

1 **INCREASING THE CAPACITY OF A LANE-FREE BELTWAY FOR CONNECTED**  
2 **AND AUTOMATED VEHICLES USING POTENTIAL LINES**

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6 **Majid Rostami-Shahrbabaki, Corresponding Author**

7 Chair of Traffic Engineering and Control, Technical University of Munich  
8 Arcisstrasse 21, 80333 Munich, Germany  
9 majid.rostami@tum.de  
10 ORCID: 0000-0002-8129-4519

11

12 **Hanwen Zhang**

13 Chair of Traffic Engineering and Control, Technical University of Munich  
14 Arcisstrasse 21, 80333 Munich, Germany  
15 hanwen.zhang@tum.de  
16 ORCID: 0000-0001-9051-3894

17

18 **Maya Sekeran**

19 Chair of Traffic Engineering and Control, Technical University of Munich  
20 Arcisstrasse 21, 80333 Munich, Germany  
21 maya.sekeran@tum.de  
22 ORCID: 0000-0002-5508-8463

23

24 **Klaus Bogenberger**

25 Chair of Traffic Engineering and Control, Technical University of Munich  
26 Arcisstrasse 21, 80333 Munich, Germany  
27 klaus.bogenberger@tum.de  
28 ORCID: 0000-0003-3868-9571

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**1 ABSTRACT**

2 Road congestion implies that there is not enough capacity to meet the demands of road users. One  
3 solution to increase traffic network capacity is to build beltways. A beltway guides commuters  
4 to use alternative routes without entering the city core. However, this approach has led to urban  
5 sprawls causing extended land use and induced traffic demand. Recent developments in automated  
6 vehicles with communication capabilities allow us to re-think traffic systems by optimizing cur-  
7 rent road infrastructure leading to the idea of lane-free traffic (LFT). LFT removes lane marking  
8 and allows vehicles to exploit the entire road's width, increasing lateral occupancy and reducing  
9 congestion on the beltways. In this paper, we introduce a new approach using potential lines for  
10 a CAV's driving strategy that significantly increases the capacity of a lane-free beltway. Poten-  
11 tial lines are described as a method to laterally organize the CAVs on a beltway based on their  
12 desired speeds. Compared to other available lane-free driving strategies, this approach reduces  
13 lateral interaction and maneuvers, leading to much higher throughput. The results show that the  
14 critical density of the considered beltway can reach up to  $250 \text{ veh/km}$  with a maximum flow of  
15  $27036 \text{ veh/h}$ , which is almost four times of a similar lane-based beltway. These promising results  
16 render more road construction or road extensions obsolete. Additionally, with the proposed idea,  
17 the width of the existing roads can be reduced, and the extra capacity can be allocated to other road  
18 users.

19

20 *Keywords:* beltway, ring road, lane-free traffic, potential lines, connected and automated vehicles  
21 (CAVs)

## 1 INTRODUCTION

2 There are two ways by which traffic network capacity can be increased - building more roads  
3 or using innovative traffic management approaches to maximize the exploitation of the current  
4 road capacity. The first approach often leads to costly investments, while the second approach  
5 may work for the short term as it caters to current traffic demand unless the long-term demand  
6 can be controlled. The number of auto vehicles on the road exploded over the years, especially  
7 after World War II. To address this huge demand, modern roads were introduced to manage traffic  
8 congestion and to better connect different places on land. This includes introducing beltways (also  
9 referred to as ring roads). Beltways are constructed with the aim of decentralizing urban areas  
10 and reducing congestion within the radius of highly dense cities by offering alternative routes for  
11 vehicles instead of entering the city core. However, the issue remains with the increasing number  
12 of cars on the road but still not enough road networks to meet this demand despite continuous road  
13 infrastructure developments.

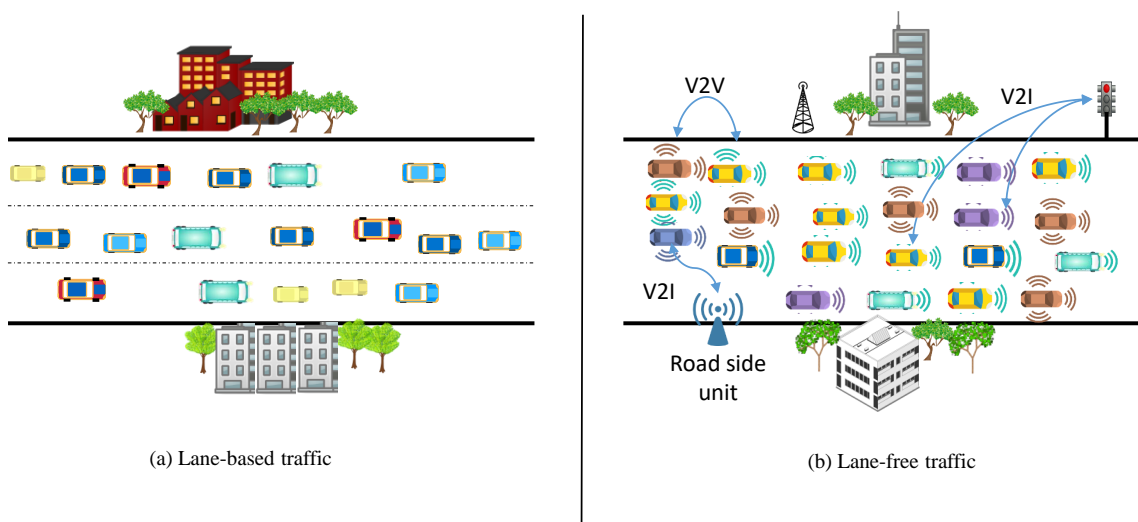
14 From the construction design aspect, beltway attributes towards a displacement of service  
15 sector and industrial activities (1, 2). This, in turn, reduces city population by 20% through moving  
16 jobs and housing away to suburban areas. In Beijing, there are currently seven ring roads but yet no  
17 sign of relief for traffic congestion and carbon emission (3). A case study on the big Almaty ring in  
18 Kazakhstan shows that ring roads are good for short-term congestion relief but not sustainable in  
19 the long run (4) as also observed in other cities around the world (4, 5). The effect is a significant  
20 increase up to 20% in traffic after only one year. It is not surprising to observe that the forecast for  
21 traffic intensity does not include induced traffic as a factor since the economical benefits outweigh  
22 the congestion and travel time factors. An earlier case study on a beltway in Virginia in 1958  
23 already showed adverse effects on the reduction of road capacity (5). These developments also  
24 causes environmental impacts such as deforestation and displacement of wildlife to make way for  
25 road expansions (6–8).

26 To manage ongoing capacity drop on beltways, traffic planners/controllers started using  
27 technology through the implementation of intelligent traffic systems (ITS) (9–11). However, de-  
28 spite these attempts, capacity drop on beltways remains a problem. Therefore, the question re-  
29 mains, how do we increase traffic capacity on current beltways and optimize existing road infras-  
30 tructure? A beltway typically has between four to six lanes (two or three lanes in each direction).  
31 The width of the lanes is maximized for large vehicles. Therefore the common car typically oc-  
32 cupies about 50% of a lane width. This fact indicates that, laterally, a three-lane beltway could  
33 theoretically span roughly six cars instead of three cars. But this is not doable with current lane-  
34 based traffic system with predefined lane markings. Even the lane detection and lane-keeping  
35 assistant algorithms in automated vehicles (AVs) will force the vehicles to adhere to lane rules.  
36 Although driving in lanes aid human drivers to focus on the traffic ahead and to ensure some level  
37 of traffic safety, it becomes complex and risky when human drivers perform lane changes that  
38 requires multiple angles of perception before performing a lateral move. This causes inefficient  
39 driving that often leads to accidents while also limiting dynamic road capacity.

