

State Diagrams and Tree Tensor Networks

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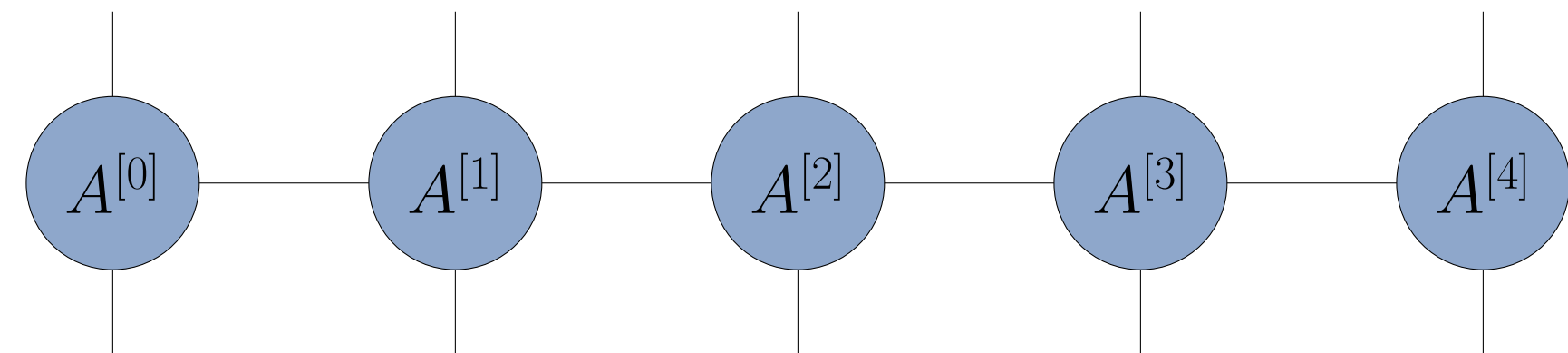
Motivation

Many relevant Hamiltonians and operators have the following form

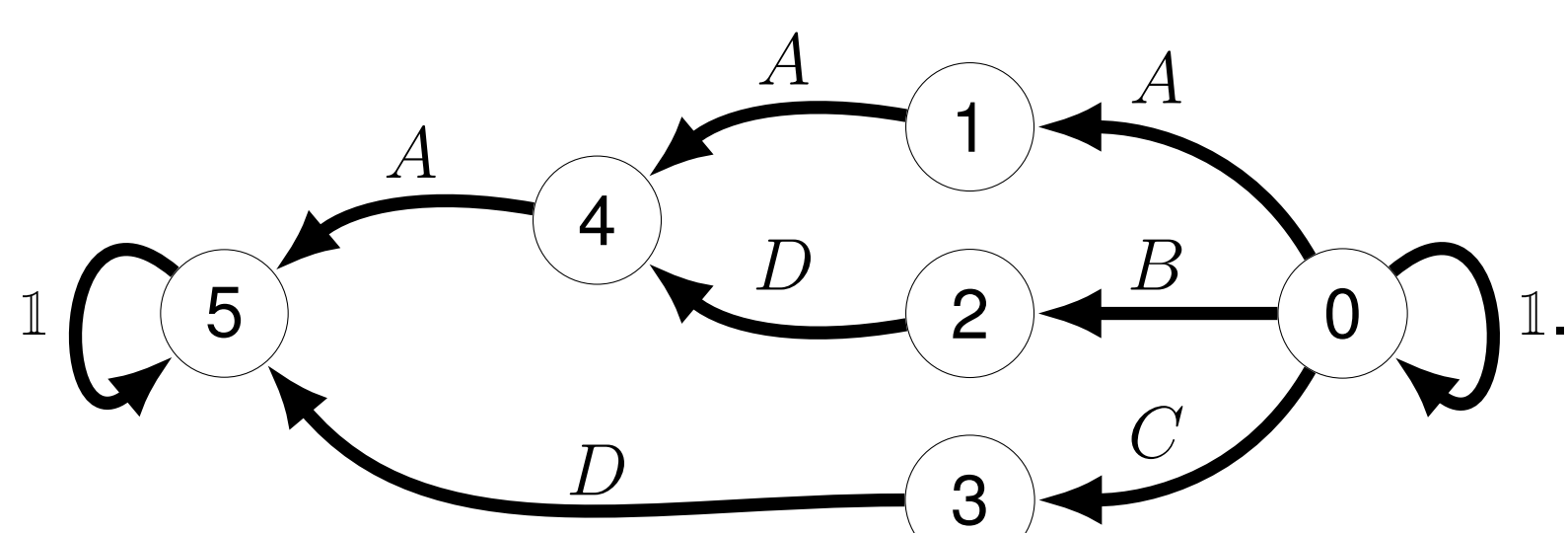
$$H = \sum_{i=1}^K \bigotimes_{s \in Q} A_i^{[s]},$$

