Ternary unitary quantum lattice models and circuits in 2+1 dimensions

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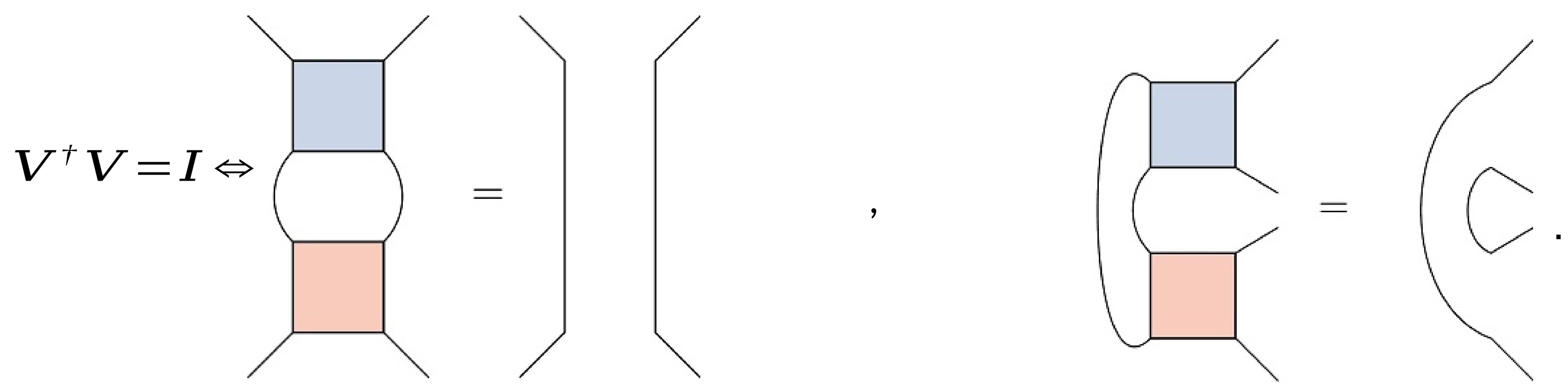
Abstract

Recently special two-particle gates dubbed dual unitary gates were introduced that admit an interpretation of unitarity in time and space when used in a brick-wall pattern(1). We extended this concept to two spatial dimensions in the form of operators called ternary unitary which are unitary in time and in both spatial dimensions. There are a variety of ways to construct such operators from the lower dimensional dual unitary operators. The most notable feature that arises from the properties of ternary unitary operators is that when used as a time-evolution they admit exact computations of quantities. For example, the dynamical correlation functions exhibit a light-ray structure.

