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Reliability for load and non-load induced strength deterioration – Derivation of partial safety factors

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ABSTRACT: Computation of structural reliability under non-stationary conditions is known for a number of important loading processes and deteriorating resistances. While there is a one to one correspondence between theoretical results and practical design rules via partial safety factors - at least ideally - no such relationship has been established for time-variant problems. Making use of the outcrossing approach together with FORM/SORM methods it can, however, be shown that time variant failure probabilities can be factored into five different terms representing the coincidence probability, the frequency of load changes, a local failure probability in the most likely failure point, a SORM correction factor and a term which accounts for the time behaviour of the structural state function around the critical point. This enables to define partial safety factors in the point in time distributions as opposed to factors for normal extreme value design where the partial safety factors for loads are related to the extreme value distributions. The concept is illustrated at three examples.

1. Introduction

Many building materials suffer load or non-load induced deterioration in time. Whereas formulations for fatigue have found large interest in the literature other deterioration phenomena such as corrosion or chemical aging have been studied only occasionally. In particular, large uncertainty exists among engineers how to design verification procedures for practical use as is documented in present codes of practice. Essentially, two dramatically different approaches can be found in practice. In the first approach a design for the ultimate limit state is performed for the deterioration state expected at the end of the intended time of service. This approach appears to be overconservative. The other approach handles deterioration in the context of serviceability limit states which implies a formally lower reliability level although failure has consequences comparable to the ultimate limit state.

Methods for time-dependent reliability, especially the outcrossing approach in the context of FORM/SORM have been developed and are now ready for use. They will be reviewed briefly. However, whereas there is a one-to-one correspondence between reliability results and practical safety elements such as partial safety factors in time-invariant reliability problems - at least ideally-, no such relation exists in time-variant reliability with or without deteriorating strength and/or non-stationary loading. It will be shown that asymptotic concepts can help to clarify the nature of this relationships and a proposal will be made to define partial safety also for time-dependent problems.

2. Time-dependent reliability

A general upper bound to the failure probability of a time-variant system is

$$P_f(t) \leq E_R\left[E_Q\left[E\left[N_S^+(t,r,q)\right]\right]\right] \qquad (1)$$

where R is a vector of non-ergodic random variables which, possibly, depend deterministically on time, Q a vector of slowly varying ergodic sequences defining the parameters of S and S a rapidly varying stochastic process. This subdivision into three categories of variables is useful from a computational and from a modeling point of view. N_S^+ is the number of crossings from the safe domain $\overline{F}$ into the failure domain F. Assuming that the process of outcrossings is a regular process the mean number of outcrossings in the interval $[0,t]$ is

$$E\left[N_S^+(t,r,q)\right] = \int_0^t \nu^+(F,\tau,r,q)d\tau \quad (2)$$

where omitting now reference to the conditioning variables r and q

$$\nu^+(F,\tau,r,q) = \lim_{\Delta \to 0} \frac{1}{\Delta} P\{S(\tau) \in \overline{F}(\tau)\} \cap \{S(\tau+\Delta) \in F(\tau+\Delta)\} \quad (3)$$

is the outcrossing rate. The computation of the outcrossing rate is usually not straightforward but a number of results for both renewal jump processes and differentiable processes are available. The expected number of outcrossings can be computed by FORM/SORM for typical models for loads as well as the expectation operations in eq. (1) with respect to R and Q. If the processes are intermittent, i.e., the set k of active processes has probability $p_k^{i,\dots,k}$ the mean number of outcrossings is

$$\begin{aligned} E\left[N_S^+(t,r)\right] &= \sum_{i=1}^{s} p_1^{(i)} E_Q\left[E\left[N_i^+(t,r,q)\right]\right] \\ &+ \sum_{i=1}^{s}\sum_{j=1}^{s} p_2^{(i,j)} E_Q\left[E\left[N_{i+j}^+(t,r,q)\right]\right] \\ &\dots + p_s E_Q\left[E\left[N_{1+2+\dots+n}^+(t,r,q)\right]\right] \quad (4) \end{aligned}$$

The coincidence probabilities can be calculated according to various models the simplest being exponential interarrival and exponential durations truncated at the next arrival (Shinozuka 1981). In the following the derivations will be made for rectangular wave renewal processes only but parallel results can also be obtained for differentiable processes.

