$D^0 - \bar{D}^0$ mixing: theory basics

Diego Guadagnoli
Physik-Department, Technische Universität München, D-85748 Garching, Germany
E-mail: diego.guadagnoli@ph.tum.de

Abstract. I discuss how the novel experimental data on $D^0 - \bar{D}^0$ mixing can be combined to provide information on the fundamental theoretical quantities describing the mixing itself. I then discuss the theoretical impact of the new data, focusing in particular on the MSSM.

For times much longer than the strong interaction time scale, the flavor eigenstates $D^0 = (c\bar{u})$ and $\bar{D}^0$ mix into each other. Mixing is a purely quantum effect and the $D^0 - \bar{D}^0$ system is the only one featuring it among 'up-type-quark' mesons, since the top quark decays before forming a bound state with an antiquark. The time evolution producing the mixing is calculated through

$$i\frac{d}{dt}\begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix}, \quad \text{with } \hat{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}, \tag{1}$$

where the explicit form for the hermitian $\hat{M}$ and $\hat{\Gamma}$ matrices holds assuming CPT invariance. Mass eigenstates, with masses $m_{1,2}$ and widths $\Gamma_{1,2}$, are defined through

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \ , \quad \text{with } \left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \ , \tag{2}$$

allowing in turn to define the basic mixing observables as $x = (m_2 - m_1)/\Gamma$ and $y = (\Gamma_2 - \Gamma_1)/(2\Gamma)$ [1]. If $|q/p| = 1$ in eq. (2), then $|D_{1(2)}\rangle$ is CP even (odd), since one can choose phases so that $|D^0\rangle \overset{\mathrm{CP}}{\leftrightarrow} |\bar{D}^0\rangle$ (see Y.Nir in [1]). If, on the other hand, $|q/p| \neq 1$, then mass eigenstates cannot be chosen as CP eigenstates and there is CP violation in mixing.

Within the Standard Model (SM), meson mixings are well described, in the $B_{d,s}$ and K cases, by box diagrams with loop-exchange of W-bosons and up-type quarks. The flavor structure of the contributions is the product of a CKM factor and an Inami-Lim function $S_0(m_{q_1}^2, m_{q_2}^2)$, summed over the quark flavors q_1, q_2 running in the loop. If $m_{q_1}^2, m_{q_2}^2 \ll M_W^2$, as is the case for all the down-type quarks, one has e.g. $S_0(m_{q_1}^2, m_{q_1}^2) \simeq m_{q_1}^2/M_W^2$, showing a very effective GIM suppression. One should also note that in the D-mixing case, the third family contribution, which would be enhanced by the relatively large m_b mass, is accidentally suppressed by a very small CKM factor, resulting in a relative box contribution from the third family of $\mathrm{O}(10^{-3})$, and a correspondingly suppressed amount of CP violation within the SM. This is also in sharp contrast with the $K, B_{d,s}$ cases, where the third family (top) contribution is always important or dominant. Due to the above reasons, for D-mesons SM (box) contributions are tiny, in principle making the mixing ideal room for New Physics to show up. On the other hand, the charm mass is accidentally of the same order of the hadronic scale. Hence, K, π intermediate states are likely to dominate the mixing amplitude contributions. Generic predictions are $x_{\mathrm{box}} \leq 10^{-5}$ and $x_{\mathrm{long\ dist.}} \leq O(10^{-3})$ (Burdman & Shipsey in [1],[2]). The poor control over the long distance contributions presently impairs an effective use of D-mixing as a test of the SM.

Parameter	Value	Ref.
x'^2_+	$(-0.24 \pm 0.43 \pm 0.30) \cdot 10^{-3}$	[4]
x'^2_-	$(-0.20 \pm 0.41 \pm 0.29) \cdot 10^{-3}$	[4]
y'_+	$(9.8 \pm 6.4 \pm 4.5) \cdot 10^{-3}$	[4]
y'_-	$(9.6 \pm 6.1 \pm 4.3) \cdot 10^{-3}$	[4]
x	$(8.1 \pm 3.5) \cdot 10^{-3}$	[5]
y	$(3.7 \pm 2.9) \cdot 10^{-3}$	[5]
ϕ	$(-14 \pm 19)^\circ$	[5]
$\lvert q/p \rvert$	0.86 ± 0.32	[5]
$y_{\rm CP}$	$(13.1 \pm 3.2 \pm 2.5) \cdot 10^{-3}$	[6]
A_Γ	$(0.1 \pm 3.0 \pm 1.5) \cdot 10^{-3}$	[6]
$\cos\delta_{K\pi}$	1.09 ± 0.66	[7]
τ_D	(0.4101 ± 0.0015) ps	[8]

