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TIME-VARIANT RELIABILITY OF STRUCTURAL SYSTEMS SUBJECT TO FATIGUE

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ABSTRACT
A model is presented for the reliability calculation of so-called time variant redundant systems subject to a Gaussian load-process accounting for the possibility of failure under fatigue reduced resistances. The upcrossing approach together with the well-known Poisson limit theorem is used for the derivation of the distribution function of individual times to failure. The numerical part is facilitated by modern FORM/SORM techniques. Especially informative are parameters computed for the most likely failure state. The times up to and between individual failures resulting from these calculations can be used for the design of appropriate inspection and repair strategies.

1. INTRODUCTION

Fatigue-induced deterioration of offshore platforms and other structures is an important source of diminished structural performance and can even cause structural collapse. A reliability analysis of deteriorating structures is especially interesting for redundant structures as it can help to design suitable inspection and repair strategies and/or to quantify the remaining time of safe use of the structure. However, reliability models so far have primarily been developed for failure of structural components and systems under extreme loading and only a few studies are directed towards the reliability analysis of structural components subject to fatigue (see, for example, Martindale/Wirsching, 1983).

Recently, a reliability formulation has been proposed (Guers et al.,1987) which leads to approximate "time-variant" reliabilities of redundant structures whose componential resistance properties are still time-invariant. It models structural deterioration by considering explicitely the consecutive states of degradation in time as descibed by the sequence of componential failures. It is particularly suited for high reliable structures.

In this paper that formulation is generalized to include fatigue deterioration. It is based on some earlier studies on structural component reliability under fatigue where the so-called upcrossing approach (Guers/Rackwitz, 1985) has been found to be feasible and sufficiently accurate, especially in the context of modern FORM/SORM techniques. It enables a unified reliability approach for both extreme-value failure and fatigue rupture. It will be shown that those concepts directly carry over to structural systems.

2. BASIC MECHANICAL AND STOCHASTIC ASSUMPTIONS

Assume a linear-elastic, statically reacting, redundant structural system where M control points (bars, cross sections, hot spots, etc.) have been preselected as prone to failure. For simplicity of presentation but also sufficiently realistic in many cases, the load on the structure is taken as a stationary scalar Gaussian process $L(\tau)$ with mean m_L and standard derivation σ_L. Further, the load process obeys certain regularity conditions, that is continuous differentiability of sample paths and ergodicity (see Cramer et al.1967). The load-effects in the control points are then given by $S_m(\tau) = a_m L(\tau)$ where the a_m's are certain determin-istic coefficients. The m-th control point is said to fail when a large load "wave" causes S_m to exceed a possibly deteriorated control point resistance R_m for the first time.

This event (upcrossing of $R_m(\tau)$ by $S_m(\tau)$) is followed by a more or less abrupt change of the mechanical properties of the control point. The change is assumed as perfectly brittle as concerns the remaining resistance after failure. The corresponding load re-distribution, however, is supposed to occur sufficiently slowly such that, firstly, no dynamic effects need to be considered at the surviving hot spots and, secondly, the new stress regime in the structure is reached at the earliest in the next loading cycle. This means that the same resistance thresholds are relevant during the very wave that caused control point failure or, potentially, multiple control point failure resp. multiple threshold crossings. It is then possible in any state of the system to assign a critical threshold $R_m(\tau)/a_m$ to each non-failed control point and failure of one or more elements in a large wave can be treated by the out-crossing approach formulated in the same load space.

The resistance parameters in the control points, in general, must be assumed as uncertain. Therefore, control point failure is first considered conditional on a realisation of the vector $\underline{Q}=\underline{q}$ of un-certain resistance parameters.In a second step these conditions must be removed by integration according to the total probability law.

The other limiting case of immediate stress redistribution can be dealt with in a similar manner at the price of slightly more invol-ved computational operations (see Guers, 1987). With some restrict-ions it is also possible to consider dynamic overshooting of load effects in non-failing control points (Rackwitz/Guers,1986) and even damped load redistribution. These and other refinements of the approach are presently still under study.