Applying formula (3) for the m-th load case and performing the probability distribution transformation into the standard space leads to (Breitung/Rackwitz, 1982; Breitung, 1993)

$$\begin{aligned} &\nu_m^+(F,\tau,r,q) = \\ &\sum_{i=1}^{k_m} \lambda_{mi}(\tau) P\{S(\tau) \in \overline{F}(\tau)\} \cap \{S_i^+(\tau) \in F(\tau)\} \\ &\sim \sum_{i=1}^{k_m} \lambda_{mi}(\tau) P\{S_i^+(\tau) \in F(\tau)\} \\ &\approx \sum_{i=1}^{k_m} \lambda_{mi}(\tau)\Phi(-\beta_m(\tau)) \prod_{j=1}^{k_m-1} (1+\beta_m(\tau)\kappa_{m,j}(\tau))^{-1/2} \end{aligned} \quad (5)$$

where $\lambda_i(\tau)$ is the jump rate of the i-th component of the load processes $S(\tau)$ active in the $m-th$ load case. In eq. (5) $S(\tau)$ denotes the state of the process just before a jump and $S_i^+(\tau)$ the state of the process right after a jump of the i-th component $S(\tau)$. The product term is the usual SORM correction factor with $\kappa_{m,j}(\tau)$ the main curvatures in the most likely failure point. The first formula is the exact result and the second is Breitung's asymptote (Breitung, 1993). In the stationary case none of the quantities in eq. (5) depends on time. In the time-variant case the critical point (most likely failure point) must be sought in the basic variable space spanned by (r,q,s) but also with respect to time, i.e. τ. This critical point will either be a boundary point of the interval $[0,t]$ or in the interior of this interval. The final time integration in eq. (2) can be performed numerically or approximately again making use of asymptotic considerations. If it is assumed that the critical point is the upper boundary point it is finally

$$\begin{aligned} P_{f,m}(t) &\le p_m E_R\left[E_Q\left[\int_0^t \nu_m^+(F,\tau,r,q)d\tau\right]\right] \\ &\sim p_m \sum_{i=1}^{k_m} \int_{t(1-p_m)}^{t} \lambda_i(\tau)\Phi(-\beta_m(\tau)) \times \\ &\quad \prod_{j=1}^{k-1}(1+\beta_m(\tau)\kappa_{m,j}(\tau))^{-1/2} d\tau \\ &\approx p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*) \prod_{j=1}^{k_m-1} (1+\beta_m(\tau^*)\kappa_{m,j}(\tau^*))^{-1/2} \times \\ &\quad \int_{t(1-p_m)}^{t} \Phi(-\beta_m(\tau))d\tau \\ &= p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*) \prod_{j=1}^{k_m-1} (1+\beta_m(\tau^*)\kappa_{m,j}(\tau^*))^{-1/2} \times \\ &\quad \int_{t(1-p_m)}^{t} \exp\left[\ln(\Phi(-\beta_m(\tau)))\right] d\tau \\ &= p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*) \prod_{j=1}^{k_m-1} (1+\beta_m(\tau^*)\kappa_{m,j}(\tau^*))^{-1/2} \times \\ &\quad \int_{t(1-p_m)}^{t} \exp\left[f(\tau)\right] d\tau \\ &\approx p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*) \times \\ &\quad \prod_{j=1}^{k_m-1} (1+\beta_m(\tau^*)\kappa_{m,j}(\tau^*)^{-1/2}\Phi(-\beta_m(\tau^*)) \times \\ &\quad \left[\frac{1-\exp\left[f'(\tau^*))(\tau^* p_m)\right]}{|f'(\tau^*)|}\right] \\ &\approx p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*) \cdot \left[\frac{1}{|f'(\tau^*)|}\right] \times \end{aligned}$$

$$\prod_{j=1}^{k_m-1} (1+\beta_m(\tau^*)\kappa_{m,j}(\tau^*))^{-1/2}\Phi(-\beta_m(\tau^*))$$

or

$$P_{f,m}(t) \le$$

$$p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*)\, C_{SORM,m}\Phi(-\beta_m(\tau^*))\, C_{T,m} \quad (6)$$

