on the observation that in non-convex electricity markets where no Walrasian equilibrium exists, a market operator needs to consider multiple incentives to deviate from the outcome. These include the incentives to deviate from the overall allocation (leading to "global" lost opportunity costs, GLOCs),[2] but also incentives because bidders might incur a loss (leading to make-whole payments, MWPs), and incentives to deviate from the output under fixed commitment (leading to "local" lost opportunity costs, LLOCs, and biased congestion signals). The latter two types of incentives have received little attention so far.

Minimal MWPs matter because in most electricity spot markets the market operator only pays MWPs such that the market participants do not incur a loss, but they do not compensate GLOCs. Deviations from the welfare-maximizing dispatch are avoided by imposing penalties. As a result,

[1]https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation, https://www.nemo-committee.eu/assets/files/NEMO_CACM_Annual_Report_2020_deliverable_1_pub.pdf

[2]Global lost opportunity costs describe the difference between each participant's profits under the welfare-maximizing allocation and the individual profit maxima each participant could obtain through self-dispatch given the prices.

reducing MWPs has been a concern by market operators, and it has been the focus of recent research [Bichler et al., 2022, O'Neill et al., 2019]. However, low MWPs do not necessarily lead to low GLOCs or good congestion signals, as we will show.

LLOCs assume that the commitment decisions[3] have been made but that generators can deviate from their cleared volumes in an attempt to improve their payoff. Such deviations would not be profitable if there was a Walrasian equilibrium. These local incentives to deviate are in general a strict subset of global incentives, where generators may also alter their commitment decisions. Low LLOCs constitute another important but less obvious design goal, that of adequate congestion signals. Electricity markets are based on a network with possibly thousands of spatially distributed nodes connected via transmission lines. If transmission lines are congested, the marginal costs of electricity on two different nodes should differ. As transmission operators have convex costs, zero LLOCs imply that the congestion price indeed describes the value of additional transmission capacity. Unfortunately, current pricing heuristics might lead to different prices on neighboring nodes although there is no congestion [Schiro et al., 2016], creating unrealized arbitrage opportunities for transmission operators. It is important to have pricing rules that adequately signal congestion in the electricity grid, for risk hedging of participants, but also for adequate investment decisions.

1.2 Contributions

All three types of incentives matter, but there are trade-offs as we show. In this paper, we establish these trade-offs for market operators and formalize the pricing problem as a multi-objective optimization problem. Our *main contribution* is then the formulation of a parameter-free pricing rule which balances trade-offs between reducing MWPs and LLOCs to minimize local incentives of participants to deviate from the efficient outcome or incentives to exit the market. As discussed above, this also guarantees adequate congestion signals. Our technique does not require elicitation of weights or other parameters for individual objectives by the market operator, avoiding a fine-tuning problem. Numerical evidence shows that it outperforms existing pricing rules in reducing MWPs while maintaining adequate congestion signals.

To formalize our pricing rule, we begin via an in-depth theoretical analysis of pricing in coupled markets. As our *first contribution*, we make explicit the link between previously proposed pricing rules and the classes of incentives they optimize. For this purpose, we study pricing in coupled and (non-)convex markets through the lens of convex optimization. As a *second contribution*, we establish that in the presence of non-convexities, pricing is inherently a multi-objective optimization problem. This is different to prior literature that primarily aims to minimize GLOCs. Of course, when Walrasian prices exist, they eliminate all incentives for a market participant to deviate from their allocation. However, in the presence of non-convexities, the optimization of a class of incentives is inherently contradictory to the optimization of another, even when one class is a strict subset of another. Our *third contribution* introduces alternative pricing rules based on multi-objective optimization. We propose a pricing rule which minimizes a weighted sum of GLOCs, LLOCs, and MWPs. This weighing of objectives is referred to as scalarization. The freedom in determining the weights causes a fine-tuning problem: without information on the bids, the impact of weights is thus difficult to judge and it can be challenging to settle on a pricing rule. To rectify this issue, we introduce a join of two established pricing rules (Minimum-MWP and Integer Programming (IP) pricing) as an alternative approach. This method minimizes the maximum of MWPs and LLOCs, i.e. incentives for each participant to deviate from the allocation locally (given the commitment decision) or incentives to exit the market entirely. We prove that the join always achieves lower

[3] Commitment decisions on spot markets determine whether a generator is scheduled to produce electricity during a market time unit (a binary decision variable in the allocation problem), but not the production quantity (in Megawatt hours (MWh)).

MWPs than IP pricing, and lower LLOCs than any minimal-MWP solution if zero MWPs are attainable. In addition, we show that prices computed via the join result in a participant-wise Pareto-optimal outcome, such that different prices cannot jointly reduce MWPs and LLOCs of this participant. As a practical advantage, the join does not require to specify weights and is thus a parameter-free pricing rule. In extensive numerical experiments we show that prices computed via this join require significantly less MWPs than traditional IP pricing and retain good congestion signals at the same time. Besides, the approach can be computed efficiently and it requires no fine-tuning of objective weights, as pointed out earlier.

1.3 Organization of the Paper

The remainder of this paper is structured as follows. Section 2 summarizes relevant literature on competitive equilibrium theory and on electricity market design. In Section 3, we introduce a generic market and revise central findings for convex and non-convex settings. In Section 4, we outline how current pricing rules each optimize different objectives, and how these objectives can be in substantial conflict with each other. To that end, we propose a multi-optimization perspective in Section 5 and introduce principled ways to balance the trade-offs. Subsequently, we tailor these pricing rules to an exemplary electricity market in Section 6. We present explicit formulations and numerical findings that illustrate the advantages of our pricing rules. Section 7 provides a summary and conclusions.

2 RELATED LITERATURE

The literature on competitive equilibrium has a long history. In this section, we summarize the main theoretical findings before we discuss the related literature on electricity market design. These streams of literature are often considered separately, while we aim to connect the contributions of economics and engineering.

2.1 Competitive Equilibrium Theory

Early in the study of markets, general equilibrium theory was used to understand how markets could be explained through the demand, supply, and prices of multiple commodities. The Arrow-Debreu model shows that under convex preferences, perfect competition, and demand independence there must be a set of competitive equilibrium prices [Arrow and Debreu, 1954]. Market participants are price-takers, and they sell or buy goods in order to maximize their total utility. General equilibrium theory assumes divisible goods and convex preferences and the well-known welfare theorems do not carry over to markets with indivisible goods and non-convex preferences and constraints.

More recently, competitive equilibria with indivisible objects were studied and the idea of a quasilinear utility function was widely adopted [Baldwin and Klemperer, 2019, Bikhchandani and Mamer, 1997, Bikhchandani and Ostroy, 2002]. In these competitive equilibrium models, buyers and sellers with a quasilinear utility maximize their respective payoffs at the prices, resulting in an outcome that is stable (i.e. no participant wants to deviate from their resulting trade). A large part of the literature focuses on Walrasian equilibria, i.e. efficient market outcomes with linear (i.e. item-level) and anonymous prices, where all participants maximize payoff. If such prices exist, then the outcome maximizes welfare, as can be shown via linear programming duality. In general, Walrasian equilibria for markets with indivisible goods only exist for restricted valuations such as strong substitutes [Baldwin and Klemperer, 2019, Bikhchandani and Mamer, 1997, Kelso and Crawford, 1982]. These conditions lead to a concave aggregate value function, the allocation problem can be solved in polynomial time, and linear and anonymous (Walrasian) competitive equilibrium prices clear the market [Bichler and Waldherr, 2019]. Importantly, under these conditions the welfare theorems hold for markets with indivisible objects [Bichler et al., 2020, Blumrosen and Nisan, 2007].

Unfortunately, these conditions are very restrictive and in most markets goods can be substitutes and complements such that no Walrasian equilibria exist. This has led to significant research on non-convex combinatorial markets, which allow bidders to specify package bids, i.e. a price is defined for a subset of the items [Bichler and Goeree, 2017, Milgrom, 2017]. The specified bid price is only valid for the entire package and the package is indivisible such that bidders can express complex (quasilinear) preferences for general valuations including complements and substitutes.

The generality of these markets comes at a price. First, the winner determination problem becomes an NP-hard optimization problem. Second, competitive equilibrium prices need to be non-linear and personalized to allow for full efficiency [Bikhchandani and Ostroy, 2002]. Bichler and Waldherr [2017] show that the core of the game can even be empty such that no competitive equilibrium prices exist. However, non-linear and personalized prices would convey little information other than that a bidder lost or won. Besides, if prices should serve as a baseline for futures trading, this is hardly possible with non-linear prices that differ among participants. In other words, anonymity and linearity are important requirements for many markets, electricity markets being the prime example.

2.2 Pricing on Electricity Markets

In this work, we focus on central wholesale electricity spot markets that are based on auctions. Here, market participants submit supply and demand bids according to a certain *bid language* which translates into a central allocation problem. As market participants often exhibit start-up costs, minimum generation requirements, or other technical constraints, bid languages typically imply some form of non-convexities [Herrero et al., 2020]. Price-sensitive demand [Bichler et al., 2022], demand response [Papavasiliou and Oren, 2014] or the multi-period nature of the clearing problem [Cho and Papavasiliou, 2022] add further complexity. Market operators around the world use mixed-integer programming (MIP) to address these non-convexities and to determine the efficient allocation or dispatch [Hobbs et al., 2001]. However, computing (electricity) prices in the presence of non-convexities remains a fundamental problem.

If Walrasian equilibrium prices exist, no market participant will have an incentive to deviate from the optimal allocation. In other words, no participant would bear GLOCs. A natural extension to non-convex markets – where Walrasian equilibrium prices do not exist in general – is to minimize these GLOCs but maintain linearity and anonymity of prices. This is referred to as *Convex Hull (CH) pricing*, originally explored by Gribik et al. [2007] based on Hogan and Ring [2003]. Convex Hull pricing replaces the non-convex feasible region of the combinatorial allocation problem by its convex hull, and obtains prices from the dual of the resulting convex problem. We refer to Schiro et al. [2016] for a comprehensive and critical overview. However, obtaining exact Convex Hull prices is computationally expensive. A common approach involves solving the (convex but non-smooth) Lagrangian dual of the original non-convex problem, which – under mild assumptions – is equivalent to optimizing the convex envelope of the original cost functions over the convex hull of the feasible region [Falk, 1969, Hua and Baldick, 2017]. While there has been significant progress in this field [Knueven et al., 2022, Stevens and Papavasiliou, 2022], such methods are not yet used in practice and computational complexity remains a major concern.

In practice, market operators resort to different heuristics in order to price electricity on real-world markets. Most practical implementations are based on *Integer Programming (IP) pricing*, where the non-convex allocation problem is first solved to optimality, and then solved again with integer variables being fixed to their optimal values. The IP prices are derived from the dual solution of the latter convex program [O'Neill et al., 2005]. IP pricing has become popular as it follows the notion of marginal cost pricing in non-convex markets and furthermore provides accurate congestion signals when applied on an electricity network. However, IP prices do not constitute competitive equilibrium prices and can come with high lost opportunity costs. Based on the ongoing debate about high MWPs discussed in the introduction, new pricing rules have been proposed in an attempt to reduce them [Bichler et al., 2022, O'Neill et al., 2019], either via price differentiation or by minimizing MWPs directly. Recently, Yang et al. [2019] proposed to consider congestion signals in the design of pricing rules, which we do in our proposal. A comprehensive discussion of pricing rules as they have been proposed in the academic literature is provided by Liberopoulos and Andrianesis [2016].

3 DUAL PRICING IN CONVEX MARKETS

In this section, we introduce a model for a non-smooth and coupled market which allows for Walrasian equilibria. Our paper draws in large parts on the more general field of convex optimization; in Appendix A, we provide a brief introduction on the relevant notions from convex and non-smooth optimization.

Our goal in this paper is to gain an understanding of previously proposed pricing rules through the lens of convex optimization to reveal the corresponding design objectives they optimize. Towards this end, in this section, we first introduce our market model and then formalize the notion of a dual pricing problem via analysis of a convex market, where there exists a canonical pricing rule for optimal outcomes – Walrasian equilibria.

In what follows, we work with functions that take values in the extended real line, $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$. Addition and multiplication are commutative binary operations on the extended real line, defined as usual for any real number, and $x + \infty = \infty$, $x - \infty = -\infty$ for any $x \in \mathbb{R}$.

Definition 3.1. A **coupled market** consists of a set of goods M, a set of commodity flow parameters F and a set of market participants $L = B \cup S \cup R$, partitioned into the set of buyers B, set of sellers S and the set of transmission operators R. Each market participant $\ell \in L$ has preferences over bundles in $\mathbb{R}^{M \cup F}$, i.e. each buyer b has a valuation function $v_b : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$, each seller s has a cost function $c_s : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$, and each transmission operator r has a cost function $d_r : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$.

We encode any feasibility constraint for market participants in the domain of cost functions, i.e. buyers have valuation $-\infty$ and sellers / transmission operators have cost $+\infty$ for an infeasible bundle. We write x_b (y_s) for a bundle purchased by buyer b (seller s) and f_r for an exchange enacted by transmission operator r. To make the notation more concise, we write z_ℓ for the allocation of an arbitrary market participant $\ell \in L$. We assume that buyers and sellers only have values for the consumption or supply of goods in M while commodity flows are performed exclusively by transmission operators, i.e. only bundles in $\mathbb{R}^M \times \{0\}^F$ are feasible for buyers and sellers.

As a result of encoding feasibility in costs, allocations to market participants are assumed to be constrained only by the set of supply-demand equivalence constraints

$$\sum_{s \in S} y_s - \sum_{b \in B} x_b + \sum_{r \in R} B_r f_r = 0, \tag{1}$$

where B_r is some matrix specifying how commodity flows interact with the supply-demand balance for the good at each market.

We are concerned with supporting an *optimal allocation* with *prices*. An optimal allocation is a solution $(z_\ell^*)_{\ell \in L}$ of the *welfare maximization problem*

$$\max_{x,y,f} \sum_{b \in B} v_b(x_b) - \sum_{s \in S} c_s(y_s) - \sum_{r \in R} d_r(f_r) \text{ subject to (1).} \tag{2}$$

Then, in a coupled market, *prices* $p \in \mathbb{R}^{M \cup F}$ correspond to the per-unit cost of purchase of each unit of a good or flow. Utilities are assumed to be quasilinear in payment – thus each market participant has utility

$$\begin{aligned} u_b(x|p) &= v_b(x) - p^T x && \forall\, b \in B, \\ u_s(y|p) &= p^T y - c_s(y) && \forall\, s \in S, \\ u_r(f|p) &= p^T B_r f - d_r(f) && \forall\, r \in R. \end{aligned} \tag{3}$$

By optimizing over x, y, f in (3), each market participant has an *indirect utility function*, denoting the utility they would have from consuming / providing the bundle that maximizes their utility given prices,

$$\begin{aligned} \hat{u}_b(p) &= \max_x v_b(x) - p^T x && \forall\, b \in B, \\ \hat{u}_s(p) &= \max_y p^T y - c_s(y) && \forall\, s \in S, \\ \hat{u}_r(p) &= \max_f p^T B_r f - d_r(f) && \forall\, r \in R. \end{aligned} \tag{4}$$

Thus indirect utility function is simply the *convex conjugate* of the preferences of buyers and sellers, $\hat{u}_b(p) = (-v_b)^*(-p)$ and $\hat{u}_s(p) = c_s^*(p)$ for any price vector p and any buyer b or seller s. Similarly for transmission operators, $\hat{u}_r(p) = d_r^*(B_r^T p)$.

An optimal allocation $(z_\ell^*)_{\ell \in L}$ and prices p together are then said to form a *Walrasian equilibrium* if the allocation of each market participant is utility-maximizing at the given prices.

Definition 3.2 (Walrasian equilibrium in coupled markets). A price vector p and a feasible allocation $(z_\ell^*)_{\ell \in L}$ in $\mathbb{R}^{M \cup F}$ form a *Walrasian equilibrium* if:

(1) (Market clearing) The supply-demand equivalence constraints (1) are satisfied.
(2) (Envy-freeness) The allocation of every agent maximizes their utility at the prices – i.e. for any market participant ℓ, $u_\ell(z_\ell^*|p) = \hat{u}_\ell(p)$.
(3) (Budget balance) The sum of payments equals zero, $p^T \left(\sum_{s \in S} y_s^* - \sum_{b \in B} x_\ell^* + \sum_{r \in R} B_r f_r\right) = 0$.

At a Walrasian equilibrium, envy-freeness implies that no agent's utility can be less than that for consuming / supplying zero goods. Therefore, it is also *individually rational* for each buyer and seller to participate in the market, as no agent earns a negative payoff as a result of their market participation. A market is *convex* if $-v_b, c_s, d_r$ are all closed convex functions. It is the case that the First and Second Welfare Theorems hold for coupled and convex markets; here we present its proof as it provides a derivation of the *convex hull* pricing rule of [Gribik et al., 2007, Hogan and Ring, 2003] and motivates our discussion on different pricing rules.

Theorem 3.3 (The Welfare Theorems for Coupled and Convex Markets). *Let price vector $p^* \in \mathbb{R}^{M \cup F}$ and the allocation $(z_\ell)_{\ell \in L}^*$ be a Walrasian equilibrium, then this allocation maximizes social welfare. Conversely, if $(z_\ell)_{\ell \in L}^*$ is a welfare-maximizing allocation, then it can be supported by a Walrasian price vector p^* that forms a Walrasian equilibrium.*

Proof. The theorem follows by first considering the *welfare function*, defined as the value of the welfare maximization problem as its linear constraints are perturbed (cf. Rockafellar [2015] for a detailed discussion). Assuming strong supply-demand equivalence is required,[4] the social welfare $\omega : \mathbb{R}^{M \cup F} \to \overline{\mathbb{R}}$ is defined as a function of excess supply such that

$$\begin{aligned} \omega(\sigma) = \max_{x,y,f} & \sum_{b \in B} v_b(x_b) - \sum_{s \in S} c_s(y_s) - \sum_{r \in R} d_r(f_r) \\ \text{subject to} & \sum_{s \in S} y_s - \sum_{b \in B} x_b + \sum_{r \in R} B_r f_r = \sigma. \end{aligned}$$

[4]The discussion generalizes easily to the case of weak supply-demand equivalence

Thus ω is a convolution, and by Proposition A.4.4 the convex conjugate of $-\omega$ is given by

$$(-\omega)^*(p) = \sum_{b\in B}(-v_b)^*(p) + \sum_{s\in S} c_s^*(p) + \sum_{r\in R} d_r^*(B_r^T p) = \sum_{\ell\in L} \hat{u}_\ell(p).$$

Given that valuations are concave and costs are convex, ω is concave in all arguments and thus $-\omega$ is convex. As all constraints are linear, constraint qualification is satisfied and $-\omega$ is closed. Therefore, $-\omega$ admits a subdifferential $\partial(-\omega)(0)$ at $\sigma = 0$. Any element of the subdifferential provides prices that correspond to the per-unit value of the provision of an additional constraint violation. As the constraint fixes excess supply to 0, this is precisely the value of the provision of an additional unit of each good to the market.

By the Fenchel-Young inequality, in general we have $(-\omega)^*(p) - \omega(0) \geq 0$, with equality holding if and only if $p \in \partial(-\omega)(0)$. Therefore, for an optimal solution p of the *subgradient problem*

$$\min_p \sum_{\ell\in L} \hat{u}_\ell(p) - \omega(0), \tag{5}$$

we note that $p^T\left(\sum_{s\in S} y_s^* - \sum_{b\in B} x_b^* + \sum_{r\in R} B_r f_r\right) = 0$ and $\omega(0) = \sum_{b\in B} v_b(x_b^*) - \sum_{s\in S} c_s(y_s^*) - \sum_{r\in R} d_r(f_r^*)$. Then by the Definition (3) of utilities and by re-arranging terms, this subgradient problem may be rewritten

$$\begin{aligned}
&\min_p \sum_{\ell\in L} \hat{u}_\ell(p) - \omega(0)\\
=&\min_p \sum_{b\in B} \hat{u}_b(p) + p^T x_b^* - v_b(x_b^*) + \sum_{s\in S} \hat{u}_s(p) + c_s(y_s^*) - p^T y_s^* + \sum_{r\in R} \hat{u}_r(p) + d_r(f_r^*) - p^T B_r^T f_r^*\\
=&\min_p \sum_{\ell\in L} \hat{u}_\ell(p) - u_\ell(z_\ell^*|p). \qquad (6)
\end{aligned}$$

For any market participant ℓ and for any price vector p, $\hat{u}_\ell(p) \geq u_\ell(z_\ell^*|p)$. Therefore as the value of the problem equals zero, at an optimal solution p these must all hold with equality, which implies that an optimal solution to (5) p together with a welfare-maximizing allocation $(z_\ell^*)_{\ell\in L}$ form a Walrasian equilibrium. Likewise, if p, $(z_\ell^*)_{\ell\in L}$ form a Walrasian equilibrium then the value of (6) is zero, which implies that $(z_\ell^*)_{\ell\in L}$ maximizes social welfare. □

We do not need differentiability, and convexity of $-v_b$, c_s, d_r is sufficient for this proof. As convex optimization is also computationally efficient so long as each valuation and cost function is tractable to compute, the subgradient provides a natural way of computing prices. This motivates us to define **dual pricing functions** for an optimal outcome $\left((x_b^*)_{b\in B}, (y_s^*)_{s\in S}, (f_r^*)_{r\in R}\right)$,

$$\lambda_b(p|x_b^*) = \hat{u}_b(p) - u_b(x_b^*|p),\ \lambda_s(p|y_s^*) = \hat{u}_s(p) - u_s(y_s^*|p),\ \lambda_r(p|f_r^*) = \hat{u}_r(p) - u_r(f_r^*|p). \tag{7}$$

Furthermore, we call (5) the **dual pricing problem** associated with the welfare maximization problem (2).

Although the connections between convexity and the existence of a Walrasian equilibrium are well-known [Bikhchandani and Mamer, 1997, Liberopoulos and Andrianesis, 2016], the version of the welfare theorems for coupled markets clearly delineates when we can expect Walrasian prices in coupled markets. This provides a foundation for our discussion of pricing rules in non-convex electricity markets.

4 PRICING NON-CONVEX AND COUPLED MARKETS

In the absence of convexity, the negative welfare function $-\omega$ is non-convex in general and the subgradient problem (5) has value > 0, pointing to a duality gap. The representation (6) of the convex pricing problem then suggests that for any welfare-maximizing outcome $(z_\ell^*)_{\ell \in L}$ and any price vector p^*, Definition 3.2.2 is not satisfied and market participants incur *lost opportunity costs* (LOCs). This raises the question of how to price these markets.

In this section, we review some proposals for pricing optimal outcomes of such non-convex markets via the formalism through which we established the existence of Walrasian equilibria in coupled and convex markets. We will see that these pricing rules consist of convex models for the dual pricing problem (5). Moreover, they assert a convexified model of the original welfare maximization problem corresponding to minimization of a *class* of LOCs, given prices, as a central design goal. *global* LOCs (GLOCs) which correspond to unrealized profit as participants deviate to any feasible outcome, *local* LOCs (LLOCs) which correspond to unrealized profit as participants deviate to another outcome under fixed commitment, and *make-whole payments* (MWPs) as the required amount of compensation to market participants to ensure they do not make a loss. We conclude the section by illustrating that the minimization of each class of LOCs results in prices causing market participants to incur large LOCs in other classes. This motivates framing pricing in non-convex markets as a multi-objective optimization problem.

4.1 Minimizing Global Lost Opportunity Costs

While an optimal solution to (5) of the subgradient problem does not provide prices that form a Walrasian equilibrium in the presence of non-convexities, the possibility of using such prices was nevertheless investigated by Hogan and Ring [2003] and Gribik et al. [2007]. Noting that the term in the objective for each market participant is the difference between their payoff if they were to choose an allocation of their choice and the payoff they receive from their current allocation, given the prices, a solution to (5) minimizes GLOCs. This pricing rule is known as CH pricing [Schiro et al., 2016] because the associated welfare maximization problem is obtained by convexifying the preferences of market participants in the original welfare maximization problem

$$\max_{x,y,f} \sum_{b \in B} \overline{conc}[v_b](x_b) - \sum_{s \in S} \overline{conv}[c_s](y_s) - \sum_{r \in R} \overline{conv}[d_r](f_r) \text{ subject to (1).} \qquad \text{(Primal CH)}$$

Given the optimal allocation $(z_\ell^*)_{\ell \in L}$, we thus relabel the contribution $\lambda_\ell(p|z_\ell^*)$ of market participant $\ell \in L$ to the dual pricing problem as $\lambda_\ell^{CH}(p|z_\ell^*)$ and call this contribution the *Convex Hull pricing function* of ℓ at z_ℓ^*. We infer that if p is a solution to (5) with objective value 0, then

$$-p \in \partial(-v_b)^{**}(x_b^*) \ \forall\, b \in B, \quad p \in \partial c_s^{**}(y_s^*) \ \forall\, s \in S, \quad B_r^T p \in \partial d_r^{**}(f_r^*) \ \forall\, r \in R.$$

In other words, when the objective of the CH pricing problem equals zero, prices form supragradients of the concave closures of buyers' valuation functions and subgradients of the convex closure of sellers' and transmission operators' cost functions. Furthermore, in this case the duality gap equals zero and prices reflect the marginal valuations and costs for any extra unit of a good or flow. In general, CH pricing provides a supragradient of the *concave closure of the welfare function* at $\sigma = 0$, though these prices might not necessarily belong to the subdifferential of market participants at an optimal outcome when there exists a positive duality gap due to non-convexities.

