Advanced Material Models and Dynamic Rupture in SeisSol

SeismoChat, Scripps Institute of Oceanography

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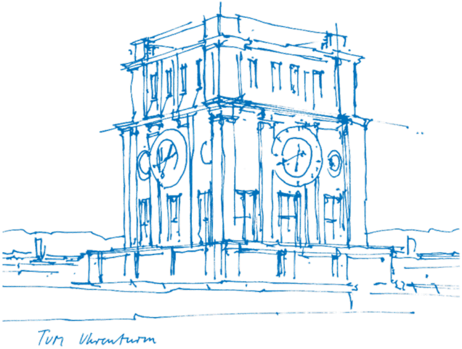


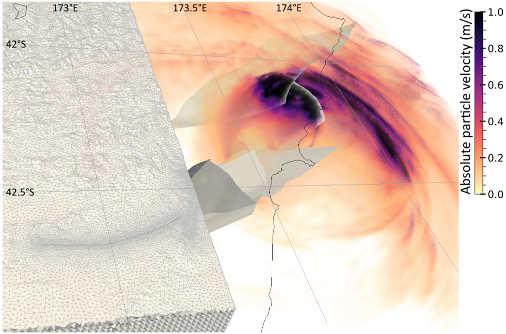
Earthquake modeling

Wave propagation: Solve the elastic wave equation

- $\partial_t q + A\partial_x q + B\partial_y q + C\partial_z q = 0$
- q contains stresses and velocities, A, B and C contain material information.

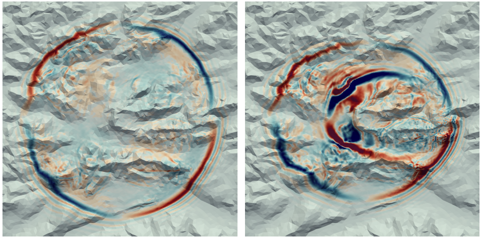
Source Dynamics: Prestressed fault and frictional failure criterion

(Ulrich et al., 2019): "Dynamic viability of the 2016 Mw 7.8 Kaikōura earthquake cascade on weak crustal faults"



Anisotropic materials

- Directional dependent material behaviour, e.g. cracked or layered media
- Jacobian A, B, C are more densely populated, but can reuse the numerical scheme from the elastic wave equation (Wolf et al., 2020)



Left: isotropic material, Right: anisotropic material

Isotropic:

$$\sigma = \lambda \mathrm{tr}(\epsilon) I + 2\mu\epsilon$$

Anisotropic:

$$\sigma_{ij} = \sum_{k,l=1}^{3} c_{ijkl}\epsilon_{kl}$$

Poroelastic wave equation

- Elastic waves (velocity–stress formulation)
- Fluid phase (pressure and relative fluid velocities)
- Possible applications: Geothermal energy production, fault zone effects
- Challenge: Stiff source term

- Constitutive relation relates stress, strain, pore pressure and fluid variation

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \\ -p \end{pmatrix} = \begin{pmatrix} \lambda_M + 2\mu_M + M\alpha^2 & \lambda + M\alpha^2 & \lambda + M\alpha^2 & 0 & 0 & 0 & M\alpha \\ \lambda + M\alpha^2 & \lambda_M + 2\mu_M + M\alpha^2 & \lambda + M\alpha^2 & 0 & 0 & 0 & M\alpha \\ \lambda + M\alpha^2 & \lambda + M\alpha^2 & \lambda_M + 2\mu_M + M\alpha^2 & 0 & 0 & 0 & M\alpha \\ 0 & 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ M\alpha & M\alpha & M\alpha & 0 & 0 & 0 & M \end{pmatrix} \begin{pmatrix} \epsilon^M_{xx} \\ \epsilon^M_{yy} \\ \epsilon^M_{zz} \\ \epsilon^M_{yz} \\ \epsilon^M_{xz} \\ \epsilon^M_{xy} \\ -\zeta \end{pmatrix}$$