where Q is a set of small quantum systems or sites and the operator $A^{[s]}$ acts on site s .

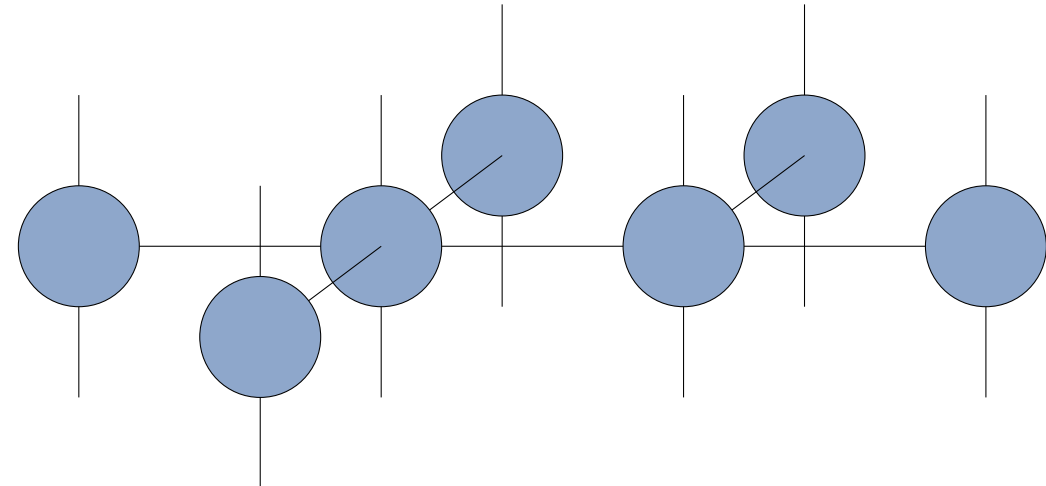
If Q represents a 1D-chain, we can bring such an operator in matrix product operator form



using cellular automata, e.g.

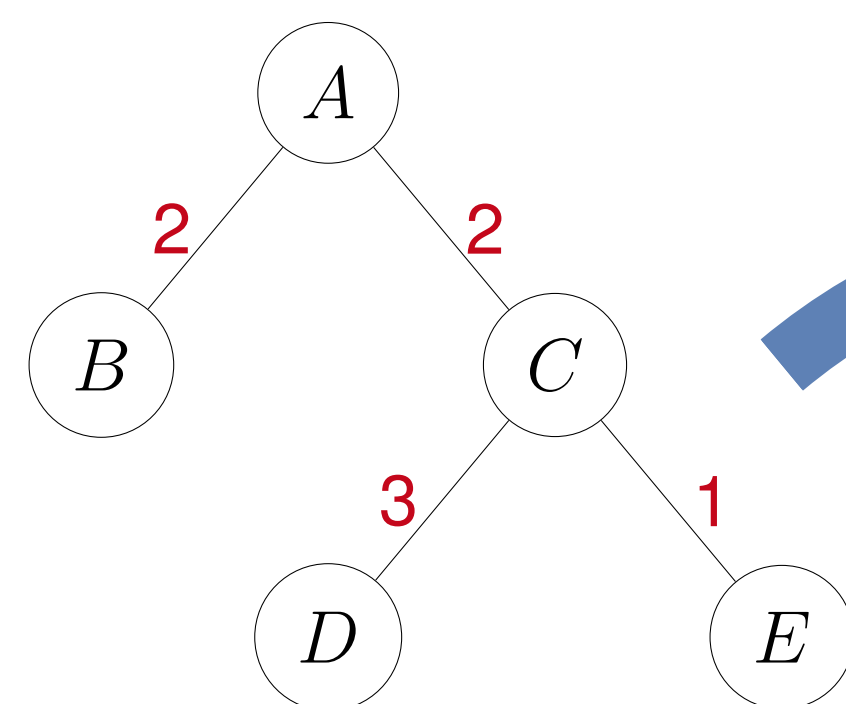


However, if Q or the operator have a tree structure it can be advantageous to use a tree tensor network operator