One issue with Convex Hull prices is that they are generally intractable to compute [Schiro et al., 2016]. There have been efforts to establish conditions under which Convex Hull prices can be computed by simple linear programs [Hua and Baldick, 2017] or to design more efficient algorithms [Andrianesis et al., 2022, Knueven et al., 2022], yet as of today CH pricing cannot be applied for practical problems.

4.2 Minimizing Local Lost Opportunity Costs

In the presence of non-convexities CH pricing may result in large LLOCs (an example is provided below). As a result, market participants have high incentives for just small deviations from the welfare-maximizing allocation, requiring large amounts of penalties to enforce the optimal outcome. This drastic instability under such pricing rules is of concern, and it merits direct minimization of LLOCs to ensure local stability.

Most U.S. and European electricity markets implement bidding languages which allow the expression of piecewise-concave valuations for buyers and piecewise-convex costs for sellers. We say that a function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ is *piecewise convex* if there exist disjoint convex sets $X_1, X_2, .., X_K \subseteq \mathbb{R}^n$ and closed convex functions $f_1, f_2, .., f_k$ such that for any $1 \leq k \leq K, f_k : X_k \to \mathbb{R}$ and $f = \min_{1\leq k\leq K} f_k$. For each $k \in K$, we let $\text{dom}(f_k) = X_k$ denote the *domain* of f_k. For any $x \in \mathbb{R}^n$, $f_{k'}$ is called *active* at x if $k' \in \arg\max_{1\leq k\leq K} f_k(x)$.

Piecewise-convex preferences may be modeled via the addition of binary variables that represent the choice of active valuation / cost functions. When each v_b, c_s, d_r is piecewise linear, this allows casting the welfare maximization problem as a mixed-integer linear program. It is shown in O'Neill et al. [2005] that marginal prices which eliminate LLOCs can be obtained by fixing binary variables to their optimal values and solving the resulting dual problem. This provides what is known as IP pricing in electricity markets. Formally, IP pricing is given by the dual pricing problem associated with the welfare maximization problem

$$\max_{x,y,f} \sum_{b\in B} \hat{v}_b(x_b) - \sum_{s\in S} \hat{c}_s(y_s) - \sum_{r\in R} \hat{d}_r(f_r) \text{ subject to (1),} \tag{Primal IP}$$

where $\hat{v}_b, \hat{c}_s, \hat{d}_r$ are the corresponding active valuation and cost functions given the optimal allocation. The dual pricing problem can be explicitly enumerated as the minimization of LLOCs of all market participants,

$$\begin{aligned}
\min_p \quad & \sum_{b\in B} \max_{x_b} \hat{v}_b(x_b) - p^T x_b - u_b(x_b^*|p) + \sum_{s\in S} \max_{y_s} p^T y_s - \hat{c}_s(y_s) - u_s(y_s^*|p) \qquad \text{(IP Pricing)}\\
& + \sum_{r\in R} \max_{f_r} p^T B_r f_r - \hat{d}_r(f_r) - u_r(f_r^*|p)\\
\text{subject to} \quad & x_b \in \text{dom}(\hat{v}_b)\ \forall\, b \in B, \quad y_s \in \text{dom}(\hat{c}_s)\ \forall\, s \in S, \quad f_r \in \text{dom}(\hat{d}_r)\ \forall\, r \in R,
\end{aligned}$$

minimizing the total LOCs incurred by market participants for deviations under fixed commitment. We then denote the contribution of market participant ℓ to the dual pricing problem as $\lambda_\ell^{IP}(p|z_\ell^*)$ and call this contribution the *IP dual pricing function* of ℓ at z_ℓ^*.

We note that in our setting, by local convexity prices, may be found that completely eliminate LLOCs. Moreover, when transmission operators are fully convex, we can conclude zero GLOCs for them as well. Therefore IP pricing provides adequate signaling of congestion in the network as prices reflect the marginal value of additional transmission capacity [Yang et al., 2019], mitigating a shortfall of congestion income that was identified for other pricing rules [Schiro et al., 2016].

4.3 Minimizing Make-Whole Payments

As discussed in the introduction, increasing levels of MWPs are a growing concern in electricity markets. They imply discriminatory pricing and can be seen as a distortion of price signals by participants. To rectify this issue, Bichler et al. [2022] introduced the following optimization problem, where MWPs are minimized directly.

$$\min_{\lambda \geq 0, p, \gamma} \quad \sum_{b \in B} \lambda_b^{MWP}(p|x_b^*) + \sum_{s \in S} \lambda_s^{MWP}(p|y_s^*) + \sum_{r \in R} \lambda_r^{MWP}(p|f_r^*) \qquad \text{(Min-MWP)}$$

$$\text{subject to} \quad -v_b(x_b^*) + p^T x_b^* - \lambda_b^{MWP}(p|x_b^*) \leq 0 \; \forall \; b \in B,$$

$$-p^T y_s^* + c_s(y_s^*) - \lambda_s^{MWP}(p|y_s^*) \leq 0 \; \forall \; s \in S,$$

$$-p^T B_r f_r^* + d_r(f_r^*) - \lambda_r^{MWP}(p|f_r^*) \leq 0 \; \forall \; r \in R.$$

The associated dual pricing functions for this problem are $\lambda_\ell^{MWP}(p|z_\ell^*) = \max\{-u_\ell(z_\ell^*|p), 0\}$ for each market participant ℓ, and we call this contribution the *Min-MWP dual pricing function* of ℓ. By definition the Min-MWP dual pricing function accounts only for the lost opportunity cost with respect to non-participation, thus the value of (Min-MWP) is precisely the minimum MWPs required to compensate participants' losses.

4.4 Contradictions in Design Goals

We established in the preceding discussion that CH pricing, IP pricing, and Min-MWP pricing each optimize a corresponding class of LOCs. While the minimization of all such LOCs is evidently desirable, it turns out to be the case that focusing on only one such design goal can lead to large LOCs in other classes.

For instance, even in settings where CH prices may be tractably computed, a minimization of GLOCs does not necessarily correspond to low MWPs and LLOCs. This is because the convexified market model (Primal CH) allows for fractional commitment. Therefore, an optimal solution of (Primal CH) might prescribe offline units to *partially activate* for gains in welfare, even when the original welfare maximization problem has no optimal allocation where these units are committed. In other words, with CH pricing offline units can set prices. For this reason, CH prices do not exhibit a clear economic interpretation [Schiro et al., 2016]. This phenomenon may manifest itself as very high MWPs and LLOCs (and thereby flawed congestion signals in coupled markets). Similarly, IP prices fix LLOCs to zero via local convexity, but discard any information on changing commitment. As a result, IP prices may fail to adequately account for GLOCs or MWPs. Finally, Min-MWP directly minimizes the make-whole payments but loses all information on subgradients in the process. As a result, while MWPs may be reduced to insignificant amounts, the solution set will be large and the computed prices will in general incur both significant LLOCs and GLOCs. We provide illustrative examples in Appendix B.

5 PRICING AS MULTI-OBJECTIVE OPTIMIZATION

In the previous section, we discussed dual pricing problems that minimize either GLOCs (CH pricing), LLOCs (IP pricing), or MWPs (Min-MWP pricing). We provided examples illustrating that a focus on one of these objectives can lead to high costs in the others. Ultimately, we are faced with trade-offs between the three conflicting objectives. Noting that the market designer can also have design goals that go beyond minimization of deviation incentives, e.g. (approximate) budget balance, we infer that the pricing problem in non-convex markets is one of multi-objective optimization rather than single-objective optimization.

While many techniques for multi-objective optimization have been proposed (see, e.g., Deb [2001], Miettinen [2012] or Emmerich and Deutz [2018]), not all of them are suitable to be used as pricing rules in electricity markets. A poor choice of a multi-objective formulation can impose a highly distorted convex model for the welfare maximization problem, resulting in a divergence between its implied optimal allocation and the true optimal allocation. Specifically, the underlying convex model might possess fictitious goods and agents, unrealistically expand the feasible sets of market participants, or consolidate market participants to a single entity. Then, as in the case of CH pricing in Example B.1 in the appendix, we are faced with the possibility of an *economic interpretability problem*. We elaborate on this issue in Appendix C.

Therefore, in order to prevent such distortions and to allow for economic interpretability, we restrict attention to pricing problems with an additively separable objective function with linear prices, i.e. pricing problems of the form $\min_{p \in \mathbb{R}^M} \sum_{\ell \in L} \lambda_\ell(p|z_\ell^*)$. We also keep in mind that a pricing rule should be simple to communicate and computationally scalable for the large problems in electricity markets.

In what follows, we first consider linear scalarization and analyze the Pareto frontiers of the multi-objective optimization problem minimizing a weighted sum of GLOCs, MWPs, and LLOCs. However, we note that linear scalarization requires fine-tuning of the weights and might be impractical for practical application. Therefore, motivated by the correspondence between the minimization of MWPs or LLOCs and convex approximations to the market participants' preferences, we propose a new approach thereafter that tightens these convex approximations and requires no additional preference information.

5.1 Linear Scalarization

Linear scalarization optimizes a weighted sum of individual objective functions. In particular, given dual pricing functions for market participants $\left(\sum_{\ell \in L} \lambda_\ell^i(\cdot|z_\ell^*)\right)_{i \in \{1,..,m\}}$ associated with m dual pricing problems and a non-negative weight vector $w \in \mathbb{R}^m_{\geq 0}$, the linear scalarization problem is given by

$$\min_p \sum_{\ell \in L} \sum_{i=1}^m w_i \cdot \lambda_\ell^i(p|z_\ell^*) \tag{8}$$

As each dual pricing objective λ^i is separable over the market participants, linear scalarization is equivalent to picking certain convex approximations to each participant's valuation or cost function, as in the case for CH, IP, and Min-MWP pricing. In fact, the resulting pricing problem is that of a convex market where each participant is replaced by an *admixture* of participants, where preferences from each dual pricing problem i are present in proportion to the weight w_i. Specifically, suppose the weights are non-negative and normalized such that $\sum_{i=1}^m w_i = 1$. Then, if buyer b has allocation x_b^* and valuation function v_b^i in the welfare maximization problem corresponding to dual pricing objective i, the weighted dual pricing function $\sum_{i=1}^m w_i \lambda^i(p|x_b^*)$ corresponds to the valuation function

$$\bar{v}_b(x) = \max \sum_{i=1}^m w_i v_b^i(\chi_b^i) \text{ subject to } \sum_{i=1}^m w_i \chi_b^i = x. \tag{9}$$

In a multi-objective optimization problem, it is generally not possible for a solution to be optimal for each objective. Therefore, the optimality criterion is, in general, *Pareto optimality*. For m dual pricing problems, a price vector p is said to be **Pareto optimal with respect to objectives** if there does not exist another price vector q such that for any pricing problem i, $\sum_{\ell \in L} \lambda_\ell^i(q|z_\ell^*) \leq \sum_{\ell \in L} \lambda_\ell^i(p|z_\ell^*)$, with one inequality holding strictly.

It is known that solutions to the linear scalarization problem yield the Pareto frontier of the multi-objective [Emmerich and Deutz, 2018]. We thus infer that if the dual pricing functions under consideration correspond to CH, IP, and Min-MWP pricing, linear scalarization allows to obtain other Pareto-efficient solutions with respect to total GLOCs, MWPs, and LLOCs.

However, linear scalarization comes with certain disadvantages for practical applications despite providing a Pareto optimal solution with respect to GLOCs, MWPs, and LLOCs. In particular, it requires preference information in order to set weights that produce a desirable outcome. In practice, however, preferences might be difficult to define in an environment with various stakeholders and without being able to study the impact of weights on the outcomes. One might want to re-calibrate and fine-tune the weights after the Pareto frontier has been computed. For this reason, linear scalarization cannot be considered a full a priori multi-objective technique as weights and therefore the pricing rule might be set only after bids have been elicited. Therefore, linear scalarization might be difficult to implement in practice.

5.2 Join

In what follows, we focus on a pricing rule that treats low LLOCs and low MWPs as first-order objectives. Together, they imply that the market allocation is as locally stable as possible against considerations of non-participation or small deviations by the market participants. Whereas linear scalarization weighing IP and Min-MWP pricing achieves a Pareto optimal outcome with respect to minimizing total LLOCs and MWPs globally over all participants, the prices obtained are not optimal for the incentives of individual market participants to deviate under fixed commitment (LLOCs) or for incentives exit the market (MWPs). Specifically, given prices p and an optimal allocation $(z_\ell^*)_{\ell\in L}$, market participant ℓ's opportunity cost under consideration of local deviations in allocation is given by $\lambda_\ell^{IP}(p|z_\ell^*)$ while their opportunity cost against exiting the market is given by $\lambda_\ell^{MWP}(p|z_\ell^*)$. Therefore, the amount of compensation required to disincentivize such deviations from ℓ is given by $\max\{\lambda_\ell^{IP}(p|z_\ell^*), \lambda_\ell^{MWP}(p|z_\ell^*)\}$. This motivates us to consider the following dual pricing problem.

Definition 5.1. For each market participant ℓ, let $\lambda_\ell^{IP}(p|z_\ell^*), \lambda_\ell^{MWP}(p|z_\ell^*)$ denote the LLOCs and MWPs of ℓ at prices p and allocation z^*. Then the **join** IP $\vee$ MWP of IP and Min-MWP pricing is the dual pricing problem

$$\min_p \sum_{\ell\in L} \max\{\lambda_\ell^{IP}, \lambda_\ell^{MWP}\}(p|z_\ell^*). \qquad \text{(IP} \vee \text{MWP)}$$

Specifically, the objective is the sum of MWPs to compensate losses and penalties against deviations required for local stability, and the join provides prices that minimizes it. IP $\vee$ MWP utilizes minimal concave closures of valuation functions and convex closures of cost functions that account for both LLOCs and MWPs, implying that the associated convex approximation is minimally distortionary for preferences. A linear programming formulation can be found in Appendix H.

We emphasize that the join is distinct from any pricing rule obtained via linear scalarization. Inspection of (9) shows that the associated convex approximations to preferences do not fully account for neither the LLOCs nor the MWPs. Formally, there do not exist weights (w_{IP}, w_{MWP}) such that the LOCs of some market participant ℓ under such deviations equals $w_{IP}\lambda_\ell^{IP}(p|z_\ell^*) + w_{MWP}\lambda_\ell^{MWP}(p|z_\ell^*)$ for every set of prices p. Based on these considerations, we introduce the sum of these LOCs over the market participants to ensure local stability as a new and *parameter-free* multi-objective. It jointly minimizes MWPs and LLOCs while properly aligning incentives, without requiring specific weights as an input.

Let us next prove some useful properties of the join. First, the join IP $\vee$ MWP is indeed guaranteed to achieve lower MWPs than IP pricing.

PROPOSITION 5.2. *Suppose that $p^\vee$ is an optimal solution of (IP $\vee$ MWP) and p^{IP} is an optimal solution of (IP Pricing). Then $\sum_{\ell\in L} \lambda_\ell^{MWP}(p^\vee|z_\ell^*) \leq \sum_{\ell\in L} \lambda_\ell^{MWP}(p^{IP}|z_\ell^*)$.*

Proof. The result follows since

$$\sum_{\ell \in L} \lambda_\ell^{MWP}(p^\vee | z_\ell^*) \leq \sum_{\ell \in L} \max\{\lambda_\ell^{MWP}, \lambda_\ell^{IP}\}(p^\vee | z_\ell^*)$$
$$\leq \sum_{\ell \in L} \max\{\lambda_\ell^{MWP}, \lambda_\ell^{IP}\}(p^{IP} | z_\ell^*) = \sum_{\ell \in L} \lambda_\ell^{MWP}(p^{IP} | z_\ell^*).$$

Here, the first inequality holds since the terms of the second sum are element-wise no less than the terms of the first sum, the second inequality holds by the minimum property of $p^\vee$. The final equality then holds since for any market participant ℓ, prices p^{IP} achieve zero LLOCs due to convexity. □

While Proposition 5.2 might appear to immediately follow from its definition, this is only guaranteed since IP pricing achieves zero LLOCs for each market participant. Trying to modify the proof to show that a solution of (IP ∨ MWP) achieves lower LLOCs than any solution of (Min-MWP), we obtain the following guarantee.

Corollary 5.3. *Suppose that $p^\vee$ is an optimal solution of (IP ∨ MWP) and p^{MWP} is an optimal solution of (Min-MWP) such that $\sum_{\ell \in L} \lambda_\ell^{MWP}(p^{MWP} | z_\ell^*) = 0$. Then $\sum_{\ell \in L} \lambda_\ell^{IP}(p^\vee | z_\ell^*) \leq \sum_{\ell \in L} \lambda_\ell^{IP}(p^{MWP} | z_\ell^*)$.*

Therefore, a solution of (IP ∨ MWP) achieves lower LLOCs than any solution of (Min-MWP) in settings where zero MWPs may be achieved with linear and anonymous prices. In general, the presence of non-convexities might render this impossible. Moreover, the following example demonstrates that the zero MWP condition in Corollary 5.3 is necessary.

Example 5.4. Consider a market with one good, one seller s and two buyers b_1 and b_2. Buyer b_1 has a block bid of \$20 for 1 MWh and buyer b_2 has a block bid of \$10 for 1 MWh, both of which must be fulfilled completely or not at all. Seller s has a cost function for generation

$$c_s(y) = \begin{cases} 0 & y = 0, \\ \frac{30}{\epsilon} \cdot (y - 2 + \epsilon) & y \in [2 - \epsilon, 2], \\ +\infty & \text{otherwise,} \end{cases}$$

with some small $\epsilon > 0$. In this case, note that only the seller s can have strictly positive LLOCs, thus she determines IP prices and LLOCs are minimized for $p^{IP} \geq 30/\epsilon$. Meanwhile, setting the price p^{MWP} equal to 15 minimizes MWPs. However, since $p^\vee \in [0, 10]$, seller s incurs a greater LLOC than for p^{MWP}.

A sufficient condition to satisfy the zero MWP condition with linear and anonymous prices is the presence of purely inelastic demand [Bichler et al., 2022]. Furthermore, we note that the example above is pathologically chosen. Specifically, in our numerical analysis in Section 6 we find that the set of optimal solutions to (Min-MWP) is large and that a solver typically picks solutions with very high GLOCs and LLOCs. Then, the solution of (IP ∨ MWP) tends to achieve lower LLOCs than the solution of (Min-MWP) picked by the solver.

The join does not necessarily exhibit Pareto optimality with respect to the objectives of total LLOCs and total MWPs. However, the join satisfies a *participant-wise* Pareto optimality criterion.

Proposition 5.5. *For an optimal outcome (z_ℓ^*) of a market, there exists some optimal solution $p^\vee$ of (IP ∨ MWP) such that deviations cannot jointly reduce MWPs and LLOCs of all market participants. Formally, there is no other price vector q, such that for any market participant ℓ and any $i \in \{IP, MWP\}$, $\lambda_\ell^i(q | z_\ell^*) \leq \lambda_\ell^i(p^\vee | z_\ell^*)$, with the inequality holding strictly for some ℓ, i.*

The solution $p^\vee$ in the statement of Proposition 5.5 may be obtained via lexicographic minimization of $\lambda_\ell^i(p | z_\ell^*)$ over the set of optimal solutions of (IP ∨ MWP). We identify this weakening of the global Pareto optimality guarantee as the trade-off for parameter-freeness and the alignment of participants' incentives, as now the individuals' incentives to deviate locally are minimized. Still, our experiments show that the join may achieve very low LLOCs and MWPs in practical settings.

Finally, we emphasize that the join indeed avoids the distortions to the welfare maximization problem exhibited by CH pricing. Specifically, Schiro et al. [2016] show that CH pricing allows offline sellers to distort both locational prices and congestion signals. As the join may only modify the dual pricing function of online units, it disallows offline units from having any effect on the price. The join can also be implemented efficiently (in poly-time) in practical electricity markets. Overall, the join provides a straightforward, *parameter-free* method of jointly minimizing LLOCs and make-whole payments, eliminating the need for fine-tuning or preference elicitation.

6 NUMERICAL RESULTS

To analyze the proposed pricing rules in an exemplary electricity market, we consider a simplified multi-period *direct current optimal power flow* (DCOPF) model [Frank et al., 2012, Molzahn and Hiskens, 2019]. This model describes a nodal electricity market with linearized transmission flows.[5] We refer to Appendix D for the notation and problem formulation. Each buyer b has a concave, piecewise linear valuation function, a certain inelastic demand and a maximum power consumption for each period t. Each seller s has some convex cost function for producing a positive amount at time t on some closed interval. Moreover, a binary commitment variable $u_{st} \in \{0, 1\}$ denotes whether the unit of seller s is active at time t (associated with certain fixed costs). Moreover, there potentially exists a set of linear minimum uptime constraints. For the transmission operators, the flow on each line is restricted by minimum and maximum flows and phase angles can be provided at any value for no cost, except for the operator at the reference node whose phase angle is fixed at 0 at each time period.

With this welfare maximization problem at hand, we implement the pricing rules discussed in Section 4. While CH pricing is intractable in general, a common heuristic in practical electricity markets is to relax the binary constraints in the DCOPF for u_{st}, $\{0, 1\} \rightarrow [0, 1]$, and to obtain prices from the resulting dual problem. It has been shown by Hua and Baldick [2017] that with the valuation / cost functions in our setting, the dual of this relaxation of the DCOPF is in fact equivalent to CH pricing. Therefore, we have an explicit and tractable dual formulation of CH prices, provided in Appendix E.

In order to compute IP prices, we follow the approach suggested by O'Neill et al. [2005]. In particular, we restrict each integer variable of the welfare maximization problem to the value it takes in the optimal allocation and solve the dual of the resulting problem. More explicitly, we set u_{st} as $\{0, 1\} \rightarrow [0, 1]$ and $u_{st} = u^*_{st}$ in the DCOPF and solve the dual problem in Appendix F.

For our implementation of Min-MWP, we note that given some prices the phase angle operators have either infinite or zero LLOCs (and hence GLOCs). To be able to obtain a solution to Min-MWP with finite LLOCs / GLOCs, we thus opt to replace the Min-MWP dual pricing functions of phase angle operators with their actual CH dual pricing functions in our implementation, thus accounting for their GLOCS. This implementation, provided in Appendix G, minimizes MWPs of buyers, sellers and flow operators under the constraint that phase angle operators suffer no GLOCs.

For linear scalarization, we combine these pricing problems and optimize a weighted average of the individual objective functions. The formulation for the join (IP ∨ MWP) is provided in Appendix H and considers each participant's maximum of the IP and Min-MWP dual pricing function.

[5]Tighter power flow relaxations exist, but are currently not applied in practical electricity markets for computational reasons. We refer to Molzahn and Hiskens [2019] for a comprehensive overview of power flow problems.

6.1 IEEE RTS System

First, we report results based on the IEEE RTS System, originally introduced by Grigg et al. [1999] and used in a variety of studies on electricity markets [Garcia-Bertrand et al., 2006, Hytowitz et al., 2020, Morales et al., 2009, Zocca and Zwart, 2021, Zoltowska, 2016]. Grigg et al. [1999] provide the stylized system topology, transmission network parameters, hourly (nodal) demand data as well as characteristics of generating units. In accordance with Zoltowska [2016], we select the single-area, 24-node topology by Grigg et al. [1999] for a representative 24-hour winter day with 32 generators (total capacity: 6.81 GW) and 17 consumers (average hourly demand: 2.60 GWh). Generators exhibit several non-convexities, such as no-load costs, minimum loads, or minimum runtimes. For data on generation costs or demand valuations we rely on the bid and offer curves provided by the case studies of Garcia-Bertrand et al. [2006] and Zoltowska [2016] on this system. Generators and consumers are embedded in a DC power flow model with 24 nodes. The optimal dispatch is computed by solving the mixed-integer DCOPF problem. All applied pricing rules constitute linear programs, with negligible computational effort compared to the initial MILP. Table 1 provides a first high-level overview of the results.

