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On conditional sampling to assess the reliability of dynamic offshore jacket structures

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ABSTRACT: In the reliability analysis of offshore jacket structures subjected to random sea waves usually only uncertainties in the loading and local resistance are taken into account. For dynamically excited jacket structures, however, stochastic analyses indicate that the influence of uncertainties in system properties can be significant both with respect to extreme value failure and with respect to fatigue in some cases. Stochastic finite element codes in conjunction with modern reliability methods such as the first-order reliability method (FORM) or its improvements can in principle be used. However, for large models and high dimension of the uncertainty vector these methods involve considerable numerical effort in the gradient-based search of the design point. Moreover, multiple most likely failure points and highly nonlinear failure surface may be present. As an alternative, simulation methods have been proposed. Among these conditional sampling appears most suitable, because it does not require a priori knowledge on the important region(s) and can retain the advantages of FORM. The suitability of the method to provide accurate estimates of failure probabilities in as few structural analyses as possible is demonstrated in a case study representative for a number of offshore structures.

1 INTRODUCTION

The reliability methodology for linear static problems is well known and has been used in many applications. First-order reliability method (FORM) using gradient-based search of the most likely failure point has been shown to be especially efficient. In most of these studies it was found that the influence of uncertainties in the system properties, for instance in member stiffnesses, is negligible in comparison to the influence of loading and local resistance uncertainties. This result was incorrectly used to simplify also the analysis of linear dynamic reliability problems for offshore jacket structures. In most of these studies uncertainties in the loading and the resistance parameters were accounted for whereas uncertainties in structural data were neglected or grossly simplified (Flint and Baker 1977, Olufsen et al. 1989, Karadeniz and Haritos 1992). Structural data, however, may have a great influence on dynamic properties of structures. Due to larger water depth, offshore structures now become increasingly more flexible and dynamically sensitive.

In the following, we consider numerical methods to assess the reliability of Finite Element models with uncertain properties under dynamic stochastic excitation. Throughout the paper, the emphasis is on the methodological aspects and it is assumed that linear stochastic analyses are accurate enough to investigate the problem. First, uncertainties in the system properties and in the model are listed. Further, fatigue and yielding failures of cross sections of jackets are discussed. Then a short review of the available reliability methods is given. Finally, the efficiency of the methods, i.e. their capability to provide accurate estimates of failure probabilities in as few Finite Element analyses as possible, is compared in a case study representative for a number of offshore structures.

2 STOCHASTIC ANALYSIS

An offshore steel jacket subjected to random waves. In order to optimize inspections of such structure, stochastic fatigue or fracture mechanics analyses will be performed to identify the most critical cross sections. These analyses are primarily concerned with the computation of the stress distribution on the circumference of cross sections. In these analyses, the wave climate is modeled as an ergodic sequence of sea states and each sea state is characterized by the significant wave height H_s, the mean period T_z and the main wave direction Θ. During a sea state, the sea elevation is assumed to be a stationary normal process with mean zero. It is then completely characterized by its power spectral density. In order to simplify analyses, linear wave theory is used to derive wave kinematics. Since structural members are slender the wave loads are modeled by the Morison equation. For each sea state, an equation of motion relates wave loads and structural motion of the jacket structure. Unfortunately there exists no closed form for the solution to the equation of motion, because of the nonlinear drag term in the Morison loading. In practice an approximate solution of the equation of motion is obtained by replacing each drag term by a linear term. Finally, a linear interpolation with so-called stress concentration factors (SCF's) is used to approximate the nominal stress in a finite number p of points ("hot spots") around any cross section. Then, each hot spot stress is the response of a linear dynamic system excited by the wave elevation. It follows from the theory of linear dynamic systems that each hot spot stress is a normal process, too. Its power spectral density is obtained by classical spectral analysis of a FE-model of the structure. In practice, spectral analyses are simplified by using modal decomposition which requires an eigenvalue analysis of the FE-model. If wave current is accounted for, the non-zero mean value of each hot spot stress is estimated by static analysis. The procedure can be considered as standard. For more details see, e.g., (Chakrabarti 1987).

