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Polynomial Approximation of Morison Wave Loading

For offshore structures with slender elements, the modeling of random wave loads by the Morison equation yields an equation of motion which has no analytical solution for response moments except in a few limiting cases. If polynomial approximations of the Morison drag loads are introduced, some procedures are available to obtain the stationary moments of the approximate response. If the response process is fitted by non-Gaussian models such as proposed by Winterstein (1988), the first four statistical moments of the response are necessary. The paper investigates how many terms should be included in the polynomial approximation of the Morison drag loading to accurately estimate the first four response moments. It is shown that a cubic approximation of the drag loading is necessary to accurately predict the response variance for any excitation. For the fit of the first four response moments, at least a fifth-order approximation appears necessary.

Introduction

For offshore structures, the main nonlinearity generally is in the wave loading consisting of potential and viscous forces. In the particular case of structures with slender elements, for example, jacket structures, the wave-induced force on any structural member usually is described by the extended Morison equation where potential and viscous forces are modeled by a linear inertia term and a nonlinear drag term, respectively. Other nonlinearities are often neglected. For example, linear structural behavior is assumed and the water elevation $\eta(t)$, water particle velocity $x(t)$, and acceleration $\dot{x}(t)$ are assumed to be zero-mean stationary normal processes during sea states with duration of a few hours. For a discussion of these assumptions, see, e.g., Chakrabarti (1987).

In the paper, we consider a time-invariant single-degree-of-freedom oscillator in unidirectional random waves. Zero current is assumed throughout. Wave loading is modeled by Morison's equation. This equation of motion is

$$m\ddot{y} + c\dot{y} + ky = K_D(x - \dot{y})|x - \dot{y}| + K_M\dot{x} - K_A\ddot{y} \quad (1)$$

where $y(t)$ is the structural displacement; $|.|$ denotes absolute value; $K_D = \frac{1}{2}\rho DC_D$, $K_M = \frac{1}{4}\pi\rho D^2C_M$, and $K_A = \frac{1}{4}\pi\rho D^2(C_M - 1)$; C_D and C_M, are the drag and the inertia coefficients; ρ is the water density; D the representative diameter, m the mass, c the damping, and k the stiffness. Equation (1) is not general. For example, it does not result from modal decomposition of a multi-degree-of-freedom because modal forcing functions are, in general, a combination of correlated Morison loads. However, interesting conclusions can be drawn from the study of Eq. (1), and it is believed that at least a part of these conclusions remain true in general. It is assumed that one and only one solution exists for Eq. (1), and that the system is stable. The existence, uniqueness, and stability of the solution must be expected on physical grounds.

If Eq. (1) is solved, the response process may be used to assess reliability or to estimate fatigue life. It is known that the results of these analyses are accurate only if a good fit of the extreme value behavior of the response process is achieved. If the response is approximated by a truncated series of Hermite polynomials, it was shown by Winterstein (1988) that for many engineering problems, accurate results can be obtained if the first four statistical moments of the response are known.

Unfortunately, the estimation of even low-order moments of $y(t)$ is not straightforward. These moments, of course, can be exactly calculated through time domain simulations, but this approach is excessively time-consuming to obtain estimates with low coefficient of variation (cov). Therefore, approximations must be introduced. It has been proposed to approximate the drag component of the Morison loading by a polynomial. The Volterra theory of nonlinear systems is used to describe the response.

In the following, Eq. (1) is first standardized as proposed by Hu et al. (1991). Further, classical methods to approximate the nonlinear drag loading are described. Then the application of the Volterra theory of nonlinear systems to estimate the response to wave loads with polynomial nonlinearities is discussed. It is emphasized that the complexity of this method increases rapidly with the order of the polynomial approximation of the drag loading and the number of terms included in the Volterra expansion of the response. Therefore, before investigating the possibility to extend the aforementioned method to larger systems and to implement such method in appropriate computer codes, it appears necessary to study low-order approximations of the drag loading. The accuracy in the moments of the response to low-order approximations is quantified by time domain analyses. It will be shown that a cubic approximation of the Morison loading yields accurate estimates for the response variance of any sea state. However, it will be observed that a fifth-order approximation of the Morison loading is necessary to ensure a good fit of the first four response moments in quasi-static cases.

Standardized Equation of Motion

Introducing the so-called hydrodynamic mass $m_{\text{hyd}} = K_A$, we rewrite Eq. (1) in the form

$$(m + m_{\text{hyd}})\ddot{y} + c\dot{y} + ky = K_D(x - \dot{y})|x - \dot{y}| + K_M\dot{x} \quad (2)$$

The left-hand side of Eq. (2) is further standardized as

$$\ddot{y} + 2\xi_s\omega_s\dot{y} + \omega_s^2y = \frac{K_D}{m + m_{\text{hyd}}}(x - \dot{y})|x - \dot{y}| + \frac{K_M}{m + m_{\text{hyd}}}\dot{x} \quad (3)$$

where ω_s denotes natural frequency with structural damping ξ_s. In the following, it is assumed that a Pierson-Moskowitz or a JONSWAP one-sided spectral density $G_{\eta\eta}(.)$ describes the sea elevation process $\eta(t)$ during each sea state. Let ω_0 denote