The second expression in eq.(6) represents the asymptotic approximation. In the third expression it is assumed that the jump rates and the curvatures vary only smoothly with time and therefore are drawn in front of the integral with their values at the critical point. It can easily be verified that the curvatures, in fact, vary very little with time. In the sixth expression the asymptotic concepts are applied and the last expression is valid for large τ^*. The function $f(\tau) = \ln(\Phi(-\beta_m(\tau)))$. and its derivative can be evaluated easily in the critical point. Note that if the "on"-probability is p_m the total "on"-time is only $p_m\tau^* = p_m t$. The important aspect of this equation is that one can identify five factors accounting for different influences as in eq. (6). The first factor is the coincidence probability, if there are any. The second factor accounts for the jump rates in the considered load case. The third factor is the second order correction factor which in practice may be taken as one in most cases. The forth factor is the local failure probability according to FORM with $\beta_m(\tau^*)$ the local geometrical reliability index. Finally, the fifth factor corrects the result with respect to time. Note that in the case considered the derivative of $f(\tau)$ is negative. From theorem 1 in Breitung/Hohenbichler (1989) one may conjecture that the formulae given above are asymptotically correct for τ^* being a boundary point. For the derivations to come it is necessary to split the result into factors. In the stationary case the time factor is equal to t. In the non-stationary case we always have $C_T \le t$. The stationary case cannot be recovered from the non-stationary case. Numerically, it is found that a certain time gradient > 0 must be assured. Otherwise one should use the stationary solution without large error. In the following the so-called stationary solution is a solution where the time gradient is taken as zero in the critical point. Even if the asymptotic time corrections are not used but numerical integration is performed, it is wise to compute the ratio

$$C_{T,m} = \frac{\int_0^t \Phi(-\beta_m(\tau))d\tau}{\Phi(-\beta_m(\tau^*))} \quad (7)$$

in order to retain the factored form of the result. Specifically, the asymptotic time factor is

$$\begin{aligned} C_{T,m} &\approx \left[\frac{1-\exp\left[f'(\tau^*)p_m\tau^*\right]}{|f'(\tau^*)|}\right] \\ &\sim \left[\frac{1}{|f'(\tau^*)|}\right] \\ &= \left[\frac{1}{\left|\beta_m(\tau^*)\,\frac{\partial\beta_m(\tau^*)}{\partial\tau}\right|}\right] \end{aligned} \quad (8)$$

If the critical point is an interior point the asymptotic time correction factor can be given as (no intermittency)

$$\begin{aligned} C_T &\approx \left(\frac{2\pi}{|f''(\tau^*)|}\right)^{1/2} \times \\ &\quad \left(\Phi(\sqrt{f''(\tau^*)}(t-\tau^*)) - \Phi(\sqrt{f''(\tau^*)}(-\tau^*)\right) \\ &\sim \left(\frac{2\pi}{|f''(\tau^*)|}\right)^{1/2} \\ &= \left(\frac{2\pi}{\left|\beta(\tau^*)\,\frac{\partial^2\beta(\tau^*)}{\partial\tau^2}\right|}\right)^{1/2} \end{aligned} \quad (9)$$

involving now the second order time derivative. For intermittent loads the expression is more complicated. Again there is always $C_T \le t$. It is observed that asymptotically the C_T–factor does not involve the time interval explicitly. However, the length of the interval may influence the magnitude of the local reliability index $\beta_m(\tau^*)$.