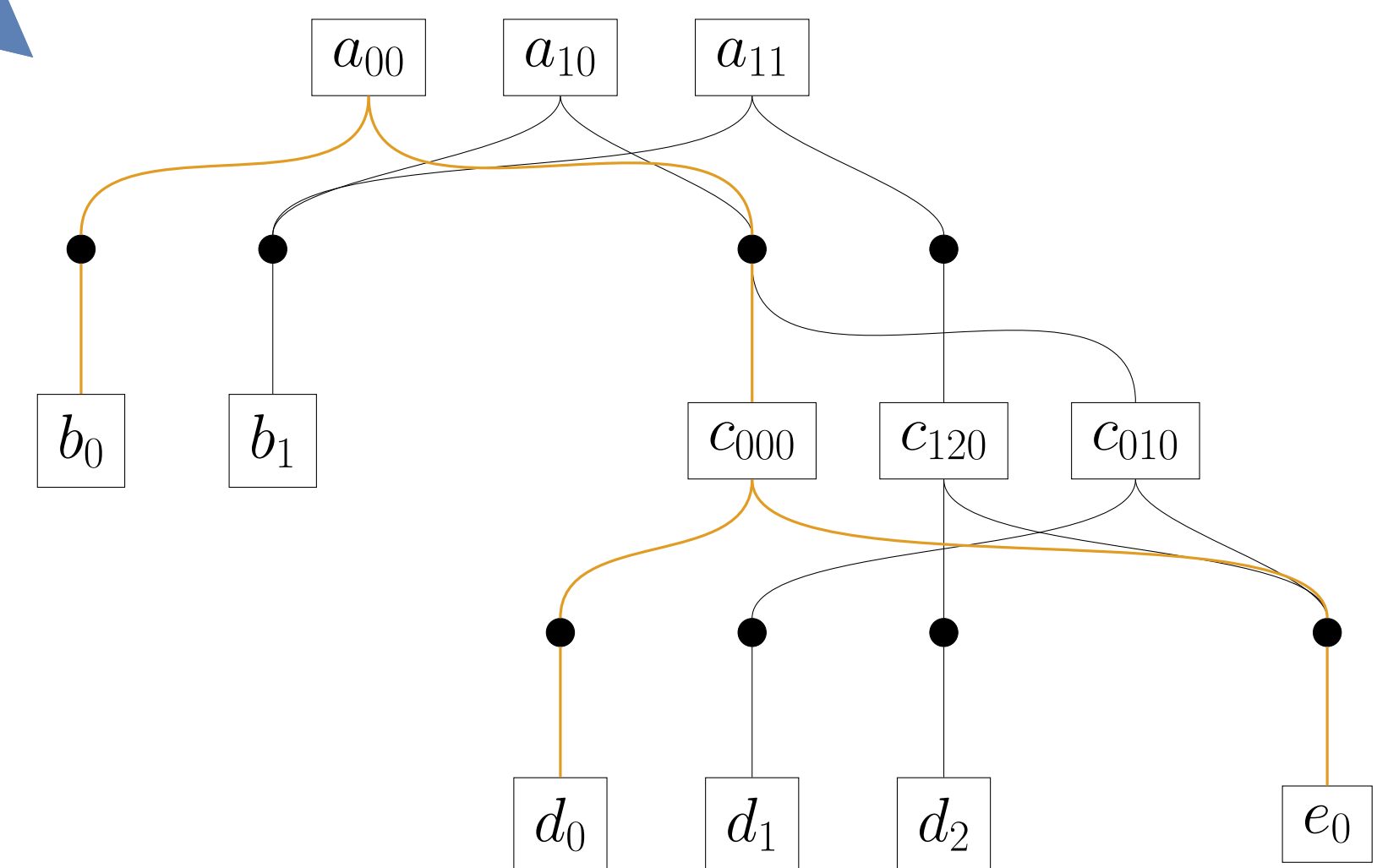


In this case the basic automaton method fails. Therefore we considered state diagrams and developed an algorithm to obtain a state diagram that corresponds to a given operator.