40 From an infrastructure standpoint, developing more beltways will be unsustainable as it  
41 also involves long-term and costly maintenance of road markings and road signs. Furthermore,  
42 there have been discussions about how AVs will deal with faded lane marking over the years. All  
43 the issues mentioned so far leads to the following research question - what if we do away with lane-  
44 markings and corresponding road signs by introducing lane-free traffic on roads? This can be made  
45 possible by leveraging CAV capabilities equipped with advanced sensors, vehicle to vehicle (V2V)

1 communications and efficient control algorithms. With these capabilities, it opens up opportunities  
 2 for new traffic systems that allows for vehicles to move seamlessly through traffic in a lane-free  
 3 manner with communication updates between vehicles and the infrastructure to guide the traffic  
 4 systems.

5 Lane-free traffic is a new and futuristic idea, and in order to realize lane-free traffic with  
 6 CAVs, there is potential to re-think traffic design. The earliest work found so far is by (12) with the  
 7 idea of rendering the lane markings obsolete and allowing the vehicles to drive on a continuous 2-d  
 8 surface, more specifically, along any desired lateral location of the road. The term lane-free traffic  
 9 was first coined in a point of view paper by Papageorgiou et al. (13). The authors suggested two  
 10 combined principles for LFT namely "lane-free" traffic and "nudging". The first principle allows  
 11 vehicles to be at any lateral position within the road boundary. While "nudging" is inspired by  
 12 the effect of fast-moving vehicles on the front slower vehicles in a lane-less traffic observed in  
 13 some countries with low lane discipline. These faster vehicles apply a pushing force by honking  
 14 the vehicles in front to make space for overtaking. This new force implies that the upstream  
 15 vehicles can impact downstream vehicles. This impact is not represented in conventional traffic  
 16 flow models. Figure 1 below shows the concept of LFT compared to lane-based traffic.



**FIGURE 1:** Lane-based traffic vs Lane-free traffic (14)

17 Since then, several control designs have been proposed considering both classical and ad-  
 18 vanced approaches for deriving strategies in the LFT. In this paper, we introduce the concept of  
 19 potential lines to uniformly assign the lateral occupancy of the road based on the desired speed  
 20 of the vehicles. This novel approach extensively increases the traffic capacity on lane-free belt-  
 21 ways. Furthermore, we consider the factor of large differences in speed that causes faster vehicles  
 22 to move in slalom. Potential lines prevent this by reducing lateral interaction and maneuvers thus  
 23 increasing traffic safety as well. To the best of our knowledge, this approach has not been con-  
 24 sidered before. The research in (13, 15, 16) shows that the capacity of a freeway with three lanes  
 25 that is generally about 6000-7000 *veh/h* would increase to 14000-15000 *veh/h* in the lane-free  
 26 condition, doubling the current lane-based capacity. However, with the developed potential lines,  
 27 the achieved flow in the ring road is almost four times of equivalent the lane-based road.

## 1 LITERATURE REVIEW

2 To manage capacity drop on beltways, innovative solutions using technology were introduced. In  
3 (17) a dynamic speed control strategy increases capacity and speed but only if there are policies  
4 in place to control demand. In (18), the authors proposed separating lane usage based on class  
5 of vehicles. However, this approach may lead to under use or over use of certain lanes. In (19),  
6 the authors proposed a beltway feedback strategy that may be implemented as a feedback system  
7 for CAVs while not requiring lane-based adherence. To manage the increasing traffic congestion  
8 along the Venice-Mestre area despite the introduction of the MARCO system, project MARCO-T3  
9 installed several traffic management mechanisms (9). Among these are traffic monitoring, video  
10 surveillance, automatic incident detection (AID), ramp metering, variable message sign (VMS),  
11 lane control sign (LCS), textual information and rerouting, and multiagency incident management  
12 (MIM). An evaluation conducted eight years later show a reduction of congestion hours by 21%.  
13 However, there was no indication of the cost to maintain this integrated system. The risk of one or  
14 more of the systems failing at any point could also cause issues where the traffic can no longer be  
15 managed for a certain period of time. The I-66 Beltway in the United States introduced a dynamic  
16 tolling system that benefited alternative route travel time but not in the opposing direction of the  
17 beltway during tolling hours (10). Another approach is to implement ramp metering as described  
18 in (11) where over 10 different ramp metering algorithms are implemented on various freeways in  
19 the United States.

20 As mentioned earlier, lane-free traffic is a new concept for a fully automated traffic envi-  
21 ronment. Therefore there is potential to design new vehicle driving strategies for CAVs to increase  
22 road capacity. Recent studies introduce several classical approaches to control vehicle movements  
23 in LFT. In (20), the authors proposed an optimal path planning algorithm using nonlinear Opti-  
24 mal Control Problem (OCP) for efficient vehicle advancement. In Levy and Haddad (21) applied  
25 the non-linear MPC framework to plan the trajectories of the vehicle in order for the vehicles to  
26 self-advance with minimal control. Decentralized controllers as proposed in (22) ensures safety of  
27 vehicle operations in LFT. Other advanced controllers uses reinforcement learning (15) and coor-  
28 dination graphs (16) to facilitate collaborative multi-agent decision making. These work show that  
29 the capacity of the lane-free road can be almost doubled compared to the similar lane-based road.

30 To further increase road capacity, especially on beltways or highways, barriers that sepa-  
31 rates bi-directional traffic can also be removed to allow vehicles to share the total road width. A  
32 flexible and dynamic internal boundary as explored in (23) and (24) is introduced which help to  
33 serve asymmetric traffic demands. In Chavoshi and Kouvelas (25), the authors proposed a dis-  
34 tributed controller within clusters of vehicles. The non-linear MPC is applied to control the move-  
35 ment of vehicles in each cluster. Other approaches such as vehicle flocking also have potentials for  
36 increasing road capacity (26).

37 These strategies show promising ways by which LFT systems can be implemented to in-  
38 crease road capacity. In this work, we introduce the idea of potential lines which allows for the  
39 uniform lateral distribution of the vehicles based on their desired speed. This approach leads to  
40 much higher flow rates mainly due to removing unnecessary lateral movement of vehicles. In the  
41 following section, our proposed methodology is elaborated.

## 42 METHODOLOGY

43 The proposed methodology consists of four parts: 1- a cruise controller allowing vehicles to reach  
44 their desired speed; 2- artificial potential fields for collision avoidance, and, most importantly, 3-

1 potential lines for uniform lateral distribution of the vehicles across the road width. In addition,  
 2 4- a boundary controller is implemented which keeps the vehicles within the road boundary by  
 3 constraining the lateral accelerations. The implementation of the potential field is the main novelty  
 4 of this present work which leads to significantly high throughput. In the following subsections,  
 5 first, the considered vehicles dynamics is explored, then the details of the proposed approach are  
 6 given.

## 7 **Vehicle dynamics**

8 For the movement of the vehicles in the lane-free environment, we used the double integrator model  
 9 to describe the vehicle dynamics. The discrete-time differential equations are used where  $T$  is the  
 10 sampling period and  $k = 0, 1, \dots$  is the discrete time index where  $t = k \cdot T$ . For a given vehicle  $i$ ,  
 11 the vehicle dynamics is described with the following state-space equations in the longitudinal and  
 12 lateral directions:

$$x_{1,i}(k+1) = x_{1,i}(k) + Tx_{2,i}(k) + \frac{1}{2}T^2u_{x,i}(k) \quad (1a)$$

$$x_{2,i}(k+1) = x_{2,i}(k) + Tu_{x,i}(k) \quad (1b)$$

$$x_{3,i}(k+1) = x_{3,i}(k) + Tx_{4,i}(k) + \frac{1}{2}T^2u_{y,i}(k) \quad (1c)$$

$$x_{4,i}(k+1) = x_{4,i}(k) + Tu_{y,i}(k) \quad (1d)$$

13 where  $x_{1,i}$  and  $x_{2,i}$  are the longitudinal position and speed of the vehicle whereas  $x_{3,i}$  and  $x_{4,i}$   
 14 represent lateral position and speed, and  $u_{x,i}$  and  $u_{y,i}$  are longitudinal and lateral accelerations,  
 15 respectively. Note that the vehicle states are measured with respect to the vehicle's center. The  
 16 motion dynamics (Equation 1) are subject to the following bound constraints.

$$-u_{x,min} < u_{x,i}(k) < u_{x,max} \quad (2a)$$

$$-u_{y,min} < u_{y,i}(k) < u_{y,max} \quad (2b)$$

17 The longitudinal and lateral acceleration of vehicles are limited due to the physical capa-  
 18 bility of the vehicles for accelerating, braking, steering, and of course due to the comfort issues of  
 19 the passengers. Such accelerations are defined based on two main forces applied to each vehicle;  
 20 target speed and inter-vehicle forces. The target speed force is induced via a cruise control loop  
 21 as the result of the vehicle's desire to drive at its desired speed. On the contrary, the inter-vehicle  
 22 force induced by an artificial potential field around each vehicle prevents collision among vehicles.  
 23 The inter-vehicle force comprises a nudging force applied to the front vehicle and a repulsive force  
 24 applied to the following vehicle. In addition, a lateral force is applied to the vehicle based on the  
 25 proposed potential lines. The details on how each force is calculated are given in the following  
 26 subsections.