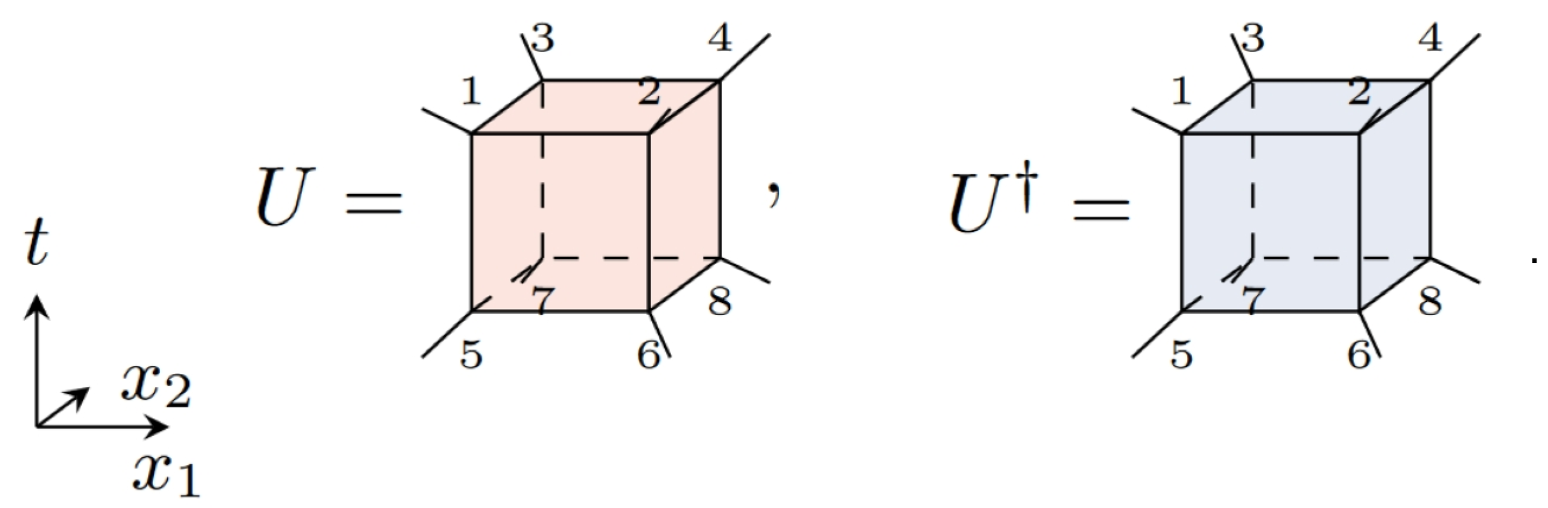
Introduction

A dual unitary gate as introduced in (1) is a unitary two-particle gate whose dual gate is also unitary. The latter arises out of a specific transformation of the legs of the former. This can be visualised as

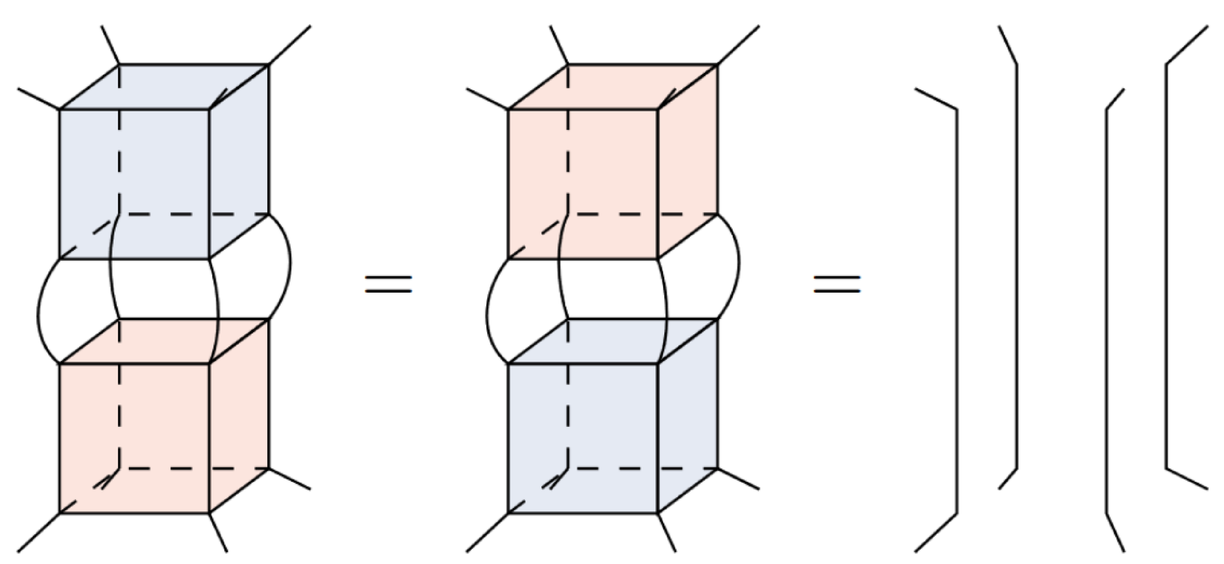


Such gates will act on a one-dimensional string of systems and can themselves be viewed as an evolution of the system. Therefore we may reinterpret the above condition to mean is unitary in both time and space.