3. Partial safety factors

Partial safety factors are defined as the ratio between design values and characteristic values. For normal design against extreme loads, i.e. under stationary conditions, the design values usually are defined in the extreme value distribution referred to the design life. This appears to be a natural definition if only one time-variant load is present. Already when several loads need to be combined corrections have to be made. However, it is not immediately straightforward how and on which basis these corrections are to be made. Whenever non-stationary conditions prevail a new concept is required. In the following we will assume for simplicity of derivation that loads are essentially stationary and all non-stationarity comes from deterioration or other phenomena affecting the resistance variables. One possible concept for the stationary case is as follows. Instead of using extreme value distributions one defines the safety factors in the point-in-time distributions. In accordance with the above considerations a local reliability index can be defined which, if the normal integral of its negative value is multiplied by the reference time, the coincidence probabilities, the SORM factor and the sum of the jump rates, gives the life time failure probability. Thus, there is

$$\Phi(-\beta_t) = \sum_{m=1}^{s} p_m \sum_{i=1}^{k_m} \lambda_i \, C_{SORM,m} \Phi(-\beta_m)\, t \tag{10}$$

If the target reliability index β_t related to the reference time (intended time of use of the structural facility) is specified the load case failure probability must be such that

$$\Phi(-\beta_m) = \frac{\Phi(-\beta_t)\,\left(1 - \sum_{j=1, j\neq m}^{s} p_m \sum_{i=1}^{k_j} \lambda_i \, C_{SORM,j}\Phi(-\beta_j)\, t\right)}{p_m \sum_{i=1}^{k_m} \lambda_i \, C_{SORM,m}\, t} \tag{11}$$

The load case reliability index must be larger than the life time reliability index. One can and should go one step further and apply eq. 11 separately for each load and load case in which case we have

$$\Phi(-\beta_{m,n}) = \frac{\Phi(-\beta_m)}{\sum_{i=1, i\neq n}^{k_m} \lambda_i \, C_{SORM,m}\, t} \tag{12}$$

which includes the frequency of load changes and consequently

$$x^*_{m,n} = F_n^{-1}(\Phi(-\alpha_{m,n}\beta_{m,n})) \tag{13}$$

where $F_n(.)$ is the point-in-time distribution and $\alpha_{m,n}$ the mean value sensitivity in the standard space. Since eq. (1) is the basis of our considerations, formula (6) may also include the R- and Q-variables and, hence, β_m and $C_{SORM,m}$ take account of all contributing variables in load case m. In this manner the design values can be made explicitely dependent on the various factors. In particular, the design values are explicitly dependent on t.

Substantial simplification is achieved if there are no coincidence probabilites involved, i.e., all load are always on. Then, the design values for the loads are simply derived from

$$x^*_n = F_n^{-1}(\Phi(-\alpha_n\beta_n)) \tag{14}$$

with

$$\Phi(-\beta_n) = \frac{\Phi(-\beta_t)}{\sum_{i=1, i\neq n}^{k} \lambda_i \, C_{SORM}\, t} \tag{15}$$

For R- and Q-variables we have

$$x^*_n = F_n^{-1}(\Phi(-\alpha_n\beta_t)) \tag{16}$$

Assume now that for a non-stationary problem the target reliability index β_t is given. Then, in analogy to (10) it holds

$$\Phi(-\beta_t) = \sum_{m=1}^{s} p_m \sum_{i=1}^{k_m} \lambda_i(\tau^*)\, C_{SORM,m}\Phi(-\beta_m(\tau^*))\, C_{T,m} \tag{17}$$

In other words, in the non-stationary case the jump rates, the local geometrical reliability index and second order correction factor are replaced by the corresponding quantities in the critical point and t is replaced by the time correction factor $C_{T,m}$. Eq. (17) makes, of course, only sense if the argument of the inverse normal integral is smaller than one. If this is no more true the domain of validity of eq. (1) is left. Since $C_{T,m}$ is equal or smaller than t the partial load safety factors (19) for the non-stationary case are equal or smaller than the corresponding factors for the stationary case. Therefore, if compared to the stationary case the "effective" local failure probability is increased by $t/C_{T,m}$ and the "effective" reliability index is

$$\beta^{red}_{m,n} = -\Phi^{-1}\left(\Phi(-\beta_{m,n})\,\frac{t}{C_{T,m}}\right) \tag{18}$$

β^{red}_m denotes the reduced reliability indices.The partial safety factors, finally, are defined as

$$\gamma_{m,n} = \frac{x^*_{m,n}}{x_{c,n}} = \frac{F_n^{-1}(\Phi(-\alpha_{m,n}\beta^{red}_{m,n}))}{x_{c,n}} \tag{19}$$

with $x_{c,n}$ the nominal or characteristic value defined as an upper/lower quantile in the respective point-in-time distribution functions. Similarly, one defines the safety factors for resistance variables.

