Reliability and Risk Function for Deteriorated Structures

Fiabilité et risque des structures endommagées

Zuverlässigkeit und Risikofunktion geschädigter Bauteile

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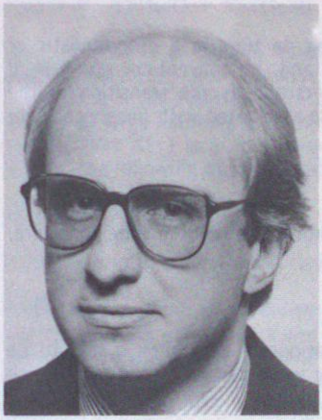

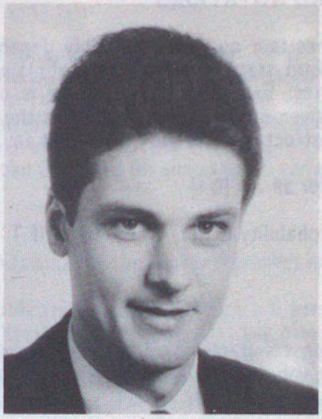

SUMMARY

Some important stochastic degradation models are reviewed and the methods for determining relevant reliability characteristics are given. The concepts for updating reliability characteristics by inspection results are reviewed. Two exemples, one for carbonation of concrete and subsequent spalling of the concrete cover due to corrosion and another for load-induced fatigue illustrate the methodology.

RÉSUMÉ

Quelques modèles d'endommagement sont rappelés et les méthodes de détermination des caractéristiques stochastiques importantes sont présentées. Les principales idées concernant l'utilisation des résultats d'inspection lors de l'analyse de fiabilité sont rappelées. Deux exemples illustrent la méthologie: l'un sur la carbonisation du béton avec prise en compte de la détérioration de la surface par corrosion; l'autre concernant la fatigue induite par des charges.

ZUSAMMENFASSUNG

Einige wichtige Schädigungsmodelle und Methoden zur Festlegung massgebender Zuverlässigkeitscharakteristiken werden dargestellt. Ein Konzept zur Berücksichtigung von Inspektionsergebnissen im Rahmen einer Zuverlässigkeitsanalyse wird erläutert. Anhand je eines Beispiels für die Karbonatisierung von Beton mit nachfolgendem Abplatzen der Betondeckung infolge Korrosion und für lastinduzierte Ermüdung wird die Methodik veranschaulicht.

1. INTRODUCTION

Explicit consideration of durability aspects of building structures is still a non–classical task of engineering, especially in a probabilistic context. Neither is the understanding of the various physical and/or chemical degradation phenomena as developed as, for example, structural mechanics nor can the classical design concepts for mechanical adverse performance states be directly applied. In fact, inspection and maintenance are integral parts of the means to achieve durability. In the following some basic terminology and notions are given first. Then a flexible model for damage accumulation and computational tools for the treatment of deteriorating components are presented with due consideration of inspection and repair. Special emphasis is given to the calculation and interpretation of the risk function.

2. BASIC TERMINOLOGY AND NOTATIONS

Let $\mathbf{X}(t)$ be the vector of uncertain quantities possibly depending on time in a deterministic or stochastic manner. Then, a limit state is defined as $g(\mathbf{x}(\tau)) = 0$ and, by convention, $g(\mathbf{x}(\tau)) \leq 0$ defines the set of failure states. Exceeding a limit state is understood as the transition of the structure into a state with a given utility loss , for example the loss associated with unserviceability or structural collapse. Hence, structural reliability is defined as:

$$R(t) = P(g(\mathbf{X}(\tau)) > 0) \quad \text{for all } \tau \in [0,t] \tag{1}$$

The time dependent failure probability is $F(t) = 1 - R(t)$. If T denotes the random time to failure, an equivalent formulation is

$$R(t) = P(T > t) \tag{2}$$

and this is the formulation most suitable for durability considerations. A structure is said to be reliable if $R(t)$ exceeds a given value $R_0(t)$. Alternatively, a limiting value can be placed on the risk or hazard function defined as:

$$\rho(\tau) = \frac{f(\tau)}{R(\tau)} \quad \text{or by} \quad R(t) = \exp[-\int_0^t \rho(\tau)\, d\tau] \tag{3}$$

Here, $f(\tau)$ is the probability density of the time to failure. $\rho(\tau)$, if multiplied by a time interval $\Delta\tau$, obviously is the failure probability related to that time interval and to the "population" of structures still existing at time τ or the interval failure probability (failure rate) conditional on the event that the structure has survived up to τ.