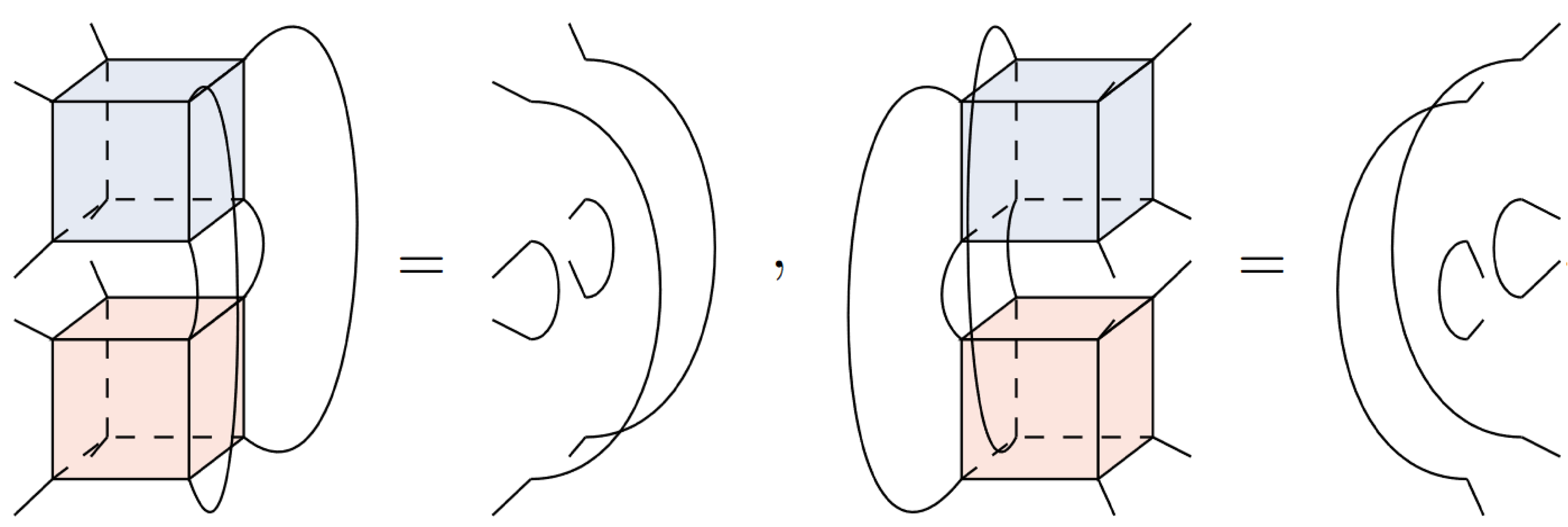
We extend this concept to two-spatial dimensions by considering a four-particle gate



acts on a two-dimensional square lattice of systems, each of which is described by a finite Hilbert space . We call a ***ternary unitary gate*** if it is unitary in time and along both spatial directions. Visually this is represented as



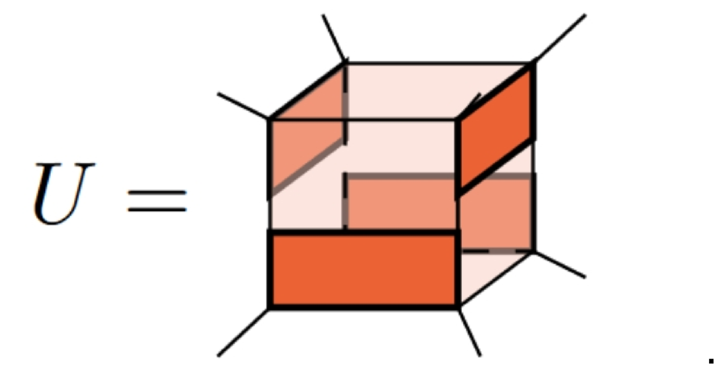
for the -direction and



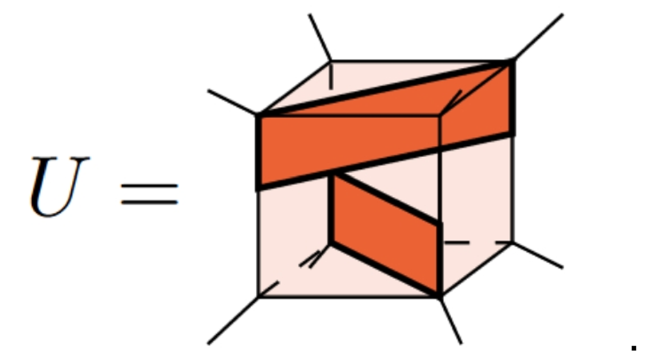
for the -direction. The condition for the -direction is analogous to the one above, but rotated by around the -axis.