$$\gamma_{m,n} = \frac{x_{c,n}}{x^*_{m,n}} = \frac{x_{c,n}}{F_n^{-1}(\Phi(-\alpha_{m,n}\beta^{red}_{m,n}))} \tag{20}$$

The reduction factor for each variable then is

$$\varepsilon_{m,n} = \frac{F_n^{-1}(\Phi(-\alpha_{m,n}\beta^{red}_{m,n}))}{F_n^{-1}(\Phi(-\alpha_{m,n}\beta_{t,m}))} \leq 1 \tag{21}$$

4. Examples

The following examples for three kinds of deterioration are made as realistic as possible but are still illustrations showing rather the methodoloy than practical applications. Therefore, the numerical values should be interpreted with care.

4.1 *Chemical deterioration of textiles for soil stabilization*

Three types of rather general relationships for chemical attack can be found in the literature and will be presented here for easy reference. According to Arrhenius the rate of reaction of chemical substances depends on temperature

$$\frac{dM}{dt} = C \exp\left[-\frac{E}{k}\frac{1}{\theta}\right] \tag{22}$$

where

$M =$ *reaction mass*
$C =$ *empirical constant*
$E =$ *activation energy* $[Joule]$
$k =$ *Boltzmann constant* $(1.38 \cdot 10^{-23}\,[\frac{Joule}{K}])$
$t =$ *time*
$\theta =$ *temperatur* $[K]$

In general the details of the physical background of this relationship are not so important and also difficult to identify but the general form of eq. (22) is reasonable can be retained. Another general law derived from quantum physics is the Eyring law

$$\frac{dM}{dt} = A\,\theta\, \exp\left[-\frac{E}{k}\frac{1}{\theta}\right] \tag{23}$$

with similar interpretation of the parameters. Integration with respect to time shows that the reaction mass (or volume) increases linearly with time in both cases. It can be seen from the Arrhenius equation that the rate of reaction is zero for zero temperature and approaches C for high temperatures. In contrast, the rate for the Eyring equation also starting from zero grows asymptotically linearly. If a certain mass M_0 is necessary to produce a critical damage a characteristic time for the Arrhenius law is

$$\tau_0(\theta) = \frac{M_0}{C\, \exp\left[-\frac{E}{k}\frac{1}{\theta}\right]} \tag{24}$$

A purely empirical relationship is the power law (the Wöhler curve)

$$\tau_0(\theta) = \frac{1}{K\,\theta^m} \tag{25}$$

with K and m being empirical constants and which is also often used. In all three relationships the parameter θ need not be temperature but can be any suitable quantity such as concentration, pH-value, etc. Further it is possible construct models which contain more than one such parameters.

Even under laboratory conditions the variability of the times to critical damage can vary considerably. Suitable models then are the exponential (k = 1) or even the Weibull distribution ($k \neq 1$)

$$F_{T_M}(t \mid \theta) = 1 - \exp\left[-\left(\frac{t}{\tau_0(\theta)}\right)^k\right] \tag{26}$$

There is generally no particular physical or probabilistic reason for selecting this distribution. It is chosen here for mere practical reasons. The parameters of those models have to estimated from tests.

In some cases these relationships can be used directly. If the parameters can be determined from the accelerated tests, which unfortunately, is not an easy task in general, and if their statistical spread can be assessed, i.e. by the residuum in least square estimation, by asymptotic maximum-likelihood estimation or by boot strapping, the failure probability is $P_f(t) = E_Q\left[F_T(t) \mid q\right]$ with t being the reference time and Q the vector of uncertain parameters in the respective model.