3. A FLEXIBLE DAMAGE ACCUMULATION MODEL

A special though flexible type of failure model is when a "demand" process causes "capacity" reductions whose magnitude typically depends on the magnitude of the demand process. These capacity reductions accumulate. Failure occurs when the total capacity reduction exceeds some preselected value or if the demand exceeds the capacity. Abrasion of the pavement on roads due to passing vehicles or the development of cracks in vessels due to variable stresses are typical examples. The simplest but practically important formulation is [1]:

$$\frac{dZ(t)}{dt} = f(Z(t), X(t)) \tag{4}$$

where $Z(t)$ is some damage indicator and $X(t)$ the demand process. Obviously, the damage increment per time unit is proportional to a function of the total damage at time t and the demand at that time. If, in particular,

$$\frac{dZ(t)}{dt} = g(Z(t))\, h(X(t)) \tag{5}$$

the differential equation can be separated and integrated

$$\int_{Z(t_0)}^{Z(t)} \frac{dz(\tau)}{g(z(\tau))} = \int_{t_0}^{t} h(X(\tau))d\tau \tag{6}$$

$$\Psi(Z(t)) - \Psi(Z(t_0)) = Y(t_0,t) \tag{7}$$

from which

$$Z(t) = \Psi^{-1}[Y(t_0,t) + \Psi(Z(t_0))] \tag{8}$$

Here, $Y(t_0,t)$ is a random variable obtained by integration of the random process $h(X(\tau))$. If $h(X(\tau))$ is strictly non–negative, the damage indicator is monotonically increasing. Of central importance is the additive character of the right–hand side of eq. (6) as it allows the application of the law of large numbers and even the central limit theorem under certain conditions. For example, assume that $X(\tau)$ is a stationary and ergodic process and $h(X(\tau))$ has finite variance. Then, for large t the following approximation can be found for the random variable $Y(t) = Y(t_0,t)$:

$$Y(t) \approx E[h(X(\tau))]\,(t-t_0) \tag{9}$$

In this asymptotic version the time–variation of the demand process is no more present.

There are a number of prominent applications a few of which are presented below with $y(0) = 0$. For example, let $g(Z(t)) = 1$ and $h(X(t)) = X(t)$ where $X(t)$ has mean μ and a covariance function described by the variance σ^2 and the correlation length τ_0. Then, $Z(t)$ is a Gaussian process with mean $t\,\mu$ and variance $t\,\tau_0\,\sigma^2$. It is clear that this model is suitable for the abrasion of a road pavement in time. Also, the corrosion depth of steel surfaces in splash zones can be described with this model. In both cases μ and σ may also be random functions of spatial coordinates. Next, let $g(Z(t)) = Z^m(t)$ and $h(X(t)) = X^n(t) = X^{m/2}(t)$. One finds in making use of eq. (9):

$$\ln(Z(t)) - \ln(Z(t_0)) \approx \mu_{X^n} t \quad \text{for } m = 1 \tag{10a}$$

$$\frac{1}{1-m}(Z^{1-m}(t) + Z^{1-m}(t_0)) \approx \mu_{X^n} t \quad \text{for } m = 2,3,. \tag{10b}$$

If one now interprets the function $Z(t)$ as crack length and $X(t)$ as the effective stress range we have, apart from some constants, precisely the formula for Paris–Erdogan's crack propagation law. For $m = 2$, $Z(t)$ has a lognormal distribution. Further, let $g(Z(t)) = C/Z(t)$ and $h(X(t)) = X(t)$. One determines:

$$\frac{Z^2(t)}{2C} \approx \mu t \tag{11}$$

If, on the other hand, $g(Z(t)) = (C_1/Z(t)+C_2)$ and $h(X(t)) = X(t)$, then:

$$\frac{Z(t)}{C_2} - \frac{C_1}{C_2^2}\ln(1 + C_2\, Z(t)/C_1) \approx \mu t \tag{12}$$

Inspection shows that the last two results describe the carbonation depth of concrete after continuous attack of carbon dioxide from the concrete surface according to [2] and [3] with $X(t)$ the randomly varying humidity of the outer concrete layer which changes the diffusion "constant" accordingly. Both models appear to have certain physical deficiencies but it is out of the scope of this paper to discuss those.