	GLOCs [$]	MWPs [$]	LLOCs [$]
CH	1436.21	202.57	1272.12
IP	10364.24	3387.00	0
Min-MWP	2.1×10^{11}	0	1.1×10^{11}
0.9 IP + 0.1 Min-MWP	11025.03	693.00	111.79
0.9 CH + 0.1 Min-MWP	1432.93	199.27	1119.85
0.5 IP + 0.5 CH	1570.13	606.21	729.29
IP $\vee$ Min-MWP	12022.38	0	191.14

Table 1. RTS System - GLOCs, MWPs, LLOCs

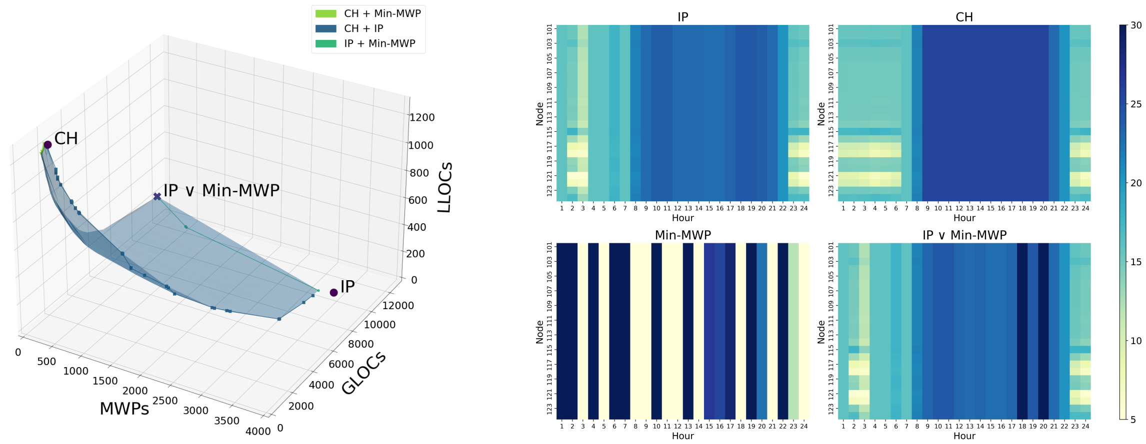
To begin with, the results underpin the finding that the optimization of individual objectives via CH, IP, and Min-MWP pricing is undesirable in at least one of the other objectives.

Result 1. *The LOC classes under consideration (GLOCs, LLOCs, MWPs) are conflicting. Pricing rules that optimize one class of LOCs lead to undesirable outcome in other classes of LOCs.*

Applying linear scalarization to IP and Min-MWP pricing leads to alternative results on the Pareto frontier (e.g., Table 1 displays a linear scalarization with 90% weight on IP and 10% weight on Min-MWP[6]), yet setting the weights is arbitrary. It should be noted that the set of outcomes implied by linear scalarization is not continuous. That is, there is only a limited number of outcomes and different weight vectors might produce the same result. The join IP $\vee$ Min-MWP requires no MWPs and LLOCs are reduced by 85% compared to CH pricing (and close to 100% compared to Min-MWP pricing). Moreover, GLOCs are significantly reduced compared to Min-MWP pricing and are only 16% higher than the GLOCs of IP pricing.

Figure 1a illustrates the GLOCs, MWPs, and LLOCs for the tested pricing rules and the Pareto frontier. Note that Min-MWP possesses very high GLOCs and LLOCs and can therefore not be meaningfully depicted. The Pareto frontier is obtained by applying linear scalarizations to the three pricing rules.

[6] A simple 50:50 weight assignment would lead to a high emphasis on MWPs and outcomes that are much worse than that of the join.



(a) GLOCs, MWPs, LLOCs [$]

(b) RTS System - Price Heatmaps [$/MWh]

Fig. 1. RTS System - GLOCs, MWPs, LLOCs, Prices

Result 2. *The Pareto frontier of GLOCs, LLOCs, and MWPs possesses a high curvature, i.e. the LOCs of a linear scalarization are well below the weighted averages of its parts. This suggests that there are significant merits to balancing the trade-offs between different classes of LOCs by applying multi-objective optimization.*

For example, with an equal-weight linear scalarization of IP and CH pricing, we obtain prices that have less GLOCs and LLOCs than the average over IP and CH. This effect is particularly prominent for Min-MWP pricing: although the original price profile is fairly extreme, a linear scalarization causes the GLOCs and LLOCs to collapse very fast.[7]

Result 3. *Pure minimization of MWPs leads to very high GLOCs and LLOCs. Market participants have high incentives to deviate from the optimal outcome and congestion signals are flawed. Accounting for other classes of LOCs, e.g., by adding a small weight of LLOCs or GLOCs by means of linear scalarization, can significantly improve the outcome while retaining very low MWPs.*

However, finding the correct weight vector for an a priori pricing rule requires information on preferences, which may require some degree of exploration and thus poses an impediment in practice. In contrast, the proposed join IP $\vee$ Min-MWP does not require such preference elicitation.

Figure 1b contains price heatmaps (in $/MWh) along the different nodes and periods. The Min-MWP pricing without additional constraints leads to high price volatility and inadequate congestion signals. However, joining it with standard IP pricing retains the congestion signals almost perfectly and at the same time reduces the substantial MWPs that IP pricing implies.

6.2 ARPA-E Grid Optimization Competition Data

Apart from the IEEE RTS System, we test our pricing rules on the network and bid data provided for the ARPA-E Grid Optimization Competition Challenge 2.[8] This competition seeks the development of modern and scalable optimization techniques for solving complex power flow problems. To that end, they provide large-scale and realistic test sets of single-period power flow problems.

[7]We also tested linear scalarizations with a non-exact ELMP approximation of CH pricing, in order to simulate a situation when CH prices are not readily available. This provided similar results, albeit with slightly increased GLOCs due to the non-exactness of the approximation.

[8]See https://gocompetition.energy.gov/challenges/challenge-2 for further details.

For our purposes, we test the five different scenarios provided for an exemplary 617-node grid. We parameterize the DCOPF with the available data (with capped susceptance values to avoid numerical instability) and report the results of exemplary linear scalarizations and the join in the table below.

	617-I			617-II		
	GLOCs	MWPs	LLOCs	GLOCs	MWPs	LLOCs
CH	1393.93	856.66	1145.55	1878.76	1364.99	1273.89
IP	7690.75	4913.05	0.0	6826.01	3783.67	0.0
Min-MWP	830053.91	0.0	506335.11	611089.04	0.0	441164.65
0.9 IP + 0.1 Min-MWP	6029.08	3219.04	38.77	6275.68	3038.20	53.77
0.9 CH + 0.1 Min-MWP	1393.97	852.46	1145.59	1879.28	1359.99	1274.33
0.5 IP + 0.5 CH	1532.48	1169.44	871.31	2174.56	1722.28	819.39
IP ∨ Min-MWP	3990.84	749.79	1093.35	5055.31	1053.66	1222.65

	617-III			617-IV			617-V		
	GLOCs	MWPs	LLOCs	GLOCs	MWPs	LLOCs	GLOCs	MWPs	LLOCs
CH	2559.29	1800.84	783.26	1873.58	1288.15	1310.83	1877.00	1364.69	1273.05
IP	19027.58	3273.93	0.0	6822.54	3775.00	0.0	6824.22	3783.09	0.0
Min-MWP	671163.75	0.0	395002.33	667209.59	0.0	459423.61	675095.75	0.0	425703.24
0.9 IP + 0.1 Min-MWP	18215.44	2318.44	65.43	6525.53	3422.37	28.88	6274.43	3037.82	53.48
0.9 CH + 0.1 Min-MWP	2559.29	1800.84	783.26	1874.17	1282.66	1311.03	1877.50	1359.86	1273.47
0.5 IP + 0.5 CH	2485.78	1797.36	537.03	2164.45	1723.96	852.58	2172.80	1721.73	818.55
IP ∨ Min-MWP	18064.77	976.18	1039.22	4958.65	1007.88	1268.75	5055.28	1053.35	1222.04

Table 2. ARPA Grid Optimization Competition - GLOCs, MWPs, LLOCs [$]

Our results confirm the observations made for the IEEE RTS system. The lost opportunity costs implied by Min-MWP pricing are very high, yet adding a small weight of make-whole payments to IP or CH can already reduce MWPs significantly. IP prices can lead to large MWPs and GLOCs, which are reduced by linear scalarization and the join, leading to only a small increase in LLOCs. As discussed earlier, linear scalarization is sensitive to the weights which are difficult to set a priori. In contrast, IP ∨ Min-MWP requires no parameterization and trades off MWPs and congestion signals (by means of LLOCs) in a meaningful way. In our experiments on the ARPA-E dataset, the MWPs are always below those of the scalarization with different weights. The join also reduces GLOCs compared to IP and Min-MWP.

Result 4. *The join IP ∨ Min-MWP significantly reduces LLOCs compared to Min-MWP pricing and MWPs compared to IP pricing. It produces a better congestion signal than CH pricing, while exhibiting lower MWPs at the same time. GLOCs are higher than those of CH pricing but below those of Min-MWP and IP pricing.*

Compared to IP ∨ Min-MWP, CH prices compromise on MWPs and LLOCs (and thus congestion signals) to reduce GLOCs. Typically, CH prices are intractable. As indicated earlier, we chose a model formulation that allows to compute CH prices in polynomial time. For more complex valuation or cost functions, convex hull formulations are not readily available, rendering CH pricing intractable [Schiro et al., 2016]. In contrast, IP ∨ Min-MWP can always be computed in polynomial time and it is practically tractable even for large problem sizes.

7 CONCLUSIONS

Pricing in non-convex markets remains a fundamental problem in market design and economics. Motivated by electricity spot markets, we study a coupled market with non-convex valuation and cost functions. We introduce a version of the Welfare theorems for coupled markets that delineates those environments for which we can expect Walrasian equilibria to exist. In the absence of Walrasian equilibria that support the efficient allocation in non-convex markets, several heuristics have been proposed to calculate prices in a computationally scalable manner. We establish that existing pricing rules minimize certain classes of lost opportunity costs, which can cause substantial increases in other relevant lost opportunity costs.

Based on this trade-off, we propose to view the design of pricing rules in non-convex electricity markets as a multi-objective optimization problem. We analyze linear scalarization as a weighted sum of individual objective functions to derive a Pareto frontier of the different design goals. As this comes with certain practical limitations, i.e. the need for preference elicitation, we propose a novel and scalable pricing rule that requires no parameterization or fine-tuning. This join of the IP and Min-MWP pricing rule addresses current policy issues in U.S. and European electricity markets, i.e. minimizes side-payments and maintains adequate congestion signals. In contrast to Convex Hull pricing, the join can be computed efficiently. We apply our pricing rules to an exemplary electricity market and demonstrate the possibilities to capitalize on the upsides of several pricing rules. In view of recent concerns by regulators regarding increasing levels of MWPs with the IP pricing rule and the desire to maintain good congestion signals, the join provides a straightforward and easy-to-implement alternative for regulators. Apart from the practical relevance of the designed artifact, this paper contributes to the fundamental problem of pricing in non-convex markets.

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A BASICS OF CONVEX OPTIMIZATION

Definition A.1 (Convexity, closedness and properness). A function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ is called

(1) **convex** if for any $x, y \in \mathbb{R}^n$, for any $\lambda \in [0,1]$, $f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$,
(2) **closed** (or **lower semi-continuous**) if for any $x \in \mathbb{R}^n$, $\liminf_{y \to x} f(y) \geq f(x)$, and
(3) **proper** if $f(x) \neq -\infty$ for any $x \in \mathbb{R}^n$, and $f \neq \infty$ identically.

In turn, a set $S \subseteq \mathbb{R}^n$ is called convex if its characteristic function χ_S is convex, where $\chi_S(x) = 1$ if $x \in S$ and 0 otherwise.

A function f might not be closed or convex, but it is always possible to consider its convex closure. The **convex closure** of a function f, $\overline{conv}(f)$, is the pointwise maximum closed and convex function that underestimates it,

$$\overline{conv}(f)(x) = \max\{g(x) | g \leq f \text{ is closed and convex}\}.$$

A closed, proper and convex function f admits information on the change in the value of the function in response to small changes at each point in its domain. This quantity is given in the form of an *affine minorant*, and is the analogue of a gradient for differentiable functions.

Definition A.2 (Affine minorants and subgradients). A function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ is said to have an **affine minorant** if $\exists p \in \mathbb{R}^n, c \in \mathbb{R}, \forall x, f(x) \geq p^T x + c$. If $c = f(x^*) - p^T x^*$ for some $x^* \in \mathbb{R}^n$, i.e. if

$$f(x) \geq f(x^*) + p^T(x - x^*) \ \forall\, x \in \mathbb{R}^n,$$

then p is said to be a **subgradient** of f at x^*. The set of all subgradients of f at a given point x is called the **subdifferential** of f at x, and is denoted $\partial f(x)$.

For a function f that has an affine minorant, a transformation of f encodes information on the set of its affine minorants. This transformation is obtained by evaluation of a maximization problem.

Definition A.3. For a function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$, the **Legendre-Fenchel transformation** (or the **convex conjugate**) of f is the function

$$f^*(p) = \sup_{x \in \mathbb{R}^n} p^T x - f(x).$$

In particular, for a proper function f, $f^*(p) < \infty$ if and only if there exists c such that $p^T x + c$ is an affine minorant of f. In this case, f^* is closed, convex and proper. Furthermore, the biconjugate f^{**} then equals $\overline{conv}(f)$, the convex closure of f [Rockafellar, 2015]. Convex conjugation provides a connection between operations on functions $f, g : \mathbb{R}^n \to \overline{\mathbb{R}}$ and their conjugates f^*, g^*. We remark that for a function f taking values in a *primal* space, its conjugate f^* has its arguments in the corresponding *dual* space. For instance, in our economic setting, as we shall see in (4), if $f(x)$ corresponds to the costs a generator incurs for supplying x MWh then $f^*(p)$ is the maximum profit attainable by this generator given a price vector p – i.e. the generator's *indirect utility.*

The following propositions are standard in convex analysis, and we need them to prove the welfare theorems for coupled markets and in the analysis of the pricing rules we consider in this paper. The first proposition lists the rules for the algebraic manipulation of functions and their conjugates, which we employ to study the link between pricing rules and their associated convex market models.

Proposition A.4 (Calculus of convex conjugation [Rockafellar, 2015]). *Let $f, g : \mathbb{R}^n \to \overline{\mathbb{R}}$, $\alpha, \beta \in \mathbb{R}$ such that $\alpha > 0$, and $v \in \mathbb{R}^n$, then the following hold:*

(1) *(Translation by a vector) Addition of a linear function to f corresponds to a constant shift in the argument of its conjugate, $(f - v^T(\cdot))^* = f^*((\cdot) + v)$.*

(2) *(Addition of a number) Addition of a constant to f corresponds to subtracting the same constant from its conjugate, $(f+\beta)^* = f^* - \beta$.*
(3) *(Multiplication by a number) Rescaling f by some positive constant rescales both the magnitude and the argument of its conjugate, $(\alpha f)^* = \alpha f\left(\frac{(\cdot)}{\alpha}\right)$.*
(4) *(Convolution) Suppose the function $\omega : \mathbb{R}^n \to \overline{\mathbb{R}}$ is defined via a convolution of f and g, i.e.*

$$\omega(\sigma) = \min_{Ax+By=\sigma} f(x) + g(y)$$

for some $m \times n$ matrices A, B. Then for any $p \in \mathbb{R}^m$ the conjugate of ω is given by

$$\omega^*(p) = f^*(A^T p) + g^*(B^T p).$$

(5) *(Partial closure) The convex closure of the minimum of two functions f, g has as its conjugate the pointwise maximum of the conjugates f^*, g^*, $(\overline{conv}\min\{f,g\})^* = \max\{f^*, g^*\}$.*

Specifically, in the next section we will see that optimal market clearing subject to supply-demand equivalence is a maximization problem, which can be interpreted as the value of a welfare function defined by a convolution. Thus Proposition A.4.4 implies that the conjugate of this welfare function additively separates over participants' indirect utility functions. Pricing problems in practice turn out to be approximations of this conjugate, and the rest of Proposition A.4 allows us study their corresponding convex models.

The next proposition shows that if f is closed and convex, then its convex conjugate encodes information on its subgradients:

Proposition A.5 (Fenchel-Young (in)equality). *Suppose that f is closed, convex and proper. Then for any $p, x \in \mathbb{R}^n$,*

$$f^*(p) + f(x) \geq p^T x.$$

Moreover, the inequality holds with equality if and only if $p \in \partial f(x)$.

Thus for any x, $\partial f(x) = \text{argmax}_p\, p^T x - f^*(p)$. In particular, if $f(x)$ is given as the solution to a linear (minimization) program, then the subdifferential $\partial f(x)$ is a polyhedron whose form may be noted down explicitly.

We conclude this section by noting that the results above may be extended immediately to concave functions. A function f is called concave if $-f$ is convex, and **upper semi-continuous** if $-f$ is closed. The **concave closure** $\overline{conc}(f)$ is then simply $-\overline{conv}(-f)$, and the definitions of *affine majorants* and *supragradients* follow analogously to the discussion above.

B EXAMPLES FOR CONTRADICTING DESIGN GOALS

In this appendix, we provide examples to illustrate that CH pricing, IP pricing and Min-MWP pricing each optimize a class of LOCs, but fail to account for the other classes of LOCs. This implies significant trade-offs and motivates to view pricing in non-convex markets as a multi-objective optimization problem.

First, we show that a minimization of GLOCs through CH pricing does not necessarily correspond to low MWPs and LLOCs.

Example B.1. Consider a single-item market with two sellers in a single hour and two coupled locations. The first (second) seller is located at the first (second) node and has a minimum sales quantity of 2 (8) units, a maximum quantity of 15 (15) units, variable per-unit costs of \$1000 (\$1) and fixed costs of \$10 (\$10) when supplying a positive amount. At the first (second) node a fixed demand of 6 (1) units needs to be satisfied, and the line capacity is 4 in either direction. The optimal solution is for the first seller to supply the entire demand. In this case, 1 unit is transmitted from the first node to the second node, and there is no congestion in the network. We record prices, GLOCs, MWPs, LLOCs for the pricing rules under consideration in Table 3. While CH pricing minimizes GLOCs, it may very well imply large MWPs and LLOCs and a false congestion signal (by the price difference across the uncongested line).

	Price 1 [$/Unit]	Price 2 [$/Unit]	GLOCs [$]	MWPs [$]	LLOCs [$]
CH	1000.67	1.67	5000.33	1004.33	5000.33
IP	1000.00	1000.00	14985.00	10.00	0
Min-MWP	1001.43	1001.43	15007.86	0	11.43

Table 3. Example for CH pricing with large MWPs and LLOCs

To illustrate the reason for this discrepancy, we compute participants' preferences and the optimal allocation for (Primal CH). After convexification, the first (second) seller can supply up to 15 units with variable per-unit costs of $1000 + 2/3 ($1), while the fixed demand and line capacity are unchanged. Therefore, the optimal allocation is for the second seller to provide 5 units while the first seller provides 2 units, and 4 units are transmitted from the second node to the first. However, in the true optimal allocation the second seller could not be committed (due to its minimum quantity of 8), and 1 unit was transmitted in the opposite direction.

IP prices fix LLOCs to zero but discard any information on changing commitment. As a result, IP prices may fail to adequately account for GLOCs or MWPs.

Example B.2. Consider an single-item market with two sellers in a single hour and two coupled locations. The first (second) seller is located at the first (second) node and has a minimum sales quantity of 2 (8) units, a maximum quantity of 8 (15) units, variable per-unit costs of $1 ($10) and fixed costs of $100 ($100) when supplying a positive amount. At the first (second) node a fixed demand of 6 (1) units needs to be satisfied, and the line capacity is 4 in either direction. The optimal solution is for the first seller to supply the entire demand, and there is no congestion in the network. We record prices, GLOCs, MWPs, LLOCs for different pricing rules in Table 4. While IP pricing minimizes LLOCs, it may very well imply large GLOCs and MWPs.

	Price 1 [$/Unit]	Price 2 [$/Unit]	GLOCs [$]	MWPs [$]	LLOCs [$]
CH	13.5	13.5	12.5	12.5	12.5
IP	1	1	100	100	0
Min-MWP	15.29	15.29	14.29	0	14.29

Table 4. Example for IP pricing with large GLOCs and MWPs

Finally, Min-MWP directly minimizes the make-whole payments but loses all information on subgradients in the process. As a result, the computed prices will in general incur significant LLOCs and GLOCs.

Example B.3. Consider a single-item market with two sellers in a single hour and two coupled locations. The first (second) seller is located at the first (second) node and has a minimum sales quantity of 2 (8) units, a maximum quantity of 50 (15) units, variable per-unit costs of $10 ($10) and fixed costs of $1000 ($10) when supplying a positive amount. At the first node there is a fixed demand of 4 units, at the second node a buyer is willing to pay $50 per unit to consume up to 3 units, and the line capacity is 2 in either direction. The optimal solution is for the first seller to supply the entire demand at the first node and 2 units to the second node, and as a consequence there is congestion in the network. We record prices, GLOCs, MWPs, LLOCs for different pricing rules in Table 5. While Min-MWP pricing minimizes MWPs, it may very well imply large GLOCs and LLOCs and the congestion is not reflected in the prices as the price is identical at both nodes. In contrast, with IP pricing the price on node 2 is higher than on node 1, indicating correctly directed congestion.

	Price 1 [$/Unit]	Price 2 [$/Unit]	GLOCs [$]	MWPs [$]	LLOCs [$]
CH	30.00	10.67	996.67	918.67	996.67
IP	10.00	50.00	1590.00	1000.00	0
Min-MWP	176.67	176.67	10076.67	253.33	7586.67

Table 5. Example for Min-MWP pricing with large GLOCs and LLOCs

C ECONOMIC INTERPRETABILITY PROBLEM

In Section 5, we had mentioned that a dual pricing problem induces a convexified model for the underlying welfare maximization problem (2), and that a poor choice of multi-objective formulation for the dual pricing may expand the feasible region of the underlying welfare maximization problem, exaggerating the gap between the dual pricing and the primal allocation problems. This, in turn, may lead to a loss of *semantic meaning*, disallowing any meaningful economic interpretation. In this section of the electronic companion, we elaborate on this phenomenon. In order to do so, we first explain how to derive the corresponding convex model of a dual pricing problem by application of results from Section A. We then illustrate how severe distortions in the welfare maximization problem may occur by considering two commonly used multi-objective solution concepts.

To derive the welfare maximization problem corresponding to some dual pricing problem $\min_p \sum_{\ell \in L} \lambda_\ell(p|z_\ell^*)$, we note that for a convex market the dual pricing functions are defined as in (7), i.e.

$$\begin{aligned}
\lambda_b(p|x_b^*) &= (-v_b)^*(-p) + p^T x_b^* - v_b(x_b^*) && \forall\, b \in B,\\
\lambda_s(p|y_s^*) &= c_s^*(p) + c_s(y_s^*) - p^T y_s^* && \forall\, s \in S,\\
\lambda_r(p|f_r^*) &= d_r^*(B_r^T p) + d_r(f_r^*) - p^T B_r f_r^* && \forall\, r \in R.
\end{aligned}$$

This implies that, given a convex dual pricing function $\tilde{\lambda}_b(p|x_b^*)$ for a buyer b, denoting by $\tilde{v}_b$ the corresponding concave valuation function for buyer b,

$$(-\tilde{v}_b)^*(p) = \tilde{\lambda}_b(-p|x_b^*) + p^T x_b^* + v_b(x_b^*), \tag{10}$$

where $v_b(x_b^*)$ is the value buyer b has for allocation x_b^* in the *original* welfare maximization problem (2). Noting that for a closed convex function f the biconjugate equals the function itself, i.e. $f = f^{**}$, we conclude that

$$\begin{aligned}
\tilde{v}_b(x) &= -(\tilde{\lambda}_b(-p|x_b^*) + p^T x_b^* + v_b(x_b^*))^*(x) \\
&= -(\max_p p^T(x - x_b^*) - v_b(x_b^*) - \tilde{\lambda}_b(-p|x_b^*)) \\
&= -\tilde{\lambda}_b^*(x_b^* - x) + v_b(x_b^*).
\end{aligned} \tag{11}$$

By a similar analysis, the corresponding cost function for a seller s is given by

$$\tilde{c}_s(y) = \tilde{\lambda}_c^*(y - y_s^*) + c_s(y_s^*). \tag{12}$$

The induced cost functions of transmission operators, in turn, need to be more carefully defined due to the presence of matrices B_r which specify the interaction of flows with the supply-demand constraints. In this case, as a first attempt one may consider a modified conjugate function

$$\tilde{d}_r(f) = \max_p p^T B_r(f - f_r^*) + d_r(f_r^*) - \tilde{\lambda}_r(p|f_r^*).$$

However, this allows for transmission operator r a feasible flow of the form $f = f' + \delta$, where $B_r\delta = 0$ (i.e. δ is in the *null-space* $\text{null}(B_r)$ of B_r) and f' is orthogonal to δ. Such flows δ do not have any effect on the supply-demand balance, and can be considered to be infeasible. Therefore, to rectify this, we instead set

$$\tilde{d}_r(f) = \begin{cases} \max_p p^T B_r(f - f_r^*) + d_r(f_r^*) - \tilde{\lambda}_r(p|f_r^*) & \text{if } f^T\delta = 0, \forall\, \delta \in \text{null}(B_r), \\ +\infty & \text{else.} \end{cases} \tag{13}$$

Then by Proposition A.4.4, we note for the welfare function

$$\begin{aligned} \tilde{\omega}(\sigma) = \max_{x,y,f} & \sum_{b\in B} \tilde{v}_b(x_b) - \sum_{s\in S} \tilde{c}_s(y_s) - \sum_{r\in R} \tilde{d}_r(f_r) \\ \text{subject to} & \sum_{s\in S} y_s - \sum_{b\in B} x_b + \sum_{r\in R} B_r f_r = \sigma, \end{aligned}$$

the conjugate of the negative welfare function $(-\tilde{\omega})^*$ equals

$$(-\tilde{\omega})^*(p) = \sum_{\ell\in L} \tilde{\lambda}_\ell(p|z_\ell^*) + \omega(0). \tag{14}$$

The pricing problem for this convex market, given by the subgradient problem (5), is therefore precisely $\min_p \sum_{\ell\in L} \tilde{\lambda}_\ell(p|z_\ell^*)$.