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some dominant, central frequency of the wave excitation which defines $G_{\eta\eta}(.)$ in a sea state. Then, dividing Eq. (3) by ω_0^2 yields a standardized equation of motion (Hu et al., 1991)

$$\bar{y}'' + 2\xi_s\bar{\omega}_s\bar{y}' + \bar{\omega}_s^2\bar{y} = (u - \alpha\bar{y}')|u - \alpha\bar{y}'| + \beta a \quad (4)$$

where $\bar{y} = (m + m_{\text{hyd}})\omega_0^2 y/K_D$ is a standardized displacement; $\bar{y}' = d\bar{y}/d(\omega_0 t)$; $\bar{\omega}_s = \omega_s/\omega_0$; $u = \dot{x}/p$ and $a = \ddot{x}/q$ are the standardized water particle velocity and acceleration, respectively, with factors p and q obtained from

$$p^2 = \omega_0^2 G_{\eta\eta}(\omega_0) \quad \text{and} \quad q^2 = \omega_0^5 G_{\eta\eta}(\omega_0) \quad (5)$$

It is easily shown that the densities of the standardized water particle velocity $u(t)$ and acceleration $a(t)$ written as functions of ω/ω_0 are independent of the sea state. The two forcing function coefficients α and β in Eq. (4) are measures of water-structure interaction and hydrodynamic ratio, respectively. They are given by

$$\alpha = \frac{p}{\omega_0} K_D/(m + m_{\text{hyd}}) \quad \text{and} \quad \beta = \frac{q}{p^2} K_M/K_D \quad (6)$$

Approximation of Drag Loading

The choice of a low-order polynomial approximation of the nonlinear term $(u - \alpha\bar{y}')|u - \alpha\bar{y}'|$ is not straightforward. First of all, at $u = 0$ the first derivative of $u|u|$ vanishes and higher derivatives do not exist. Further, the nonlinear term depends on the unknown response when hydrodynamic interaction is not negligible ($\alpha \neq 0$). Therefore, numerous procedures exist to approximate the nonlinear term $(u - \alpha\bar{y}')|u - \alpha\bar{y}'|$ by a polynomial of given order, but none of them can be shown to be optimal to fit a given low-order moment of the solution of Eq. (4).

It is assumed that the approximation order has a larger influence than the approximation criterion on the model accuracy. In this study, an approximation of $(u - \alpha\bar{y}')|u - \alpha\bar{y}'|$ which minimizes the error in the mean square sense is preferred. Assume that an accurate approximation of this nonlinear term by a polynomial of order n is sought

$$(u - \alpha\bar{y}')|u - \alpha\bar{y}'| \approx a_1(u - \alpha\bar{y}') + \ldots + a_n(u - \alpha\bar{y}')^n \quad (7)$$

Then the coefficients $(a_1, \ldots, a_n)$ in Eq. (7) are chosen so that they minimize the mean square error

$$E[((u - \alpha\bar{y}')|u - \alpha\bar{y}'| - a_1(u - \alpha\bar{y}') - \ldots - a_n(u - \alpha\bar{y}')^n)^2] \quad (8)$$

Elementary differential calculations show that this criterion yields a set of n linear equations satisfied by the coefficients $(a_1, \ldots, a_n)$

$$\sum_{i=1}^{n} a_i E[(u - \alpha\bar{y}')^{i+k}] = E[(u - \alpha\bar{y}')^{k+1}|u - \alpha\bar{y}'|], \quad k = 1 \ldots n \quad (9)$$

Since zero current is considered, the response has zero mean and any approximation of even order $n = 2p$ reduces to an approximation of odd order $2p - 1$. If wave current is taken into account, then approximations of even order need also to be considered. For this reason, Donley and Spanos (1990) considered a quadratic approximation of the drag loading.

Alternatively, the nonlinear term can be replaced by a series of Hermite polynomials

$$(u - \alpha\bar{y}')|u - \alpha\bar{y}'| = \sum_{i\geq 0} b_i \text{He}_i(u - \alpha\bar{y}') \quad (10)$$

with $\text{He}_i(.)$ the ith Hermite polynomial (Winterstein, 1988). The coefficients b_i are obtained from the orthogonal properties of the Hermite polynomials. If the series is truncated after n terms, it can be shown that this alternative method yields exactly the same polynomial approximation of order n as the mean square statistical approximation. Therefore, only the latter method is considered in the following.

In the limit case where $\alpha = 0$, the system of Eqs. (9) does not depend on the response process. If well-known expressions for the higher-order central moments of a normal process are used, it follows that for $n = 2p - 1$:

$$\sum_{i=1}^{p} a_{2i-1} \frac{(2i + 2k - 3)!}{2^{i+k-2}(i + k - 2)!}\sigma_u^{2i-2} = 2^k k!\sigma_u\sqrt{\frac{2}{\pi}}, \quad k = 1 \ldots p \quad (11)$$

with σ_u the standard deviation of u. For instance, some "optimum" low-order polynomial approximations of the drag term $u|u|$ are

$$n = 1,\ u|u| \approx 2\sqrt{\frac{2}{\pi}}\sigma_u u \quad (12)$$

$$n = 3,\ u|u| \approx \sqrt{\frac{2}{\pi}}\sigma_u^2\left[\frac{u}{\sigma_u} + \frac{1}{3}\left(\frac{u}{\sigma_u}\right)^3\right] \quad (13)$$

$$n = 5,\ u|u| \approx \sqrt{\frac{2}{\pi}}\sigma_u^2\left[\frac{3}{4}\frac{u}{\sigma_u} + \frac{1}{2}\left(\frac{u}{\sigma_u}\right)^3 - \frac{1}{60}\left(\frac{u}{\sigma_u}\right)^5\right] \quad (14)$$

These approximations (12) to (14) originally have already been proposed in Borgman's (1969) seminal paper.