3 SYSTEM AND MODEL UNCERTAINTIES

When fatigue lives estimated by a stochastic approach are compared to inspection results, large discrepancies are observed. A part of the discrepancies between theoretical estimations and observations is due to uncertainties in the modeling of wave loads and hot spot stresses. Indeed, the Morison coefficients C_d and C_m and the SCF's computed or estimated from measured data exhibit a considerable degree of scatter. This scatter is due to statistical, measurement and numerical uncertainties and also due to the idealization of complex physical phenomena. Consequently, the scatter in model parameters is essentially random in nature. In the following these parameters are modeled as random variables.

Random fluctuations intrinsic in nature also exist in system properties. It is not clear how they affect the reliability of dynamic structures. In this study, we concentrate on the influence of uncertain system properties. Following common practice, these properties are modeled as random fields and the fields are discretized on a random mesh by some kind of preprocessor. For a review and a comparison of some discretization schemes see, e.g., (Der Kiureghian and Li 1993).

4 FAILURE CRITERIA

Assume that failure occurs when $g(\mathbf{x}) < 0$ with $g(.)$ the so-called limit state function and $\mathbf{x}$ a realization of the vector $\mathbf{X} = (X_1, \ldots, X_n)$ of basic variables describing the structure. The vector $\mathbf{X}$ contains, e.g., system properties, and is thus a random vector.

In this study we consider fatigue and yielding failures of a tubular cross-section of an offshore jacket. Failure criteria are written in terms of the hot spot stress and an individual failure criterion is formulated in each hot spot. The limit state function for the cross-section is the union of all individual ones. For the case of fatigue failure, the limit state function is given by

$$g(\mathbf{X}) = \bigcup_{i=1}^{p} \Big(\log K - \log\left(\nu_{0,i}\, t\, \lambda_{0,i}^{m/2}\, (2\sqrt{2})^m\, \Gamma\left(1+\tfrac{m}{2}\right)\right)\Big) \qquad (1)$$

where, for simplicity of illustration, the damage indicator derived by Miles (1954) and a simple SN-curve are used. For the yielding failure criterion, one obtains

$$g(\mathbf{X}) = \bigcup_{i=1}^{p} (R - S_i) \qquad (2)$$

where S_i is distributed according to $F_{S_i}(s) = \exp\left(-\nu_{0,i} t e^{-\frac{1}{2}s^2/\lambda_{0,i}}\right)$ using Rice's formula. In equations (1) and (2) t is the service time; $\log K$ and m denote parameters in the SN-curve; R denotes the yield stress; $\nu_{0,i}$ and $\lambda_{0,i}$ are the zero upcrossing rate and the zero-th spectral moment of the hot spot stress S_i. It is here assumed that the structure experiences only one sea state. In practice several sea states need to be considered and the expectation of the reliability estimate over all

sea state parameters yields the reliability measure of interest.

5 RELIABILITY METHODS

Over the last twenty years, several schemes were developed to estimate the probability P_f that a realization of the random vector $\mathbf{X}$ of basic variables enters the failure domain $\{\mathbf{x} \mid \bigcup_{i=1}^{p} g_i(\mathbf{x}) < 0\}$

$$P_f = \int_{\{\mathbf{x} \mid \bigcup_{i=1}^{p} g_i(\mathbf{x})<0\}} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \qquad (3)$$

with $f_{\mathbf{X}}(.)$ the probability density of $\mathbf{X}$. Given the probabilities $P[g_i(\mathbf{X}) < 0]$ of failure in the individual modes, simple bounds on P_f are

$$\max_{1\le i\le p} P[g_i(\mathbf{X}) < 0] \le P_f \le \sum_{i=1}^{p} P[g_i(\mathbf{X}) < 0] \qquad (4)$$

Closer bounds can be obtained using joint failure probabilities but equation (4) is sufficiently accurate for most practical problems and is also used herein.