To illustrate how the calculations work, we derive the convex model corresponding to (Min-MWP):

Example C.1. In (Min-MWP), each market participant ℓ has a dual pricing function $\lambda_\ell^{MWP}(p|z_\ell^*) = \max\{-u_\ell(z_\ell^*|p), 0\}$. For any buyer b,

$$u_b(x_b^*) = v_b(x_b^*) - p^T x_b^*.$$

We know that the convex model $\tilde{v}_b$ here satisfies (11), and by Proposition A.4.5, $[\lambda_b^{MWP}(\cdot)]^*(x) = \overline{conv}\min\{(-u_b(x_b^*|\cdot))^*, 0^*\}(x)$. The convex conjugate of the identical zero function is $\chi_{\{0\}}(x)$, the indicator function for the singleton set containing 0. Meanwhile, $(-u_b(x_b^*|\cdot))^*$ is given by

$$\begin{aligned} (-u_b(x_b^*|\cdot))^*(x) &= \max_p p^T x + u_b(x_b^*|p) \\ &= \max_p p^T(x - x_b^*) + v_b(x_b^*) \\ &= \chi_{\{x_b^*\}}(x) + v_b(x_b^*). \end{aligned}$$

Therefore, substituting for the expression (11), we get

$$-\tilde{v}_b(x) = \overline{conv}\min\{\chi_{\{0\}}(x) + v_b(x_b^*), \chi_{\{x_b^*\}}\} - v_b(x_b^*),$$

which then implies that

$$\tilde{v}_b(x) = \overline{conc}\max\{-\chi_{\{0\}}, -\chi_{\{x_b^*\}} + v_b(x_b^*)\}.$$

Specifically, buyer b's valuations are modelled as the convexification of the valuation they have for their allocation, and the valuation they have for not participating in the market. Likewise, for a seller s and a transmission operator r, we have

$$\tilde{c}_s(y) = \overline{conv}\min\{\chi_{\{0\}}, \chi_{\{y_s^*\}} + c_s(y_s^*)\},$$
$$\tilde{d}_r(f) = \overline{conv}\min\{\chi_{\{0\}}, \chi_{\{f_r^*\}} + d_r(f_r^*)\}.$$

Thus it is indeed possible to extract a convex welfare maximization problem from a given dual pricing problem, and the dual pricing problem prices the optimal outcome of this welfare maximization problem. The set of participants and goods of this welfare maximization problem might in fact greatly differ for this problem; we provide two examples.

Example C.2 (Penalty functions: Fictitious goods and agents). As mentioned previously, in non-convex markets a Walrasian equilibrium need not exist. In this case, there is no price vector p such that the conditions of Walrasian equilibria (3.2) are all satisfied. While we consider budget balance in linear payments, market participants' losses still need to be compensated. To finance these uplift payments, O'Neill et al. [2016] consider imposing personalized price vectors to participants. Specifically, their approach involves finding a set of participants who require make-whole payments, and then imposing personalized prices so that their uplifts are financed via the payments of other market participants. As a measure of fairness, they minimize the magnitude of these transfers.

Here, we analyze the distortionary effect of a similar pricing problem, stylized for simplicity of analysis. We impose personalized prices to the dual pricing problem (6), with a quadratic penalty term for the magnitude of differences in prices. Furthermore, we add a budget balance constraint. Then the resulting dual pricing problem is given by

$$\begin{aligned}\min_{(p_\ell)_{\ell\in L}} \quad & \sum_{\ell\in L}\lambda_\ell^{CH}(p_\ell|z_\ell^*) + \frac{1}{2}\sum_{(\ell,\ell')\in L^2}\|p_\ell - p_{\ell'}\|_2^2 \\ \text{subject to} \quad & \sum_{\ell\in L} p_\ell^T z_\ell^* = 0.\end{aligned}$$

We can then compute the associated welfare maximization problem for this dual pricing problem. The corresponding market has set of *personalized* goods and flow parameters $M\times L$, $F\times L$. We also add a fictitious exchange operator $(\ell,\ell')\in L^2$ for each pair of participants, who can exchange goods and flows in $\{\ell\}\times(M\cup F)$ with the corresponding goods and flows in $\{\ell'\}\times(M\cup F)$ at a one-to-one ratio. Finally, we add an auctioneer A who can provide any multiple of $(z_\ell^*)_{\ell\in L}$ to the market at cost 0, clearing personalized goods in proportion to the priced optimal outcome. The resulting welfare maximization problem is thus

$$\begin{aligned}\max_{x,y,f,z_A,(z_{(\ell,\ell')})_{(\ell,\ell')\in L^2}} \quad & \sum_{b\in B} v_b(x_b) - \sum_{s\in S} c_s(y_s) - \sum_{r\in R} d_r(f_r) - \frac{1}{2}\sum_{(\ell,\ell')\in L^2}\|z_{(\ell,\ell')}\|_2^2 \\ \text{subject to} \quad & x_b - z_A x_b^* - \sum_{\ell\in L} z_{(b,\ell)} - z_{(\ell,b)} = 0 \ \forall\, b\in B \\ & y_s - z_A y_s^* - \sum_{\ell\in L} z_{(s,\ell)} - z_{(\ell,s)} = 0 \ \forall\, s\in S \\ & f_r - z_A f_r^* - \sum_{\ell\in L} z_{(r,\ell)} - z_{(\ell,r)} = 0 \ \forall\, r\in R.\end{aligned}$$

The additional exchange operators may be thought of as participants who can take advantage of personalized price differences for arbitrage, while the auctioneer attempts to enforce optimal market clearing.

Example C.3 (Chebyshev scalarization: Consolidation of agents). To obtain a more balanced outcome with respect to the GLOCs of market participants, one might consider minimizing the (weighted) maximum of participants' GLOCs instead. Such a method is known as *Chebyshev scalarization* in the literature, where for objectives to $f_1, f_2, \ldots, f_n$ to be jointly minimized, one seeks a solution of

$$\min_{q \in \mathcal{F}} \max_{1 \leq i \leq n} \frac{f_i(q)}{w_i}.$$

Here, $(w_i)_{1 \leq i \leq n}$ are the weights on each objective f_i, signifying their relative importance, and $\mathcal{F}$ is the feasible region of the problem.

Let us consider implementing Chebyshev scalarization as a dual pricing problem, where each participant's GLOCs are scaled by the 2-norm of the participant's allocation. Intuitively, this minimizes the GLOCs incurred for each unit of good purchased. This leads us to consider the dual pricing problem

$$\min_p \max_{\ell \in L} \frac{\lambda_\ell^{CH}(p|z_\ell^*)}{\|z_\ell^*\|_2}.$$

However, this dual pricing problem is not additively separable over the market participants. Assuming the market does not have transmission operators for simplicity and by evaluating the convex conjugate of the dual pricing problem equals

$$\overline{conc} \max \left\{ \frac{v_b(\|x_b^*\|_2 \cdot (\sigma + x_b^*)) - v_b(x_b^*)}{\|x_b^*\|_2} \right\}_{b \in B} \cup \left\{ -\frac{c_s(\|y_s^*\|_2 \cdot (\sigma + y_s^*)) - c_s(y_s^*)}{\|y_s^*\|_2} \right\}_{s \in S}.$$

It does not seem possible, however, to transform this expression into a utilitarian welfare function for the original set of market participants. In fact, the expression does not appear to admit a straightforward interpretation as any meaningful welfare function.

By the examples above, we see that the addition of constraints and penalty functions lead to an addition of new agents in the primal welfare maximization problem, while combining participants' lost opportunity costs under a single term consolidates them into a single entity. We expect such drastic changes in the welfare maximization problem to widen the gap between the true optimal allocation and the allocation priced by the dual pricing problem, as in the case of CH pricing. Therefore, we would like to avoid such distortions if possible.

We note that the dimension M of the price vector $p \in \mathbb{R}^M$ is precisely the number of priced goods. This is because the welfare function ω takes as its argument supply-demand constraint violations in real goods, and prices are in the corresponding dual space. To maintain the number of goods, we must thus impose linear prices – as modifying the size of the price vector (e.g. by imposing personalized prices) modifies the number of goods in the corresponding welfare maximization problem.

To maintain the number of participants, note that by Proposition A.4.4 a welfare maximization problem with a set of agents L necessarily has an associated subgradient problem (5) that is additively separable over the participants. That is, for a welfare maximization problem of the form

$$\begin{aligned} \max_{x,y,f} \quad & \sum_{b \in B} \tilde{v}_b(x_b) - \sum_{s \in S} \tilde{c}_s(y_s) - \sum_{r \in R} \tilde{d}_r(f_r) \\ \text{subject to} \quad & \sum_{s \in S} y_s - \sum_{b \in B} x_b + \sum_{r \in R} B_r f_r = 0, \end{aligned}$$

the dual pricing problem is of the form $\min_p \sum_{\ell \in L} \tilde{\lambda}_\ell(p|z_\ell^*)$. The converse implication also holds and an additively separable dual pricing function leads to a welfare maximization problem with set of participants L. This leads us to observe that, to obtain a dual pricing problem with minimal distortion, we should restrict attention to dual pricing problems with an additively separable objective function (over L) and linear prices for each good.

D DCOPF PROBLEM

Sets		
B		Buyers
S		Sellers
$T = \{1, .., \overline{T}\}$		Time Periods
V		Nodes (with some $R^* \in V$ as reference node)
$N(v)$		Neighboring nodes of a node $v \in V$
β_b^t		Bids of buyer $b \in B$ in period $t \in T$
β_s^t		Bids of seller $s \in S$ in period $t \in T$
Mappings		
$\nu(b)$	$B \rightarrow V$	Mapping of buyer $b \in B$ to its node $v \in V$
$\nu(s)$	$S \rightarrow V$	Mapping of seller $s \in S$ to its node $v \in V$
Parameters		
B_{vw}	[pu]	Susceptance of the line connecting $v, w \in V$
$v_{bt\ell}$	[$/MWh]	Value of bid $\ell \in \beta_b^t$ of buyer $b \in B$ in period $t \in T$
$q_{bt\ell}$	[MWh]	Maximum quantity of bid $\ell \in \beta_b^t$ of buyer $b \in B$ in period $t \in T$
$\underline{P}_{bt}$	[MWh]	Price-inelastic demand of buyer $b \in B$ in period $t \in T$
$\overline{P}_{bt}$	[MWh]	Maximum demand of buyer $b \in B$ in period $t \in T$
$c_{st\ell}$	[$/MWh]	Cost of bid $\ell \in \beta_s^t$ of seller $s \in S$ in period $t \in T$
h_s	[$]	No-load costs of seller $s \in S$
$q_{st\ell}$	[MWh]	Maximum quantity of bid $\ell \in \beta_s^t$ of seller $s \in S$ in period $t \in T$
$\underline{P}_{st}$	[MWh]	Minimum output of seller $s \in S$ in period $t \in T$
$\overline{P}_{st}$	[MWh]	Maximum output of seller $s \in S$ in period $t \in T$
$\underline{R}_s$		Minimum uptime of seller $s \in S$
$\underline{F}_{vw}$	[MWh]	Minimum flow on the line connecting $v, w \in V$
$\overline{F}_{vw}$	[MWh]	Maximum flow on the line connecting $v, w \in V$
Primal Variables		
$x_{bt} \geq 0$	[MWh]	Consumption of buyer $b \in B$ in period $t \in T$
$x_{bt\ell} \geq 0$	[MWh]	Consumption of buyer $b \in B$ in period $t \in T$ regarding bid $\ell \in \beta_b^t$
$y_{st} \geq 0$	[MWh]	Generation of seller $s \in S$ in period $t \in T$
$y_{st\ell} \geq 0$	[MWh]	Generation of seller $s \in S$ in period $t \in T$ regarding bid $\ell \in \beta_s^t$
$u_{st} \in \{0, 1\}$		Commitment of seller $s \in S$ in period $t \in T$
$\phi_{st} \geq 0$		Start-up indicator for seller $s \in S$ in period $t \in T$
$\alpha_{vt} \in \mathbb{R}$	[rad]	Voltage angle at node $v \in V$ in period $t \in T$
$f_{vwt} \in \mathbb{R}$	[MWh]	Flow on the line connecting $v, w \in V$ in period $t \in T$
Dual Variables		
$p_{vt} \in \mathbb{R}$	[$/MWh]	Price at node $v \in V$ in period $t \in T$
$\gamma_{vwt} \in \mathbb{R}$	[$/MWh]	Congestion price for the line connecting $v, w \in V$ in period $t \in T$
$r_t \in \mathbb{R}$		Dual of the reference node voltage angle constraint in period $t \in T$

Table 6. DCOPF Notation

$$\max \quad \sum_{b\in B}\sum_{t\in T}\sum_{\ell\in\beta_b^t} v_{bt\ell}x_{bt\ell} - \sum_{s\in S}\sum_{t\in T}\sum_{\ell\in\beta_s^t} c_{st\ell}y_{st\ell} - \sum_{s\in S}\sum_{t\in T} h_s u_{st} \qquad \text{(DCOPF-MILP)}$$

$$\text{subject to} \quad x_{bt\ell} \in [0, q_{bt\ell}] \qquad \forall b \in B, t \in T, \ell \in \beta_b^t$$

$$x_{bt} - \sum_{\ell\in\beta_b^t} x_{bt\ell} = \underline{P}_{bt} \qquad \forall b \in B, t \in T$$

$$x_{bt} \le \overline{P}_{bt} \qquad \forall b \in B, t \in T$$

$$y_{st\ell} \in [0, q_{st\ell}] \qquad \forall\, s \in S, t \in T, \ell \in \beta_s^t$$

$$y_{st} - \sum_{\ell\in\beta_s^t} y_{st\ell} = 0 \qquad \forall\, s \in S, t \in T$$

$$y_{st} - \underline{P}_{st}u_{st} \ge 0 \qquad \forall\, s \in S, t \in T$$

$$y_{st} - \overline{P}_{st}u_{st} \le 0 \qquad \forall\, s \in S, t \in T$$

$$\phi_{st} - u_{st} + u_{s(t-1)} \ge 0 \qquad \forall\, s \in S, 1 < t \le T$$

$$\sum_{i=t-R_s+1}^{t} \phi_{si} - u_{st} \le 0 \qquad \forall\, s \in S, 1 < t \le T$$

$$f_{vwt} \in [\underline{F}_{vw}, \overline{F}_{vw}] \forall\, v \in V, w \in N(v), t \in T$$

$$f_{vwt} - B_{vw}(\alpha_{vt} - \alpha_{wt}) = 0 \qquad \forall\, v \in V, w \in N(v), t \in T$$

$$\sum_{s:v(s)=v} y_{st} - \sum_{b:v(b)=v} x_{bt} - \sum_{w\in N(v)} f_{vwt} = 0 \qquad \forall\, v \in V, t \in T$$

$$\alpha_{R^*t} = 0 \qquad \forall\, t \in T$$

E ELMP PRICING PROBLEM

The continuous relaxation of the binary integer constraints in the welfare maximization problem provide an LP. The associated dual pricing problem with this relaxation of the welfare maximization problem is the pricing rule known as ELMP [MISO, 2019]. Furthermore, the class of valuation / cost functions we consider are such that the optimal solutions to the dual LP provide CH prices [Hua and Baldick, 2017].

Therefore, to formulate the Convex Hull pricing problem in our setting we consider the dual LP to the DCOPF where for each seller s and each time period t, the binary integer constraints are relaxed $u_{st} \in [0,1]$. Furthermore, we add constants to the dual objective function which do not alter the solution sets. However, the addition of these constants allows the objective function value to be sum of GLOCs. The resulting dual LP is given as:

$$\min_{\overline{\epsilon},\overline{\psi},\overline{\chi}\geq 0,\underline{\epsilon},\underline{\psi},\underline{\chi},\hat{\chi}\leq 0,p,\gamma,r,\lambda,\epsilon} \quad \sum_{b\in B}\lambda_b + \sum_{s\in S}\lambda_s + \sum_{v\in V,w\in N(v),t\in T}\lambda_{vwt} \tag{ELMP-LP}$$

$$\text{subject to} \quad \lambda_b - \sum_{t\in T}\left[\epsilon_{bt}\underline{P}_{bt} + \overline{\epsilon}_{bt}\overline{P}_{bt} + \sum_{\ell\in v_b^t}\overline{\epsilon}_{bt\ell}q_{bt\ell}\right] + v_b(x_b^*) - p_{\nu(b)}^T x_b^* \geq 0 \qquad \forall\, b\in B$$

$$\lambda_s - \sum_{t\in T}\left[\overline{\psi}_{st} + \sum_{\ell\in v_s^t}\overline{\epsilon}_{st\ell}q_{st\ell}\right] + p_{\nu(s)}^T y_s^* - c_s(y_s^*,u_s^*) \geq 0 \qquad \forall\, s\in S$$

$$\lambda_{vwt} - \overline{\epsilon}_{vwt}\overline{F}_{vw} - \underline{\epsilon}_{vwt}\underline{F}_{vw} + \gamma_{vwt}f_{vwt}^* \geq 0 \qquad \forall\, v\in V, w\in N(v), t\in T$$

$$\sum_{w|v\in N(w)} B_{wv}(p_{wt}+\gamma_{wvt}) - \sum_{w\in N(v)} B_{vw}(p_{vt}+\gamma_{vwt}) = 0 \qquad \forall\, v\in V\setminus\{R^*\}, t\in T$$

$$r_t + \sum_{w|R^*\in N(w)} B_{wR^*}(p_{wt}+\gamma_{wR^*t}) - \sum_{w\in N(R^*)} B_{R^*w}(p_{R^*t}+\gamma_{R^*wt}) = 0 \qquad \forall\, t\in T$$

$$-\gamma_{vwt} + \overline{\epsilon}_{vwt} + \underline{\epsilon}_{vwt} = 0 \qquad \forall\, v\in V, w\in N(v), t\in T$$

$$\overline{\epsilon}_{bt\ell} + \underline{\epsilon}_{bt\ell} - \epsilon_{bt} = v_{bt\ell} \qquad \forall\, b\in B, t\in T, \ell\in v_b^t$$

$$\epsilon_{bt} + \overline{\epsilon}_{bt} + p_{\nu(b)t} = 0 \qquad \forall\, b\in B, t\in T$$

$$\overline{\epsilon}_{st\ell} + \underline{\epsilon}_{st\ell} - \epsilon_{st} = -c_{st\ell} \qquad \forall\, s\in S, t\in T, \ell\in v_s^t$$

$$\epsilon_{st} + \underline{\epsilon}_{st} + \overline{\epsilon}_{st} - p_{\nu(s)t} = 0 \qquad \forall\, s\in S, t\in T$$

$$-\sum_{\ell\in v_s^1} q_{s1\ell}\overline{\epsilon}_{s1\ell} + \overline{\psi}_{s1} + \underline{\psi}_{s1} - \overline{P}_{s1}\overline{\epsilon}_{s1} - \underline{P}_{s1}\underline{\epsilon}_{s1} + \underline{\chi}_{s2} = -h_{s1}, \qquad \forall\, s\in S$$

$$-\sum_{\ell\in v_s^t} q_{st\ell}\overline{\epsilon}_{st\ell} + \overline{\psi}_{st} + \underline{\psi}_{st} - \overline{P}_{st}\overline{\epsilon}_{st} - \underline{P}_{st}\underline{\epsilon}_{st} - \overline{\chi}_{st} - \underline{\chi}_{st} + \underline{\chi}_{s(t+1)} = -h_{st} \qquad \forall s\in S, 1<t<T$$

$$-\sum_{\ell\in v_s^T} q_{sT\ell}\overline{\epsilon}_{sT\ell} + \overline{\psi}_{sT} + \underline{\psi}_{sT} - \overline{P}_{sT}\overline{\epsilon}_{sT} - \underline{P}_{sT}\underline{\epsilon}_{sT} - \overline{\chi}_{sT} - \underline{\chi}_{sT} = -h_{sT} \qquad \forall\, s\in S$$

$$\hat{\chi}_{st} + \underline{\chi}_{st} + \sum_{t'=t}^{\min\{\overline{T},t+\underline{R}_s-1\}} \overline{\chi}_{st} = 0 \qquad \forall\, s\in S, 1<t\leq T.$$

F IP PRICING PROBLEM

Note that in our setting, an optimal allocation $\left((x_b^*)_{b\in B}, (y_s^*)_{s\in S}, (f_r^*)_{r\in R}\right)$ fixes the binary integer variables u_{st}^* which indicate whether seller s is generating at period t. IP pricing then restricts the primal DCOPF problem by adding the constraints $u_{st} = u_{st}^*\ \forall\, s \in S, t \in T$. The resulting welfare maximization problem is an LP, and the dual LP provides prices which minimize LLOCs. Again, we modify the dual objective function to be the sum of LLOCs by adding constants that do not affect the optimal solution. In our setting, this dual LP is given as:

$$\min_{\overline{\epsilon}\geq 0, \underline{\epsilon}\leq 0, p,\gamma,r,\lambda,\epsilon} \quad \sum_{b\in B}\lambda_b + \sum_{s\in S}\lambda_s + \sum_{v\in V, w\in N(v), t\in T}\lambda_{vwt} \tag{IP-LP}$$

$$\begin{aligned}
\text{subject to}\quad & \lambda_b - \sum_{t\in T}\left[\epsilon_{bt}\underline{P}_{bt} + \overline{\epsilon}_{bt}\overline{P}_{bt} + \sum_{\ell\in v_b^t}\overline{\epsilon}_{bt\ell}q_{bt\ell}\right] + v_b(x_b^*) - p_{\nu(b)}^T x_b^* \geq 0 \qquad \forall\, b \in B \\
& \lambda_s - \sum_{t\in T}\left[\underline{\epsilon}_{st}\underline{P}_{st}u_{st}^* + \overline{\epsilon}_{st}\overline{P}_{st}u_{st}^* + \sum_{\ell\in v_b^t}\overline{\epsilon}_{st\ell}q_{st\ell}\right] + p_{\nu(s)}^T y_s^* - c_s(y_s^*, u_s^*) \geq -h^T u_s^* \qquad \forall\, s \in S \\
& \lambda_{vwt} - \overline{\epsilon}_{vwt}\overline{F}_{vw} - \underline{\epsilon}_{vwt}\underline{F}_{vw} + \gamma_{vwt}f_{vwt}^* \geq 0 \qquad \forall\, v \in V, w \in N(v), t \in T \\
& \sum_{w | v\in N(w)} B_{wv}(p_{wt} + \gamma_{wvt}) - \sum_{w\in N(v)} B_{vw}(p_{vt} + \gamma_{vwt}) = 0 \qquad \forall\, v \in V \setminus \{R^*\}, t \in T \\
& r_t + \sum_{w | R^*\in N(w)} B_{wR^*}(p_{wt} + \gamma_{wR^*t}) - \sum_{w\in N(R^*)} B_{R^*w}(p_{R^*t} + \gamma_{R^*wt}) = 0 \qquad \forall\, t \in T \\
& -\gamma_{vwt} + \overline{\epsilon}_{vwt} + \underline{\epsilon}_{vwt} = 0 \qquad \forall\, v \in V, w \in N(v), t \in T \\
& \overline{\epsilon}_{bt\ell} + \underline{\epsilon}_{bt\ell} - \epsilon_{bt} = v_{bt\ell} \qquad \forall\, b \in B, t \in T, \ell \in v_b^t \\
& \epsilon_{bt} + \overline{\epsilon}_{bt} + p_{\nu(b)t} = 0 \qquad \forall\, b \in B, t \in T \\
& \overline{\epsilon}_{st\ell} + \underline{\epsilon}_{st\ell} - \epsilon_{st} = -c_{st\ell} \qquad \forall\, s \in S, t \in T, \ell \in v_s^t \\
& \epsilon_{st} + \underline{\epsilon}_{st} + \overline{\epsilon}_{st} - p_{\nu(s)t} = 0 \qquad \forall\, s \in S, t \in T.
\end{aligned}$$

G MIN-MWP PRICING PROBLEM

As mentioned before, while a direct implementation of (Min-MWP) is possible, solutions to the resulting dual pricing problem may result in phase angle operators having infinite LLOCs / GLOCs. To rectify this issue, in our implementation of (Min-MWP) we still account for GLOCs of phase angle operators. This is achieved by including the dual constraints associated with primal variables α_{vt} for $v \in V, t \in T$, and the resulting dual pricing problem is given as:

$$\min_{p,\gamma,r,\lambda} \quad \sum_{b\in B}\lambda_b + \sum_{s\in S}\lambda_s + \sum_{v\in V, w\in N(v), t\in T}\lambda_{vwt} \tag{Min-MWP-LP}$$

$$\begin{aligned}
\text{subject to}\quad & -v_b(x_b^*) + p_{\nu(b)}^T x_b^* - \lambda_b \leq 0\ \forall\, b \in B \\
& -p_{\nu(s)}^T y_s^* + c_s(y_s^*, u_s^*) - \lambda_s \leq 0\ \forall\, s \in S \\
& -\gamma_{vwt}f_{vwt}^* - \lambda_{vwt} \leq 0\ \forall\, v \in V, w \in N(v), t \in T \\
& \sum_{w | v\in N(w)} B_{wv}(p_{wt} + \gamma_{wvt}) - \sum_{w\in N(v)} B_{vw}(p_{vt} + \gamma_{vwt}) = 0\ \forall\, v \in V \setminus \{R^*\}, t \in T \\
& r_t + \sum_{w | R^*\in N(w)} B_{wR^*}(p_{wt} + \gamma_{wR^*t}) - \sum_{w\in N(R^*)} B_{R^*w}(p_{R^*t} + \gamma_{R^*wt}) = 0\ \forall\, t \in T.
\end{aligned}$$