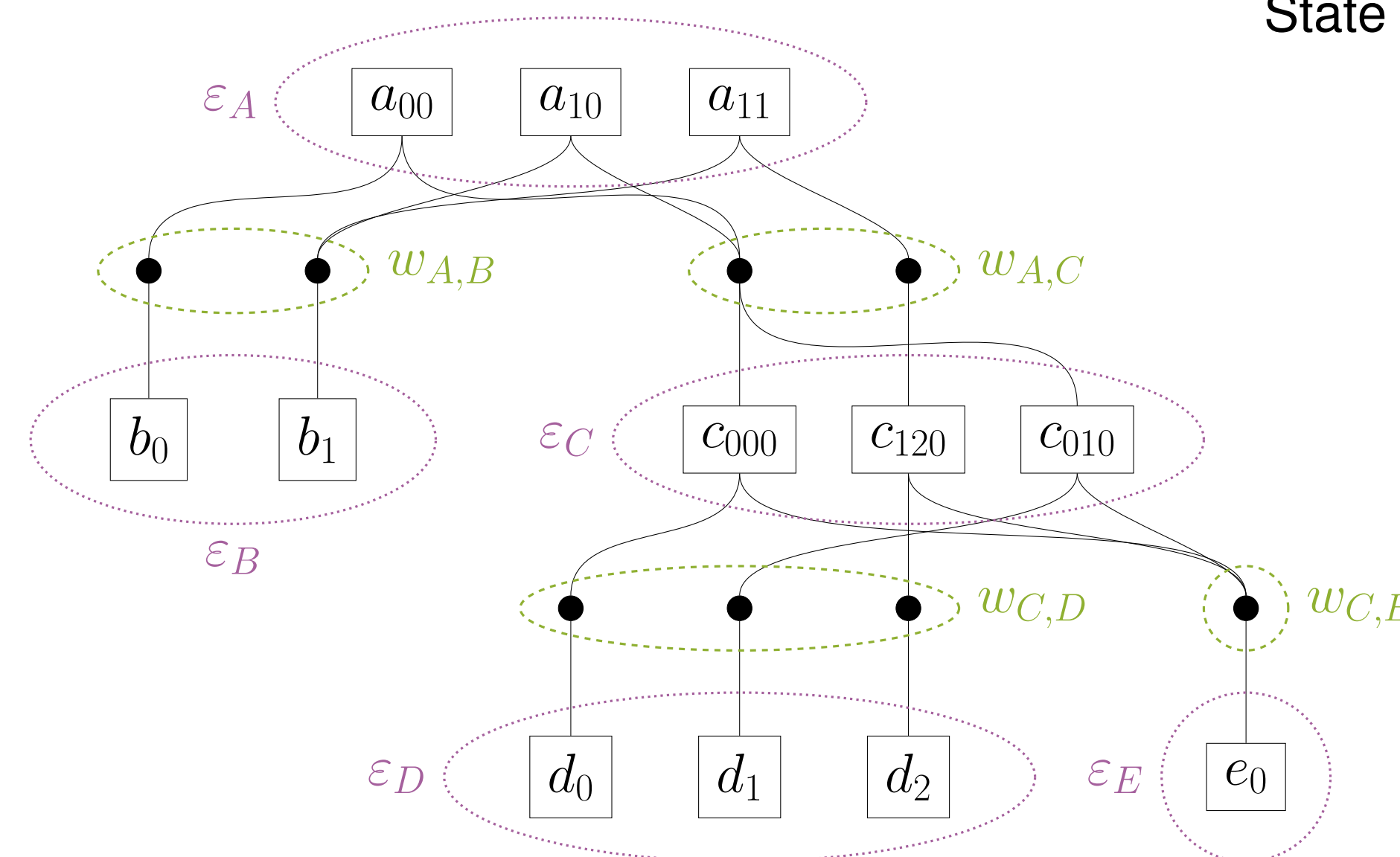
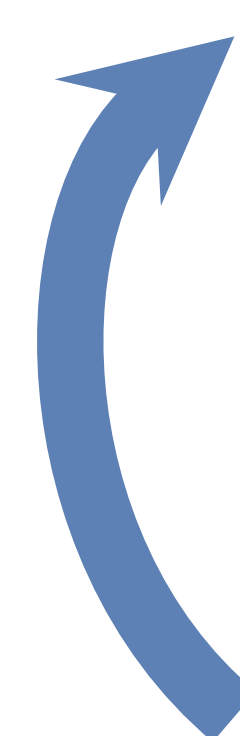
The State Diagram