In the following we shortly discuss the advantages and shortcomings of gradient-based reliability methods and simulation methods which can be used to estimate the failure probability.

5.1 *FORM and its improvements*

Each individual failure probability $P[g_i(\mathbf{X}) < 0]$ may be estimated by FORM, requiring primarily schemes for the calculation of gradients of the individual limit state function with respect to the basic random variables X_j ($j = 1, \ldots, n$). Igusa and Der Kiureghian (1988) showed how gradients can be computed analytically for simple linear dynamic problems. It leads to combinations of stochastic finite element codes and extended FORM tools. For large problems, however, it is found that these extensions result in excessively large computer codes and time consuming computations. Indeed, it is found that taking simple numerical or semi-analytical gradients is superior to analytical methods provided that the accuracy with which the derivatives can be determined is sufficient. It is clear, however, that these methods are efficient if and only if the failure surface possesses a unique most likely failure point (β-point) and if the failure surface is relatively smooth. If these conditions are fulfilled but the nonlinearity of the failure surface is non-negligible, improvements of FORM estimates can be necessary, using second order method (SORM) and/or importance sampling (Hohenbichler et al. 1987, Hohenbichler and Rackwitz 1988).

5.2 *Conditional sampling*

Simulation schemes are believed to be a potential alternative to gradient-based reliability methods especially if certain variance reduction techniques are used (Rubinstein 1981). Simulation schemes, no doubt, are less elegant than gradient-based methods but they are easy to implement and they provide estimates of the failure probability together with accuracy measures. Further, they do not imply separate analysis of each individual failure mode. In view of the problems with rigorous FORM such sampling methods become especially attractive and it is to be tested whether they are also suitable for large real structures.

In the following with consider the method of conditional expectation, which seems adequate for reliability problems where $\mathbf{X}$ is high dimensional. This method does not suffer from any of the aforementioned shortcomings if it is properly implemented as it can combine rigorous FORM/SORM with simulation (Melchers 1991, Ayyub and Chia 1992). With this method, the vector $\mathbf{X}$ is divided into two subsets $\mathbf{Y}$ and $\mathbf{Z}$. Then the failure probability P_f is calculated as

$$P_f = \int_{\mathbf{Y}} P_{f|\mathbf{Y}=\mathbf{y}} f_{\mathbf{Y}}(\mathbf{y}) d\mathbf{y} \qquad (5)$$

with

$$P_{f|\mathbf{Y}=\mathbf{y}} = \int_{\{\mathbf{z} \mid \bigcup_{i=1}^{p} g_i(\mathbf{z},\mathbf{Y}=\mathbf{y})<0\}} f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z} \qquad (6)$$

The subsets $\mathbf{Y}$ and $\mathbf{Z}$ are respectively termed the generated and the controlling set. An unbiased estimate of P_f is obtained by sampling outcomes $\mathbf{y}$ of $\mathbf{Y}$ and calculating the conditional failure probabilities $P_{f|\mathbf{Y}=\mathbf{y}}$. In practice, samples are generated until the coefficient of variation of the estimate drops down to a target value, say 30% (Melchers 1991).

Conditional sampling is particularly effective if the conditional probability of failure $P_{f|\mathbf{Y}=\mathbf{y}}$ can easily and accurately be calculated. This is for instance the case when the subset $\mathbf{Z}$ is chosen such that the failure surface possesses a unique most likely failure point in the $\mathbf{z}$-space. Any conditional failure probability may then be calculated by FORM or its improvements. However, if the generated **y**-vectors have a great influence on $P_{f|\mathbf{Y}=\mathbf{y}}$ or if the c.o.v.s of its components are not too small a great number of samples may be necessary to reach the target c.o.v. Therefore, for improvements on convergence it may be necessary to include parts of the set **Y** in **Z**.