In general, i.e., when hydrodynamic interaction is non-negligible ($\alpha \neq 0$), no closed solution exists for Eqs. (9) because the statistics of the absolute relative water velocity $|u - \alpha\bar{y}'|$ are not known beforehand. Then the coefficients $(a_1, \ldots, a_n)$ of the polynomial approximation must be evaluated iteratively. Initially, the response is assumed to be zero. Then, coefficients are 1) *evaluated* for estimated moments of $|u - \alpha\bar{y}'|$, the response to the approximate excitation is 2) *calculated*, and 3) used to *update* Eqs. (9). The analysis is then repeated from steps 1 to 3 until convergence is reached.

If Eq. (4) is solved numerically, then any moment of the absolute value of relative water velocity $|u - \alpha\bar{y}'|$ can be evaluated from the time histories. If, on the other hand, an analytical procedure is used to estimate the response, evaluation of these moments is difficult. Let us reconsider the linear approximation of the drag loading. From Eqs. (9), it follows that the moment $E[|u - \alpha\bar{y}'|^3]$ is needed to evaluate the coefficient a_1 of the linear approximation. However, the response to the linearized forcing function is assumed normal, and therefore this moment cannot be evaluated exactly. Consequently, further approximations are necessary to evaluate the coefficient a_1 iteratively. Since analytical procedures practically allow the calculation of, at most, the first four moments of the relative water velocity, the same difficulty is encountered if higher-order approximations (7) are considered. For instance, it follows from Eqs. (9) that the fifth and the ninth-order moments of $|u - \alpha\bar{y}'|$ are required to evaluate the coefficients of third and fifth-order approximations, respectively. In practice, these higher-order moments are approximated from a moment-based approximation of the probability density of the relative water velocity (Donley and Spanos, 1990). This probability density can be best approximated by using Winterstein's (1988) model. Less numerical difficulties are encountered if linearization is performed using Bolotin's criterion of equal variances (Bouyssy and Rackwitz, 1994).

Analytical Solutions

Several methods are available to estimate the moments of the solution of Eq. (4) when the Morison loading is approximated by a polynomial. For example, Ito's differentiation rule can be used to derive differential equations for the moments of the approximate response when the motion of the structure is not accounted for in the drag loading ($\alpha = 0$) (e.g., Grigoriu and Ariaratnam, 1988). If hydrodynamic interaction is nonnegligible ($\alpha \neq 0$) or if a spectral description of the response is required, however, the Volterra theory of nonlinear systems or one of its improvements provides a more convenient approach. The basic idea is to express the response to wave loads with polynomial nonlinearities as a Volterra series in which the wave elevation $\eta(t)$ is the input function (Vassilopoulos, 1967). Using this generalization of the Taylor series expansion from functions to functionals, the response is written as

$$\bar{y}(t) = \sum_{k=0}^{\infty} \int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} h_k(t_1, \ldots, t_k)\eta(t - t_1) \ldots \eta(t - t_k)dt_1 \ldots dt_k \quad (15)$$

with $h_k(t_1, \ldots, t_k)$ the kth Volterra kernel (Bedrosian and Rice, 1971).

For practical purposes, expression (15) is hardly tractable. Therefore, the Volterra series expansion (15) is generally truncated. Then an exact expression can be found for any moment of the truncated expansion. Response moments can be obtained by integration of higher-order spectra in the frequency domain if multidimensional Fourier transformations are used. This procedure was applied by Olagnon et al. (1989) to calculate the response of a free-standing conductor pipe to a cubic approximation of the Morison wave loading. Hydrodynamic interaction was taken into account and few iterations were necessary to reach convergence. Volterra series were also used by Donley and Spanos (1990) to evaluate statistics of the response of a three-degree-of-freedom tension leg platform submitted to quadratic viscous forces. For this particular case, response statistics of the second-order Volterra model can alternatively be obtained by the Kac-Siegert technique (e.g., Naess, 1985). Finally, it was shown how second-order Volterra models could be used to assess extreme and fatigue reliability of offshore structures (e.g., Winterstein et al., 1994).

Comparison of analytical and numerical results for response moments and/or response spectral density were satisfactory in some cases, but it is not clear whether proposed approximations of the analytical response are accurate for any sea state. As Eq. (4) usually must be solved for several sea states to account for the long-term variability of wave climate in fatigue or reliability analyses, the accuracy of the aforementioned approximations for arbitrary excitation is a matter of considerable interest. There are two kinds of problems. On the one hand, with present-day computers, only low-order Volterra expansions of the response can be handled in practical cases. The computational effort to estimate responses increases substantially with the number of terms included in the truncated expansion (15). If the Volterra series expansion (15) is truncated after the kth term, it follows from the normality of the sea elevation $\eta(t)$ that a k-fold and a $2k$-fold integration in the frequency domain needs to be performed in order to estimate second and fourth-order response moments, respectively. Notice that similar integrations need also to be performed to estimate the moments of the relative water velocity, which are necessary to evaluate the coefficients of the approximation (7). In addition, the derivation of high-order Volterra kernels, for example, using the harmonic input method described in Bedrosian and Rice (1971), is not straightforward. Consequently, the Volterra series expansion (15) must be truncated after the first few terms. The analysis can further be simplified by using approximate higher-order transfer functions (e.g., Newman, 1977). On the other hand, it is not clear whether approximations of the drag loading by polynomials of low order yield accurate responses for all possible sea states. This problem is crucial, because at least the k first terms of the Volterra series expansion must be taken to capture the effect of the kth-order viscous force, when the drag loading is approximated by a polynomial of order k.