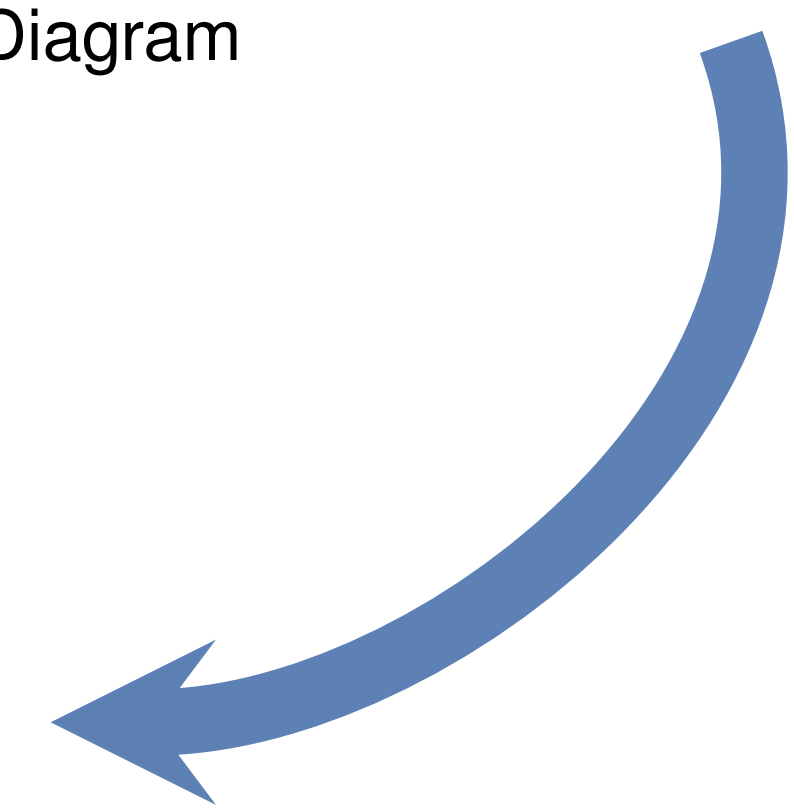
Tree Tensor Network