H IP ∨ MIN-MWP PRICING PROBLEM

As discussed, the join of IP and Min-MWP pricing considers each participant's maximum of the IP and Min-MWP dual pricing function. For the DCOPF, we need not introduce new decision variables and the addition of only $|B \cup S|$ constraints to IP-LP is sufficient. Then, by choice of the constraints, each participant's contribution to the objective function λ_ℓ will equal the maximum of their MWPs and LLOCs. The dual LP is given as:

$$\min_{\overline{\epsilon}\geq 0,\underline{\epsilon}\leq 0,p,\gamma,r,\lambda,\epsilon} \quad \sum_{b\in B}\lambda_b + \sum_{s\in S}\lambda_s + \sum_{v\in V, w\in N(v), t\in T} \lambda_{vwt} \qquad \text{((IP} \vee \text{Min-MWP)-LP)}$$

$$\begin{aligned}
\text{subject to} \quad & \lambda_b - \sum_{t\in T}\left[\epsilon_{bt}\underline{P}_{bt} + \overline{\epsilon}_{bt}\overline{P}_{bt} + \sum_{\ell\in v_b^t}\overline{\epsilon}_{bt\ell}q_{bt\ell}\right] + v_b(x_b^*) - p_{v(b)}^T x_b^* \geq 0 \qquad \forall\, b \in B \\
& v_b(x_b^*) - p_{v(b)}^T x_b^* + \lambda_b \geq 0 \qquad \forall\, b \in B \\
& \lambda_s - \sum_{t\in T}\left[\underline{\epsilon}_{st}\underline{P}_{st}u_{st}^* + \overline{\epsilon}_{st}\overline{P}_{st}u_{st}^* + \sum_{\ell\in v_b^t}\overline{\epsilon}_{st\ell}q_{st\ell}\right] + p_{v(s)}^T y_s^* - c_s(y_s^*, u_s^*) \geq -h^T u_s^* \qquad \forall\, s \in S \\
& p_{v(s)}^T y_s^* - c_s(y_s^*, u_s^*) + \lambda_s \geq 0 \qquad \forall\, s \in S \\
& \lambda_{vwt} - \overline{\epsilon}_{vwt}\overline{F}_{vw} - \underline{\epsilon}_{vwt}\underline{F}_{vw} + \gamma_{vwt}f_{vwt}^* \geq 0 \qquad \forall\, v \in V, w \in N(v), t \in T \\
& \sum_{w|v\in N(w)} B_{wv}(p_{wt} + \gamma_{wvt}) - \sum_{w\in N(v)} B_{vw}(p_{vt} + \gamma_{vwt}) = 0 \qquad \forall v \in V \setminus \{R^*\}, dt \in T \\
& r_t + \sum_{w|R^*\in N(w)} B_{wR^*}(p_{wt} + \gamma_{wR^*t}) - \sum_{w\in N(R^*)} B_{R^*w}(p_{R^*t} + \gamma_{R^*wt}) = 0 \qquad \forall\, t \in T \\
& -\gamma_{vwt} + \overline{\epsilon}_{vwt} + \underline{\epsilon}_{vwt} = 0 \qquad \forall\, v \in V, w \in N(v), t \in T \\
& \overline{\epsilon}_{bt\ell} + \underline{\epsilon}_{bt\ell} - \epsilon_{bt} = v_{bt\ell} \qquad \forall\, b \in B, t \in T, \ell \in v_b^t \\
& \epsilon_{bt} + \overline{\epsilon}_{bt} + p_{v(b)t} = 0 \qquad \forall\, b \in B, t \in T \\
& \overline{\epsilon}_{st\ell} + \underline{\epsilon}_{st\ell} - \epsilon_{st} = -c_{st\ell} \qquad \forall\, s \in S, t \in T, \ell \in v_s^t \\
& \epsilon_{st} + \underline{\epsilon}_{st} + \overline{\epsilon}_{st} - p_{v(s)t} = 0 \qquad \forall\, s \in S, t \in T.
\end{aligned}$$

5. Part III: Pricing and Optimal Power Flow Problems

Peer-Reviewed Journal Paper

Title: Getting Prices Right on Electricity Spot Markets: On the Economic Impact of Advanced Power Flow Models.

Authors: Martin Bichler, Johannes Knörr.

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Abstract: As the share of variable renewable energy increases, adequate prices on electricity spot markets become increasingly important as they set signals for scarcity, investment, or demand response. Market prices are derived from the underlying welfare maximization problem. On electricity spot markets, this optimization problem is based on the non-convex and non-linear Alternating Current Optimal Power Flow (ACOPF) model. Since the ACOPF is intractable, electricity markets around the world use a linear approximation, the Direct Current Optimal Power Flow (DCOPF) model. Recent research has led to better non-linear relaxations of the ACOPF. We show that these non-linear relaxations increase welfare and imply significantly lower redispatch costs and side-payments. Most importantly, we show that the price signals obtained from non-linear relaxations are much improved. The DCOPF often yields high price differences between nodes when there is no line congestion in the AC-feasible solution or vice versa. Such biased price signals pose a significant problem in practice as they lead to inefficient demand response, distorted investment signals, and incorrect congestion incomes. The use of non-linear relaxations mitigates this problem and provides an important advantage of the resulting prices over prices based on the DCOPF.

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Getting prices right on electricity spot markets: On the economic impact of advanced power flow models☆

Martin Bichler, Johannes Knörr *

School of Computation, Information and Technology, Technical University of Munich, Boltzmannstrasse 3, Garching, 85748, Germany

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ABSTRACT

As the share of variable renewable energy increases, adequate prices on electricity spot markets become increasingly important as they set signals for scarcity, investment, or demand response. Market prices are derived from the underlying welfare maximization problem. On electricity spot markets, this optimization problem is based on the non-convex and non-linear Alternating Current Optimal Power Flow (ACOPF) model. Since the ACOPF is intractable, electricity markets around the world use a linear approximation, the Direct Current Optimal Power Flow (DCOPF) model. Recent research has led to better non-linear relaxations of the ACOPF. We show that these non-linear relaxations increase welfare and imply significantly lower redispatch costs and side-payments. Most importantly, we show that the price signals obtained from non-linear relaxations are much improved. The DCOPF often yields high price differences between nodes when there is no line congestion in the AC-feasible solution or vice versa. Such biased price signals pose a significant problem in practice as they lead to inefficient demand response, distorted investment signals, and incorrect congestion incomes. The use of non-linear relaxations mitigates this problem and provides an important advantage of the resulting prices over prices based on the DCOPF.

1. Introduction

Electricity spot markets collect bids from buyers and sellers and solve a welfare maximization problem to determine the optimal economic dispatch and prices. With an accurate representation of the transmission network and the underlying physics, this can be described as a non-linear and non-convex optimization problem referred to as Alternating Current Optimal Power Flow (ACOPF) problem (Molzahn and Hiskens, 2019). This ACOPF is computationally intractable for the problem sizes that we observe in real-world electricity markets. As a result, linearized network models are used for market clearing and pricing. The standard linearized model is known as Direct Current Optimal Power Flow (DCOPF) problem. The DCOPF or variations thereof are used in most jurisdictions today, but an optimal solution for DCOPF is generally neither AC-optimal nor AC-feasible. This requires market operators or transmission system operators (TSOs) to adjust the dispatch after the market clearing to reach a physically feasible outcome.

This issue is magnified by the transition to renewable energy sources (Lété et al., 2022). A decreasing amount of thermal generators can supply reactive power and the consideration of reactive power on the transmission level is therefore crucial (Hemmati et al., 2013; Karmakar and Bhattacharyya, 2020). Larrahondo et al. (2021) found that high integration of wind power contributes to the inaccuracies of the DCOPF, e.g., by disregarding reactive power. There have been significant efforts to obtain tighter and more accurate relaxations of the ACOPF problem in an effort to leverage recent advances in convex optimization for real-world markets, which culminated in the ARPA-E Grid Optimization Competition.[1] However, errors in the optimization models due to linearization or simplifying assumptions remain a concern in virtually all electricity markets. For example, the revision of the European Capacity Allocation and Congestion Management (CACM) regulation requires accounting for “linearization errors” while calculating available capacities for trading (ACER, 2021). A key concern of linearized models is the welfare loss arising from a poor approximation of ACOPF.

In recent years, substantial research has been devoted to finding tighter convex relaxations and approximations for the ACOPF problem (Molzahn and Hiskens, 2019). This research is driven by advances in convex non-linear optimization algorithms. A goal of the ARPA-E Grid Optimization Challenge is to develop algorithms for the next

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* Corresponding author.
E-mail addresses: bichler@cit.tum.de (M. Bichler), knoerr@cit.tum.de (J. Knörr).

[1] See https://gocompetition.energy.gov/ for further details.

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generation of solvers providing tighter relaxations of ACOPF and to better reflect the physics of the power grid. This research focuses on the optimality of the solution and computational costs to obtain it.

However, even if one could solve ACOPF to optimality, getting adequate price signals is challenging. Competitive equilibrium (aka. Walrasian) prices in economic theory leverage duality theory in convex optimization. Walrasian prices in a convex market are such that no participant would want to deviate from the efficient outcome (aka. envy-freeness), and the market is budget-balanced. This means that market operators neither need to subsidize the market, nor do they make a profit. Unfortunately, a well-known problem in economics is that Walrasian prices generally exist only when the allocation problem is a convex optimization problem (Bichler et al., 2020). Many real-world markets are based on non-convex optimization problems with electricity spot markets as a prime example. Such non-convexities can even affect transmission network expansion planning (García-Cerezo et al., 2021)

The literature on electricity market pricing developed a number of proposals how to price electricity in the presence of such non-convexities (Liberopoulos and Andrianesis, 2016). The most prominent examples are Integer Programming (IP) pricing (O'Neill et al., 2005) and Convex Hull (CH) pricing (Gribik et al., 2007; Hogan and Ring, 2003; Bichler et al., 2023). IP pricing is used in a number of U.S. markets and so are versions of CH pricing. Both pricing rules require side-payments by the market operator to compensate losses of market participants (make-whole payments, MWPs) or even to compensate all incentives to deviate from the efficient allocation (lost opportunity costs, LOCs, for buyers and sellers; potential congestion revenue shortfalls, PCRS, for TSOs). However, the high MWPs are a concern of the U.S. Federal Energy Regulatory Commission (FERC).[2] The U.S. FERC also found that the size and concentration of side-payments can be affected by the inability to accurately model ACOPF, suggesting that the benefit-to-cost ratio of more accurate power flow representations is at least 100-fold (Cain and O'Neill, 2012).

We focus on the improvement of price signals we can expect from better relaxations of ACOPF. Ideally, prices reflect marginal costs at a network node to provide adequate incentives for investment or demand reduction to aid overall system stability (Herrero et al., 2015). If DCOPF does not adequately reflect the physics of the power grid, the prices retrieved from the DCOPF solution will not be accurate. Thus, tighter relaxations of ACOPF could lead to prices that better reflect scarcity in the physical network. However, the magnitude of these improvements is unclear. So far, only very few articles study pricing based on non-linear convex relaxations of ACOPF (Ndrio et al., 2019; Papavasiliou, 2018; Garcia et al., 2020; Winnicki et al., 2020). All of these papers focus on small networks with a few nodes and they assume convex cost functions of sellers and fixed demand of buyers. Non-convexities of sellers' cost or buyers' valuation functions are important and have been the central problem that sparked the literature on electricity market pricing (Liberopoulos and Andrianesis, 2016; Zoltowska, 2016; Kuang et al., 2019).

We study the impact of non-linear convex market clearing models on prices, MWPs, LOCs, and PCRS. In contrast to prior work, we consider non-convexities in the preferences of buyers and sellers and analyze realistic problem sizes, which should provide a good estimate of the prices and welfare gains we can expect from tighter power flow relaxations in the field. More precisely, we consider the linearized DCOPF, a second-order conic (SOC) relaxation, and a quadratic convex (QC) relaxation (Molzahn and Hiskens, 2019).[3] For each of these relaxations, we compute Integer Programming (IP) prices (O'Neill et al., 2005) and Convex Hull (CH) prices (Gribik et al., 2007; MISO, 2023; Hua and Baldick, 2017) as the two pricing rules primarily used in U.S. markets.[4] To our knowledge, this is the first paper to simultaneously consider non-convexities arising from power flow models as well as non-convexities arising from bidding formats. We leverage the large problem instances from the ARPA-E Grid Challenge II[5] as they were designed to reflect real-world problems.

The results of our extensive numerical experiments show that different power flow models lead to substantial differences in the allocation. This of course has an impact on both welfare and prices. If the dispatch computed on the market is not AC-feasible, redispatch is required. Therefore, we report the costs of redispatch as well as welfare, MWPs, LOCs, and PCRS after computing a near AC-feasible solution.

As one would expect, tighter convex relaxations require substantially less redispatch compared to the standard DCOPF approximation, and MWPs, LOCs and PCRS are, on average, lower for the final AC-feasible outcome. Moreover, the tighter convex relaxations lead to higher welfare of the final dispatch. However, welfare gains are not the most important argument for tighter convex relaxations of ACOPF. Our numerical experiments show that the prices obtained from DCOPF are often substantially different from those of the non-linear relaxations, leading to biased scarcity signals that distort effective demand response and investment decisions. In contrast, the results of the SOC and QC relaxation, which both model reactive power and line losses more accurately, are almost identical. Prices obtained from DCOPF might be excessively high at some of the nodes, even though there is no congestion in the AC-feasible solution. In other words, DCOPF leads to unnecessary price peaks at some of the nodes, with prices being multiples of the average price, even though there is no congestion in the physical grid at all. Similarly, we might encounter congestion in the AC-feasible solution that the DC prices of adjacent nodes do not reflect. Overall, DCOPF can expose some of the market participants to unjustifiably high prices and further leads to inefficient demand response, biased investment signals, and wrong congestion incomes. In a world with 100% renewables, where adequate demand response is even more important, this is a decisive disadvantage of standard DCOPF solutions compared to tighter non-linear relaxations.

2. Power flow models and electricity pricing

In this section, we describe the setup of the electricity market we consider. We outline the optimization problem to be solved by the market operator and discuss competitive equilibrium theory with respect to pricing on non-convex markets.

2.1. Setup

We consider a coupled electricity market with a set of buyers (consumers) B and a set of sellers (generators) S of electricity located at interlinked nodes N in an electricity grid.[6] The set of power lines L is encoded by the pairs of nodes (i,k) that are directly connected by lines. We consider a single period such that the set of goods corresponds to units of (real or reactive) power at the locations N. Each buyer $b \in B$ possesses a valuation function $v_b : \mathbb{C}^{|N|} \to \mathbb{R}$ and each seller $s \in S$ possesses a cost function $c_s : \mathbb{C}^{|N|} \to \mathbb{R}$ that encodes feasibility constraints and preferences.

A market operator collects buy and sell bids and provides a feasible allocation (x, y) as well as locational prices λ. The variables $x \in \mathbb{C}^{|N|\times|B|}$

[2] See https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation for further details.

[3] We also considered a semidefinite programming (SDP) relaxation, but it was not possible to solve large mixed-integer SDPs to optimality for the problem sizes at hand.

[4] CH pricing is currently used for fast-start pricing by MISO and ISO-NE (MISO, 2023; PJM, 2018).

[5] See https://gocompetition.energy.gov/challenges/challenge-2 for further details and to access the data.

[6] One may aggregate several nodes into zones as it is done in European markets, but we assume a fully nodal setting.

(with column vectors $x_b \in \mathbb{C}^{|N|}$) and $y \in \mathbb{C}^{|N|\times|S|}$ (with column vectors $y_s \in \mathbb{C}^{|N|}$) describe the allocated bundles to buyers and sellers, respectively. In essence, the market operator is faced with the following welfare maximization problem:

$$\max_{x,y} \sum_{b\in B} v_b(x_b) - \sum_{s\in S} c_s(y_s) \tag{1}$$
$$\text{subject to} \quad x, y \in \Psi.$$

Here, Ψ describes the set of feasible power flows. In other words, the market operator seeks the welfare-maximizing allocation that ensures feasible power flows with strict supply–demand equivalence at each node. However, with an accurate representation of the underlying physics, Ψ is highly non-linear and non-convex, and solving such problems to optimality is not viable from a practical perspective. Therefore, various relaxations and approximations have been introduced that reduce Ψ to a convex set. We discuss possible power flow models in Section 2.2.

Given an optimal solution (x^*, y^*), the market operator proceeds by determining adequate prices $\lambda \in \mathbb{R}^{|N|}$ (we assume non-negative prices for real power and prices of zero for reactive power). However, by design of the bidding formats, valuation functions v_b and cost functions c_s are typically non-convex. In this case, Walrasian equilibrium prices do not exist and market operators need to resort to non-uniform prices and consider incentives to deviate. We provide an overview of these issues in Section 2.3. It should be noted that some market operators (e.g., in European markets) deviate from the welfare-maximizing allocation in order to ensure better price properties. Since this comes with computational challenges (All NEMO Committee, 2022), we focus on pricing optimal outcomes in this paper.

Both the complexity of power flows and of pricing in non-convex markets has been studied extensively, but typically separated. Practical electricity markets employ fully linearized versions of Ψ and pricing rules that require side-payments to some market participants. The interdependence between alternative power flow models, different pricing rules, and the welfare of the resulting outcome has not yet been investigated.

2.2. Power flow

In essence, power flow models describe the relationship between power injections/withdrawals at each node and the resulting line flows and voltage phasors. Due to their complexity and fundamental importance for power systems, there is a huge strand of literature dealing with power flow problems in various forms and aspects. We refer to Cain and O'Neill (2012), Frank et al. (2012) Castillo and O'Neill (2013), Low (2014a) and Molzahn and Hiskens (2019) for comprehensive overviews.

Modeling the physics of a power grid results in a highly non-linear and non-convex optimization problem, the Alternating Current Optimal Power Flow (ACOPF) problem. As the intention of this paper is to connect the literature on power models and pricing with non-convexities, we will neither discuss the physics of electric circuits that lead to these types of constraints, nor the numerous representations of ACOPF that have been proposed. We refer interested readers to Bienstock et al. (2020) and Molzahn and Hiskens (2019). For our purposes, we provide a bus injection formulation of ACOPF (Molzahn and Hiskens, 2019) below. We use $\mathrm{Re}(x_b)$ to denote the real power consumed by buyer b and $\mathrm{Im}(x_b)$ to denote the reactive power (and analogously for sellers s). Parameters that are given include a complex nodal admittance matrix $Y \in \mathbb{C}^{|N|\times|N|}$ with $Y = G + jB$ consisting of a real conductance part G and an imaginary susceptance part B, as well as minimum/maximum voltages V_i^{min}, V_i^{max} at each node $i \in N$. With $V_i = V_{di} + jV_{qi}$ as complex voltage phasor variable, the set of feasible power flows Ψ^{AC} can be written as

$$\begin{aligned}
\Psi^{AC} = \{& x \in \mathbb{C}^{|N|\times|B|}, y \in \mathbb{C}^{|N|\times|S|} : \\
&\sum_{s\in S} \mathrm{Re}(y_{is}) - \sum_{b\in B} \mathrm{Re}(x_{ib}) = \sum_{k\in N} V_{di}(G_{ik}V_{dk} - B_{ik}V_{qk}) \\
&\quad + V_{qi}(B_{ik}V_{dk} + G_{ik}V_{qk}) \quad \forall i \in N, \\
&\sum_{s\in S} \mathrm{Im}(y_{is}) - \sum_{b\in B} \mathrm{Im}(x_{ib}) = \sum_{k\in N} V_{di}(-B_{ik}V_{dk} - G_{ik}V_{qk}) \\
&\quad + V_{qi}(G_{ik}V_{dk} - B_{ik}V_{qk}) \quad \forall i \in N, \\
&V_{di}^2 + V_{qi}^2 = |V_i|^2 \quad \forall i \in N, \\
&(V_i^{min})^2 \le |V_i|^2 \le (V_i^{max})^2 \quad \forall i \in N, \\
&V_i \in \mathbb{C} \quad \forall i \in N \quad \}.
\end{aligned}$$

The set of constraints may further include limits on line flows or the definition of a reference bus. In any case, the quadratic and non-convex constraints implied by this system of equations make the ACOPF problem notoriously hard to solve and it is considered intractable in spite of advances in global optimization. The desire to obtain good solutions to ACOPF has sparked competitions such as the ARPA-E Grid Optimization Competition.[7] In this context, new reformulations and decomposition techniques have been proposed (Petra and Aravena, 2021).

Overall, there has been significant research on convex relaxations and approximations of ACOPF to provide tractable programs and high quality solutions for practical application. These relaxations make use of advances in convex optimization, i.e. they leverage increasingly powerful solvers for linear programs (LPs), second-order cone programs (SOCPs), and semidefinite programs (SDPs). We briefly outline the main ideas of the relaxations used in this paper and refer to more comprehensive summaries (Zohrizadeh et al., 2020; Molzahn and Hiskens, 2019) for further details. Molzahn and Hiskens (2016) provide an accessible illustrative example of different convex relaxations.

SDP relaxation

The SDP-relaxation, dating back to the works of Shor (1987), is based on lifting decision variables of the canonical ACOPF to capture non-convexities in a single constraint. In particular, defining $W = VV^T$ as the product of voltages, we can rewrite the quadratic terms in Ψ^{AC} in linear terms of entries of W. Constraining W to be a positive semidefinite and rank-one matrix, i.e. $W \succcurlyeq 0$ and $\mathrm{rank}(W) = 1$, is a sufficient condition for the exactness of this reformulation. Since the rank-one constraint now contains all non-convexities resulting from power flows, dropping this constraint yields a convex SDP relaxation.

With $\langle \cdot, \cdot \rangle$ as Frobenius inner product and e_i as ith unit vector, the relaxed set of feasible power flows Ψ^{SDP} can be written as (Bai et al., 2008; Molzahn and Hiskens, 2019)

$$\begin{aligned}
\Psi^{SDP} = \{& x \in \mathbb{C}^{|N|\times|B|}, y \in \mathbb{C}^{|N|\times|S|} : \\
&\sum_{s\in S} \mathrm{Re}(y_{is}) - \sum_{b\in B} \mathrm{Re}(x_{ib}) = \langle L_{P_i}, W \rangle \quad \forall i \in N, \\
&\sum_{s\in S} \mathrm{Im}(y_{is}) - \sum_{b\in B} \mathrm{Im}(x_{ib}) = \langle L_{Q_i}, W \rangle \quad \forall i \in N, \\
&|V_i|^2 = \langle M_i, W \rangle \quad \forall i \in N, \\
&W \succcurlyeq 0 \quad \}.
\end{aligned}$$

Here, the matrices L_{P_i}, L_{Q_i}, and M_i are defined as

$$L_{P_i} = \frac{1}{2}\begin{pmatrix} \mathrm{Re}(Y^T e_i e_i^T + e_i e_i^T Y) & \mathrm{Im}(Y^T e_i e_i^T - e_i e_i^T Y) \\ \mathrm{Im}(e_i e_i^T Y - Y^T e_i e_i^T) & \mathrm{Re}(Y^T e_i e_i^T + e_i e_i^T Y) \end{pmatrix}$$

$$L_{Q_i} = -\frac{1}{2}\begin{pmatrix} \mathrm{Im}(Y^T e_i e_i^T + e_i e_i^T Y) & \mathrm{Re}(e_i e_i^T Y - Y^T e_i e_i^T) \\ \mathrm{Re}(Y^T e_i e_i^T - e_i e_i^T Y) & \mathrm{Im}(Y^T e_i e_i^T + e_i e_i^T Y) \end{pmatrix}$$

$$M_i = \begin{pmatrix} e_i e_i^T & 0 \\ 0 & e_i e_i^T \end{pmatrix}.$$

[7] See https://gocompetition.energy.gov/ for further details.

While SDP solvers are not as mature as classical LP software, many commercial packages can approximately solve these continuous SDPs in polynomial time using interior point algorithms (Ben-Tal and Nemirovski, 2001; Vandenberghe and Boyd, 1996). Exploiting the sparsity of the network (and thus of matrix W) can further alleviate computational challenges, i.e. by using the matrix completion theorem to impose semidefiniteness constraints only on smaller submatrices of W (Grone et al., 1984).

The SDP relaxation was much celebrated due to its capability to solve several IEEE test cases exactly (Lavaei and Low, 2012).[8] However, the relaxation fails to be exact in general and exactness can even depend on the specific formulation of the problem (i.e. the exactness does not merely depend on physical characteristics of the network) (Kocuk et al., 2016a; Bukhsh et al., 2013; Lesieutre et al., 2011; Molzahn and Hiskens, 2019; Low, 2014b). As a consequence, there are efforts to further strengthen the relaxation, e.g., using the Lasserre hierarchy (Lasserre, 2001, 2009; Josz and Henrion, 2016; Molzahn and Hiskens, 2014, 2015), bound tightening, or lifted non-linear cuts (Coffrin et al., 2015). However, the computational hardness of the problem grows quickly in this case. It should further be noted that mixed-integer SDP solvers, which become relevant when we consider non-convexities in the valuation and cost functions, are still in early stages of development (Gally et al., 2018) and cannot be reliably used for global optimization.[9] Therefore, we could not obtain meaningful results from the SDP relaxation and refrain from reporting approximate results that have little significance.