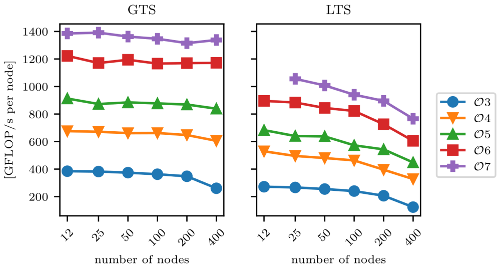
- Algebraic source term couples fluid and solid movement

$$\partial_t q + A\partial_x q + B\partial_y q + C\partial_z q = \underbrace{Eq}$$

HPC aspects

- Global time stepping (GTS)
    - Every element has the same time step
    - regular update scheme
- Local time stepping (LTS)
    - Every element can have its own time step
    - complicated scheduling, but less computational work

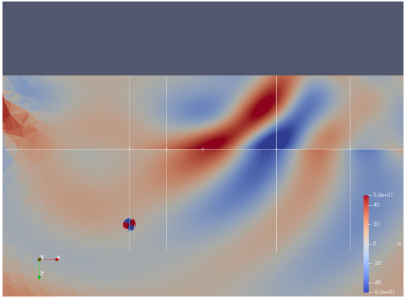
⇒ LTS reduces time-to-solution, but parallel efficiency decreases



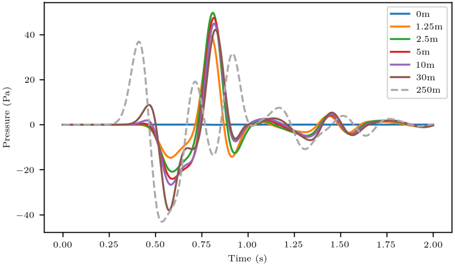
Parallel efficiency of a poroelastic setup with 7.3 million elements for global (GTS) and local (LTS) time-stepping on SuperMUC-NG.

Challenge: rapidly changing pressure at material interfaces

Layer over halfspace with double-couple point source



Pressure field in the $x - z$ plane

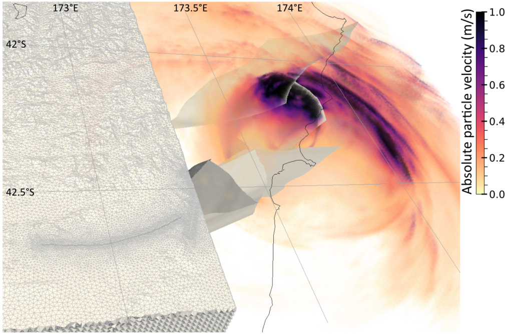


Pressure time history at $x = 0$.

Dynamic Ruputure

Internal boundary condition as earthquake source:

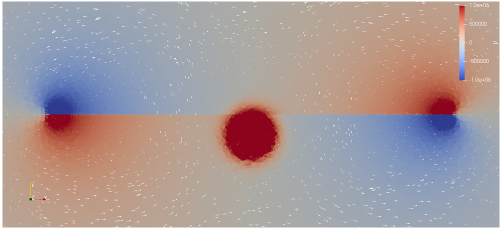
- Compute normal stress σ_n and traction τ at the fault.
- Evaluate fault strength $\mu\sigma_n$.
- If $\tau > \mu\sigma$: evaluate slip rate.
- Impose discontinuous velocity field at the interface



Kaikoura earthquake (Ulrich et al., 2019)

Poroelastic Dynamic Rupture

- Internal boundary condition: slip rate based normal stress and tangential traction.
- Pore pressure weakens fault: $\widehat{P}_n = \sigma_n - P_f$.
- Pressure pertubation as initial condition.



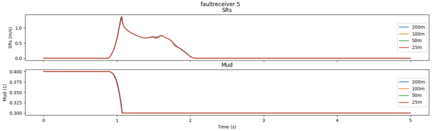
Pressure during rupture

Project at Scripps

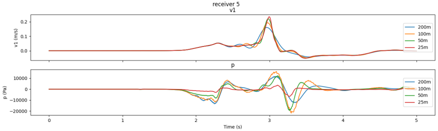
- Convergence analysis
- Parameter study: How does the biot coefficent/porosity change the rupture
- Branched faults