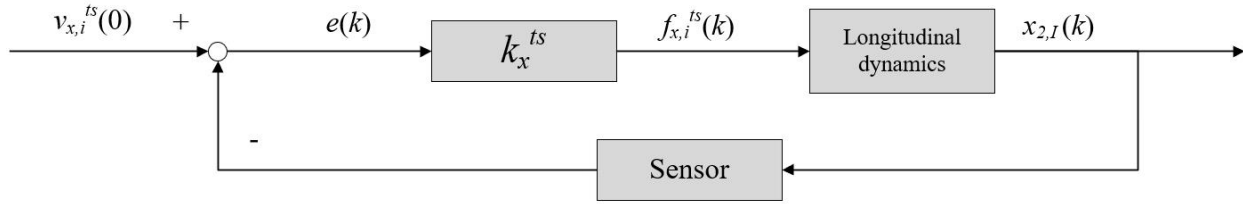
## 27 **Cruise controller for achieving desired longitudinal speed**

28 One goal in designing driving strategies for CAVs in the lane-free environment is to allow them to  
 29 drive as closely as possible, whenever possible, to their desired speed. To this end, a cruise con-  
 30 troller with a target speed set-point is used. The longitudinal target speed  $v_{x,i}^{fs}$  is defined as the mini-  
 31 mum of the vehicle desired speed  $v_{x,i}^d$  and a multiple of its current speed as  $\min \left\{ 1.3 \times x_{3,i}(k), v_{x,i}^d \right\}$ .  
 32 This allows for a smooth increase in the vehicle speed and less desire to accelerate during the con-

1 gestion. Note that the longitudinal desired speed of the vehicle may differ from one vehicle to  
 2 the other, whereas, for comfort and safety reasons, the desired lateral speed of each CAV is zero.  
 3 A feedback P-controller (Equation 3) is used as the cruise controller to regulate the longitudinal  
 4 speed of the vehicles using the target speed forces  $f_{x,i}^{ts}$  as follows:

$$f_{x,i}^{ts}(k) = k_x^{ts}(v_{x,i}^{ts} - x_{3,i}(k)) \quad (3)$$

5 where  $k_x^{ts}$  and  $k_y^{ts}$  are controller gains. As just mentioned, the lateral target speed is zero.  
 6 The implemented cruise controller is shown in Figure 2 for regulating the longitudinal speed. The  
 7 lateral speed is controlled using the potential lines and will be explained later.



**FIGURE 2:** Feedback loop for the cruise controller.

8 At each time step  $k$ , the longitudinal (lateral) speed  $x_{2,i}(k)$  ( $x_{4,i}(k)$ ) of the vehicles  $i$  is  
 9 compared with the corresponding target speed. Then the resulting error  $e(k)$  is the input value for  
 10 the control system. The achieved longitudinal (lateral) target speed force  $f_{x,i}^{ts}(k)$  ( $f_{y,i}^{ts}(k)$ ) is then  
 11 used as a part of vehicle dynamics input. i.e., acceleration.

## 12 Artificial Potential Fields

13 The second goal of the driving strategies for CAVs is collision avoidance. In conventional lane-  
 14 based driving environment, drivers tend to maintain a certain comfort and safety distance from  
 15 the front vehicles. This depends on the acceptable time gap for the drivers. Hence, the space gap  
 16 between the vehicles also depends on the vehicles' speed. This prevents rear-end collisions.

17 In the lane-free environment, we maintain the proper distance between CAVs using arti-  
 18 ficial potential fields and corresponding inter-vehicle forces. To this end, and to avoid collision,  
 19 we assume that each vehicle is surrounded by an artificial potential field that applies repulsion or  
 20 nudging forces to other vehicles within its field. We follow the approach proposed in (20), with  
 21 some modifications, to create the potential functions. First, we assume a safety ellipse around  
 22 each vehicle  $j$ , which acts as an obstacle. The major and minor axes of this ellipse,  $d_x$  and  $d_y$   
 23 respectively, depend on the time-gap-like parameters and the speed of vehicle  $j$  and the vehicle  $i$ ,  
 24 as defined in Equation 4.

$$d_x^j(k) = s_x l_i + t_{1,x} (x_{3,i}(k) + x_{3,j}(k)) + t_{2,x} |x_{3,i}(k) - x_{3,j}(k)| \quad (4a)$$

$$d_y^j(k) = s_y w_i + t_y \left[ \tanh \left( \frac{x_{2,i}(k) - x_{2,j}(k)}{x_{4,j}(k) - x_{4,i}(k) + \epsilon} \right) + \sqrt{\left\{ \tanh \left( \frac{x_{2,i}(k) - x_{2,j}(k)}{x_{4,j}(k) - x_{4,i}(k) + \epsilon} \right) \right\}^2 + \epsilon} \right] \quad (4b)$$

25 The first terms in Equation 4a and Equation 4b provide a minimum safety gap between ve-

1 hicles in the stationary mode where  $l_i$  and  $w_i$  are the length and width of the vehicle  $i$ , respectively,  
 2 and  $s_x$  and  $s_y$  are safety factors. The second term in Equation 4a maintains additional space-gaps  
 3 behind and in front of the obstacle vehicle  $j$ . The third term in Equation 4a, takes the relative  
 4 speeds into account and induces lower space-gaps when two vehicles are driving with the same  
 5 speed, e.g., in a flock. The lateral ellipse axis depends on the relative lateral speed and distance  
 6 of two vehicles as represented in the second term in Equation 4b and is applied only if two vehi-  
 7 cles are laterally approaching each other as suggested in (20). This term implies that the greater  
 8 the lateral speed deviation or the smaller the lateral space-gap between two vehicles cause larger  
 9 lateral axis of the ellipse and, hence, a bigger repulsion force.  $t_{1,x}$ ,  $t_{2,x}$ , and  $t_y$  are time gaps which  
 10 are selected appropriately to provide a collision-free driving behavior. Using the ellipse axes 4, the  
 11 potential function by which the repulsion and nudging force are calculated is defined as follows:

$$F(i, j, k) = \frac{M}{\left[ \left( \frac{x_{1,i}(k) - x_{1,j}(k)}{0.5 \cdot d_x^j(k)} \right)^{f_1} + \left( \frac{x_{3,i}(k) - x_{3,j}(k)}{0.5 \cdot d_y^j(k)} \right)^{f_2} \right]^{f_3} + 1} \quad (5)$$

12 where  $M$  is the maximum value of the function,  $f_1$  and  $f_2$  are positive even integers and  
 13  $f_3$  is a positive integer. The resulting inter-vehicle force from Equation 5 is applied to the front  
 14 vehicle as the nudging force and to the rear vehicle as a repulsive force and are in the direction  
 15 of the line connecting two vehicles' centers as shown in Figure 3, where in this representation  $M$   
 16 equals 2. These forces are then projected into the longitudinal and lateral directions, which are  
 17 used in 6 for updating the vehicle's acceleration.