SOC relaxation

In contrast to SDPs, second-order cone programs (SOCPs) can be efficiently solved by a variety of solvers, even in a mixed-integer form, using interior point methods (Andersen et al., 2003; Ben-Tal and Nemirovski, 2001; Alizadeh and Goldfarb, 2003). However, since SDPs represent a generalization of SOCPs, SOCP relaxations are usually less tight. In this paper, we consider two second-order cone relaxations of ACOPF: Jabr's second-order cone (SOC) relaxation and Hijazi's quadratic convex (QC) relaxation.

Jabr's SOC relaxation (Jabr, 2006; Molzahn and Hiskens, 2019) is similar to the SDP relaxation in that first lifted decision variables are defined from squared voltage magnitudes and the product of voltage phasors, i.e. $c_i = V_{di}^2 + V_{qi}^2$, $cr_{ik} = V_{di}V_{dk} + V_{qi}V_{qk}$, and $ci_{ik} = V_{di}V_{qk} - V_{qi}V_{dk}$. Then, in a radial network, which was the original setting considered by Jabr (2006), an exact representation of Ψ^{AC} can be formulated using these variables.[10] Similar to the SDP relaxation, all non-convexities are contained in a single quadratic equality constraint $cr_{ik}^2 + ci_{ik}^2 = c_i c_k$. Replacing this by an inequality constraint yields a convex SOC relaxation, i.e.,

$$
\begin{aligned}
\Psi^{SOC} = \{ & x \in \mathbb{C}^{|N|\times|B|}, y \in \mathbb{C}^{|N|\times|S|} : \\
& \sum_{s\in S} \mathrm{Re}(y_{is}) - \sum_{b\in B} \mathrm{Re}(x_{ib}) = G_{ii}c_i + \sum_{k\in N\setminus\{i\}} G_{ik}cr_{ik} - B_{ik}ci_{ik} \quad \forall i \in N, \\
& \sum_{s\in S} \mathrm{Im}(y_{is}) - \sum_{b\in B} \mathrm{Im}(x_{ib}) = -B_{ii}c_i + \sum_{k\in N\setminus\{i\}} -B_{ik}cr_{ik} - G_{ik}ci_{ik} \quad \forall i \in N, \\
& cr_{ik} = cr_{ki} \quad \forall (i,k) \in L, \\
& ci_{ik} = -ci_{ki} \quad \forall (i,k) \in L, \\
& cr_{ik}^2 + ci_{ik}^2 \le c_i c_k \quad \forall (i,k) \in L, \\
& c_i, cr_{ik}, ci_{ik} \in \mathbb{R} \quad \forall (i,k) \in L \quad \}.
\end{aligned}
$$

This SOC relaxation can also be obtained from the SDP relaxation by replacing the positive semidefiniteness constraint of W by second-order conic constraints on 2×2 submatrices thereof. As this constitutes a necessary but not sufficient condition for $W \succcurlyeq 0$, the SDP relaxation is at least as tight as the SOC relaxation. Therefore, the SOC relaxation is exact only under restrictive conditions (Jabr, 2006; Gan et al., 2012; Low, 2014b), and tightening techniques can be used to improve the quality of the solution (Kocuk et al., 2016b).

QC relaxation

Hijazi's QC relaxation (Hijazi et al., 2017; Coffrin et al., 2016) is derived from a polar representation of ACOPF. In essence, it uses convex envelopes for various non-convex terms to yield a second-order cone program. By accounting for voltage angle constraints in loops, it is explicitly designed to be applicable to mesh networks (Molzahn and Hiskens, 2019), unlike the SOC relaxation. In fact, the QC relaxation simply tightens Ψ^{SOC} by adding more constraints on voltage angles and magnitudes. We will write $\langle\cdot\rangle$ to denote convex envelopes of non-convex square, bilinear, sine, or cosine terms and refer to Hijazi et al. (2017) and Molzahn and Hiskens (2019) for explicit formulations of these convex envelopes. With this notation and θ_i as voltage angle at node i, the set of power flows can be written as

$$
\begin{aligned}
\Psi^{QC} = \Psi^{SOC} \cap \{ & x \in \mathbb{C}^{|N|\times|B|}, y \in \mathbb{C}^{|N|\times|S|} : \\
& c_i \in \langle |V_i|^2 \rangle \quad \forall i \in N, \\
& cr_{ik} \in \langle \langle |V_i||V_k| \rangle \langle \cos(\theta_i - \theta_k) \rangle \rangle \quad \forall (i,k) \in L, \\
& ci_{ik} \in \langle \langle |V_i||V_k| \rangle \langle \sin(\theta_i - \theta_k) \rangle \rangle \quad \forall (i,k) \in L, \\
& c_i, cr_{ik}, ci_{ik}, \theta_i \in \mathbb{R} \quad \forall (i,k) \in L \quad \}.
\end{aligned}
$$

Note that phase angle differences $\theta_i - \theta_k$ are assumed to be bound between zero and $\frac{\pi}{2}$ (Coffrin et al., 2016). By design, the QC-relaxation is at least as tight as the SOC relaxation while still maintaining computational efficiency as a second-order cone program. Neither the SDP nor the QC relaxation dominate the other.

DC approximation

Finally, we consider the standard linear approximation for ACOPF as it is currently applied in practical electricity markets. The Direct Current Optimal Power Flow (DCOPF) problem (Stott et al., 2009; Li and Bo, 2007; Overbye et al., 2004; Molzahn and Hiskens, 2019; Eldridge et al., 2018) makes three simplifying assumptions: (i) line resistances and reactive power are ignored, (ii) voltage magnitudes $|V_i|$ at each bus are set to 1, and (iii) voltage angle differences between nodes are assumed to be small such that $\sin(\theta_i - \theta_k) \approx \theta_i - \theta_k$ and $\cos(\theta_i - \theta_k) \approx 1$. As a result, the set of power flows simply formulates as a set of linear constraints, i.e.,

$$
\begin{aligned}
\Psi^{DC} = \{ & x \in \mathbb{C}^{|N|\times|B|}, y \in \mathbb{C}^{|N|\times|S|} : \\
& \sum_{s\in S} \mathrm{Re}(y_{is}) - \sum_{b\in B} \mathrm{Re}(x_{ib}) = \sum_{k\in N:(i,k)\in L} -B_{ik}(\theta_i - \theta_k) \\
& \quad - \sum_{k\in N:(k,i)\in L} -B_{ki}(\theta_k - \theta_i) \quad \forall i \in N, \\
& \theta_i \in \mathbb{R} \quad \forall i \in N \quad \}.
\end{aligned}
$$

Of course one may impose an admissible range on voltage angles, e.g. $\theta_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. We also note that many system operators in practice use refined linear approximations that go beyond the standard DCOPF (Li et al., 2022).[11] In any case, the set of constraints constitute a polyhedron, and mature and powerful linear programming solvers can be applied (Bertsimas and Tsitsiklis, 1997). At the same time, Ψ^{DC} may not approximate Ψ^{AC} sufficiently well. While there are ambiguous empirical findings on the accuracy of DCOPF (Molzahn and Hiskens, 2019), recent developments in research and practice alike suggest that tighter relaxations would be preferable as long as tractability can be ensured. This motivates to examine the introduced non-linear network models in the context of pricing on wholesale electricity markets.

[8] Note that for the SDP relaxation the rank-one condition of W provides a simple way to verify exactness.

[9] See http://www.opt.tu-darmstadt.de/scipsdp/ for an example of a solver.

[10] Note that for mesh networks, this representation constitutes a relaxation as voltage angles in a loop may not sum to zero (Molzahn and Hiskens, 2019).

[11] For example, approximation models in practice include more constraints that reflect specific knowledge of the system. We therefore remark that practical implementations of DCOPF are likely to be tighter than Ψ^{DC} presented here and may yield better numerical results.

Redispatch

As discussed, the introduced relaxations and approximations are generally not tight, i.e., $\Psi^{AC} \subseteq \Psi^{SDP} \subseteq \Psi^{SOC}$, $\Psi^{AC} \subseteq \Psi^{QC}$ and for Ψ^{DC} we cannot make any statement. Solutions obtained from these power flow models may thus not be physically feasible. It is therefore necessary to reconstruct a feasible solution from Ψ^{AC} given the optimal solution to the relaxation/approximation. As noted by Petra and Aravena (2021), this reconstruction may be as complex as finding a solution from Ψ^{AC} from scratch. Ensuring AC-feasibility often involves iterative processes with metaheuristics which are ineffective when the system is stressed (Castillo, 2016). Thus, the economic costs of reconstructing an AC-feasible solution is another important factor to consider. We will refer to this reconstruction as *redispatch*.[12]

For our purposes, we will consider a cost-based redispatch. When the market operator detects that the obtained solution is not in Ψ^{AC}, the dispatch of certain market participants will be adjusted upward or downward to ensure AC-feasibility. The affected market participants are compensated for their additional costs or lost profits and are thus indifferent to the initially assigned dispatch. We refer to the sum of these compensation payments as *redispatch costs*. The redispatch costs serve as additional benchmarks for efficiency losses of the different relaxations and approximations.

In order to find a near AC-feasible solution, we construct an AC power flow model $x, y \in \Psi^{AC}$, albeit with no objective function. In order to speed up the calculations, we use a warm-start procedure and set starting values for various variables. In particular, the voltage magnitudes, active and reactive power generation, as well as active and reactive power consumption of the solution of the relaxation/approximation are provided. Moreover, we add constraints that restrict the active power generation and consumption to only deviate by some specified tolerance from the initial solution. If the tolerance is chosen too restrictive, the problem may become infeasible, yet an appropriately chosen tolerance helps to keep the redispatch of active power within reasonable limits. Moreover, due to the computational complexity of this problem, the commitments of the generators cannot be altered. As a result, redispatch only changes the dispatch of market participants, but not their commitment status. The AC power flow problem is thus of a continuous rather than discrete nature. The problem is then solved using commercial software, i.e., interior-point methods that try to find a local solution to the large-scale, non-linear, non-convex problems at hand Nocedal et al. (2009), Wächter and Biegler (2005a,b) and Wächter and Biegler (2006). The obtained allocation (x^{AC}, y^{AC}) is AC-feasible, i.e., $(x^{AC}, y^{AC}) \in \Psi^{AC}$, but it is not AC-optimal, i.e., (x^{AC}, y^{AC}) is not a solution to (1) with Ψ^{AC}.

2.3. Competitive equilibrium theory and pricing

After computing an optimal allocation (x^*, y^*) based on some convex power flow representation, the market operator needs to calculate real power prices $\lambda \in \mathbb{R}^{|N|}$ for electricity at each node.[13] We assume prices of zero for reactive power and only assign non-zero prices to real power. The motivation to allocate and price goods through markets is based on competitive equilibrium theory and has a long history. As we will see, however, the design of v_b and c_s as non-convex valuation or cost functions implies a fundamental problem for pricing.

The promise of markets rests on well-known theoretical results such as the welfare theorems (Arrow and Debreu, 1954). Assuming convex preferences v_b and c_s, demand independence, and perfect competition with divisible goods, there is a set of competitive equilibrium prices that will maximize social welfare. Further, every welfare-maximizing allocation can be supported by a set of equilibrium prices. An equilibrium implies that no market participants wants to deviate from the assigned allocation, and that the outcome is thus envy-free and budget-balanced.

However, many real-world markets exhibit indivisible goods and non-convex preferences. In this context, the concept of a quasilinear utility function was widely adopted to study competitive equilibria (Bikhchandani and Mamer, 1997; Bikhchandani and Ostroy, 2002; Baldwin and Klemperer, 2019). A large part of the literature focuses on Walrasian equilibria, i.e., efficient market outcomes with linear (i.e., item-level) and anonymous prices such that every market participant maximizes their utility. Unfortunately, Walrasian equilibria with indivisible goods generally only exist under very restrictive assumptions on the preference functions (e.g., strong substitutes) (Kelso and Crawford, 1982; Bikhchandani and Mamer, 1997; Baldwin and Klemperer, 2019). Under these assumptions, Walrasian prices can be obtained from the Lagrangian dual of the welfare maximization problem and the welfare theorems can be generalized to markets with indivisibilities (Blumrosen and Nisan, 2007; Bichler et al., 2020). However, when these assumptions do not hold, these properties are generally lost. In general non-convex combinatorial markets, determining the allocation is NP-hard and competitive equilibrium prices need to be non-linear and personalized (Bikhchandani and Ostroy, 2002) or not exist at all (Bichler and Waldherr, 2019).

Electricity markets are a prime example of such markets. In the absence of competitive equilibrium prices, different heuristics have been proposed to serve as alternative pricing rules (Liberopoulos and Andrianesis, 2016). The resulting prices are not unique, but virtually all established pricing rules produce linear and anonymous prices such that they can serve as a baseline for derivatives contracts. Given that Walrasian equilibria are impossible to obtain, these prices compromise on some of the properties thereof. We will discuss these issues along two established pricing rules in electricity markets: Integer Programming (IP) pricing (O'Neill et al., 2005) and Convex Hull (CH) pricing (Hogan and Ring, 2003; Gribik et al., 2007).

To ease our reasoning, we will impose a specific form of valuation and cost functions for the remainder of this paper. Buyers b have concave, piecewise-linear valuation functions v_b based on their real power consumption, represented by a set of bids β_b and upper bounds $q_{b\ell}$. Moreover, for both real and reactive power, they possess an inelastic demand $\underline{P}_b$, $\underline{Q}_b$ and a maximum consumption of $\overline{P}_b$, $\overline{Q}_b$, respectively.

$$
\begin{aligned}
v_b(x_b) = \quad & \max_{x_b, x_{b\ell}} \sum_{\ell \in \beta_b} v_{b\ell} x_{b\ell} \\
\text{s.t.} \quad & \sum_{\ell \in \beta_b} x_{b\ell} = \mathrm{Re}(x_b) \\
& \underline{P}_b \le \mathrm{Re}(x_b) \le \overline{P}_b \\
& \underline{Q}_b \le \mathrm{Im}(x_b) \le \overline{Q}_b \\
& 0 \le x_{b\ell} \le q_{b\ell} \quad \forall \ell \in \beta_b
\end{aligned}
$$

Buyers are thus assumed to have fully convex preferences. On the other hand, for sellers s we define a binary commitment variable u_s that signals if a generator is supplying a positive amount and thereby introduces non-convexities. When $u_s = 1$, seller s has a convex, piecewise-linear variable cost function in analogy to buyers' valuation functions. Moreover, fixed costs h_s occur when a positive amount is produced, and sellers have minimum and maximum production quantities $\underline{P}_s$, $\underline{Q}_s$, $\overline{P}_s$, and $\overline{Q}_s$, respectively.

$$
\begin{aligned}
c_s(y_s) = \quad & \max_{y_s, y_{s\ell}, u_s} \sum_{\ell \in \beta_s} v_{s\ell} y_{s\ell} + h_s u_s \\
\text{s.t.} \quad & \sum_{\ell \in \beta_s} y_{s\ell} = \mathrm{Re}(y_s) \\
& \underline{P}_s u_s \le \mathrm{Re}(y_s) \le \overline{P}_s u_s
\end{aligned}
$$

[12] Note that redispatch also refers to the process of adjusting a dispatch in zonal markets due to the neglect of intra-zonal transmission constraints. In this paper, by redispatch we refer to the process of reconstructing an AC-feasible solution from a solution obtained from a relaxation/approximation.

[13] The prices obtained from the convexified allocation problem also apply to the allocation after redispatch, i.e., for (x^{AC}, y^{AC}).

$$\underline{Q}_s u_s \leq \mathrm{Im}(y_s) \leq \overline{Q}_s u_s$$
$$0 \leq y_{s\ell} \leq q_{s\ell} \quad \forall \ell \in \beta_s$$
$$u_s \in \{0,1\}$$

As discussed, with such non-convex cost functions, Walrasian equilibrium prices can generally not be obtained. As such, some market participants do not maximize their utility at the prices λ. We call this forgone payoff *lost opportunity costs* (LOCs) of a market participant, i.e.,

$$LOC_b = \hat{u}_b(\lambda) - u_b(x_b^*|\lambda) = (\max_{x_b} v_b(x_b) - \lambda^T \mathrm{Re}(x_b)) - (v_b(x_b^*) - \lambda^T \mathrm{Re}(x_b^*)),$$

$$LOC_s = \hat{u}_s(\lambda) - u_s(y_s^*|\lambda) = (\max_{y_s} \lambda^T \mathrm{Re}(y_s) - c_s(y_s)) - (\lambda^T \mathrm{Re}(y_s^*) - c_s(y_s^*)).$$

We call $\hat{u}_b(\lambda)$, $\hat{u}_s(\lambda)$ the *indirect utility* function that measures the individual payoff maximum given the prices λ. In contrast, the *direct utility* $u_b(x_b^*|\lambda)$, $u_s(y_s^*|\lambda)$ measures the actual payoff given the optimal allocation (x^*, y^*) and prices λ.

A natural choice for a pricing rule in non-convex markets is to minimize these lost opportunity costs. This is the main idea of Convex Hull (CH) pricing. CH pricing replaces each non-convex cost function $c_s(y_s)$ by its convex envelope $\overline{\mathrm{conv}}(c_s(y_s))$. Assuming a convex power flow representation, prices can be obtained from the dual of the resulting convex problem. Determining convex envelopes and thus calculating CH prices is generally computationally hard (Schiro et al., 2016). However, under restricted bidding languages, as applicable to our form of c_s, CH prices can be tractably obtained by relaxing the binary constraints $u_s \in \{0,1\}$ to $u_s \in [0,1]$ and solving the dual problem (Hua and Baldick, 2017). The resulting prices λ^{CH} then equal the dual variables of the active power balance constraints. They minimize LOCs, and if a Walrasian equilibrium is attainable, λ^{CH} constitutes such Walrasian prices.

Since CH prices cannot always be computed efficiently, most market operators in practice implement Integer Programming (IP) pricing. This involves (i) obtaining the optimal commitment variables u_s^* from the welfare-maximizing solution, (ii) fixing the commitment variables to these optimal values, i.e. relaxing $u_s \in \{0,1\}$ to $u_s \in [0,1]$, $u_s = u_s^*$, and (iii) retrieving prices from the dual of the active power balance constraints of the resulting convex problem. IP pricing is considered to follow the notion of marginal pricing in non-convex markets (O'Neill et al., 2005).

Recently, practical pricing rules that are based on IP pricing have come under scrutiny. They have been criticized for invoking too high *make-whole payments* (MWP) to some of the market participants, distorting the market price signal (Bichler et al., 2023; O'Neill et al., 2019). Again, this criticism refers back to the absence of Walrasian prices. In particular, in non-convex markets with linear and anonymous prices, there may be no price vector that allows all market participants to break even (Bichler et al., 2023). We call this property of non-negative payoffs *individual rationality*. If this property is violated for a market participant, they need to be compensated by an individual side-payment. This MWP amounts to

$$MWP_b = \max(-u_b(x_b^*|\lambda), 0) = \max(-(v_b(x_b^*) - \lambda^T \mathrm{Re}(x_b^*)), 0),$$

$$MWP_s = \max(-u_s(y_s^*|\lambda), 0) = \max(-(\lambda^T \mathrm{Re}(y_s^*) - c_s(y_s^*)), 0).$$

This observation has motivated the development of pricing rules with reduced levels of MWPs (Bichler et al., 2023; O'Neill et al., 2019; Ahunbay et al., 2022). Note that while $MWP_b \leq LOC_b$ (and for sellers alike), the objectives to minimize MWPs and LOCs can be conflicting (Ahunbay et al., 2022). It is unclear how CH prices, IP prices, as well as the LOCs and MWPs they imply behave depending on the choice of the power flow representation Ψ.

In addition, we aim to benchmark the quality of the congestion signals provided by the different pricing rules. In particular, congestion on network elements should be reflected in the prices, and if no congestion occurs, prices should be identical throughout the network. Flawed congestion signals have an impact on congestion income distribution, financial hedging, and investment decisions, and are thus of essential importance in practical electricity markets. In order to measure the accuracy of congestion signals, we rely on a metric known as *financial transmission right* (FTR) *uplift* or *potential congestion revenue shortfall* (PCRS) (Garcia et al., 2020). This metric represents lost opportunity costs for the transmission operators. It constitutes the difference between the maximum possible FTR payoffs (respecting the set of feasible power flows Ψ) and the actual congestion income given the optimal allocation.

Let us denote $x_N \in \mathbb{C}^{|N|}$ as the vector that holds the aggregate consumption at each node $i \in N$ as components, i.e. $x_N = (\sum_{b \in B} x_{ib})_{i \in N}$. Analogously, we define $y_N \in \mathbb{C}^{|N|}$ as the aggregate supply at each node, i.e. $y_N = (\sum_{s \in S} y_{is})_{i \in N}$. Then, the PCRS is defined as

$$PCRS = \max_{x,y \in \Psi} \lambda^T (\mathrm{Re}(x_N) - \mathrm{Re}(y_N)) - \lambda^T (\mathrm{Re}(x_N^*) - \mathrm{Re}(y_N^*)).$$

FTRs are allocated based on some set of feasible flows which may not correspond to the actual flows. The notion of the PCRS metric is that prices should be set in a way that the congestion revenues can cover payoffs from a *worst-case* FTR allocation (Garcia et al., 2020). If this were not the case, the resulting shortfall has to be distributed among market participants, which is not trivial (Hogan, 2013). Similar to LOCs and MWPs, the behavior of PCRS under different power flow representations Ψ is unclear and one focus of this study.

3. Data and processing

We apply the different power flow relaxations and pricing rules on the network and bid data provided for the ARPA-E Grid Optimization Competition Challenge II.[14] This competition seeks the development of modern and scalable optimization techniques for solving complex power flow problems. To that end, they provide large-scale and realistic test sets of single-period power flow problems. We consider the power flow models and valuation/cost functions as described in the previous section (for which ARPA-E provides all necessary data), including thermal line limits, and therefore abstract from some of the available data (e.g., switched shunts, transformers, contingencies). In the Appendix, we provide an overview of the notation.

We consider a set of five synthetic networks and 20 scenarios. The problem sizes range from 617 nodes, 94 generators, and 404 consumers to 3970 nodes, 391 generators, and 2744 consumers. This matches the size of realistic power networks, e.g., those of Germany (roughly 438 nodes (DIW Berlin, 2022)) or the United Kingdom (roughly 300 nodes (National Grid, 2022)). The dataset includes even larger networks (up to 31,777 nodes), yet solving tighter non-linear convex relaxations beyond DCOPF turned out to be intractable for these problem sizes. For example, a nodal model of Continental Europe and Ireland would encompass roughly 25,000 nodes and 25,000 generators (ENTSO-E, 2022), and the Californian market consists of roughly 9700 nodes (California ISO, 2018). Note that we ran our experiments on an Intel Xeon E312xx 2.0 GHz machine with 20 cores. European-scale models might be tractable on a super-computer, but scalability is not the focus of our research. We instead aim for an estimate of the impact of different relaxations on prices, MWPs, LOCs, and PCRS. In terms of external validity, our problem instances are large enough and representative of those in the field.

We implemented the models in the Julia programming language. For the different power flow models, we worked with the PowerModels library,[15] which offers readily available implementations of all common power flow representations. We built on this library to enrich the

[14] See https://gocompetition.energy.gov/sites/default/files/Challenge2_Problem_Formulation_20210531.pdf for the problem statement.

[15] See https://lanl-ansi.github.io/PowerModels.jl for the package documentation.

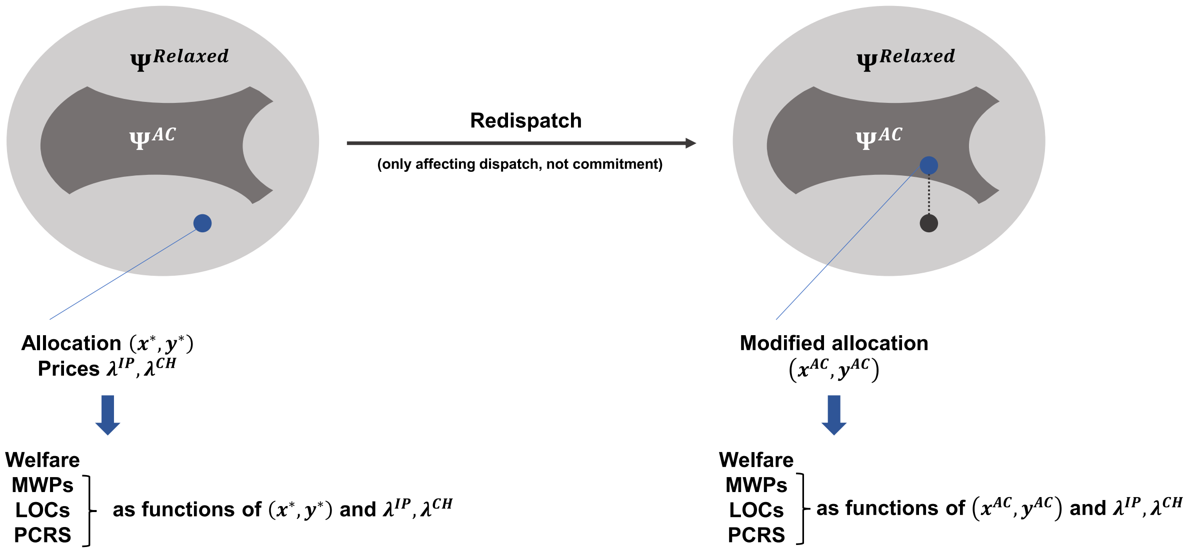

Fig. 1. Visualization of the approach.