Convergence analysis

- Homogeneous halfspace, vertical fault (4 km wide)
- Rupture nucleation by fluid stress pertubation (5 MPa, $r \approx 600$ m)
- Record wave field for 5 s
- Fault: almost perfect results, wave field: could be better



Fault receiver at $(1200, 0, -700)$



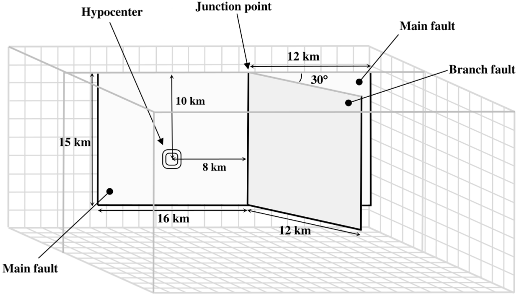
Surface receiver at $(-3000, 3000, 0)$

How do poroelastic effects influence branching?

Geometry

- vertical fault (28 km × 15 km)
- branch at 16 km from the *left* end, at 15°
- nucleation at 8 km from the *left* end, at 10 km depth
- prestress: right lateral rupture:

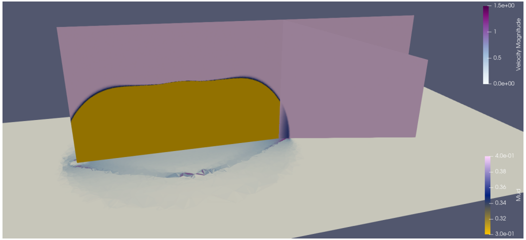
$$\sigma = \begin{pmatrix} -29.5 & 10.0 & 0.0 \\ 10.0 & -25.5 & 0.0 \\ 0.0 & 0.0 & -25.5 \end{pmatrix} \text{MPa}$$



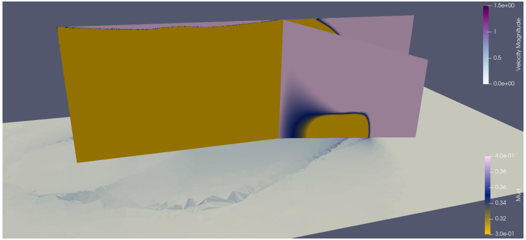
TPV24 geometry with 30° branch angle

What happens?

Use a material with Biot's coefficient $\alpha = 1 - \frac{K_M}{K_s} = 0.95$.



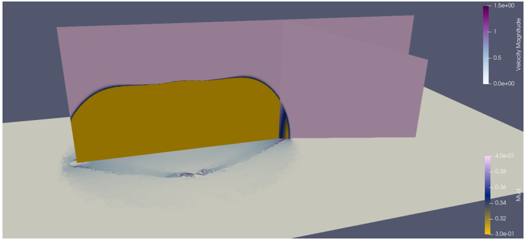
Friction coefficient and wave field after 2.3 s



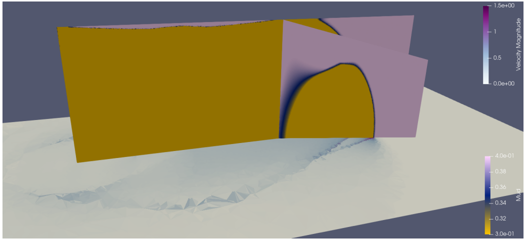
Friction coefficient and wave field after 4.5 s

How’s the elastic case?

Use the *Gassmann* equivalent elastic material

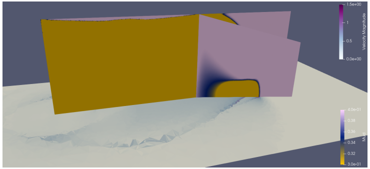


Friction coefficient and wave field after 2.3 s

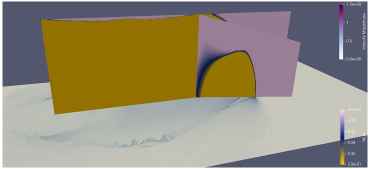


Friction coefficient and wave field after 4.5 s

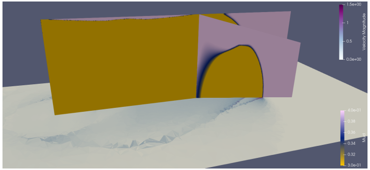
Parameter study



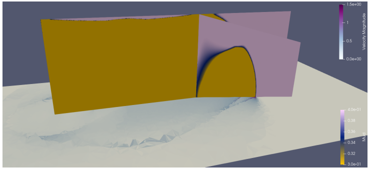
Large poroelastic effect $\alpha = 0.95$