It follows from the foregoing discussion that, before investigating the possibility to take additional terms into account in the Volterra expansion of the response, to introduce approximations of the transfer functions, and to implement such methods in appropriate computer codes, it is mandatory to find how accurate the drag loading needs to be approximated in order to obtain a good fit of the response moments for all sea states. The answer to this question will provide conclusions on the feasibility of fatigue and reliability analyses when wave loads are modeled by the Morison equation.

Fit of Response Moments

In the following, a numerical study is performed to assess the ability of first, third, and fifth-order approximations of the drag loading to yield approximate solutions with accurate first four moments. As zero current is considered, the response has zero mean and zero skewness. Therefore, only the fit of second and fourth-order response moments is considered.

Time histories of the stationary normal standardized water particle velocity $u(t)$ and acceleration $a(t)$ are simulated by using the spectral representation method. A Pierson-Moskowitz spectrum is assumed for the sea elevation process. Harmonics with random phase and deterministic amplitude are used providing ergodic time series. Time histories with duration of 4096 s and time step $\Delta t = 0.125$ s are simulated with $N = 16384$ harmonics so that they are almost perfectly normal according to the central limit theorem.

The constant average acceleration step-by-step method proposed by Newmark is used to solve Eq. (4) numerically. For $\alpha \neq 0$, the forcing function in Eq. (4) depends on the response process and the "exact" response at the end of each time step is computed iteratively. The time history of the response to each approximate forcing function is also computed with the $\beta = 0.25$-Newmark method. The displacement at the end of each time step is the unique real root of a first, third, or fifth-order algebraic equation whose coefficients depend on data available at the beginning of the time step. Iterations are necessary to evaluate the coefficient of the approximation according to Eqs. (9). Zero response is used as initial guess for the coefficient evaluation and convergence is reached in less than six iterations. Finally, statistics of the response time histories are calculated. However, it is well known that higher moments of simulated time histories of water velocity and acceleration exhibit some scatter (Tucker et al., 1984). Therefore, the aforementioned procedure is repeated several times until convergence is reached in response moments. It is found that less than 150 time histories are sufficient to reach a cov of 15 percent in fourth-order response moments.

The analysis is performed for several values of the frequency ratio ω_0/ω_s and structural data (α, β, ξ_s). The frequency ratio ω_0/ω_s is taken in (0; 1) ranging from static to dynamic excitation. The relative errors in second and fourth-order response moments for the first, third, and fifth-order approximations of the drag loading are reported as a function of the frequency ratio ω_0/ω_s in Figs. 1 to 3 for $\alpha = 0$. Errors measured for $\alpha = 1$ are reported in Figs. 4 to 6. In each figure, three curves are reported which correspond to hydrodynamic ratio $\beta = 0.0$, 0.5, and 1.0. Note the different ordinate scales in those figures. A damping ratio $\xi_s = 2$ percent was assumed which appears reasonable for steel structures. Finally, the kurtosis of the response is shown in Fig. 7 as a function of the frequency ratio for $\alpha = 0$ and $\alpha = 1$.

- It is first observed that a first-order approximation of the drag load provides good fits of both second and fourth-order response moments for high values of the frequency ratio ω_0/ω_s (see Figs. 1 and 4). This confirms findings by other authors

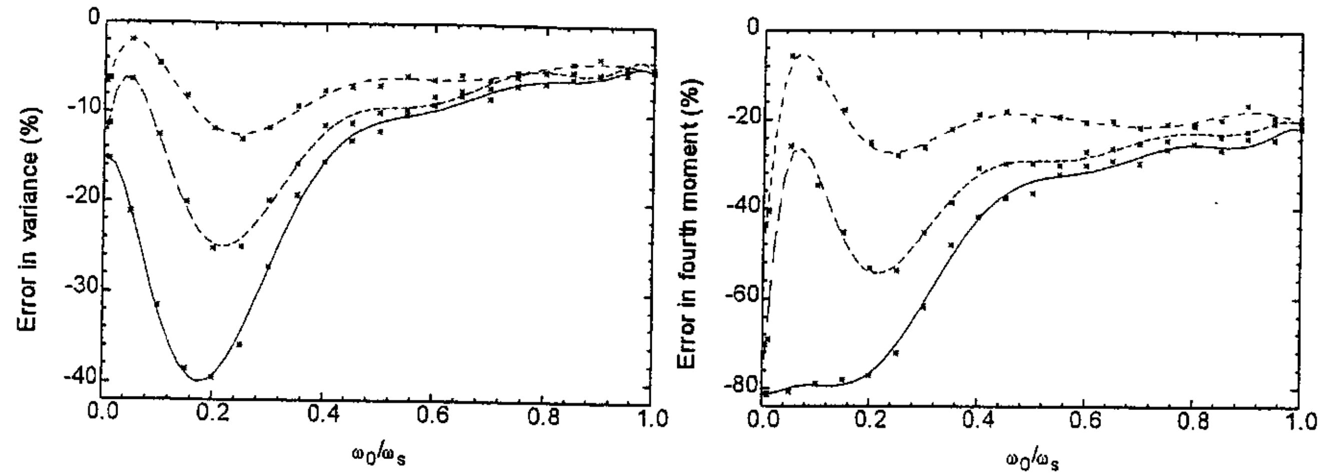

Fig. 1 Error in second (left) and fourth-order (right) response moments for first-order approximation of the drag loading for $\alpha = 0$ (——) $\beta = 0.0$, (- - -) $\beta = 0.5$, (----) $\beta = 1.0$