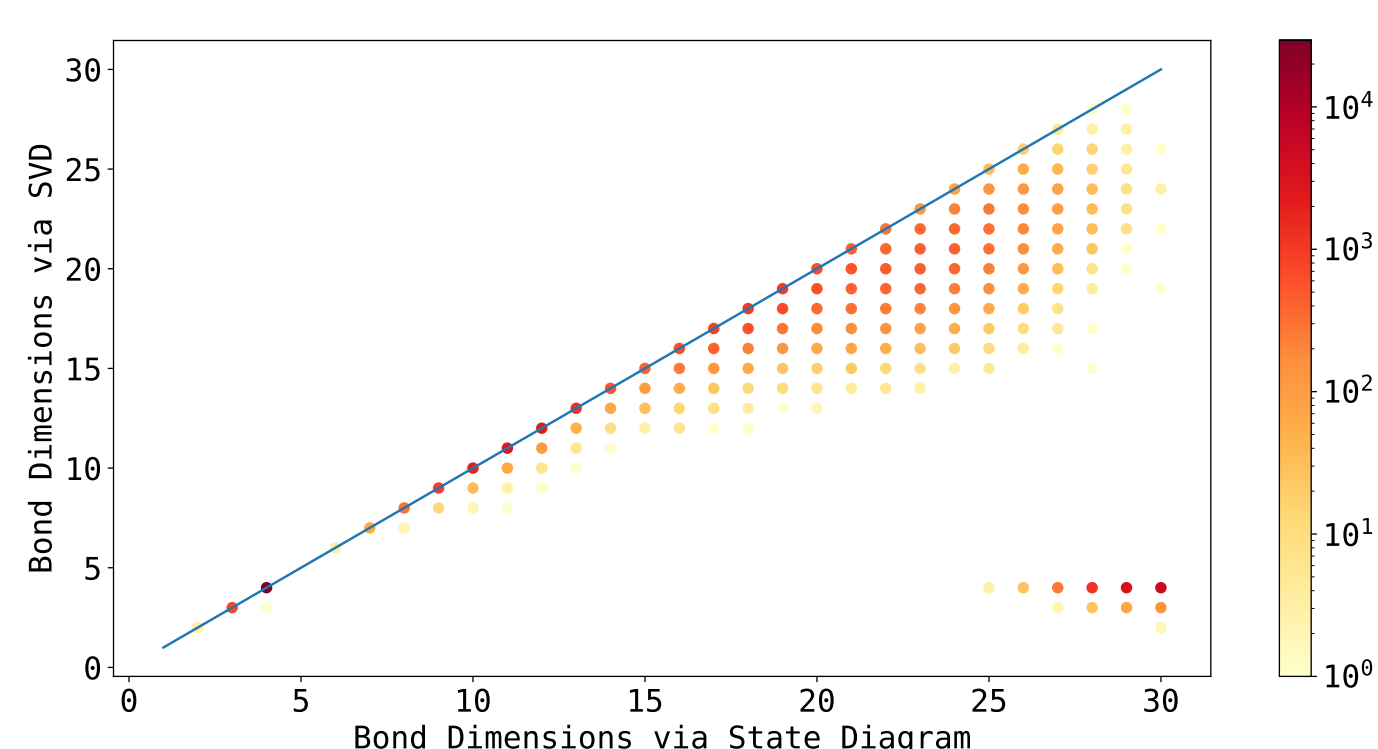
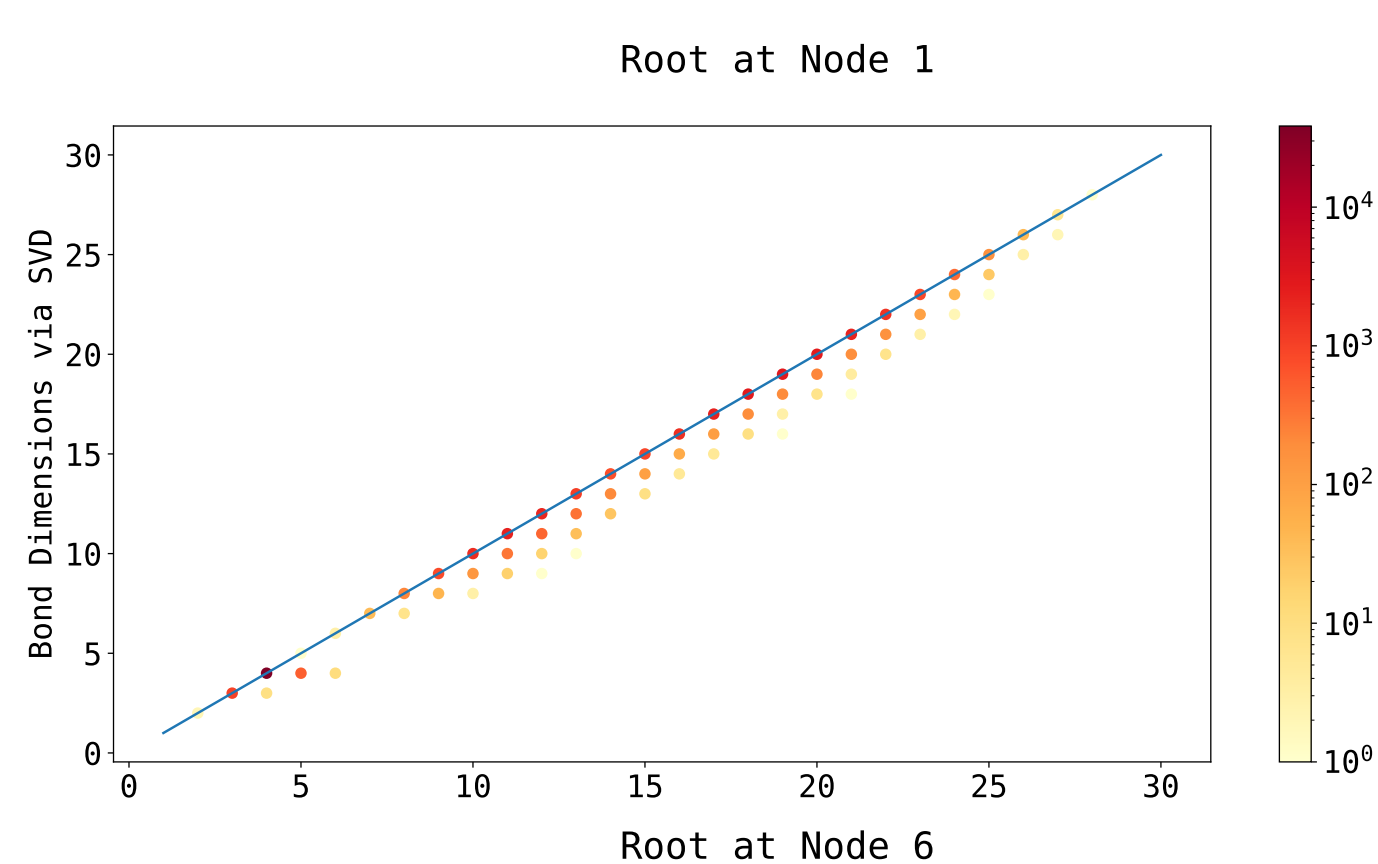
State Diagram