Parameter	68% prob.	95% prob.
x	$(6.2 \pm 2.0) \cdot 10^{-3}$	$[0.0022, 0.0105]$
y	$(5.5 \pm 1.4) \cdot 10^{-3}$	$[0.0027, 0.0084]$
$\delta_{K\pi}$	$(-31 \pm 39)^\circ$	$[-103^\circ, 28^\circ]$
ϕ	$(1 \pm 7)^\circ$	$[-15^\circ, 17^\circ]$
$\lvert \frac{q}{p} \rvert - 1$	-0.02 ± 0.11	$[-0.27, 0.25]$
$\lvert M_{12} \rvert$	$(7.7 \pm 2.4) \cdot 10^{-3}$ ps^{-1}	$[0.0030, 0.0127]$ ps^{-1}
Φ_{12}	$(2 \pm 14)^\circ \cup (179 \pm 14)^\circ$	$[-30^\circ, 36^\circ] \cup [144^\circ, 210^\circ]$
$\lvert \Gamma_{12} \rvert$	$(13.6 \pm 3.5) \cdot 10^{-3}$ ps^{-1}	$[0.0068, 0.0207]$ ps^{-1}

Table 1. Left: Recent measurements related to $D^0 - \bar{D}^0$ mixing. Right: Global fit to the mixing parameters. See Ref. [9].

A channel that very simply illustrates how to experimentally access D-mixing is that of "wrong sign" $D \to K\pi$ decays. The amplitude for the decay $D^0 \to K^+\pi^-$ proceeds in fact through the sum of a tree-diagram, which is however doubly Cabibbo-suppressed (DCS) and indicated henceforth as $\mathcal{D}_{\rm DCS} \propto \sin^2\theta_C$, and a diagram in which the D^0 oscillates first into a $\bar{D}^0$ whose Cabibbo-favored (CF) final state is then exactly $K^+\pi^-$. The latter diagram behaves as $\mathcal{D}_{\rm mix+CF} \propto \cos^2\theta_C$ but is suppressed by the loop factor of the mixing, which is what one wants to access. Hence the two diagrams are competitive and the mixing measurable [1].

A special comment deserves CP violation in the D-system. A reasonable assumption is to consider CP violation in decay amplitudes ('direct' CP violation) negligible, since the latter are dominated by the tree-level CP conserving SM contributions. On the other hand, non-negligible CP violation can occur in the mixing amplitude, due to non-SM short-distance contributions. In the case of the wrong sign $D \to K\pi$ decays, CP violation in mixing should however be hard to observe, while likely to be accessible is CP violation in the *interference* between the decay with ($\mathcal{D}_{\rm mix+CF}$) and without ($\mathcal{D}_{\rm DCS}$) mixing [1]. The latter is related to the phase $\phi = \arg(q/p)$. Recalling the definition of q/p, eq. (2), and parameterizing $M_{12} = |M_{12}|\exp(-i\Phi_{12})$, $\Gamma_{12} = |\Gamma_{12}|$, with Φ_{12} small, one easily recognizes that $\phi \approx +\Phi_{12} \times 4|M_{12}|^2/(4|M_{12}|^2 + |\Gamma_{12}|^2)$. Thereafter, a naive estimate of Φ_{12} from the SM box contributions to the mixing gives $\Phi_{12} \leq 10^{-2}$. Observation of (large) CP violation would then be a clear NP signature, immune to hadronic uncertainties. For a recent critical analysis on this issue, see Ref. [3].