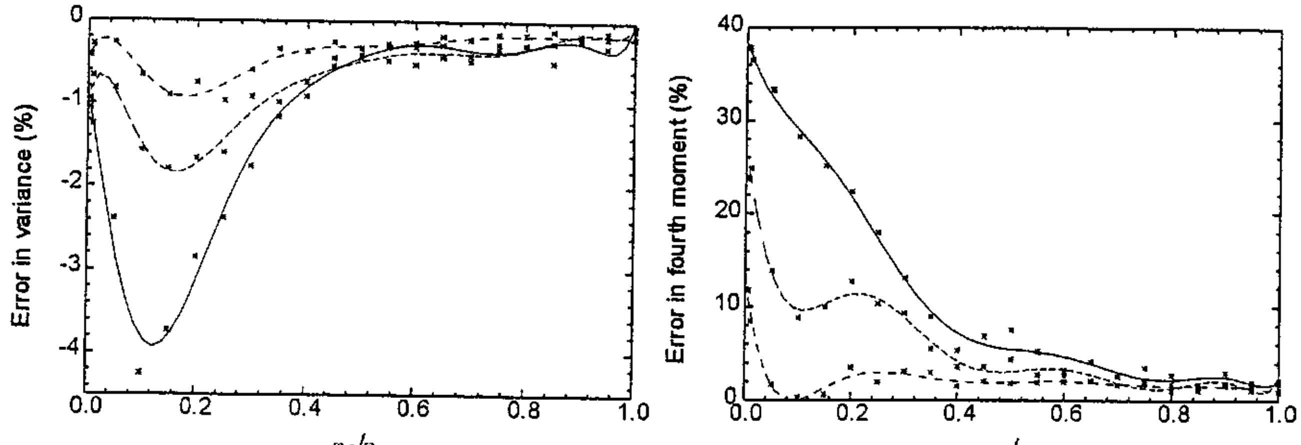

Fig. 2 Error in second (left) and fourth-order (right) response moments for third-order approximation of the drag loading for $\alpha = 0$ (see caption of Fig. 1 for legend)

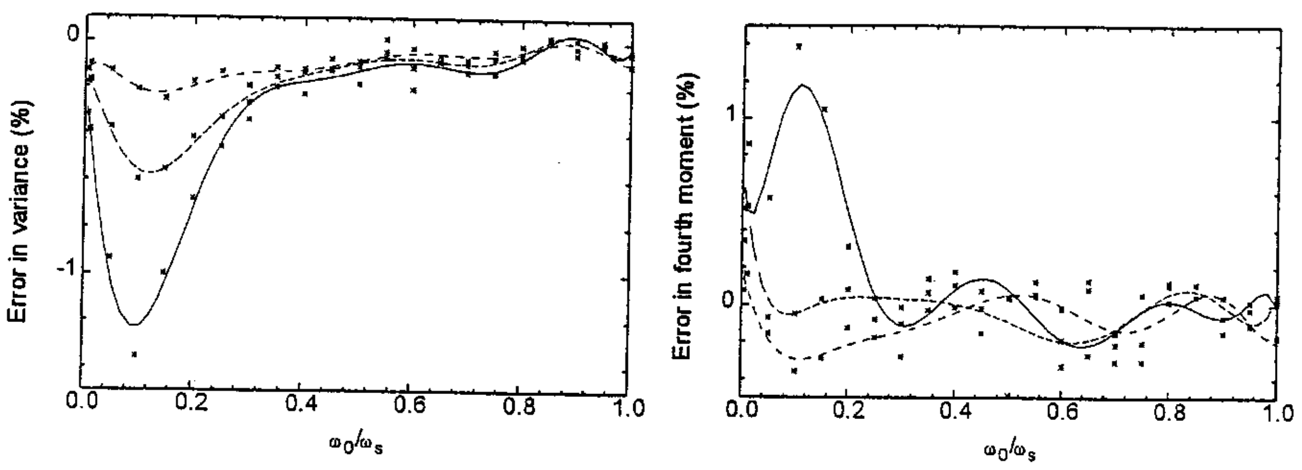

Fig. 3 Error in second (left) and fourth-order (right) response moments for fifth-order approximation of the drag loading for $\alpha = 0$ (see caption of Fig. 1 for legend)

(e.g., Manuel and Cornell, 1993). In fact, a limit theorem by Rosenblatt proves that the response process asymptotically becomes normal as ω_0/ω_s increases to infinity (e.g., Kotulski and Sobczyk, 1981).

Most structures, however, are designed to behave essentially in a static manner. Then the response can no longer be assumed normal (see Fig. 7). In the limit $\omega_0/\omega_s = 0$, the response is the forcing function multiplied by a constant. Consequently, fourth-order response moments predicted with linear—i.e., normal—approximation of the loading are excessively inaccurate in quasi-static cases. This is confirmed by the results shown in Figs. 1 and 4. Our results further show that a linear approxima-