$i$ : ego vehicle

$j$ : obstacle

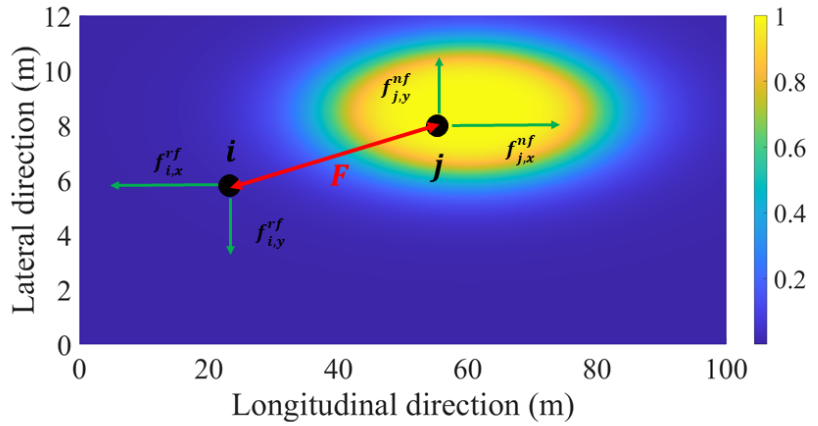
$F$ : inter-vehicle force

$f_x^{nf}$ : longitudinal nudging force

$f_y^{nf}$ : lateral nudging force

$f_x^{rf}$ : longitudinal repulsive force

$f_x^{rf}$ : lateral repulsive force



**FIGURE 3:** The repulsive and nudging forces affect the vehicles' movement.

18 To achieve the collision avoidance goal, the derived forces are combined with the longitu-  
 19 dinal and lateral target speed forces derived from the cruise controller at given discrete time  $k$  and  
 20 are computed via the following respective equations:

$$u_{x,i}(k) = \left[ f_{x,i}^{ts}(k) + c_x^{vf} \sum_{j=1}^J \left( f_{x,j}^{rp}(k) + c_x^{nf} f_{x,j}^{nf}(k) \right) \right] / m \quad (6a)$$



$$u_{y,i}(k) = \left[ f_{y,i}^{ts}(k) + c_y^{vf} \sum_{j=1}^J \left( f_{y,j}^{rp}(k) + c_y^{nf} f_{y,j}^{nf}(k) \right) \right] / m \quad (6b)$$

1 where  $m$  is the vehicle mass,  $c^{vf}$  and  $c^{nf}$  are scalar coefficients that regulate the effect of  
 2 virtual forces and the nudging force, respectively.  $f_j^{rp}$  is the repulsive force and is applied due to  
 3 vehicle  $j$  in front to avoid collision and  $f_j^{nf}$  is the nudging force applied from the vehicle  $j$  behind  
 4 and aims to facilitate the advancement of the faster vehicles where  $J$  is the total number of neigh-  
 5 boring vehicles in the detection zone of the vehicle  $i$ . The subscripts  $x$  and  $y$  in Equation 6 refer to  
 6 the corresponding coefficients and force in the longitudinal and lateral directions, respectively. In  
 7 addition to the terms in Equation 6b, there is another force which maintains the lateral location of  
 8 the vehicles based on their desired speed and is explained in the following section.

### 9 Potential lines for lateral distribution of vehicles

10 In a lane-free environment, as vehicles can choose any arbitrary lateral location, the relatively in-  
 11 tense interactions between CAVs may cause some negative chain effects in the traffic flow. For  
 12 example, the interactions generated between two CAVs with larger speed differences are more  
 13 drastic compared to that of two CAVs with smaller speed differences. Such interactions imply  
 14 greater forces and thus bigger accelerations which lead to speed fluctuations for other vehicles.  
 15 This condition increases the chaos in the traffic flow and is especially significant at higher vehicle  
 16 densities. Hence, if CAVs with larger differences in their desired speed are laterally distanced, the  
 17 mentioned turbulent flow can be weakened to a great extent.

18 Inspired by the road traffic rules in most countries around the world, where vehicles must  
 19 overtake from the left side and slow moving vehicles take the right lanes of the road to drive in,  
 20 we proposed the concept of potential line to uniformly distribute the vehicles across the road width  
 21 based on their desired speed.

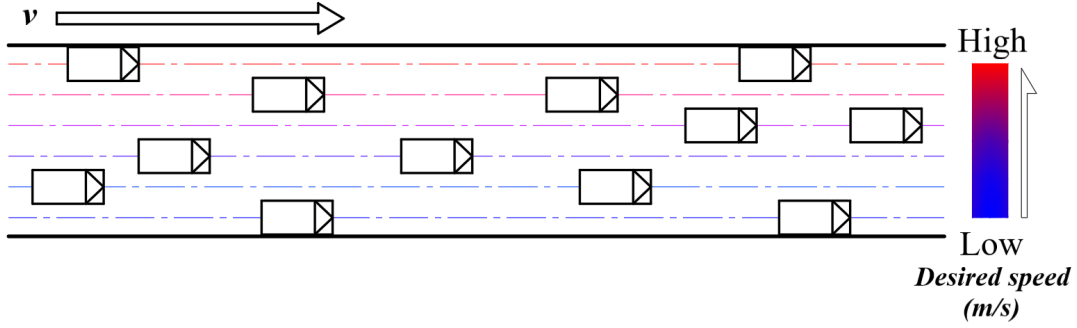
22 On a given section of the road, for each vehicle  $i$ , a potential line  $y_{pl,i}(x, v_{x,i}^d)$  is allocated  
 23 that changes with regards to the vehicle's longitudinal target speed and road geometry. For a  
 24 vehicle  $i$ , at longitudinal coordinate  $x$ , its potential line is obtained via the following equation:

$$y_{pl,i}(x, v_{x,i}^d) = (Y_{left}(x) - Y_{right}(x) - w_i) \frac{v_{x,i}^d - v_{min}^d}{v_{max}^d - v_{min}^d} \quad (7)$$

25 where  $Y_{left}(x)$  and  $Y_{right}(x)$  are the left and right boundary functions, respectively.  $v_{x,i}^d$  is the longi-  
 26 tudinal desired speed of vehicle  $i$ ,  $v_{max}^d$  and  $v_{min}^d$  represents the maximum and the minimum desired  
 27 speeds of all vehicles. A stretch of beltway segment marked with a series of potential lines is  
 28 illustrated in Figure 4.

29 The dash lines shown in Figure 4 represent the different potential lines and the color of the  
 30 dash lines indicates the desired speed to which they correspond. The induced potential lines have  
 31 the following characteristics:

- 32 1. Vehicles with the same longitudinal desired speed  $v_x^d$  have the same potential line  $y_{pl}$ .
- 33 2. The desired speeds corresponding to the potential lines decrease from the left boundary  
 34 to the right road boundary. Therefore, the leftmost potential line corresponds to the  
 35 highest desired speed and the rightmost potential line matches the lower desired speed



**FIGURE 4:** Potential lines on a beltway section.

1 of the CAVs in the section.

2 3. As the number of the vehicles and their corresponding desired speed is not limited,  
3 theoretically there are infinite numbers of the potential lines.