Tree Structure of State Diagram



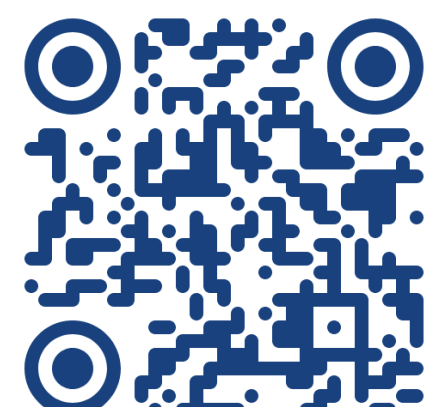
Bond Dimensions



Plots of the tree tensor network operator bond dimension for randomly generated Hamiltonians with 30 terms.

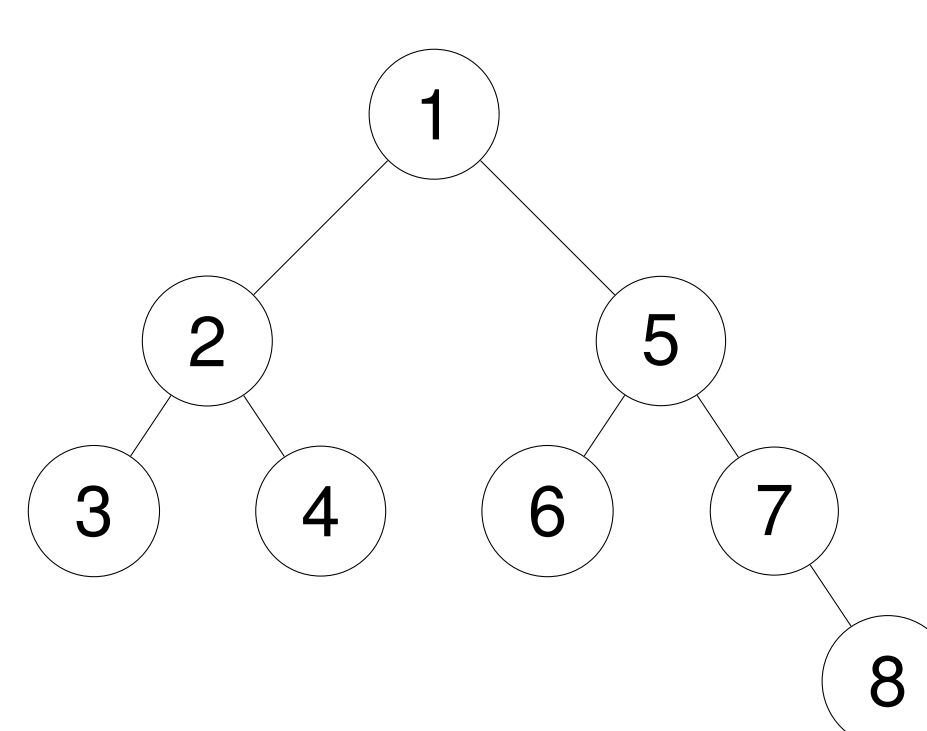
Reference

Richard M. Milbradt, Qunsheng Huang, Christian B. Mendl; *State Diagrams to determine Tree Tensor Network Operators*; arxiv: 2311.13433



Adding State Diagrams