More general models can be generated by solving less specialized stochastic differential equations but we can not pursue this any further. Experience shows that it frequently is not the randomness of the time–variant demand process but the (time–invariant) uncertainty in the parameters in these equations, at least if t can be considered as large. Therefore, it is admissible to ignore the variability of the right–hand side of the equations in many cases.

4. FAILURE CRITERIA AND FAILURE EVENTS

The computation of R(t) under sufficient general conditions for the process $\mathbf{X}(t)$ and the shape of g(.) is by no means trivial and considerably more involved than simple time–invariant reliability problems. The same is true for the risk function. The state function most frequently is formulated in the so–called damage indicator space but it is also possible and sometimes necessary to use other formulation spaces. If damage accumulation is strictly positive and the damage indicator formulation is chosen one has to solve:

$$R(t) = P(T \leq t) = P(g^{-1}(\mathbf{X}(t);Z(t)) - t \leq 0) \tag{13}$$

Application of FORM/SORM [4] yields

$$R(t) \sim \Phi(\beta_E(t)) \tag{14}$$

where $\beta_E(t)$ is the so–called equivalent safety index defined by $\Phi(-\beta_E(t)) = P(\mathbf{X}(t) \in V)$ where V is the failure domain and Φ is the standard normal distribution function. The risk function can be determined by:

$$\rho(t) = -\frac{\varphi(\beta_E(t))}{\Phi(\beta_E(t))}\frac{\partial\beta_E(t)}{\partial(t)} \sim -\frac{\varphi(\beta(t))}{\Phi(\beta(t))}\frac{\partial\beta(t)}{\partial(t)} \tag{15}$$

The last derivative term is nothing else than the so–called parametric sensitivity factor available in most FORM/SORM computation schemes [5]. φ is the standard normal density.

The reliability calculation is much more involved if the failure criteria cannot be formulated in the damage indicator space. A typical example is failure due to instable crack propagation. Changing notations to the ones usual in this area and assuming linear–elastic fracture mechanics a crack grows "stable" as long as there is $K_{IC} > K(\tau) = C\, S(\tau)\, \sqrt{(\pi a(\tau))}$ with K_{IC} the fracture toughness, $S(\tau)$ the far field stress in the component and $a(\tau)$ the actual crack length which grows proportional to the effective stress ranges $\Delta S(\tau)$ raised to the power of m according to eq. (10). C and m are material constants. It is clear that failure, i.e. crack instability can also occur when $a(\tau)$ is still moderate but $S(\tau)$ is large. The difficulty lies in the fact that one is not interested that the component is in a failure state at some time but in the event when this occurs for the first time. Unfortunately, very few solutions exist for this problem and those are widely of asymptotic nature. A relatively general method is the so–called outcrossing approach for which certain regularity conditions concerning the disturbance and the damage accumulation process must be assumed. Let

$$\nu^+(\tau) = \lim_{\vartheta \to 0} 1/\vartheta\, P(\{g(X(\tau),Z(\tau),\mathbf{q}) > 0\} \cap \{g(X(\tau+\vartheta),Z(\tau+\vartheta),\mathbf{q}) \leq 0\}) \tag{16}$$

be the outcrossing rate with $g(X(\tau),Z(\tau),\mathbf{q})$ the structural state function and $\mathbf{q}$ an uncertain time–invariant parameter vector. If the disturbance process is sufficiently mixing, i.e. becomes independent for two times τ and $\tau + \vartheta$ when $\vartheta \to \infty$, the reliability function can be shown to be:

$$R(t|\mathbf{q}) \sim \exp[-\int_0^t \nu^+(\tau|\mathbf{q})\, d\tau] \tag{17}$$

For the technical details of the calculation of the outcrossing rate we must refer to the literature [6].

5. UPDATING BY INSPECTION OBSERVATIONS

The above failure models are as mentioned distinct from the failure in classical reliability as they directly adhere to the physical damage accumulation process. For the estimation of their parameters not only failure times can be used but also measurable damage indicators and the disturbance (loading) parameters as well as material parameters which frequently can be measured independent of the damage state of the component. This enables reliability updating after inspection by use of Bayes' theorem. Let t_1 be the first inspection time and denote by B the set of observations collected up to and during inspection. Then, the updated reliability is:

$$R(t|t_1, B, \ldots) = \frac{P(\{T > t\} \cap \{T > t_1\} \cap B)}{P(\{T > t_1\} \cap B)} \tag{18}$$

B contains events of the type $\{X(t_1) \leq \hat{x}(t_1) + \epsilon\}$ or $\{Q \leq \hat{q} + \delta\}$, where $\hat{x}$ and $\hat{q}$ are the observations and ϵ and δ the corresponding measurement errors (error vectors). Again FORM/SORM techniques facilitate numerical calculations [7].