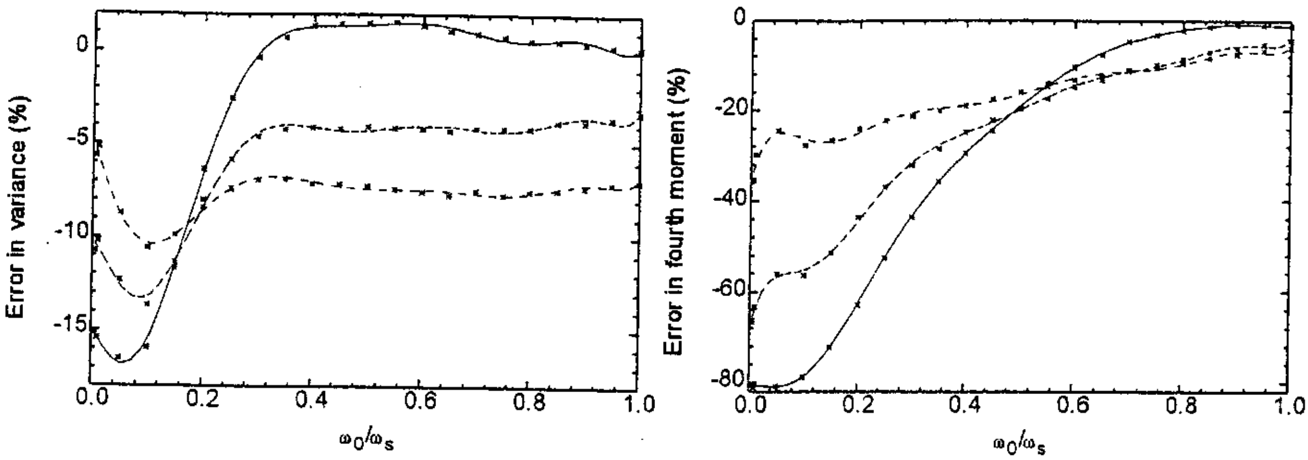

Fig. 4 Error in second (left) and fourth-order (right) response moments for first-order approximation of the drag loading for $\alpha = 1$ (see caption of Fig. 1 for legend)

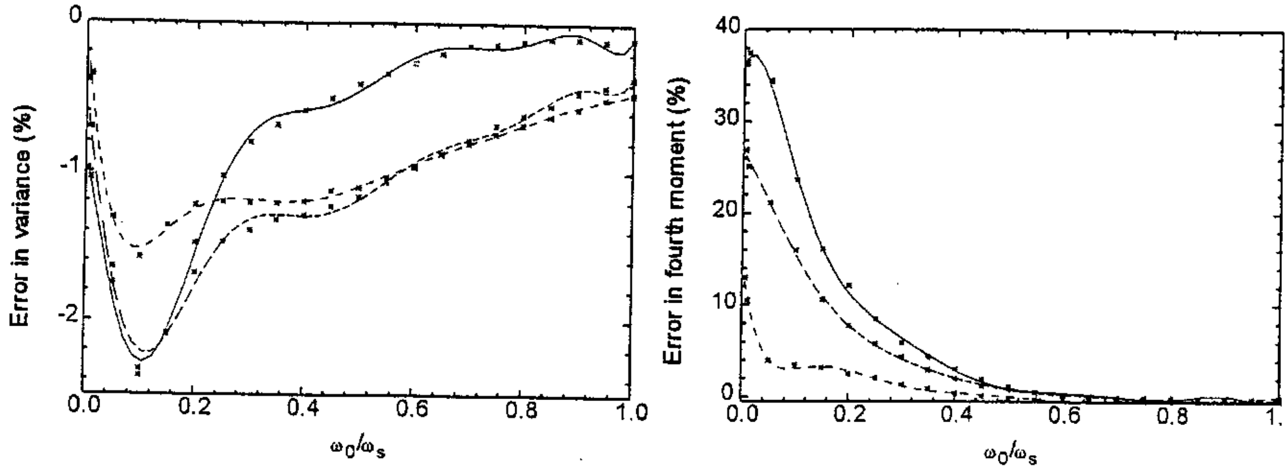

Fig. 5 Error in second (left) and fourth-order (right) response moments for third-order approximation of the drag loading for $\alpha = 1$ (see caption of Fig. 1 for legend)

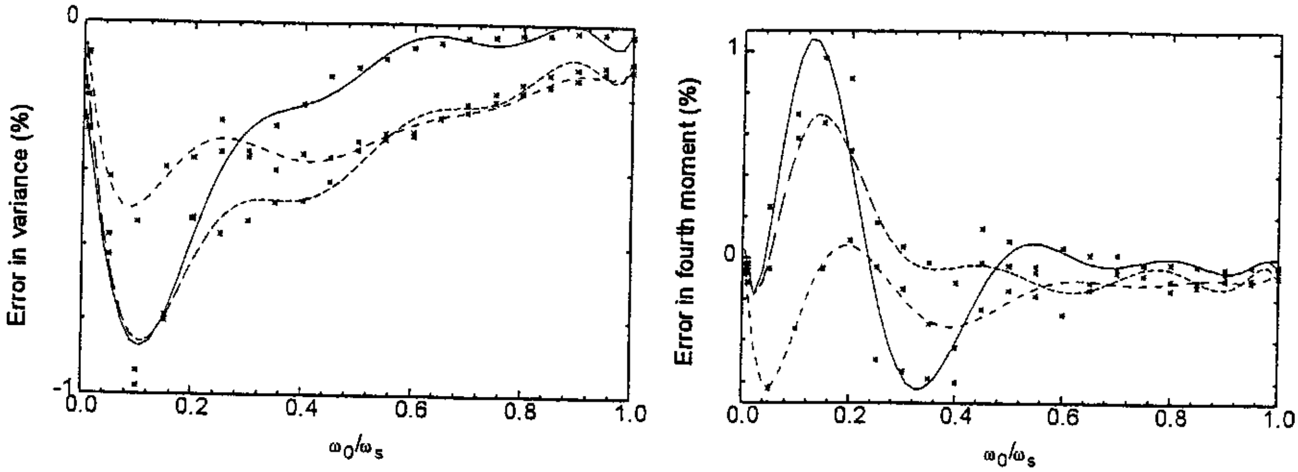

Fig. 6 Error in second (left) and fourth-order (right) response moments for fifth-order approximation of the drag loading for $\alpha = 1$ (see caption of Fig. 1 for legend)