4 Once the potential line for a vehicle is defined, a corresponding force is calculated and  
5 applied laterally on the vehicle to maintain its lateral location. Therefore, when the vehicle  $i$   
6 deviates from its potential line  $y_{pl,i}$ , it will be affected by a lateral force  $f_{y,i}^{pl}$  towards its potential  
7 line. This force is calculated similar to the cruise controller approach based on the Equation 8.

$$f_{y,i}^{pl}(k) = K_{pl}(y_{pl,i}(k) - x_{3,i}(k)) \quad (8)$$

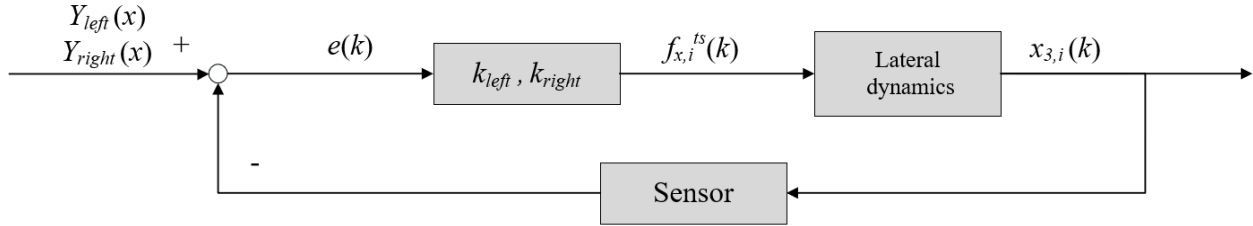
8 where  $K_{pl}$  is the controller gain. Using this new force, the Equation 6b will be updated as follows:

$$u_{y,i}(k) = \left[ f_{y,i}^{ts}(k) + c_y^{yf} \sum_{j=1}^J \left( f_{y,j}^{rp}(k) + c_y^{nf} f_{y,j}^{nf}(k) \right) + f_{y,i}^{pl}(k) \right] / m \quad (9)$$

9 Hence, the CAVs driving on the road width are laterally kept away from those vehicles that  
10 have greater differences in their desired speeds. Furthermore, CAVs driving with potential forces  
11 prefer overtaking from the left side and giving way on the right. These features allow vehicles to  
12 drive in an orderly manner in a lane-free environment similar to the current lane-based traffic. This  
13 approach reduces the chaotic driving behavior of CAVs and leads to much higher throughput as  
14 will be shown in the simulation section.

## 15 Road boundary control

16 All the vehicles must remain within the road boundary, i.e.,  $Y_{right}(x) \leq x_{3,i}(k) \leq Y_{left}(x)$ . There-  
17 fore, the need for efficient boundary control for the flock is essential. To this end, additional lateral  
18 acceleration constraints are required. This task may be addressed as a feedback control problem  
19 similar to the cruise controller as shown in Figure 5, where the left (right) road boundary is con-  
20 sidered as a reference value for all vehicles' lateral movement and  $e(k)$  indicates the error between  
21 the current lateral position of the vehicle and the left (right) road boundary (27). This control com-  
22 mand specifies how much lateral acceleration is needed to lead the vehicle towards the boundary.  
23 This value is then assumed as the maximum acceleration and ensures that the vehicles never cross  
24 the boundary.



**FIGURE 5:** Feedback loop for boundary control.

Therefore the lateral bound constraint 2b is updated as follows:

$$u_{y,i}(k) \leq \min(u_{y,max}(k), k_{left}(Y_{left}(x) - x_{3,i}(k))) \quad (10a)$$

$$u_{y,i}(k) \geq \max(u_{y,min}(k), k_{right}(Y_{right}(x) - x_{3,i}(k))) \quad (10b)$$

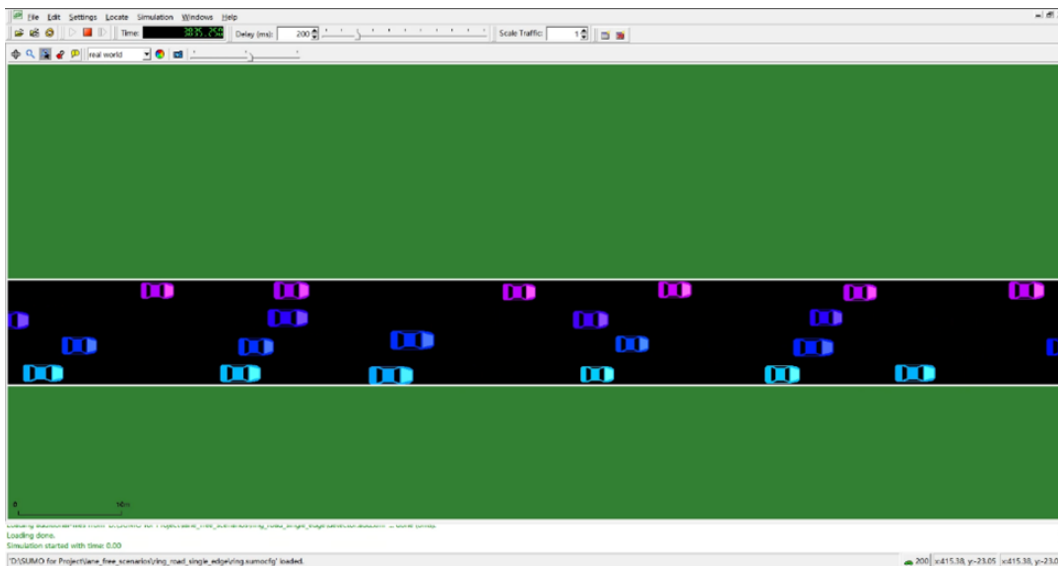
1 where  $k_{left}$  and  $k_{right}$  are controller gains for left and right boundaries, respectively.

## 2 SIMULATION RESULTS AND DISCUSSIONS

### 3 Simulation environment

4 To test the proposed algorithm, a traffic network is developed in the traffic simulator SUMO (Sim-  
5 ulation of Urban Mobility) (28). The simulation is executed using a custom extended version of  
6 SUMO called TrafficFluid-Sim. It extends the open-source codebase of SUMO to meet the needs  
7 of LFT simulation environments (29).

8 An external application programming interface (API) is used for controlling vehicular dy-  
9 namics and defining vehicle driving strategies. It allows users to set the desired speed of the vehi-  
10 cles, scanning vehicles, getting their states, and defining new longitudinal and lateral acceleration  
11 for vehicles in the given time step.



**FIGURE 6:** Simulation environment.

12 The considered traffic network is a 1 km long freeway. To emulate the beltway effect, we  
13 let the vehicles leaving from the end of the network to be entered from the other side of the road

1 at exactly the same lateral position and with the same longitudinal and lateral speeds. Since the  
 2 goal for this paper is to evaluate the capacity of the beltway in an LFT environment, the width of  
 3 the beltway is defined based on the considered scenarios. In this work, we define three scenarios;  
 4 1- a lane-free beltway as wide as a three-lane conventional beltway, i.e., with the width of 10.2  
 5  $m$ ; 2- half lane reduction of the width, i.e., with the width of 8.5  $m$ , and 3- one full lane reduction  
 6 of the width, i.e., with the width of 7  $m$ . In the simulation, five types of vehicles are considered,  
 7 each with its own dimensions (length, width) in meters: (3.2, 1.6), (3.3, 1.7), (3.4, 1.7), (3.5, 1.8),  
 8 (3.6, 1.82). A snapshot of the developed traffic network is shown in Figure 6, where different sizes  
 9 of vehicles and the lane-free feature are evident. At each scenario, the distribution of the vehicle  
 10 types is considered fixed with ratios of [0.27 0.23 0.20 0.17 0.13].

## 11 Results and discussion

12 For each scenario, we run simulations with various density of vehicles starting from 50  $veh/km$ .  
 13 The maximum density of each scenario depends on the limitation of the beltway capacity. The  
 14 maximum densities are 250, 350, and 450  $veh/km$  for scenario 1, 2, and 3, respectively. Vehicles  
 15 are identically inserted at simulation time 0  $s$  with an initial speed of 0  $m/s$  in both lateral and  
 16 longitudinal directions. The vehicles are also positioned within the considered road boundaries.  
 17 Each vehicle is equally assigned with a predefined desired speed in the range of 25 to 35  $m/s$ . Each  
 18 scenario is simulated for 20 minutes.