More often the damage produced with either of these models must be related to residual strength. This is so because it is usually not possible to assess a critical reaction mass. What is measured is the residual strength. It makes sense to assume a certain relationship, not necessarily proportionality, between the reaction volume $M(t,\theta)$ and some damage indicator. Then, the residual strength could, for example, be expressed in the form

$$R(t) = R(0)\,(1 - D(t)) \tag{27}$$

where R(0) is initial strength. The question now is how $D(t)$ develops in time. It is proposed to use a relation originally proposed by Kachanov for the damage development in long-duration tests.

$$\frac{dD(t)}{dt} = \left(\frac{1}{1 - D(t)}\right)^n \frac{1}{T_M(\theta)} \tag{28}$$

n is another parameter to be determined experimentally. Integration for D(0) = 0 yields

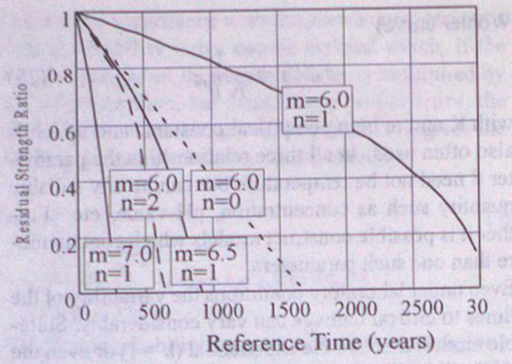

Figure 1: Typical Residual Strength Deterioration (Model 30)

$$D(t) = 1 - \left(1 - (n+1)\frac{t}{T_M(\theta)}\right)^{\frac{1}{n+1}} \quad (29)$$

Therefore, the residual strength is given by

$$R(t) = R(0)\left(1 - (n+1)\frac{t}{T_M(\theta)}\right)^{\frac{1}{n+1}} \quad (30)$$

Note that for n = 0 the damage increases linearly and thus the residual strength decreases linearly. n >0 lets the residual strength go down progressively. If $T_M(\theta)$ is treated as a constant $R(t)$ varies proportional to $R(0)$. The general behavior of eq. 30 is illustrated in figure 1.

From 6 high temperature tests the parameters of the models (22), (23) and (24) have been determined. It turned out that for the relatively short life time envisaged the models do not differ very much. With this small number of tests a discrimination of any of the three alternative models, of course, is not possible. Therefore, only the empirical power model is studied further. The parameters and the stochastic model in appropriate units are given below.

Variable	Distr.	Mean	Std.Dev.
Initial Strength	LN	120	12
m	N	6.5	0.1
lnK	N	-30.0	0.4
n	-	3	-
T_M	Weibull	$\tau_0(\theta)$	7,7%
Dead load	N	20	2
Live load	N	35	7.5
Temperature $[{}^\circ C]$	N	10	1
$\lambda\,[1/year]$	-	1	-

lnK and m are highly correlated with negative correlation coefficient $\rho_{\ln K,m} = -0.998$. For a life time of 100 years one computes an upper bound failure probability of $1.83{\cdot}10^{-5}$ ($\beta_t = 4.128$), $C_{SORM} = 1.006$ and a local reliability index of $\beta_g = 5.049$. The time correction factor is determined to be 81.65 instead of 100. In this case the degree and rate of deterioration is extremely small. Almost the same result would have been obtained if the stationary solution for time t=100 is computed.

For another data set for a similar material and somewhat more severe environmental conditions as given below

Variable	Distr.	Mean	Std.Dev.
Initial Strength	LN	80	4
m	N	6.5	0.1
lnK	N	-30.0	0.4
n	-	3	-
T_M	Weibull	$\tau_0(\theta)$	7,7%
Dead load	N	20	2
Live load	N	35	7.5
Temperature $[{}^\circ C]$	N	13	0.5
$\lambda\,[1/year]$	-	1	-

one computes again for a design life of 100 years a local geometrical reliability index of $\beta_g = 3.837$, an upper bound failure probability of $2.06{\cdot}10^{-3}$ ($\beta_t = 2.869$), $C_{SORM} = 1.134$ and a time correction factor of $C_T = 29.14$ implying that there is already significant deterioration. The analysis shows that initial strength and the two loads are most important whereas all other factors have negligible effect on the reliability. This is somewhat surprising because the parameters of relation (31) are obtained with a very small data set. It can, however, be shown that at much larger life times these variables together with the operating temperature would become more important and finally dominant. The partial safety factors given below are related to the means.