The separation $(\mathbf{Y}, \mathbf{Z})$ usually is dictated by the mechanical context. For instance, in our application it is important to reduce FE-analyses as much as possible. From the mean number of state function calls required in a typical FORM analysis, it is chosen to reduce the subset **Z** to R and $\log K$. All other properties, i.e. structural data, hydrodynamic coefficients and SCF's are regrouped in **Y** and generated by random sampling. Then, on the one hand, any conditional failure probability of individual failure given $\mathbf{Y} = \mathbf{y}$ is exactly calculated with FORM but, on the other hand, the simulation scheme may be slow in convergence. It may be improved by using the fact that the knowledge of the important region(s) increases during simulation. This improvement can for instance be achieved with so-called adaptive schemes (Karamchandani and Cornell 1991). However, it was noticed in many problems where little is known a priori on the important region(s) that the efficiency of adaptive schemes depends strongly on the starting conditions if **Y** is high dimensional (see, e.g., Bouyssy and Rackwitz 1994). Indeed, in many cases there exists a non-negligible bias in the c.o.v. of the failure probability estimate and hence this coefficient should not be used to measure estimate accuracy (Walsh, 1956). Therefore, adaptive schemes can hardly be recommended in real practical problems.

6 CASE STUDY

The efficiency of the aforementioned methods was assessed by analyzing a realistic jacket structure. The example jacket structure is 60 m high. The deck is 20 m above the mean water level. A sketch of the structure is given in Figure 1. The computer model consists of 30 beam elements and 22 nodes, resulting in 132 degrees of freedom. The first two eigenperiods are around 2 s. The midpoint method is used to discretize the random meshes modeling structural properties. The FE-mesh is used as random mesh. Since each structural member is discretized with only one beam element, the random variables describing the Young's moduli and the mass densities of the elements are uncorrelated.

The stochastic model is shown in Table 1. It yields $n = 97$ random variables if the limit state function (1) or (2) is used.

The dynamic and spectral analyses of the structure were performed with the Finite Element code NASCOM (1994) and an improved version of the code SAPOS originally written by Karadeniz (1989). All reliability computations were performed with COMREL (1994).

As was already reported (Bouyssy and Rackwitz 1994), it was found by conditional sampling that the influence of system uncertainties on fatigue failure is negligible. The reliability against fatigue failure depends primarily on the service time t and the local resistance parameters $\log K$ and m.

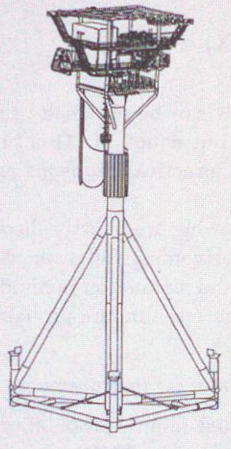

Figure 1: Example structure

Table 1: Stochastic model

Variable	Distribution	Mean value	C.o.V
Young's modulus	Normal	2.1e+11 MPa	0.05
Mass density	Normal	7800 kg/m^3	0.05
Deck load	Normal	630 tons	0.05
Morison C_d	Uniform	1.25	0.25
Morison C_m	Uniform	1.50	0.19
SCF for N	Normal	2.00	0.10
SCF for IPB	Normal	2.00	0.10
SCF for OPB	Normal	2.00	0.10
Yield stress R	Normal	360 MPa	0.10
SN-curve $\log K$	Normal	12	0.20
SN-curve m	Constant	4	-
service time t	Constant	10 years	-

Table 2: Results obtained by FORM, by FORM with importance sampling and by conditional sampling for a cross section (* analysis interrupted after 5000 FE-analyses)