tion may be excessively inefficient to fit response variance for low values of the frequency ratio. For example, when $\alpha = 0$, the approximation (12) strongly underestimates response variance (by 40 percent in the drag-dominated case $\beta = 0$) in the case of quasi-static excitation ($0 < \omega_0/\omega_s < \frac{1}{3}$). Results reported in Bouyssy and Rackwitz (1994) showed that Bolotin's linearization scheme also underestimates the variance for quasi-static excitations, but yields exact variance estimates in the ideal static case. More generally, as already reported by Hu et al. (1991), it is found that for $\alpha = 0$, linearization procedures strongly

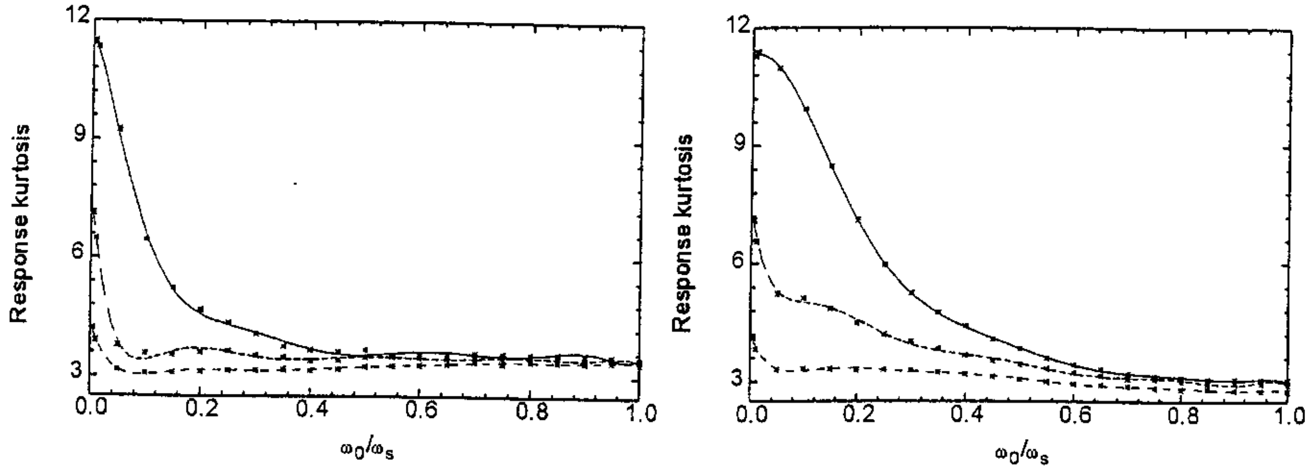

Fig. 7 Average kurtosis of the "exact" response versus frequency ratio for $\alpha = 0$ (left) and $\alpha = 1$ (right) (see caption of Fig. 1 for legend)

underestimate response variance at the subharmonic $\frac{1}{5}$, and not at the subharmonic $\frac{1}{3}$ as usually expected (e.g., Eatock Taylor and Rajagopalan, 1982).

As the water-structure interaction increases, variance predictions improve substantially for quasi-static cases (see Fig. 7). At the same time, the fit of fourth-order response moment remains poor. From these results or the evolution of the response kurtosis with ω_0/ω_s shown in Fig. 7, it is concluded that an increased non-Gaussianity of the response results in quasi-static cases when the structural motion is accounted for in the Morison loading. This phenomenon was already noticed by Manuel and Cornell (1993).

• Results reported in Figs. 2 and 5 indicate that the response variance still is underestimated when a third-order approximation of the drag loading is used. The fit, however, appears sufficiently satisfying as the relative error does not exceed 4 percent in the drag-dominated case. On the other hand, the estimates of the fourth-order response moments remain inaccurate in quasi-static cases. As noticed for the first-order approximation, it appears that the fit of response moments improves for increasing hydrodynamic interaction. For low values of the frequency ratio ω_0/ω_s and of the hydrodynamic ratio β, it seems that more terms need to be included in the approximation of the drag loading in order to obtain accurate fourth-order response moments.

• Results obtained with a fifth-order approximation of the Morison loading are accurate for any kind of excitation (see Figs. 3 and 6). The improvement in fourth-order moment predictions is impressive, and even a better fit of the response moments is achieved as α increases. Some oscillations are observed in the evolution of the error in fourth-order response moment as a function of ω_0/ω_s. This indicates that more than 150 samples would be necessary to reach a low cov in the first four moments of the response to fifth-order viscous forces. This confirms that it is excessively time-consuming to estimate response moments by simulations.

Conclusions

To the authors' knowledge, this is the first time that the ability of low-order approximations of the drag loading to predict first four response moments is investigated. In a sense, this completes the study of the drag load nonlinearity performed by Gudmestad and Connor (1983). From an extensive numerical analysis, it is concluded that:

- A linear approximation is not capable to accurately predict response moments in quasi-static cases.
- A cubic approximation of the Morison loading yields accurate estimates of the response variance for any kind of excitation. Fourth-order moments of the response, however, cannot be predicted accurately in drag-dominated quasi-static cases.
- A fifth-order approximation of the Morison loading yields accurate estimates of the response first four moments for any kind of excitation.