6. EXAMPLES FOR RELIABILITY AND RISK FUNCTIONS

As a first approach the time–variant carbonation process according to eq. (12) with constants $C_1 = b_s/a$ and $C_2 = D_{A \cdot B}\, c_o/a$ is studied where $D_{A \cdot B}$ is the diffusion coefficient of carbon dioxide for concrete, c_o the concentration of carbon dioxide in the air, a the amount of carbon dioxide for complete carbonation and b_s a parameter which collects the retarding effects. $D_{B \cdot A}$ and b_s are taken as uncertain with given distributions. With the exception of c_o the parameters can be related to concrete strength and the specific exposure conditions. The limit state function is formulated according to eq. (13) by assuming that regional carbonation is a necessary condition for longitudinal cracks and subsequent spalling of the concrete cover due to corrosion. Failure is assumed to occur when a certain percentage α of the reinforcement is reached by the carbonation front. In the following α is chosen to be 0.3. Furthermore, concrete cover and a model uncertainty parameter are considered as random variables [8].

Fig. 1 shows results of the reliability calculations. The risk function $\rho(t)$ is given for a concrete C15 under outdoor conditions but not subjected to rain with cover of 25 mm and 30 mm respectively . The dotted line represents the probability of failure $P_F(t)$. It is seen that up to a certain time the risk function is essentially zero. At this time the carbonation front reaches the reinforcement and failure is most probable. Beyond this time the risk function decreases reflecting the fact that the carbonation front has not reached the reinforcement before for a reduced population. Therefore inspections are most effective if they are performed just before this "discontinuity point". It follows that the planning of inspections must be affected by the characteristics of the risk function. Further on the quantification of the actual degradation state is of special importance. As visual inspections rarely are reliable sampling strategies should be developed on the basis of an optimization of the amount and the timing of inspections.

If structural components experience cumulative damage due to fatigue they have to be inspected and if necessary repaired. The risk function shows a somewhat similar behavior as shown in figure 2 which is based on Paris–Erdogan's crack propagation law and the crack instability criterion mentioned just below eq. (15). Again it is first increasing and then moderately decreasing beyond a certain point in time. It is worth noting that cost considerations specify about the same time as the optimal first inspection time (see [9]). The inspection results can be used to update the knowledge about the structural state resulting in new risk and failure probability functions (dashed lines). In the example the observed crack length was larger than estimated a priori which results in a more rapid increase of both functions. However, at this optimal inspection time the risk function and failure probability have reached rather large values (i.e. $P_F(t_1) \approx 0.3$) which may be considered as to high so that earlier inspections might be required for safety reasons. It thus is shown that both the risk and the failure probability function provide the necessary information for planning inspection times and possible maintenance actions.

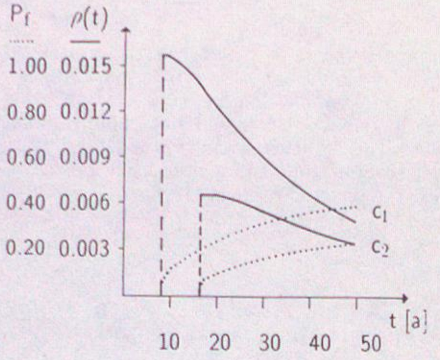

Figure 1: Hazard rate and failure probability for carbonation of concrete C15, cover c_1= 25 mm and c_2= 30 mm

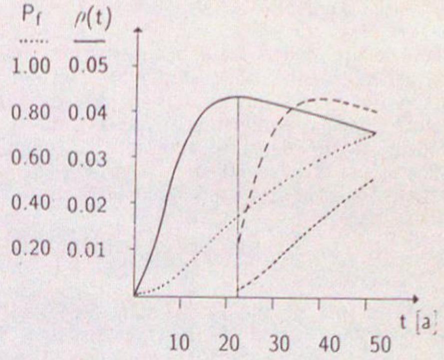

Figure 2: Hazard rate and probability of fatigue failure of a component in a steel structure, reliability updating after 22 years

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