	FORM		FORM+Importance Sampling		Conditional Sampling	
	P_f	FE - analyses	P_f	FE - analyses	P_f	FE - analyses
$P[g_2 < 0]$	$1.64 \cdot 10^{-6}$	1095	$1.02 \cdot 10^{-6}$	1150	$2.90 \cdot 10^{-8}$	888
			$9.43 \cdot 10^{-7}$	1150	$3.28 \cdot 10^{-7}$	*
			$6.73 \cdot 10^{-7}$	1150	$5.78 \cdot 10^{-7}$	*
$P[g_4 < 0]$	$1.64 \cdot 10^{-6}$	1095	$1.03 \cdot 10^{-6}$	1150	$1.40 \cdot 10^{-7}$	3014
			$1.09 \cdot 10^{-6}$	1148	$2.79 \cdot 10^{-7}$	*
			$6.86 \cdot 10^{-7}$	1156	$7.49 \cdot 10^{-7}$	*
$\max_{1 \le i \le 8} P[g_i < 0]$	$4.43 \cdot 10^{-3}$	796	$3.10 \cdot 10^{-3}$	851	$8.75 \cdot 10^{-4}$	147
			$3.13 \cdot 10^{-3}$	851	$2.19 \cdot 10^{-3}$	628
			$2.47 \cdot 10^{-3}$	855	$4.82 \cdot 10^{-3}$	673
$\sum_{i=1}^{8} P[g_i < 0]$	$8.59 \cdot 10^{-3}$	9356	$6.02 \cdot 10^{-3}$	9810	$3.63 \cdot 10^{-3}$	285
			$6.08 \cdot 10^{-3}$	9790	$4.16 \cdot 10^{-3}$	267
			$4.79 \cdot 10^{-3}$	9838	$6.22 \cdot 10^{-3}$	557

A different conclusion is drawn for reliability against yielding failure. For the extreme sea state $T_z = 2$ s, $H_s = 6$ m, failure probabilities estimated by conditional sampling for a cross section on the central column are reported in Table 2. The number of FE-analyses performed is shown, too. Estimates were obtained by conditional sampling with three different starting seeds. The target coefficient of variation was 30%. Table 2 reports also estimates obtained by FORM and corrections of these estimates with importance sampling, as suggested by Hohenbichler and Rackwitz (1988). The bounds on the failure probability are $(7.52 \cdot 10^{-8}, 1.38 \cdot 10^{-7})$ when the variability of model parameters and system properties is neglected. Finally, the algorithm of FORM provides omission sensitivity factors for each failure mode (Madsen 1985). These factors are a measure of the change in each FORM-estimate when the variability of one or some basic variables is neglected. The omission sensitivity factors are shown in Table 3. The columns of the table are ordered by decreasing failure probability. For simplicity, the omission sensitivity factors of all member Young's moduli, mass densities and Morison coefficients are reported. Sensitivity factors were also obtained by repeated conditional sampling. Figure 2 shows the effect of changes in the standard deviation of random basic variables on the upper bound on P_f. These sensitivity measurements are in good agree-

Table 3: Omission sensitivity factors

Random variables	g_3, g_7	g_2, g_4, g_6, g_8	g_1, g_5
Young's moduli	1.01	1.01	1.27
Mass densities	0.00	0.00	0.00
Deck load	1.05	1.06	1.04
Morison C_d's	1.10	1.05	1.02
Morison C_m's	1.07	1.04	1.02
SCF for N	0.00	0.00	0.00
SCF for IPB	1.15	1.15	0.00
SCF for OPB	0.00	0.00	1.08
Yield stress R	1.19	1.27	1.15

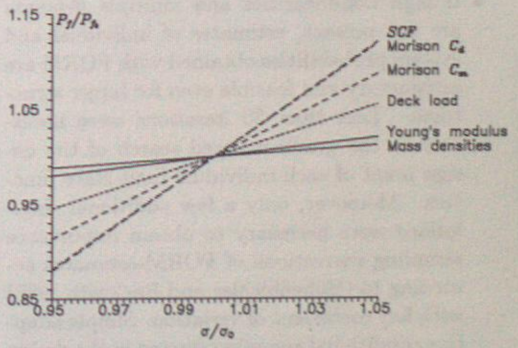

Figure 2: Sensitivity factors obtained by repeated conditional sampling

ment with the omission sensitivity factors obtained for the modes with largest failure probability.