A collection of (only) the most recent experimental progress on $D^0 - \bar{D}^0$ mixing can be found in Table 1, where $y_{CP} = \frac{\tau(D^0 \to K^-\pi^+)}{\tau(D^0 \to f_{CP})}$, A_Γ is the CP asymmetry in $D^0 \to KK$, $\delta_{K\pi}$ is the relative strong phase between wrong sign and right sign $K\pi$ decays and $x_\pm, y_\pm$ are related to x, y by a rotation through the phase $\delta_{K\pi}$ and a subsequent one through the phase ϕ (detailed formulae can be found in [4]-[9]). The relevant point here is that all the quantities listed in Table 1 (left) can be expressed in terms of $x, y, \delta_{K\pi}, \phi$ and $|q/p|$ [10], from which one calculates the fundamental mixing parameters through ($\delta = |p|^2 - |q|^2$)

$$|M_{12}|\tau_D = \sqrt{\frac{x^2 + \delta^2 y^2}{4(1-\delta^2)}},\ |\Gamma_{12}|\tau_D = \sqrt{\frac{y^2 + \delta^2 x^2}{1-\delta^2}},\ \sin\Phi_{12} = \frac{|\Gamma_{12}|^2 + 4|M_{12}|^2 - (x^2+y^2)|q/p|^2/\tau_D^2}{4|M_{12}\Gamma_{12}|}. \tag{3}$$

The determination of $|M_{12}|, \Gamma_{12}$ and Φ_{12} can proceed through a global fit [9], reported in Table 1 (right). In particular, the M_{12} determination can then be used to place constraints on any extension of the SM. To this end, one can parameterize $M_{12} = |M_{12}|\exp(-i\Phi_{12}) = (A_{\rm SM} + A_{\rm NP}\exp(i\Phi_{\rm NP}))/\tau_D$ with the SM part, real, assumed to be flatly distributed in the range $A_{\rm SM}/\tau_D \in [-0.015, 0.015]$/ps, and obtain the implied distribution on ($A_{\rm NP}, \Phi_{\rm NP}$). The latter, displayed in Fig. 1 (left), shows how the lack of knowledge of the SM contribution

[1] One should note that in practice the term most easily allowing access to the mixing variables is the interference between $\mathcal{D}_{\rm DCS}$ and $\mathcal{D}_{\rm mix+CF}$.

largely dilutes the information on the NP contribution, especially if the NP phase is aligned (or antialigned) with the SM (null) one.

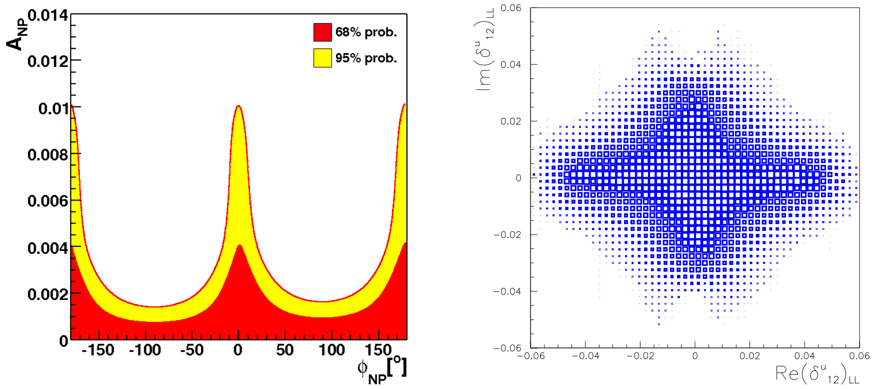

Figure 1. Left: $A_{\rm NP}$ vs $\phi_{\rm NP}$ probability density function of the combined fit from Tab. 1 (left). Right: Selected region for the mass insertion $(\delta^u_{12})_{LL}$, assuming $m_{\tilde q} = m_{\tilde g} = 350$ GeV. See [9].

It is clear that the information on $(A_{\rm NP}, \Phi_{\rm NP})$ constrains effectively only NP models producing in general large effects, as is the case for the MSSM with generic flavor violation. In this instance, one can assume gluino dominance and use the results of [11] to place constraints on the mass-insertions (normalized off-diagonal entries of the up-squark mass matrix) $(\delta^u_{12})_{AB}$, with AB the four possible chiralities. In Fig. 1 (right) the case $AB = LL$ is reported [9]. The latter has interesting consequences for models with quark-squark alignment, which tend to predict $(\delta^u_{12})_{LL} \sim 0.2$ [12]. Since the bound implied by Fig. 1 (right) is $|(\delta^u_{12})_{LL}| = 0.037$ (95% prob.), squark and gluino masses need to be raised above ~ 2 TeV, probably beyond the LHC reach.

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