19 In Figure 7, the fundamental diagram for each scenario is illustrated. It reveals the fact that  
 20 the capacity of the 10.2  $m$  beltway is around 30000  $veh/h$  which is much higher than the equivalent  
 21 lane-based beltway. In addition, comparing this flow with similar LFT strategies ((15, 27)) show  
 22 that the implementation of the potential field approach considerably affects the throughput of a  
 23 lane-free road. As one may expect, with road narrowing, the capacity of the beltway decreases  
 24 accordingly. The important point is that, even in the third scenario, where the road is as wide as  
 25 a conventional two-lane road, the maximum flow is higher than the original three-lane road. This  
 26 finding is crucial and highlights the effectiveness of lane-free traffic.

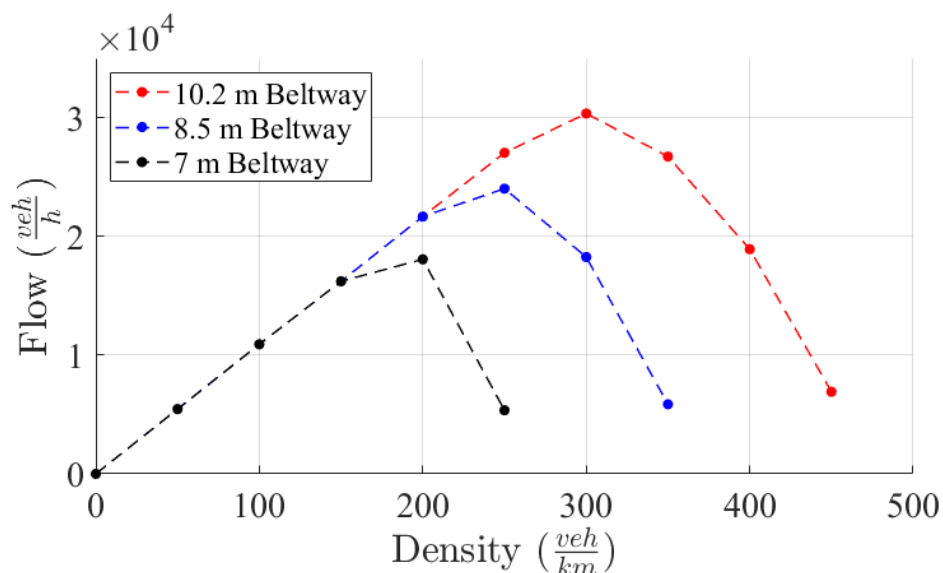


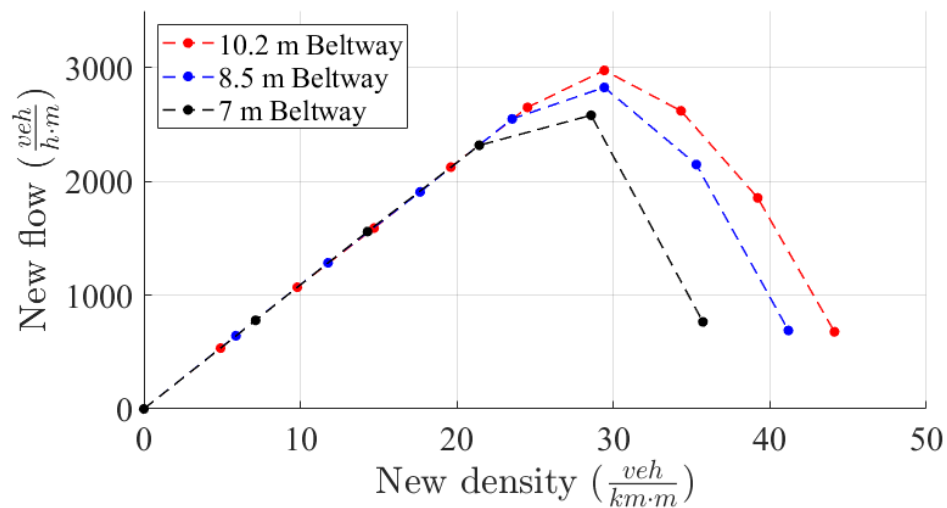
FIGURE 7: Fundamental diagrams for the three scenarios

1 However, once we change the width of the considered beltway, the corresponding flows  
 2 and densities are affected, especially in the congested situation. To have a uniform comparison of  
 3 a fundamental diagram regardless of the road width, we suggest to normalize the flow and densities  
 4 with respect to the road width.

$$q_n = \frac{q}{R_w} \quad (11a)$$

$$\rho_n = \frac{\rho}{R_w} \quad (11b)$$

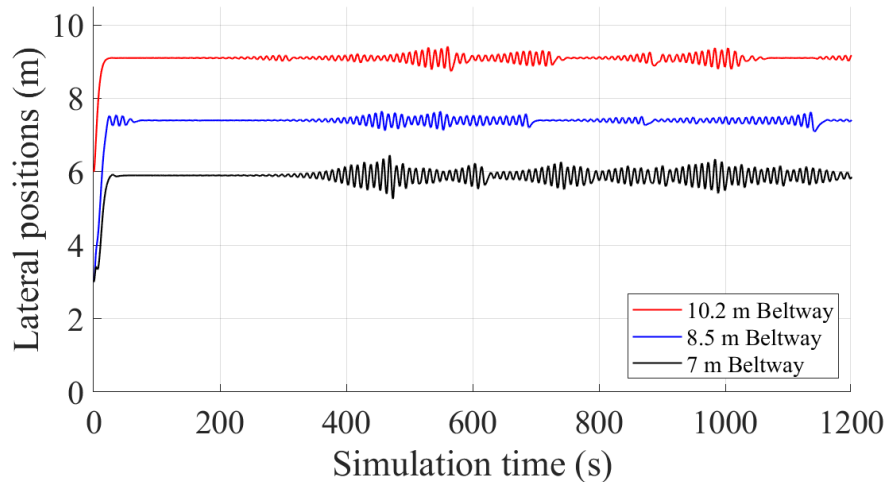
5 In (11),  $q$  and  $\rho$  are the common flow and density, whereas,  $q_n$  and  $\rho_n$  are the normal-  
 6 ized flow and densities, respectively. The new normalized fundamental diagrams with the new  
 7 definition of flow and density is shown in Figure 8.



**FIGURE 8:** Normalized fundamental diagrams for the three scenarios

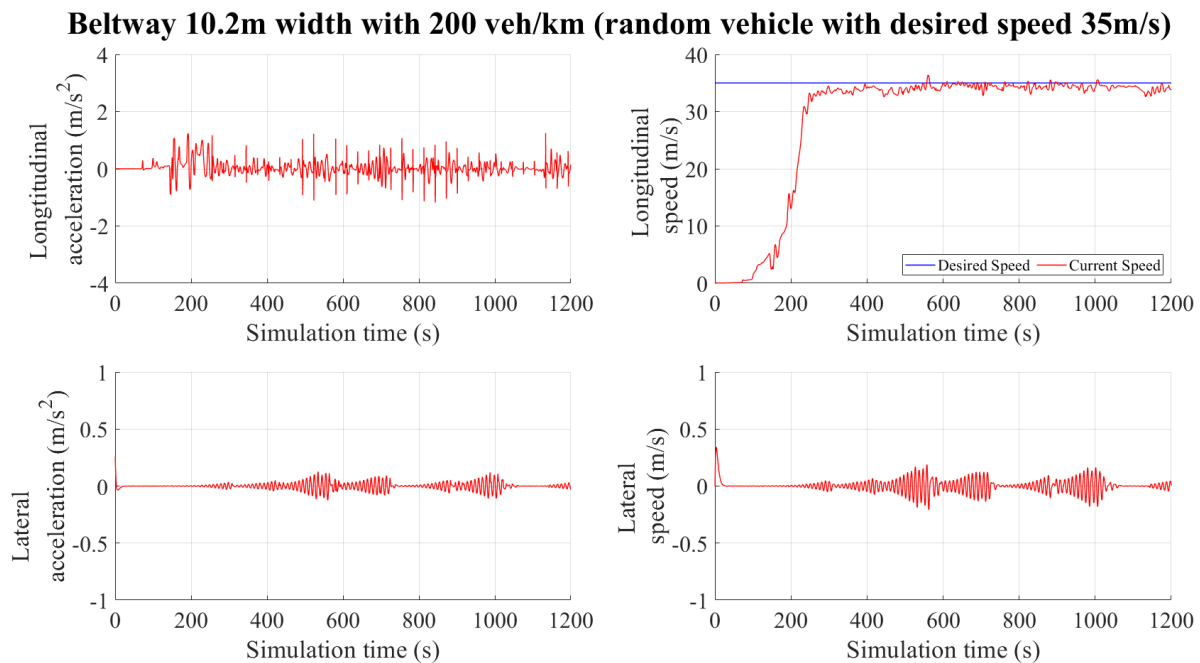
8 The diagrams in Figure 8 show that different scenarios demonstrate relatively a similar crit-  
 9 ical density and maximum flow. In addition, the wider the road the higher the normalized density.  
 10 This fact indicates that LFT is highly effective on wider roads, where vehicles have more free-  
 11 dom for lateral movement. In other words, the lateral occupancy of the wider roads in congested  
 12 situations are higher, compared to narrower roads.