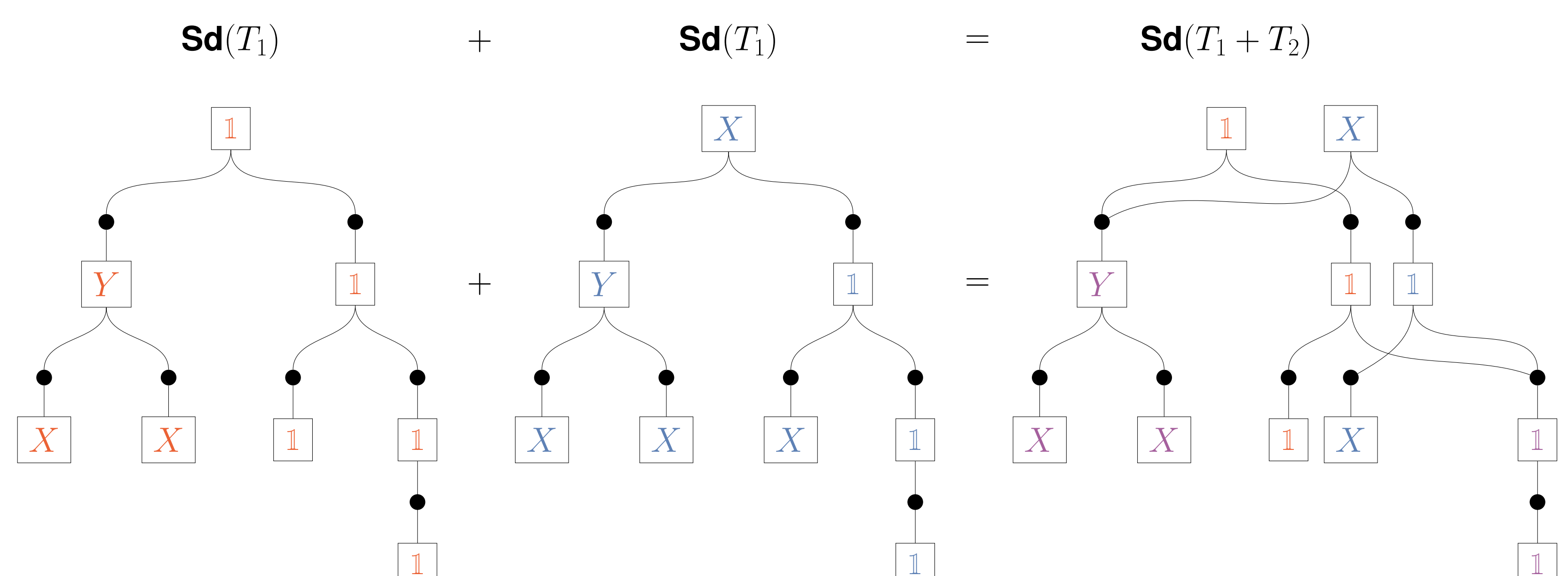
Tree Structure:



Terms:

- $T_1 = 1_1 \otimes Y_2 \otimes X_3 \otimes X_4 \otimes 1_5 \otimes 1_6 \otimes 1_7 \otimes 1_8$
- $T_2 = X_1 \otimes Y_2 \otimes X_3 \otimes X_4 \otimes 1_5 \otimes X_6 \otimes 1_7 \otimes 1_8$

Given the state diagrams of T_1 and T_2 , how can we find the state diagram of their sum $T_1 + T_2$ with a low bond dimension?



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