Table 1
Overview of experiments.

Network sizes	Power flow models	Pricing rules	Metrics[a]
617 (5 scenarios)	DC	IP	Welfare
2020 (3 scenarios)	SOC	CH	Prices
2312 (5 scenarios)	QC		MWPs
3288 (5 scenarios)			LOCs
3970 (2 scenarios)			PCRS
			Redispatch costs
			Runtime

[a] Welfare, MWPs, LOCs, and PCRS can be analyzed before and after the redispatch is conducted.

models with the non-convex valuation and cost functions described above, and further added functionalities to determine prices, LOCs, MWPs, and PCRS. We used different commercial packages to solve the optimization problems, including Gurobi (DC, SOC, QC), Ipopt (DC, SOC, QC, AC), and Mosek (DC, SOC, QC). With standard constraint qualification conditions satisfied, we can conclude strong duality (Cao et al., 2022) and obtain prices from the dual solutions of our pricing problems. Table 1 summarizes the factors of our experimental design and the metrics analyzed.

As illustrated in Fig. 1, we first compute the allocation and prices for each convexified model. This yields a certain welfare, MWPs, LOCs, and PCRS. Since the computed allocation may not be AC-feasible, we then conduct cost-based redispatch to obtain a physically AC-feasible outcome. The now modified allocation yields a different welfare and – together with the initially computed prices – different MWPs, LOCs, and PCRS. In the next section, we only report MWPs, LOCs, and PCRS for the AC-feasible solution, since this is the outcome that is physically implemented.

4. Results

Below, we report the main results from our analysis.

4.1. Welfare & prices

For each test case, we solve the welfare-maximization problem for each power flow model. This provides the optimal generator dispatch, buyer consumption, and power flows that are valid under the chosen relaxation/approximation.

Result 1. *While the welfare of different power flow models is similar, generator commitments between different power flow models vary significantly. These differences in allocation impact the prices, the level of required redispatch, and the final AC-feasible outcome.*

In terms of welfare, each relaxation/approximation yields a similar outcome. In all tested scenarios, choosing a different power flow model impacts the welfare by, at most, 0.5%. This hinges on the fact that the valuations of the buyers in the data set are very high. Large welfare losses are thus not to be expected under any power flow model, even though minor welfare improvements can already create significant savings (Cain and O'Neill, 2012). At the same time, this does not imply that the allocations are similar for all power flow models. Fig. 2 illustrates the differences in committed generation capacities between Ψ^{DC}, Ψ^{SOC}, and Ψ^{QC}. In the Appendix, we include a similar figure that visualizes differences in the dispatched capacities. In most cases, the conic relaxations commit and dispatch more generators compared to the DC approximation, in order to account for the additional network constraints (e.g., reactive power constraints). The differences can be significant and impact prices and redispatch, as we will see.

Result 2. *While the average prices of different power flow relaxations are similar, prices under Ψ^{DC} exhibit significantly more outliers. The DC solution may contain line congestions that do not occur under Ψ^{SOC} and Ψ^{QC}, leading to potentially high and unjustified price peaks in the network. In contrast, prices for Ψ^{SOC} and Ψ^{QC} are very similar, more stable, and better reflect the physics of the network. CH prices usually exhibit fewer discrepancies between power models than IP prices, but this comes at the expense of distortions in the congestion signals.*

After computing the welfare-maximizing allocation based on the respective power flow representations, we determine IP and CH prices by solving a convexified version of the original allocation problem. We present exemplary boxplots of the computed prices for the networks with 2020 and 3970 nodes (aggregated over all scenarios) in Fig. 3. Especially in large networks, IP and CH prices under Ψ^{DC} exhibit substantially more outliers. We provide small examples that illustrate how such deviations can occur with DCOPF in the Appendix. For example, the very high DC prices up to \$743/MWh for test case C2FEN02020/121 are caused by a single congested line in the respective part of the network, which implies high marginal costs and leads to high prices at all surrounding nodes. Importantly, this line congestion vanishes under Ψ^{SOC} and Ψ^{QC} and prices range at stable levels at around \$75/MWh. While we cannot compute the AC-optimal

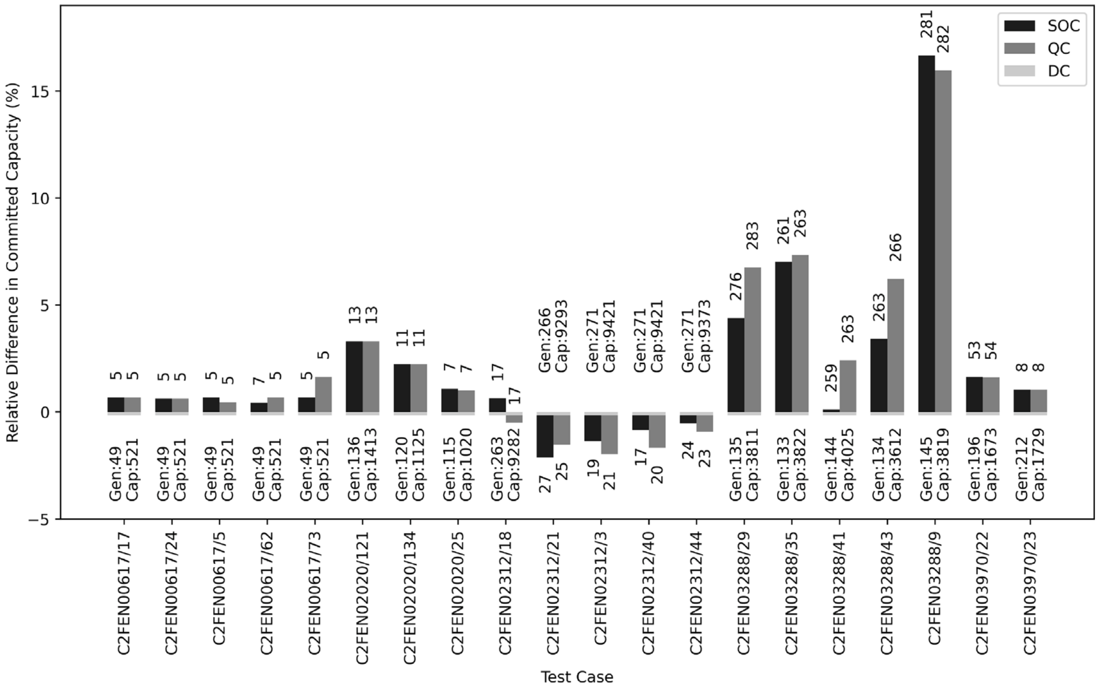

Fig. 2. Committed generation capacity: The figure denotes the number of committed generators ("Gen") and their total capacity ("Cap") for Ψ^{DC} (in MW) at the bottom. The bars indicate the relative difference (in %) in committed capacity for Ψ^{SOC} and Ψ^{QC}, respectively. The labels of each bar indicate the number of generators with a different commitment status compared to Ψ^{DC}. For example, the first bar in the first test case C2FEN00617/17 indicates that 5 units are committed differently under Ψ^{SOC} as compared to the 49 committed units under Ψ^{DC}.

Table 2
IP price statistics.

	μ_{DC}	σ_{DC}	μ_{SOC}	σ_{SOC}	μ_{QC}	σ_{QC}	$\rho_{DC,SOC}$	$\rho_{DC,QC}$	$\rho_{SOC,QC}$
617	112.36	0.00	116.11	2.15	115.29	2.18	0.73	0.73	1.00
2020	76.63	24.86	71.63	3.60	72.16	3.63	0.49	0.49	1.00
2312	9.27	2.44	9.38	2.04	9.40	1.99	0.80	0.79	0.94
3288	18.71	3.04	17.48	0.19	17.46	0.19	0.11	0.14	0.98
3970	28.92	3.72	27.92	0.29	27.92	0.29	0.23	0.23	1.00

Table 4
Redispatch costs [\$] and their proportion of welfare [%].

	DC	SOC	QC
617	14 709.70 (1.93%)	7452.95 (0.98%)	8269.10 (1.08%)
2020	70 468.60 (2.77%)	42 905.50 (1.73%)	43 580.50 (1.75%)
2312	20 624.10 (0.69%)	390.50 (0.01%)	376.75 (0.01%)
3288	48 896.30 (0.91%)	16 993.70 (0.32%)	12 461.90 (0.23%)
3970	1489.50 (1.25%)	998.41 (0.80%)	992.46 (0.83%)

Table 3
CH price statistics.

	μ_{DC}	σ_{DC}	μ_{SOC}	σ_{SOC}	μ_{QC}	σ_{QC}	$\rho_{DC,SOC}$	$\rho_{DC,QC}$	$\rho_{SOC,QC}$
617	112.87	0.12	115.00	2.07	115.00	2.06	0.21	0.21	1.00
2020	74.27	23.34	74.58	4.44	74.58	4.44	0.56	0.56	1.00
2312	9.28	2.38	9.34	2.05	9.34	2.05	0.82	0.82	1.00
3288	17.94	2.37	17.59	0.19	17.59	0.19	0.20	0.20	1.00
3970	28.96	3.43	27.94	0.19	27.94	0.29	0.24	0.24	1.00

allocation in this case, we argue that the tighter SOC and QC relaxations better reflect the actual physical flows in the network. Moreover, when computing a near AC-feasible solution, the line congestion vanishes even for the DC outcome. We therefore argue that the high price peaks of the DC network are unjustified and that the conic relaxations provide a better price signal.

The price peaks for Ψ^{DC} lead to a higher volatility (σ) of DC prices, as evident from Tables 2 and 3. While average prices (μ) are similar for different power flow relaxations, DC prices exhibit very little correlation (ρ) with SOC or QC prices. Correlations tend to be higher for CH prices compared to IP prices, with the exception of the 617-node network where DC prices tend to be constant throughout the network.

This is further illustrated by the median prices for each tested scenario in Fig. 4. They are depicted in terms of their relative deviation to the median IP and CH price under Ψ^{DC}, respectively. Generally, price spreads between different power flow relaxations tend to be higher for IP pricing. This can be explained by the design of the convexified model underlying IP and CH pricing. In particular, IP pricing fixes the commitment variables u_s of each generator to their optimal values u_s^* and computes marginal prices based on this assumption. In contrast, CH pricing computes marginal prices based on the convex envelopes of the non-convex cost functions. As a result, generators under the convexified model for CH pricing can be dispatched more flexibly than under the convexified model for IP pricing. Differences in generator commitments thus do not impact CH prices, and hence CH prices are more similar across different power flow representations. Notably, in all tested scenarios the conic relaxations do not increase prices significantly compared to the DC benchmark, and often even decrease median prices by several percent.

4.2. Redispatch costs and AC-feasibility

Next, we consider cost-based redispatch to obtain an AC-feasible solution according to the methodology described in Section 2.2. Note that redispatch refers to the allocation alone, and the choice of pricing rule therefore does not matter. Redispatch only changes the dispatch quantities of buyers and sellers, but not their commitment status.

Result 3. *Redispatch costs are substantial for the DC approximation. The required adjustments of the allocation to obtain AC-feasibility correlate with the tightness of the power flow model. The high redispatch costs of the linear DC approximation can be significantly reduced by using second-order conic power flow relaxations.*

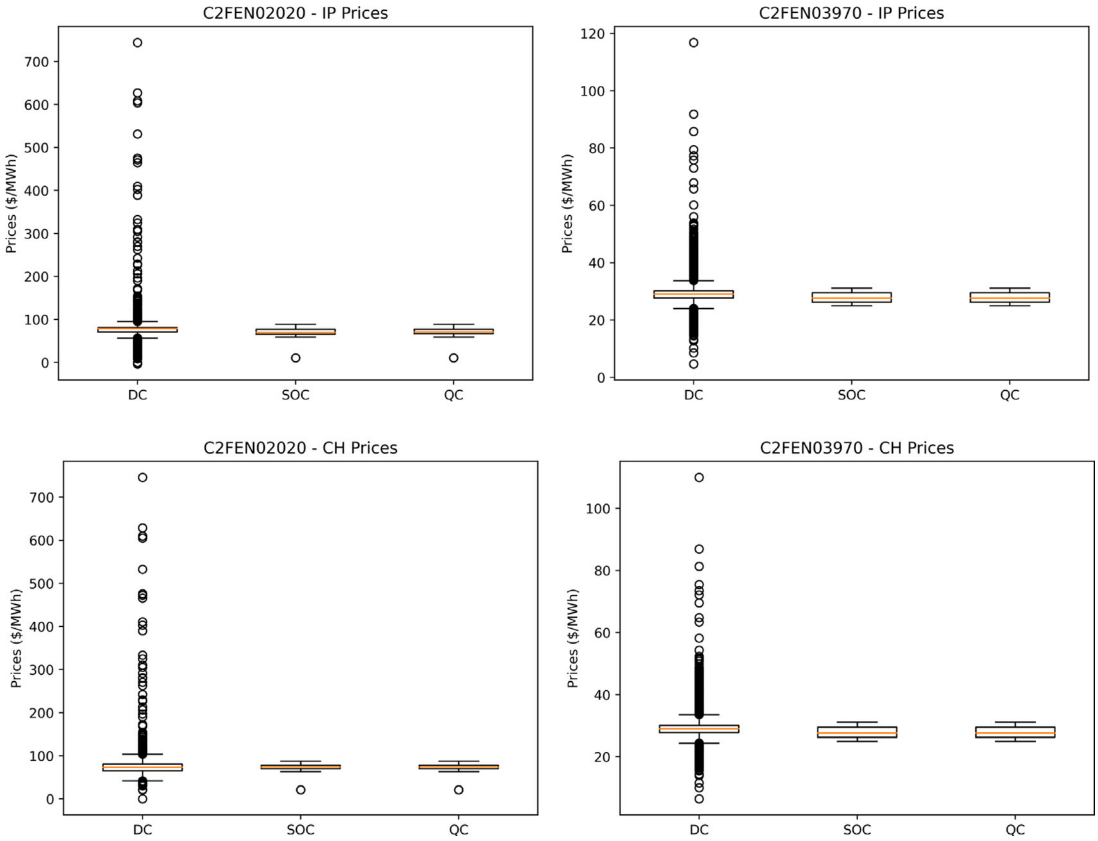

Fig. 3. Price boxplots [$/MWh].

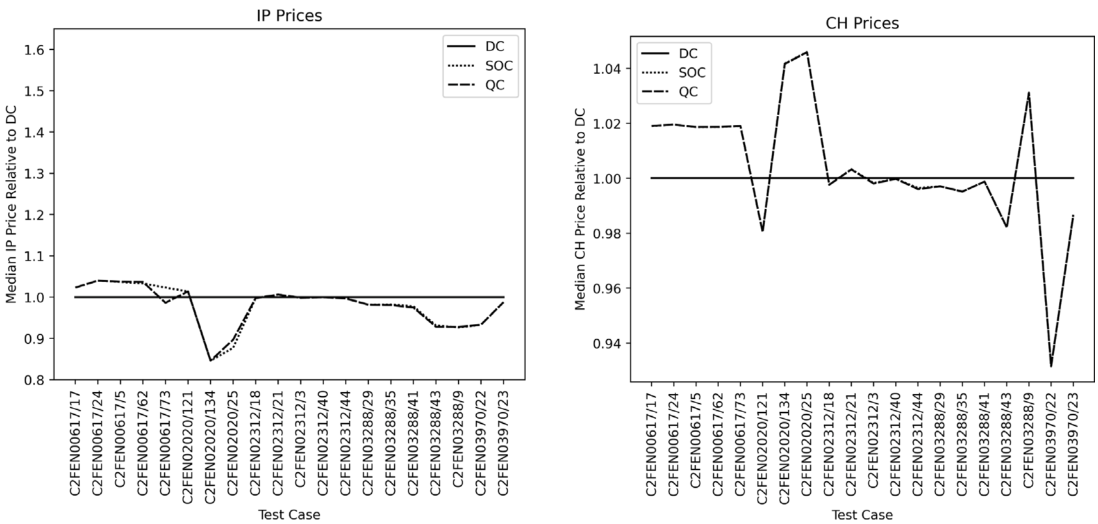

Fig. 4. Median prices.

Table 5
Average welfare, MWPs, LOCs, and PCRS for the AC-feasible solutions.

	DC		SOC		QC	
	IP	CH	IP	CH	IP	CH
Welfare vs. Ψ^{DC}_{AC}						
617	–	–	+0.91%	+0.91%	+0.83%	+0.83%
2020	–	–	+1.10%	+1.10%	+1.09%	+1.09%
2312	–	–	+0.72%	+0.72%	+0.73%	+0.73%
3288	–	–	+0.40%	+0.40%	+0.50%	+0.50%
3970	–	–	+0.53%	+0.53%	+0.53%	+0.53%
MWPs						
617	1.55	0.00	0.00	0.00	44.39	0.64
2020	5375.92	504.37	1927.20	400.76	1789.93	400.76
2312	51 766.40	50 824.40	51 029.00	50 302.70	50 750.70	50 121.60
3288	36.09	7.54	9536.83	9527.38	9624.19	9611.17
3970	49.53	24.87	16.94	16.93	16.12	16.12
LOCs						
617	13 801.00	13 779.10	7187.81	7041.99	7682.68	7536.17
2020	75 277.60	68 553.00	43 440.30	41 936.50	43 361.20	41 960.10
2312	91 967.90	90 953.10	72 330.60	71 160.00	72 648.50	70 975.40
3288	47 815.60	47 309.10	25 898.20	25 887.50	21 538.00	21 522.10
3970	692.68	672.05	457.66	457.70	456.49	456.55
PCRS						
617	1106.21	1109.01	169.16	200.15	268.47	298.14
2020	2437.87	2245.06	899.50	1042.09	1008.50	1054.98
2312	425.05	450.97	49.34	77.67	76.53	247.45
3288	1023.04	988.94	401.89	415.09	167.15	182.74
3970	526.27	523.25	112.99	114.19	112.99	114.20

Table 4 lists the redispatch costs for all tested scenarios. As expected, the DC approximation requires the highest degree of redispatch to obtain an AC-feasible outcome. On average, the redispatch costs for the DC approximation exceed those of the SOC relaxation by a factor of 2.49, and those of the QC relaxation by a factor of 2.67. The linearized DCOPF abstracts too much from physical reality, and large additional costs arise from correcting the dispatch. Even if these costs are not passed on to consumers,[16] the significant change in allocation implies that the prices do not reflect accurate scarcity signals. In contrast, the two conic relaxations are tighter and thus approximate Ψ^{AC} much better. As a result, on average, significantly less redispatch is necessary.

Result 4. *As redispatch modifies the underlying allocation, the originally computed prices lead to a different outcome. Considering final AC-feasible solutions, tighter convex power flow relaxations imply higher welfare and fewer incentives to deviate, as measured by MWPs, LOCs, and PCRS. The price signals obtained from the SOC or QC relaxation are much more suitable for the AC-feasible outcomes than standard DC prices. This results from the fact that the SOC and QC relaxations are better representations of the physical grid than the DC approximation.*

After conducting the redispatch, we have obtained an AC-feasible dispatch that satisfies grid constraints and can be physically realized. Table 5 summarizes the final welfare, MWPs, LOCs, and PCRS as a result of redispatch.

As discussed above, the redispatch is most substantial for the DC approximation, resulting in the highest welfare losses from ensuring AC-feasibility. In Table 5, we compare the welfare of the AC-feasible outcomes of the conic relaxations relative to the welfare of the AC-feasible outcome of the DC approximation (denoted Ψ^{DC}_{AC}). The welfare based on the conic relaxations outweighs that of the DC approximation in all tested scenarios. Thus, even though we cannot compute the welfare-maximizing AC-optimal outcome, tighter non-linear relaxations provide an allocation that enables higher welfare than the standard linearized DC approximation. Using estimates from the past, these efficiency gains between 0.40% and 1.10% could translate into billions of dollars in annual savings (Cain and O'Neill, 2012), especially with the recent increase in energy prices.

Importantly, while the originally computed prices are not modified, the underlying allocation changes as a result of the redispatch and implies certain MWPs and LOCs for market participants. The highest MWPs are necessary for IP pricing and Ψ^{DC} in 14 of the 20 tested scenarios. With CH pricing the MWPs are consistently lower, yet the DC approximation again requires the highest MWPs in the majority of scenarios when CH pricing is applied. One exception is the 3288 node network, where the set of generators that require MWPs under Ψ^{DC} and under Ψ^{SOC}/Ψ^{QC} are entirely disjunct. However, even though the tighter conic relaxations require higher MWPs, their welfare exceeds that of Ψ^{DC} even more (e.g., by an average of $26,611 for the QC relaxation).

Similarly, we observe a strong increase in LOCs for the DC approximation as a result of the redispatch. In particular, LOCs are highest for Ψ^{DC} in 19 of 20 cases. We thus conclude that the redispatch of the DC approximation can lead to substantial changes in the allocation (e.g., for reactive power balancing). As a result, prices based on Ψ^{DC} tend to be more distorted for the final AC-feasible outcome than prices based on tighter convex relaxations. They imply strong incentives for market participants to deviate from the AC-feasible dispatch. Large amounts of penalties are necessary to prevent market participants to deviate. In contrast, price signals obtained from non-linear relaxations maintain better incentives for the final AC-feasible dispatch.

Moreover, PCRS increase over-proportionally for Ψ^{DC}, implying that prices based on Ψ^{DC} do not reflect congestion properly and can imply significant PCRS for transmission operators. In contrast, the PCRS for the SOC and QC relaxation are significantly smaller, and the respective prices admit a much higher quality in terms of congestion signals for the final AC-feasible dispatch than the DC prices.

4.3. Computational costs

Finally, we consider the computational costs to determine allocation and prices.

[16] In decentralized European electricity markets, redispatch costs are passed on to consumers. In U.S. central dispatch markets, the independent system operator can mandate an AC-feasible dispatch as the final allocation.

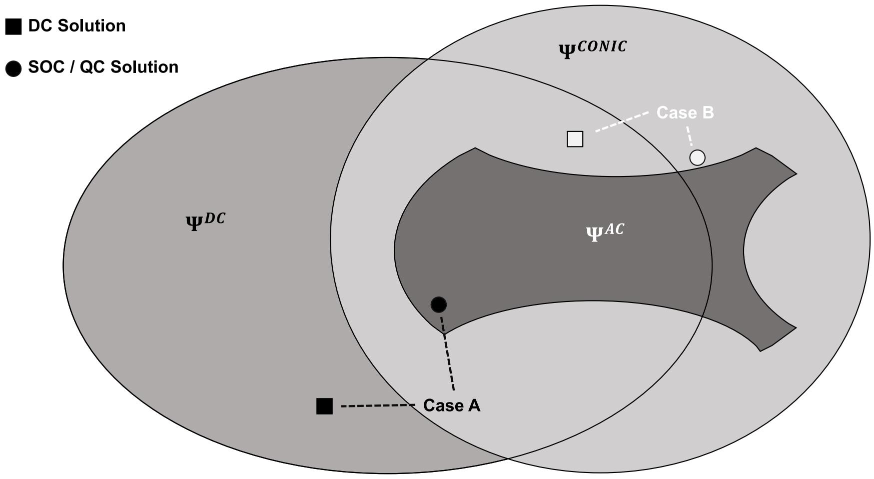

Fig. 5. Visualization of the examples.

Result 5. *Despite advances in non-linear optimization, computational effort is still a limiting factor when employing tighter power flow relaxations. We could solve all mixed-integer second-order conic and quadratic convex relaxations to optimality for the selected problem sizes. Semidefinite programs, however, could not be solved on our test installation as mixed-integer problems, and could not reliably be solved as continuous problems for larger problem sizes.*

When computing the original allocation, mixed-integer programs have to be solved in order to determine generator commitments. While such problems are NP-hard in general, the mixed-integer linear programs underlying the DC approximation can be solved effectively by modern solver technology for the problem sizes at hand. On our machine, the median runtime over all scenarios is only 2.22 s (maximum 12.16 s). Although second-order cone program solvers have matured, solving the mixed-integer SOCPs means more computational effort. As such, the median runtime for the SOC relaxation is 9.1 min (maximum 382.4 min) and for the QC relaxation it is 11.4 min (maximum 564.4 min).

Naturally, the runtimes for the continuous-type pricing problems decrease compared to the respective mixed-integer problems. In particular, the runtimes for the second-order cone programs decrease significantly to a median runtime of 81.5 s (307.4 s) over all pricing problems for the SOC (QC) relaxation, respectively. We note that we only consider a single-period problem, and that more pronounced runtime discrepancies are to be expected in a multi-period context with intertemporal constraints (e.g., ramping constraints).