13 To have a better insight on the designed potential lines, the lateral trajectories of three  
 14 random selected vehicles at each scenario are plotted in Figure 9. Each vehicle is initiated at a  
 15 random lateral location whereas it receives a potential line at a different lateral location due to its  
 16 desired speed. Just after a few seconds, all vehicles reaches their assigned lateral location, i.e.,  
 17 their potential lines, and keep such lateral locations in the rest of the simulation run. However,  
 18 the vehicles may deviate at some point from their desired lateral positions due to the inter-vehicle  
 19 forces. But it is important to see that they try to maintain their positions regardless of the applied  
 20 forces.



**FIGURE 9:** Lateral movement of three selected vehicles

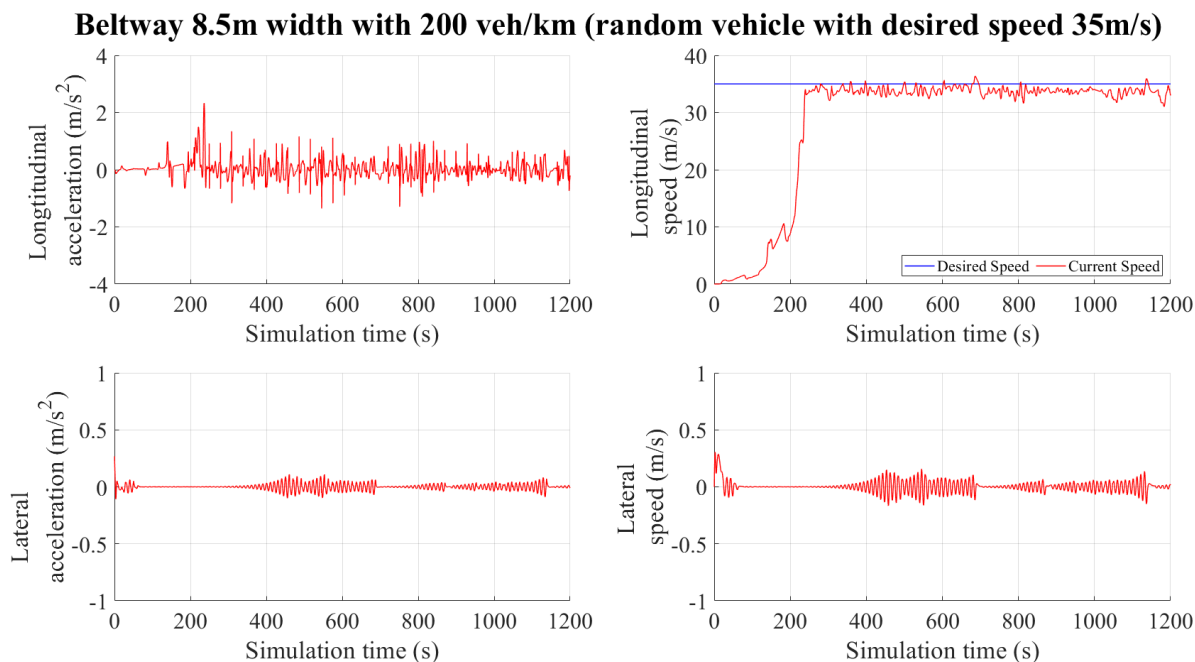
1 In order to evaluate the behavior of the vehicles, the speed and acceleration of three vehicles  
 2 in each scenario are shown in Figure 10-12. The longitudinal speeds show that all vehicles reach  
 3 their desired speed and the speed is maintained for the rest of the simulation run. The acceleration  
 4 phase in the narrower road (Figure 12) is longer since the interference of the vehicles is higher.  
 5 Therefore, it takes more time for the vehicle to reach its desired speed. The lateral speed diagrams  
 6 also demonstrates the fact that all the vehicles initially take a lateral speed to align themselves with  
 7 their potential lines. Following the initial phase, the lateral speed fluctuates around zero.



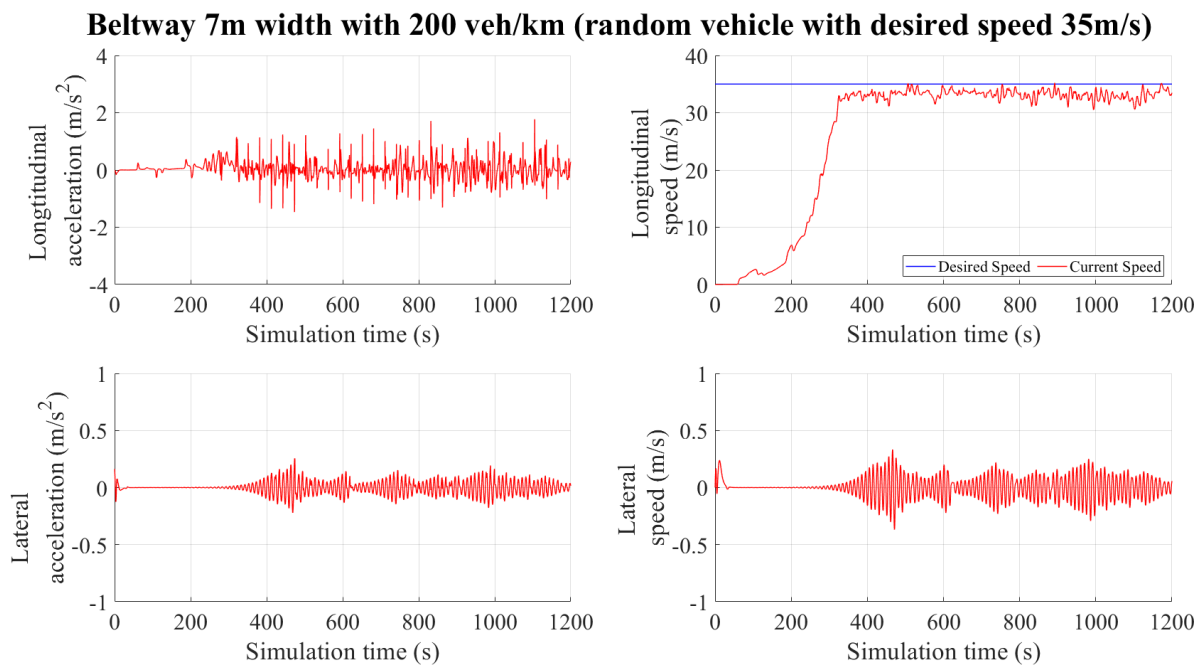
**FIGURE 10:** Acceleration and speed of a vehicle in 10.2 m scenario

8 The lateral and longitudinal speed fluctuation on the wider road is less than the narrower  
 9 ones. This issue is predictable as vehicles, with similar densities, on a wider road have more

- 1 freedom to maneuver. However, the lateral acceleration of all selected vehicles are less than  $0.5 \text{ m/s}^2$
- 2  $\text{m/s}^2$  which fulfills comfort considerations of CAV driving strategy.