Variable	α	$\gamma_{stationary}$	$\gamma_{non-stationary}$
Initial Strength	0.53	1.24	1.10
m	0	1.0	1.0
lnK	0	1.0	1.0
T_M	-0.05	1.0	
Dead load	-0.31	1.42	1.27
Live load	-0.786	1.63	1.26
Temperature $[{}^\circ C]$	0	1.0	1.0

The table shows a significant reduction in the partial safety factors which, as explained before is a consequence of the reduction in the C_T−factor.

4.2 *Corrosion of sheet pile walls*

The average annual corrosion of sheet pile walls in sea water in temperate climates in the splash zone is

about 0.12 [mm/y] and about half of that value in a normal fresh water environment. In soil it is about 0.03 [mm/y]. A little higher are the average rates at ships (Yamamoto/Ikegami, 1996). This means that for a wall thickness of 10-20 [mm] such structures have a very real but unavoidable aging problem. Melchers (1995) summarized several data sets for marine environment and found that the corrosion depth grows with mean $m_{Corr} \simeq 0.0844\ t^{0.82}$ with standard deviation $\sigma_{Corr} \simeq 0.0565\ t^{0.82}$. This is almost linear which is what is usually assumed. An appropriate failure criterion is

$$g(\mathrm{X}) = (R - A\,t) - (D - S(t)) \leq 0 \quad (31)$$

where R is the resistance, A a coefficient, D the stress due to dead load and $S(t)$ a time variant process modeling the live loads (storage loads) just behind the sheet pile wall. The renewal rate for $S(t)$ is assumed to be $\lambda = 1$ [jump/y]. The mean resistance has been adjusted to the corrosion rates such that there is a target reliability index of $\beta \simeq 3.8$. The stochastic model is given below.

Variable	Distr.	Mean	Std.Dev.
R	N	14.75(21.25)	1.475(2.125)
A	N	0.06 (0.12)	0.015 (0.03)
D	N	2.0	0.2
S	N	2.0	0.8

The C_T-factor has been evaluated according to eq. (8) and "exactly" by eq. (7). The value of β^{red}implies that all partial safety factors can be reduced by about 0.1 when compared to the factors determined for stationary conditions at τ^*. The results are given in the following table:

Case	local β	$g(u,\tau)$	C_T	β_t	β^{red}
Marine	4.123	-0.069	3.515 (3.343)	3.836	3.082
Fresh water	4.192	-0.069	5.186 (4.905)	3.816	3.196

4.3 *Instability of fatigue cracks*

In this example we study the reliability against crack instability. For the purpose of illustration a simple edge crack and ergodic Gaussian rectangular wave renewal loading is assumed. The crack propagates according to Paris/Erdogan

$$\frac{da}{dt} = C_P\,\Delta K^m \quad (32)$$

with $\Delta K = Y(a)\ \Delta S\ \sqrt{\pi\,a}$ and where a is crack length, ΔS is stress range,$Y(a) = const = 1$ a geometry factor and m a material parameter. For m = 2 and for m > 2 the crack propagation equation can be integrated analytically. Making use of the ergodicity property the right hand side of the equation can be written as

$$\int_0^t \Delta S(t)\ dt = C_P\,\lambda\,\nu_0\,E\left[\Delta S^m\right]\,t \quad (33)$$

λ is the stationary jump rate of the loading process and ν_0 is the cycle rate within a mission. For m > 2, we have with a_0 the initial crack length

$$a(t) = \left(a_0^{1-\frac{m}{2}} + (1\text{-}\frac{m}{2})\ C_P\,\lambda\,\nu_0\,E\left[\Delta S^m\right]\,t\right)^{1/(1-m/2)} \quad (34)$$

Instability occurs if

$$\frac{K_{1C}}{\sqrt{\pi\,a(t)}} - S(t) \leq 0 \quad (35)$$

where K_{1C} is the fracture toughness and $S(t)$ the loading process. This is a notoriously difficult problem in FORM/SORM because the time gradient of this state function varies very little for some time and then becomes excessively large. Also, the parameters have to be selected such that the reference time is well below the "explosion time", i.e. where eq.(34) becomes singular. The details of the stochastic model are given below.