7 CONCLUSIONS

Some conclusions can be drawn from Tables 2-3 and Figure 2 regarding the effect of uncertainties in system properties on the reliability of the example cross section. First of all, it is noticed that the influence of uncertainties in structural data is large on few individual yielding failure modes but minor on the bounds (4) on P_f. A partial explanation for this result is that small changes in structural data have a substantial effect on eigenforms but not on eigenfrequencies. Since only one severe sea state with unidirectional waves is considered, it follows that the hot spot stress distribution on the cross section is considerably modified but not the hot spot stress amplitude. Then large changes are observed in some individual failure probabilities whereas the bounds on P_f remain nearly constant. Similar results are expected if finer FE- and random meshes are used or if the randomness of properties is more accurately modeled. In fact, it appears that in reliability analyses of the example cross section uncertainties in structural data can be neglected. The results further show that uncertainties in the deck load and in the modeling of wave loading have a comparable effect on yielding failure — in agreement with other studies reported in the literature and another example structure (Bouyssy and Rackwitz 1994).

Regarding the methods it can be concluded from Table 2 and other results reported in (Bouyssy and Rackwitz 1994) that

- If high nonlinearities and multiple β-points are not present, estimates of individual and system probabilities obtained with FORM are satisfactory and feasible even for larger structures. Less than 20 iterations were necessary in the gradient-based search of the design point of each individual limit state function. Moreover, only a few additional simulations were necessary to obtain importance sampling corrections of FORM-estimates according to (Hohenbichler and Rackwitz 1988) with low coefficient of variation. Simple adaptive conditional sampling started in the design point was also found very efficient. However, the number of FE-analyses performed during the gradient-based search of the design point is large. Therefore, the method is time consuming, especially if the upper bound on the failure probability of the series system needs to be estimated.
- The method of conditional expectation is suitable if and only if the conditional failure probability is not too sensitive with respect to the generated variables. Conditional sampling appears to be slow in convergence when estimating some individual failure probabilities sensitive to the generated set. Then more FE-analyses than with FORM may be required. This case was for instance observed with all individual limit state functions except for $g_3(.)$ and $g_7(.)$ which have largest failure probabilities. Conditional sampling turned to be extremely efficient when several hot spots are investigated simultaneously. For instance, for the upper bound (4) on P_f the target c.o.v. was always reached with less than 600 samples. No gradient-based reliability method can provide an upper bound on P_f with such few FE-analyses.
- The studied example as well as other simpler ones and more complex ones clearly indicate that conditional sampling methods can be unreliable, especially if adaptive schemes are implemented. Their efficiency depends strongly on the starting conditions for the simulations if the dimension of the simulated vector is high. That conditional sampling schemes with or without adaptive sampling densities can be unreliable is most disturbing. It must therefore by recommended that such analyses are repeated several times in order to assess their validity.

Therefore, it seems that conditional sampling can be a valid alternative but it can also be rather inefficient for practical reliability calculations. First-order reliability methods (FORM) with gradient-based search of the design point appear more robust although time consuming. In this sense, our findings are somewhat inconclusive. For much larger structures one must expect excessively more numerical effort with rigorous FORM. Conditional sampling schemes may require not much more FE-analyses than for the example in Table 2, but the results will also be increasingly less reliable. So far the authors conclusion is that conditional sampling schemes, in part so fiercely advocated in

the literature, are valid alternatives only if good prior knowledge of the uncertainty structure and the dominating failure modes is available, so that the sampling density can be set appropriately. If this is not the case large numerical effort is necessary. These conclusions are considered to hold even stronger if uncertain system properties are larger than in the considered example.

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