**FIGURE 11:** Acceleration and speed of a vehicle in 8.5 *m* scenarios

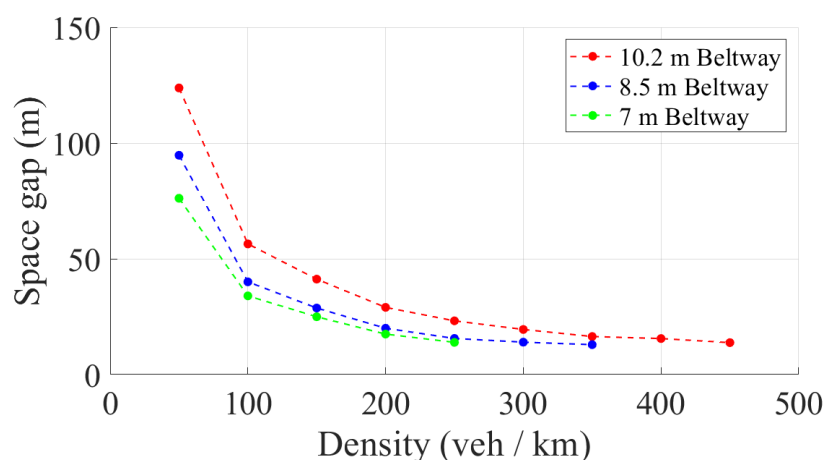


**FIGURE 12:** Acceleration and speed of a vehicle in 7 *m* scenarios

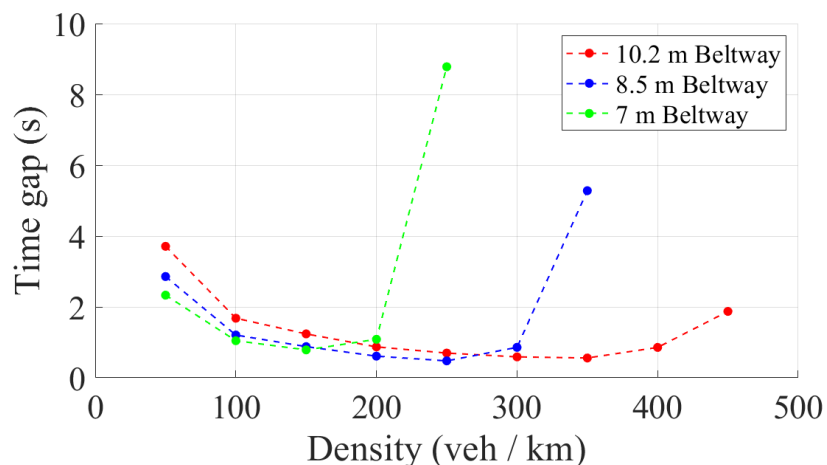
- 3 In summary, by adopting lane-free traffic with potential lines as a vehicle driving strategy,

1 we show that the width of the beltway can be shrunk appropriately to save land resources without  
 2 losing road capacity. The extra resources may be more efficiently used by other road users such as  
 3 cyclists or public transportation. At the same time, this idea satisfies not only capacity requirements  
 4 but also comfort considerations.

5 To further investigate the safety and comfort issues associated with the designed lane-free  
 6 driving strategy, we placed a virtual detector at the leftmost section of the road, i.e., at the longitu-  
 7 dinal location of the 940 *m* and the lateral location covering the lateral width of the road between  
 8 8.4 and 10.2 *m*. This detector measures the speed, space-gap, and time-gap of the passing vehicles.  
 9 Due to the designed potential lines and the location of the detector, we expect that this detector  
 10 measures the variables associated with fast-moving vehicles. The average space-gap and time-gap  
 11 between vehicles and the average speed at different scenarios are recorded and shown in Figures  
 12 13-15.



**FIGURE 13:** Space-gap diagram

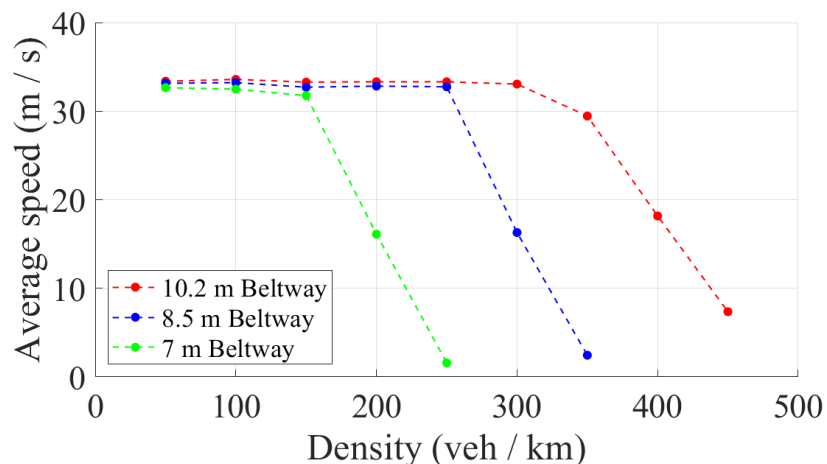


**FIGURE 14:** Time-gap diagram

13 It can be seen that the space and time gaps between vehicles decreases as the density in-  
 14 creases. However, after the critical density, as the speed of the vehicles starts to reduce, the cor-



- 1 responding time-gap increases. The shortest space-gap between vehicles is around 10 *m* and the
- 2 shortest time-gap is about 0.5 *s*.



**FIGURE 15:** Average speed

### 3 CONCLUSION AND FUTURE WORKS

4 In this paper we developed a new approach to further increase road capacity on lane-free beltways  
 5 by using potential lines. In the lane-free traffic environment, the throughput is higher compared  
 6 to the lane-based conventional traffic systems. However, with the proposed approach, due to the  
 7 harmonic lateral distribution of the vehicles across the road width, the achieved flow and densities  
 8 are considerably higher than already available lane-free driving strategies.

9 The simulation results show, even for the cases where we reduced the road width, we ob-  
 10 serve a comparable capacity to lane-based traffic with full width. In addition, the results also show  
 11 that the potential lines allow vehicles to drive at a speed closer to the predefined desired speed even  
 12 at a very high traffic demand, resulting in no delay time. This is indeed a great achievement and  
 13 implies that to resolve road congestion, we can do away with constructing new roads or extending  
 14 existing one. Rather by exploiting CAV technology, the capacity of the current road infrastructure  
 15 will be maximized. In addition, we propose that, with regards to high capacity of the lane-free  
 16 roads, the width of the current roads can be reduced and the remaining road space can be allocated  
 17 to other road users such as cyclists or for public transport.

18 In future works, we aim to focus on the safety and comfort aspects of LFT. Although this  
 19 speed-dependent lateral distribution leads to very high throughput, the smoothness of the vehicle  
 20 trajectories and safety analysis must be carefully taken into account. To this end, future works will  
 21 test such driving strategies in a driving simulator. In addition, it is also important to consider the  
 22 measurement or communication error and to investigate how they may affect the designed control  
 23 algorithms. The potential lines could also be combined with flocking, i.e., a group of vehicles  
 24 driving together with the same speed. In the future, advanced control theories could also be used  
 25 for diminishing the transition of shock waves by reducing the fluctuation of vehicle dynamics.

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1 **AUTHOR CONTRIBUTIONS**

2 The authors confirm contribution to the paper as follows. Study conception: M. Rostami-Shahrb-  
3 abaki, H. Zhang, M. Sekeran; methodology development: M. Rostami-Shahrbabaki, H. Zhang;  
4 implementation and simulation: H. Zhang; Analysis and interpretation of results: M. Rostami-  
5 Shahrbabaki; draft manuscript preparation: M. Rostami-Shahrbabaki, H. Zhang, M. Sekeran, K.  
6 Bogenberger; All authors reviewed the results and approved the final version of the manuscript.

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