Construction of ternary-unitaries

We found constructions of ternary-unitaries from dual unitary gates for which many different constructions are known (2). Our first construction uses four dual unitary gates, where two are parallel to each other



For the second construction we only require two and they connect opposite corners of the underlying square:

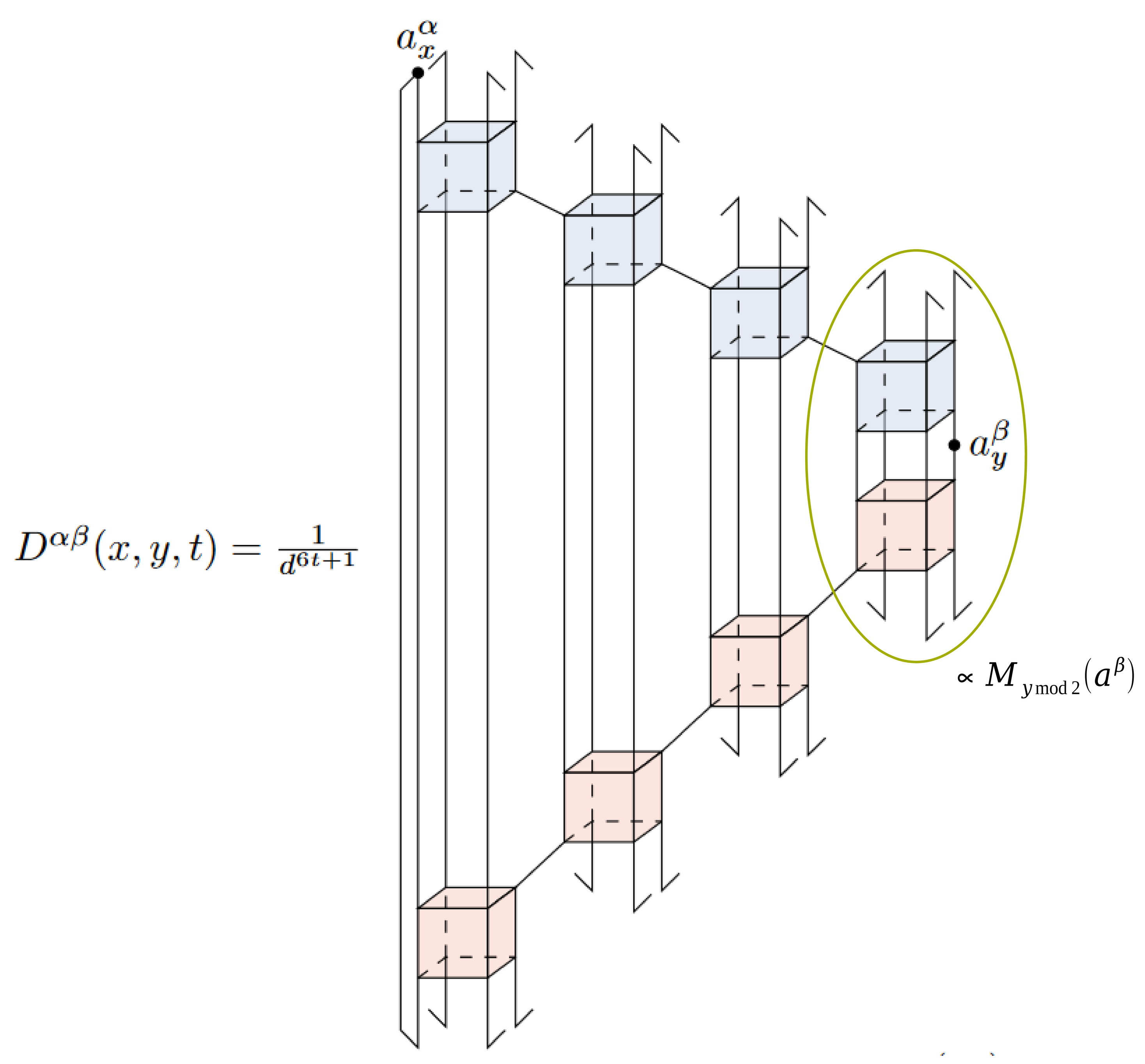


Dynamic correlations

For a basis of single site operators we define the dynamic correlation function as

$$D^{\alpha\beta}(x, y, t) = \frac{1}{d^{L^2}} \mathrm{Tr}\left[a_x^\alpha \mathbb{U}^{-t} a_y^\beta \mathbb{U}^t\right] ,$$

where $\mathbb{U}$ is the time evolution consisting of two layers of ternary unitary gates in a brick-wall pattern. We showed that all non-zero correlation functions have the two single-site operators connected along a cross-diagonal „light ray“. The tensor diagram then simplifies to the following form (for):



$$¿ \frac{1}{d} \mathrm{Tr}\left[M_{y \bmod 2}^{2t}(a^\beta) a^\alpha\right]$$

Acknowledgements

This poster is based on publication (3), which contains more details and explicit derivations. The research is part of the Munich Quantum Valley, which is supported by the Bavarian state government with funds from the Hightech Agenda Bayern Plus.

References

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