Variable	Distr.	Mean.	St. Dev.
$C_P\ [mm^{5/3}/N]$	LN	$1.3\ 10^{-10}$	$1.3\ 10^{-11}$
$a_0\ [mm]$	Rayleigh	1.75	0.66
$K_{1C}\ [N/mm^{3/2}]$	Weibull	2200	220
$\lambda\,[1/time\ unit]$	-	1	-
$S\,[N/mm^2]$	Normal	35	35
$\nu\,[1/time\ unit]$	-	8	-
m$[no\ dimension]$	-	2.7	-

Computation for a number of missions of 3000 with 8 cycles each yields an upper bound failure probability of $2.37{\cdot}10^{-3}$ ($\beta_t = 2.824$), $C_{SORM} = 1.039$ and a local reliability index of $\beta_g = 4.155$. The time correction factor is determined to be 140.3 instead of 3000 if a stationary solution would have been applied or $C_T/t = 0.0468$ determined by eq.(8). If the stationary solution would have been applied a reliability index of 1.639 would have been obtained which clearly is overconservative. The non-stationary solution yields a reduced reliability index of 3.39. Because the α−vector at the final mission number is (-0.637,-0.586,0.171,-0.471)^T which is shown to vary a little over the intended mission number in figure 2, the safety factors related to the mean values are as follows $\gamma = (1.23, 1.13, 1.05, 2.59)^T$

In fact, for mission numbers of about 2500 crack propagation begins to accelerate and the variables which dominate change as can be seen from figure 3 where the reliability index is plotted over the mission number. It can still be shown that for each mission number,

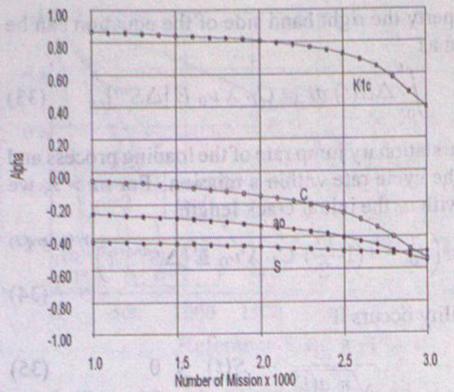

Figure 2: Sensitivities versus Mission Number

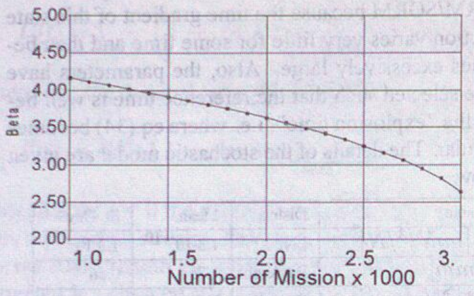

Figure 3: Reliability Index versus Mission Number

even larger ones, the α's do not change much along the time axis from $\tau = 0$ to $\tau = t$.

5. Summary and Discussion

A concept to define partial safety factors for non- stationary reliability problems under rextangular wave renewal process loading with or without intermittencies is proposed. It is based on the possibility to factor the failure probability into five terms each covering a different influence, i.e., the first being the coincidence probability, the second the frequency of load changes in time, the third the local FORM failure probability at the critical point, the forth a SORM correction factor and the fifth a factor taking account of the local behavior of the failure function in time. This last factor is shown to be equal or smaller than the reference time interval. Partial safety factors are then defined and related to the point-in-time distributions for the loads in the stationary and the non-stationary case. The factors for the non-stationary case are equal or smaller than for the stationary case. The concept is illustrated at three examples without intermittencies, the first where geotextiles are deteriorating under chemical attack, the second where corrosion takes place at sheet pile walls and the third where a crack grows and becomes unstable. However, consideration of intermittencies is possible and will lead to interesting results because now the magnitude of the "on"-probabilities also affect the partial safety factors significantly. A similar factored form of the result can also be obtained for differentiable normal or non-normal translation load processes. Because the time correction factor is essentially the same as for the case with rectangular wave renewal processes quite general load combination rules can be set up and the corresponding partial safety factors can be set up making use of a result by Ditlevsen/Madsen (1981) saying that the outcrossing rate of a mixture of rectangular wave renewal processes and differentiable processes is just their sum with the other type of processes at the point-in-time value..

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