Small poroelastic effect $\alpha = 0.1$



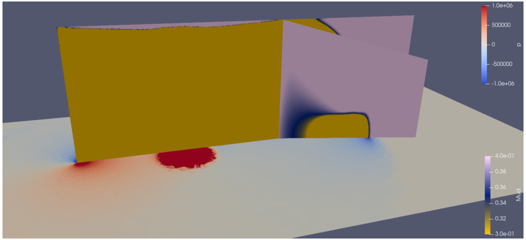
Elastic equivalent



Elastic equivalent

Why?

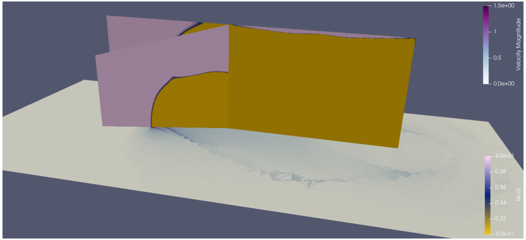
- Branch in the extensive quadrant of the main fault
- $\Delta P < 0 \Rightarrow$ increases $\widehat{P}_n$
- On the other side ($\phi = -15°$), prestress on branch is stronger
- $\Rightarrow$ no rupture on pressure weakening side.



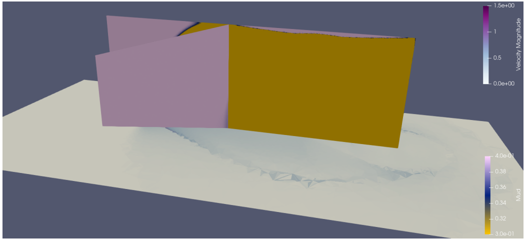
Poroelastic material ($\alpha = 0.95$) with with pore pressure.

Is there a pressure weakening configuration?

Yes! Make -15° branch weaker: $\sigma = \begin{pmatrix} -32.8 & 8.7 & 0.0 \\ 8.7 & -22.2 & 0.0 \\ 0.0 & 0.0 & -25.5 \end{pmatrix}$ MPa



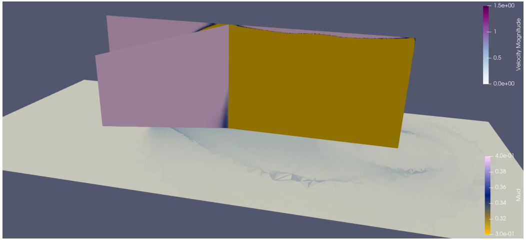
Pressure weakening breaks fault



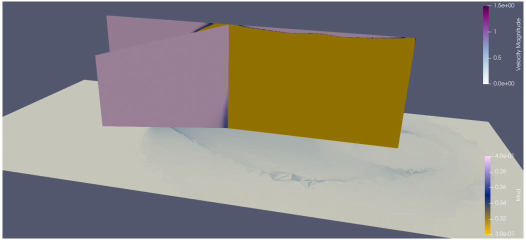
Elastic equivalent doesn’t break

Does this change with α?

Yes!, consider $\alpha = 0.1$: the branch never breaks



Pressure weakening doen’t break



Elastic equivalent doesn’t break

Poroelastic effects can inhibit or facilitate fault branching!

Take home message

- HPC enables seismological research, but it is hard.
- Domain scientists and computer scientiests have to work closely together.
- SeisSol supports complicated material models, where analytic possibilities end.
- Dynamic rupture in poroelastic materials is possible in 3D!
- Pore pressure variation can inhibit or facilitate fault branching

Try SeisSol: `https://github.com/SeisSol/Training`

Ask me: `wolf.sebastian@cit.tum.de` or in the barnyard until June 2.