5. Conclusion

Clearing electricity spot markets is a complex problem. A feasible dispatch has to consider non-linear transmission constraints and non-convex preferences of the market participants. As a result, the ACOPF clearing problem is intractable to solve and market operators need to rely on convex relaxations or approximations of network constraints. Besides, buyers and sellers typically possess non-convex preference functions, preventing the existence of Walrasian equilibrium prices. As a consequence, market participants can have incentives to deviate or even incur losses that need to be compensated by individual side-payments. In this paper, we aim to bring together the literature of optimal power flow and competitive equilibrium theory and study clearing and pricing in non-convex electricity markets with non-linear network models.

Crucially, prices based on the standard linearized DC approximation can be fundamentally flawed and insufficiently reflect actual network flows. Prices can wrongly signal congestion that does not comply with actual scarcities in the AC-feasible outcomes. This is important since consumers may be exposed to unjustifiably high prices, and investment and demand response signals as well as congestion rents are fundamentally distorted.

One reason for diverging prices is the fact that the DC approximation does not price reactive power, even though it affects the final AC-feasible allocation. Given these findings, one direction for future research could be an assessment of whether reactive power prices on the transmission level might be a reasonable addition for a future market design. Reactive power sources are currently compensated as ancillary services (Yu et al., 2019), but if reactive power constraints are binding for the allocation, a locational reactive price signal might better reflect the physical reality.

Moreover, our experiments reveal a number of additional insights. The allocations and committed generation capacities can differ significantly between different power flow models. This has an impact on the properties of prices and the required redispatch to enable AC-feasibility. Typically, more capacity is committed and dispatched with tighter convex relaxations, as this is required to accommodate the more complex transmission constraints.

In terms of pricing rules, CH prices are less dependent on the underlying power flow model. They exhibit less MWPs and minimize LOCs of market participants. CH prices are generally intractable to compute and the prices might not reflect congestion accurately, resulting in larger PCRS for the transmission operator. In contrast, IP prices provide better congestion signals and lower PCRS. However, they are more volatile and depend on the chosen power flow model.

The solution obtained from a DC approximation is often far away from an AC-feasible outcome, requiring significant redispatch. Tighter convex relaxations reduce redispatch costs substantially. Moreover, they lead to higher welfare and much improved price signals. The computational costs for dispatch and pricing remains an advantage for the DC approximation. The underlying (mixed-integer) linear programs scale better than equivalent second-order conic (or even semidefinite) programs. However, the second-order conic relaxations provide a viable alternative already today. An open question for future research is the evaluation of convex power flow relaxations for very large (e.g., European-scale) grids and multi-period problems.

Overall, we conclude that tighter power flow relaxations can provide significant upsides for electricity spot markets. Apart from welfare gains and low redispatch costs, we argue that unbiased price signals are a major advantage of advanced power flow models. This is even more relevant for future electricity markets with many decentralized renewable energy sources, variability in supply and demand, and less predictable congestion patterns.

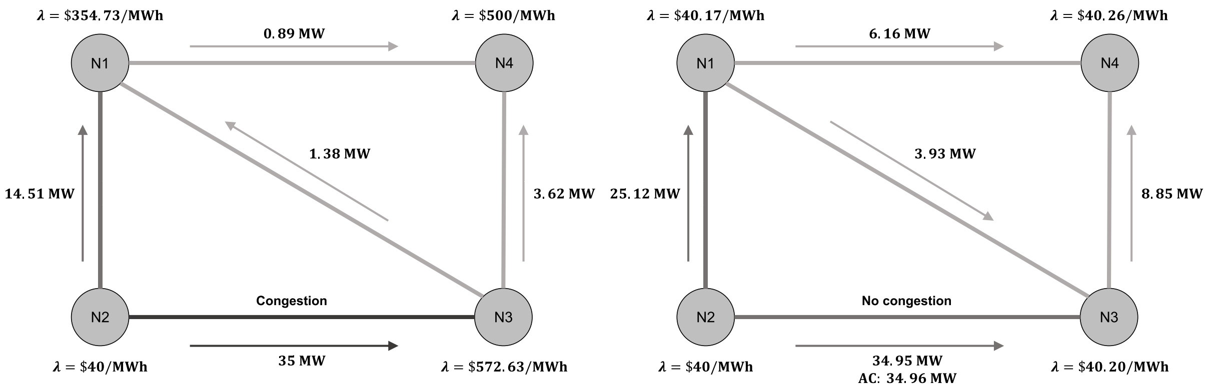

Fig. 6. DC (Left) and SOC (Right) Prices and line flows.

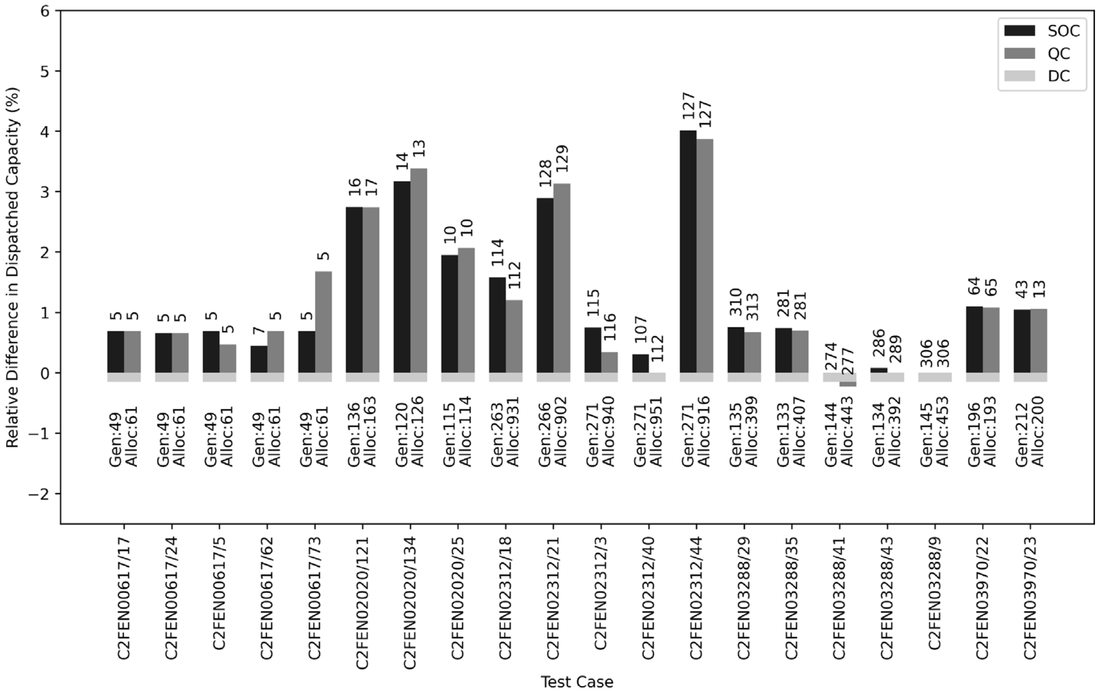

Fig. 7. Dispatched generation capacity: The figure denotes the number of dispatched generators ("Gen") and their total dispatched capacity ("Alloc") for Ψ^{DC} (in MW) at the bottom. The bars indicate the relative difference (in %) in dispatched capacity for Ψ^{SOC} and Ψ^{QC}, respectively. The labels of each bar indicate the number of generators with a different dispatch compared to Ψ^{DC}. For example, the first bar in the first test case C2FEN00617/17 indicates that 5 units were dispatched differently under Ψ^{SOC} as compared to the 49 dispatched units under Ψ^{DC}.

CRediT authorship contribution statement

Martin Bichler: Conceptualization, Validation, Resources, Writing – original draft, Writing – review & editing, Supervision, Funding acquisition. **Johannes Knörr:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Project administration.

Appendix A. Biased prices from the DCOPF approximation

Currently, prices on electricity markets rely on the linearized DC approximation, regardless of the chosen pricing rule (e.g., IP or CH). A main finding of our numerical experiments concerned the large price discrepancies at some nodes that appeared even though there was no congestion in the AC-feasible solution.

Large networks as those analyzed in the paper are difficult to study. In this section, we illustrate how such problems can arise in very small networks with two or four nodes. Even in these small examples, DC prices may signal congestion where there is none, or fail to signal congestion even though it occurs.

Fig. 5 visualizes Ψ^{AC}, Ψ^{DC}, and one exemplary conic relaxation (either Ψ^{SOC} or Ψ^{QC}). Note that Ψ^{SOC} and Ψ^{QC} represent *relaxations*, i.e., $\Psi^{AC} \subseteq \Psi^{SOC}$ and $\Psi^{AC} \subseteq \Psi^{QC}$, while Ψ^{DC} represents an *approximation*, i.e., $\Psi^{AC} \setminus \Psi^{DC}$ might be non-empty. Generally, we are unable to determine the AC-optimal allocation (due to computational hardness) and AC-optimal prices (due to computational hardness and the non-existence of competitive equilibria). For the small examples below, however, we can at least compute the AC-optimal allocation and analyze the quality of different price signals based thereupon.

Case A: DC-optimal solution being SOC/QC-Infeasible

We consider a simple market with two connected nodes N1 and N2 and a single time period. Both nodes have a base voltage of 1 p.u. and a voltage angle of 25 degrees. A single generator at N1 can supply up to 100 MW at a cost of \$40/MWh. A single buyer at N2 wishes to consume up to 30 MWh for \$3000/MWh. Both market participants

Table 6
Lines.

ID	From-node	To-node	Resistance [p.u.]	Reactance [p.u.]	Thermal limit [MW]
L1	N1	N2	0.01	0.03	35
L2	N1	N3	0.01	0.03	25
L3	N1	N3	0.02	0.04	30
L4	N2	N3	0.005	0.01	35
L5	N3	N4	0.01	0.02	30

Table 7
Mapping of notation.

Notation in this work	Description	Equivalent notation for ARPA-E
Sets		
B	Buyers	J
S	Sellers	G
N	Nodes	I
L	Lines	E and F
Parameters		
V_i^{min}	Minimum voltage at node $i \in N$	$\underline{v}_i$
V_i^{max}	Maximum voltage at node $i \in N$	$\overline{v}_i$
G_{ik}	Conductance at line $(i,k) \in L$	$\mathrm{Re}((R + jX)^{-1})$
B_{ik}	Susceptance at line $(i,k) \in L$	$\mathrm{Im}((R + jX)^{-1})$
$v_{b\ell}$	Valuation for bid $\ell \in \beta_b$ of buyer $b \in B$	c_{jn}
$q_{b\ell}$	Upper bound for bid $\ell \in \beta_b$ of buyer $b \in B$	$\overline{p}_{jn}$
$\underline{P}_b$	Minimum real power of buyer $b \in B$	$p_j^0 \underline{t}_j$
$\overline{P}_b$	Minimum real power of buyer $b \in B$	$p_j^0 \overline{t}_j$
$\underline{Q}_b$	Minimum reactive power of buyer $b \in B$	$q_j^0 \underline{t}_j$
$\overline{Q}_b$	Minimum reactive power of buyer $b \in B$	$q_j^0 \overline{t}_j$
$v_{s\ell}$	Variable cost for bid $\ell \in \beta_s$ of seller $s \in S$	c_{gn}
h_s	Fixed cost of seller $s \in S$	c_g^{on}
$q_{s\ell}$	Upper bound for bid $\ell \in \beta_s$ of seller $s \in S$	$\overline{p}_{gn}$
$\underline{P}_s$	Minimum real power of seller $s \in S$	$\underline{p}_g$
$\overline{P}_s$	Minimum real power of seller $s \in S$	$\overline{p}_g$
$\underline{Q}_s$	Minimum reactive power of seller $s \in S$	$\underline{q}_g$
$\overline{Q}_s$	Minimum reactive power of seller $s \in S$	$\underline{q}_g$
Variables		
x_b	Consumption of buyer $b \in B$	p_j
$x_{b\ell}$	Consumption of buyer $b \in B$ regarding bid $\ell \in \beta_b$	p_{jn}
y_s	Generation of seller $s \in S$	p_g
$y_{s\ell}$	Generation of seller $s \in S$ regarding bid $\ell in \beta_s$	p_{gn}
u_s	Commitment of seller $s \in S$	x_g^{on}
$\lvert V_i \rvert$	Voltage magnitude at node $i \in N$	v_i
θ_i	Voltage angle at node $i \in N$	θ_i

have no preferences or restrictions with respect to reactive power. They both have fully convex preference functions, and thus IP and CH prices are equivalent. The line connecting N1 and N2 has a resistance of 0.02 p.u., a reactance of 0.01 p.u., and a thermal line capacity of 30.1 MW.

The optimal solution based on Ψ^{DC} is to let the generator supply 30 MWh to the buyer. The lossless DC line transmits 30 MWh and is thus uncongested. The price at both nodes is $40/MWh.

In this example, the SOC and QC solutions are identical. Since these power flow models consider line losses, the DC-optimal solution is not SOC/QC-feasible. In particular, under either conic model, the generator produces 30.1 MWh and transmits this to N2, yet due to line losses, the buyer only obtains 29.95 MWh. The transmission line is congested due to its thermal line limit, and the prices are $40/MWh at N1 and $3000/MWh at N2.

Importantly, the SOC/QC-solution is equivalent to the AC-optimal allocation. In other words, the welfare-maximizing physically feasible AC-solution implies a congested transmission line, and a price difference between the two nodes is justified or even necessary. The prices based on Ψ^{DC} significantly distort the congestion income of the transmission operator and send wrong investment or demand response signals. Prices based on Ψ^{SOC} or Ψ^{QC} are much better suited for the actual AC-optimal outcome.

Case B: SOC/QC-optimal solution being DC-Infeasible

We now consider a market with four nodes and five transmission lines. All nodes have a base voltage of 1 p.u. and a voltage angle of 25 degrees, and the line data are shown in Table 6.

A generator at N2 can supply up to 100 MW at a cost of $40/MWh, and a generator at N4 can provide up to 30 MW at $500/MWh. At both N1 and N4, a buyer is willing to consume up to 15 MWh for $3000/MWh. At N3 a buyer can consume up to 30 MWh for $3000/MWh. Similar to above, there are no restrictions regarding reactive power, IP and CH prices are equivalent due to fully convex market participants, and the SOC and QC relaxations produce the same prices.

Under all allocations, all buyers consume their maximum quantities. However, the implied power flows are different. Under Ψ^{DC}, the thermal capacity of line L4 is fully exhausted and the network is thus congested. This leads to large price differences, ranging from $40/MWh at N2 to $572.63/MWh at N3. In contrast, under the optimal SOC and QC allocation, the critical line L4 is not fully exhausted. As a result, the network is uncongested, and prices are roughly $40/MWh at each location (minor price differences occur due to line losses). This is illustrated in Fig. 6.[17] The power flows of the SOC/QC-relaxation are not DC-feasible.

Importantly, under the AC-optimal outcome, none of the transmission lines are congested, including the critical line L4. We argue that in this case only minor price differences due to line losses should occur in the network, as implied by the conic relaxations. In contrast, the linearized DC approximation sets unjustifiably high prices at some of the nodes, which comes at the expense of electricity consumers at these locations.

While the test cases above are illustrative, we observe the same patterns in Section 4. While we cannot verify the AC-optimal outcome for these large networks, we argue that the price signals obtained from tighter conic relaxations reflect the physical network flows much better, avoiding the highly distorted prices, congestion incomes, or demand response signals that DC prices can imply.

Appendix B. Mapping of notation to ARPA-E documentation

In the following, we provide a mapping of the notation used in this paper to the notation used in the problem formulation for the ARPA-E Grid Optimization Competition Challenge II. We refer to the following document: https://gocompetition.energy.gov/sites/default/files/Challenge2_Problem_Formulation_20210531.pdf. Note that some parameters from ARPA-E are normalized (see Table 7).

Appendix C. Differences in dispatched capacities

Fig. 2 in the main part of this paper illustrates the differences in *committed* capacity during unit commitment. In contrast, the following Fig. 7 illustrates the difference in *dispatched* capacity. In particular, the figure visualizes the differences in allocation across different power flow models. As Fig. 2 already implied, allocations can vary significantly depending on the choice of power flow model, with up to 4.01% differences in dispatched capacities. This, of course, has an impact on pricing and the required redispatch.

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6. Conclusion

This dissertation has investigated the complexities of pricing in non-convex and coupled electricity markets, aiming to address the challenges posed by non-convex preferences and the coupled nature of power systems.

An essential economic concept related to electricity markets is the Walrasian equilibrium, consisting of an allocation and linear and anonymous prices that satisfy budget balance and envy-freeness. The welfare theorems posit that Walrasian equilibria maximize social welfare, assuming perfect competition, demand independence, and convex preferences.

However, as elaborated in this dissertation, the critical assumption of convexity does not hold in electricity markets. Bid languages and physical transmission constraints introduce non-convexities into the market clearing problem, rendering existence conditions for Walrasian equilibria invalid. Moreover, these non-convexities affect the computational scalability of the allocation problem.

Present-day electricity markets employ various methodologies to determine allocation and prices, often resorting to linearized models. This dissertation aims to contribute to the advancement of allocation and pricing rules in electricity markets. This research has resulted in three publications, each contributing to the understanding and improvement of market design in the context of day-ahead electricity markets.

6.1. Summary of Contributions

The first part of this dissertation, presented in Bichler et al. (2023a), focused on the impact of demand response on pricing in electricity markets. Recognizing the absence of Walrasian equilibrium prices, the paper proposed alternative pricing rules founded on principles of convex optimization. It demonstrated that, in the presence of price-elastic demand, achieving both efficiency and individual rationality while maintaining budget balance and linear and anonymous prices becomes an impossible task. The proposed pricing rule runs in polynomial time and addressed concerns about increasing make-whole payments, offering a potential solution to regulated day-ahead electricity markets.

The second part, detailed in Ahunbay et al. (2023b), extended the perspective to a multi-objective optimization framework. By identifying conflicting design goals in established pricing rules using duality theory, the paper framed pricing in non-convex markets as a multi-objective optimization problem. It introduced the novel Join pricing rule, optimizing multiple classes of lost opportunity costs simultaneously. This work highlighted the trade-offs inherent in pricing rules and provided a new perspective for designing rules that strike a balance between competing objectives.

The third part, discussed in Bichler and Knörr (2023), shifted the focus to the impact of different power flow models on market outcomes. Investigating the representation of the transmission network in the context of optimal power flow problems, the paper emphasized the economic implications of using simplified models such as the widespread DCOPF. It revealed substantial deviations in allocation and redispatch when employing different power flow models, with significant economic consequences. This work highlighted the importance of accurate representations of power flows for deriving price signals that reflect the locational scarcity of electricity.

6.2. Discussion and Outlook

Electricity markets are characterized by profound complexities. They encompass vast and complex physical systems with interconnected grids, where power flows adhere to non-convex and non-linear laws of physics. Electricity markets operate across various time-frames (forward, day-ahead, intraday, real-time, balancing) to achieve an efficient, physically feasible, and sustainable allocation of power. Prices serve to indicate the short-run marginal value of electricity as well as efficient long-run investment signals. At the same time, the electricity sector is undergoing a major transition with substantial investments in low-carbon technologies to meet net-zero carbon emission targets in the coming decades. As a result, electricity market design must evolve to align with these changes, and it is crucial to obtain a deep understanding of power systems and market dynamics.

The contributions made in this dissertation aim to enhance our understanding of pricing in non-convex and coupled day-ahead electricity markets. The presented publications proposed scalable pricing rules with a robust economic justification, including minimizing side-payments and providing effective congestion signals. Numerical analyses demonstrated favorable outcomes achievable through these pricing rules. Furthermore, the dissertation prompted consideration of how power flow models impact allocation and pricing, suggesting a potential shift from linearized transmission models to more sophisticated second-order conic power flow models as optimization software advances.

At the same time, these contributions raise new questions regarding clearing and pricing in day-ahead markets. Given the complexity of electricity markets and fundamental transformations in power systems and markets, numerous avenues for future research emerge, necessitating further exploration and analysis.

For example, a current issue in European markets concerns the scalability of the EUPHEMIA algorithm. With the continual expansion of renewable energy capacities, the SDAC market is set to introduce 15-minute products, thereby increasing the size of the market clearing problem and significantly impacting the runtime of EUPHEMIA (NEMO Committee, 2023). Even as the transition from MIC orders to Scalable Complex Orders occurs and PUN orders are phased out, scalability issues are anticipated to persist. As a consequence, SDAC considers non-uniform pricing approaches as an alternative (SDAC, 2023). In contrast, U.S. ISO markets, where non-uniform pricing is already in place, face fewer scalability concerns, although the mixed-integer allocation problems lack polynomial time scalability.

Similarly, all presented non-uniform pricing rules in this dissertation require to obtain the optimal, welfare-maximizing allocation as a first step. Recently, there have been promising theoretical advances in tractable, near-optimal clearing and pricing in non-convex markets (Milgrom and Watt, 2021). Exploring this line of research and its applications to electricity markets (Ahunbay et al., 2023a) could yield promising scalable solutions for practical application. Other directions could involve machine learning applications to reduce the computation costs of power flow problems (Hasan et al., 2020; van Hentenryck, 2021). Improving the scalability of second-order conic and semidefinite programs (Vandenberghe and Andersen, 2015; Yuan and Hesamzadeh, 2019) could facilitate an implementation in real-world markets, leading to enhanced allocation and price signals, as explored in the third publication of this dissertation.

Scalability concerns and the representations of transmission networks are also tied to the discussion surrounding bidding zone configurations in European markets. The current zonal market clearing process has faced scrutiny due to the substantial increase in redispatch volumes and associated costs. To address concerns related to efficient market clearing and congestion management, the EU Commission has initiated a Bidding Zone Review (BZR) to reassess the configuration of European bidding zones (ACER, 2022). However, determining appropriate bidding zone delineations (Dobos et al., 2024) or even implementing nodal pricing in Europe (PSE, 2017; Knörr et al., 2024) requires thorough analysis and investigation concerning market efficiency, security of supply, stability, carbon emission objectives, and other pertinent factors. Despite initial reports (ENTSO-E, 2022a), there remains a notable absence of large-scale empirical studies using real-world data to comprehensively understand the behavior of different market clearing and pricing rules in practical applications.

In auction theory, the concepts of *incentive compatibility* and the *core* assume an important role. Incentive compatibility asserts that every bidder achieves their best outcome by bidding truthfully, while the core comprises a set of allocations that cannot be improved by any coalition of buyers with sellers and/or the auctioneer. Due to the size and complexity of electricity markets, these concepts are often overlooked in the development of clearing and pricing rules. However, the strategic behavior of bidders and their incentives to manipulate the system merit closer examination (Garcia et al., 2022). This also includes examining gaming strategies across different timeframes, such as strategic overbidding in day-ahead markets to increase profits in subsequent redispatch markets, known as inc-dec gaming (Hirth and Schlecht, 2020).

As electricity systems continue to decarbonize, the volatility of supply increases. However, due to non-convexities, even minor bid adjustments can considerably impact allocation and prices. Therefore, a potential area for future research could involve exploring robust optimization methods for market clearing and assessing their influence on prices (Zugno and Conejo, 2015; Xiong and Singh, 2017). As a result, market clearing and pricing rules could be enhanced to provide more stable outcomes in a future energy system with higher uncertainty and volatility.

Looking ahead in the medium to long term, electricity markets may necessitate significant reforms to integrate extensive renewable energy capacities, storage resources, and demand response mechanisms. For instance, in European markets, both the European Commission (European Commission, 2023) and ENTSO-E (ENTSO-E, 2022b) have outlined their visions for future market designs. While these visions offer guidance on necessary reforms and changes in market design, numerous unanswered questions remain regarding the precise formulation of market rules and policy instruments.

Moreover, reforms in spot markets extend beyond the scope of day-ahead markets, which forms the main focus of this dissertation. Although the majority of energy is cleared in day-ahead markets, intraday, real-time, and balancing markets assume increasing importance in decarbonized electricity systems, where changing weather conditions can affect power supply close to physical delivery. For example, current European intraday markets require continuous trading, efficient utilization of cross-border capacity, and effective congestion pricing (European Commission, 2019). Reconciling these requirements with current regulations (NEMO Committee, 2019a) presents challenges, prompting active discussions to amend or replace continuous trading with frequent intraday auctions (Graf et al., 2022).

In the balancing timeframe, ancillary services are presently co-optimized with energy in U.S. spot markets. In contrast, European markets feature separate markets for distinct types of reserves, such as PICASSO (for automatic frequency restoration reserve), MARI (for manual frequency restoration reserve), and TERRE (for replacement reserves). However, to enhance the use of cross-zonal capacity, SDAC is currently exploring co-optimization approaches as well (SDAC, 2022). These developments and related aspects introduce numerous new research questions concerning electricity market design closer to real-time operations.

In conclusion, the research presented in this dissertation contributes to the ongoing discourse on market design in non-convex and coupled electricity markets. By addressing the challenges posed by demand response, non-uniform side-payments, and power flows, the findings offer insights and potential solutions for improving the efficiency, fairness, and sustainability of day-ahead electricity markets. As the energy landscape continues to evolve, future research endeavors can build upon these contributions to create innovative and adaptive market designs that align with the goals of a modern and sustainable power sector.

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