The aforementioned analytical technique to calculate the response and its first four moments become excessively complex if a fifth-order approximation of the forcing function is used. Indeed, at least a fifth-order Volterra series of the response is necessary to account for the effect of the fifth-order viscous forces. Then, the evaluation of the first four response moments involves up to 10-fold integrations and, when $\alpha \neq 0$, rough approximations of the moments up to ninth order of the relative water velocity are necessary for the evaluation of the coefficients in the drag loading approximation. Therefore, it is not sure that response moments evaluated by the approximate Volterra approach will be accurate, especially in drag-dominated quasi-static cases. As the influence of additional terms on the results cannot be quantified, the quality of the truncated Volterra expansion is unknown, and it is expected that this technique can match simulation results only in favorable cases.

Thus, and this is probably the most serious aspect of our conclusions, it is questionable whether analytical methods could provide accurate estimates of the first four response moments for all sea states (i.e., frequency ratios). It appears not worth improving the analytical tools as long as large uncertainties exist in the modeling of viscous forces. From recent studies suggesting that low-order Volterra series expansion can be superior to the Morison equation in predicting nonlinear wave forces (Worden et al., 1994), it is expected that a large part of the numerical difficulties encountered in estimating response to the nonlinear drag loading will be overcome in the future. Then, realistic fatigue and reliability analyses accounting for long-term variations in wave climate would become feasible.

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References

Bedrosian, E., and Rice, S. O., 1971, "The Output Properties of Volterra Systems (Nonlinear Systems With Memory) Driven by Harmonic and Gaussian Inputs," *Proceedings of the IEEE*, Vol. 59, pp. 1688–1707.

Borgman, L. E., 1969, "Ocean Wave Simulation for Engineering Design," ASCE *Journal of the Waterways and Harbors Division*, Vol. 95, pp. 557–583.

Bouyssy, V., and Rackwitz, R., 1994, "Approximate Non-Normal Response of Drag Dominated Jacket Structures," *Proceedings, 6th International W.G. 7.5 IFIP Conference*, Chapman and Hall.

Chakrabarti, S. K., 1987, *Hydrodynamics of Offshore Structures*, Springer Verlag, Berlin, Germany.

Donley, M. G., and Spanos, P. D., 1990, *Dynamic Analysis of Non-Linear Structures by the Method of Statistical Quadratization*, Springer Verlag, Berlin, Germany.

Eatock Taylor, R., and Rajagopalan, A., 1982, "Dynamics of Offshore Structures, Part I: Perturbation Analysis," *Journal of Sound and Vibration*, Vol. 83, pp. 401–416.

Grigoriu, M., and Ariaratnam, S. T., 1988, "Response of Linear Systems to Polynomials of Gaussian Processes," ASME *Journal of Applied Mechanics*, Vol. 55, pp. 905–910.

Gudmestad, O. T., and Connor, J. J., 1983, "Linearization Methods and the Influence of Current on the Nonlinear Hydrodynamic Drag Force," *Applied Ocean Research*, Vol. 5, pp. 184–194.

Hu, S.-L. J., Tsiatas, G., and McGrath, J. E., 1991, "Optimal Linearization of Morison-Type Wave Loading," ASCE *Journal of Engineering Mechanics*, Vol. 117, pp. 1537–1553.

Kotulski, Z., and Sobczyk, K., 1981, "Linear Systems and Normality," *Journal of Statistical Physics*, Vol. 24, pp. 359–373.

Manuel, L., and Cornell, C. A., 1993, "Sensitivity of the Dynamic Response of a Jack-Up Rig to Support Modelling and Morison Force Modelling Assumptions," *Proceedings, 12th International Conference on Offshore Mechanics and Arctic Engineering*, Vol. 2, pp. 243–250.

Naess, A., 1985, "Statistical Analysis of Second-Order Response of Marine Structures," *Journal of Ship Research*, Vol. 29, pp. 270–284.

Newman, J. N., 1974, "Second Order Slowly Varying Forces in Irregular Waves," *Proceedings, International Symposium on Dynamics of Marine Vehicles and Offshore Structures in Waves*, University College London, U.K., pp. 182–186.

Olagnon, M., Prevosto, M., and Joubert, P., 1988, "Nonlinear Spectral Computation of the Dynamic Response of a Single Cylinder," ASME JOURNAL OF OFFSHORE MECHANICS AND ARCTIC ENGINEERING, Vol. 110, pp. 278–281.

Tucker, M. J., Challenor, P. G., and Carter, D. J. T., 1984, "Numerical Simulation of a Random Sea: A Common Error and Its Effect Upon Wave Group Statistics," *Applied Ocean Research*, Vol. 6, pp. 118–122.

Vassilopoulos, L. A., 1967, "The Application of Statistical Theory of Nonlinear Systems to Ship Motion Performance in Random Seas," *International Ship Building Progress*, Vol. 14, pp. 54–65.

Winterstein, S. R., 1988, "Nonlinear Vibration Models for Extremes and Fatigue," ASCE *Journal of Engineering Mechanics*, Vol. 114, pp. 1772–1790.

Winterstein, S. R., Ude, T. C., and Marthinsen, T., 1994, "Volterra Models of Ocean Structures: Extreme and Fatigue Reliability," ASCE *Journal of Engineering Mechanis*, Vol. 120, pp. 1369–1385.

Worden, K., Stansby, P. K., Tomlison, G. R., and Billings, S. A., 1994, "Identification of Nonlinear Wave Forces," *Journal of Fluids and Structures*, Vol. 8, pp. 19–71.