3. OUTCROSSING APPROACH FOR FATIGUE-INDUCED DETERIORATION

3.1 INDIVIDUAL FAILURE TIMES

It has been shown (Guers, et al.,1987) that a probabilistic de-scription of the time-variant reliability of redundant systems can be based on the distributions of the conditional times to individu-al failures. For the highly reliable structures under consideration the distribution of the time $T_i^{(1)}$ to failure of (time) component i under a single large "wave" is well approximated by an exponential distribution. In each structural state (l) such a time component is the result of the almost simul-taneous failure of j control points, i.e. it comprises control points $i = (i_1,\ldots,i_j)$. The following well-known asymptotic formula is valid

$$F_{T_i^{(1)}}(T) = 1 - \exp[-\textstyle\int_0^T \nu_i^{(1)}(\tau)d\tau] = 1 - \exp[-I_i^{(1)}(T)] \qquad (3.1)$$

where $\nu_i^{(1)}(\tau)$ is the time-dependent upcrossing rate of $L(\tau)$ of the considered thresholds and $I_i^{(1)}(T)$ denotes the integral over $\nu_i^{(1)}(\tau)$ in the time interval $[0,T]$. For simplification of notation, the index (l) is omitted. It is further well known that (Madsen et al., 1985)

$$\nu_i(\tau) = \omega_o \varphi(r_i(\tau))\, \Psi(\dot{r}_i(\tau)/\omega_o) \qquad (3.2)$$

where

$$r_i(\tau) = (R_i(\tau) - a_i m_L)/(a_i \sigma_L) \qquad (3.3)$$

is the standardised threshold, $\dot{r}(\tau)$ its derivative, ω_o^2 the variance of the derivative of the normalized load process, and $\Psi(x) = \varphi(x) - x\Phi(-x)$ with φ the density and Φ the distribution function of a standard normal variable, respectively. The function Ψ frequently can be approximated by the constant $(2\pi)^{-1/2}$. By in-troducing an auxiliary standard normal variable U_{T_i} by the identity $F_{T_i}(T) = F_{U_{T_i}}(u) = \Phi(u)$, the variable T_i can be expressed as

$$T_i = I_i^{-1}[-\ln \Phi(-U_{T_i})] \qquad (3.4)$$

Herein, $I_i^{-1}[.]$ represents the inversion of the integral in eq. (3.1) with respect to the upper integration limit. A simple, fairly accurate formula for the evaluation of the upcrossing rate integral which makes use of Laplace's approximation (Copson, 1965), and a suitable numerical inversion algorithm has been derived in (Guers/Rackwitz ,1986). It is recognized that the failure pro-bability can now be determined from

$$P_f = P(T_i - T \le 0) \qquad (3.5)$$

which is precisely the formulation needed for the application of FORM/SORM-techniques.

3.2 SYSTEM FORMULATION

Structures can fail in one of K failure sequences which are formed by a series of individual random failure times. At the end of each time step a time component is said to fail. Each time component is associated with a single failure or multiple failures of control points. The total time to structural failure is obtained by adding up the time component failure times. Hence, structural collapse occurs in a given sequence during the reference period $[0,T]$, if

$$\sum_{i=1}^{i_m} T_i - T \le 0 \qquad (3.6)$$

where the complete set of consecutive failures of time components i=1 to $i=i_m$ imply collapse. In contrast to the case of time-inde-pendent resistances, the values of the thresholds of the non-failed control points at the beginning of every time step are dependent on the previous time-step lengths and the already failed control points due to the different deterioration regimes of the accumulation of damage along that sequence. This implies a dependency between the T_i's. It is conveniently represented by the sequence of conditional distribution functions of failure times. The corresponding Rosenblatt-transformation necessary for application of FORM/SORM for these failures times given a realisation of the resistance variables can be written as (Hohenbichler/Rackwitz, 1981):

$$F_{T_1}(T_1) = 1 - \exp[-\int_0^{T_1} \nu_1(\tau)d\tau] = \phi(U_{T_1})$$

$$F_{T_2}(T_2|T_1) = 1 - \exp[-\int_0^{T_2} \nu_2(\tau|T_1)d\tau] = \phi(U_{T_2})$$

$$F_{T_{i_m}}(T_{i_m}|T_1,\ldots,T_{i_m-1}) = 1 - \exp[-\int_0^{T_{i_m}} \nu_{i_m}(\tau|T_1,\ldots,T_{i_m-1})d\tau]$$
$$= \phi(U_{T_{i_m}}) \qquad (3.7)$$

Again, performing the integrations in eq. (3.7) and inverting them as described above and inserting the failiure times into eq. (3.6) yields a representation of the failure event in the standard space.

Consider now a general system with K possible failure paths to system collapse. The number of time steps in the k-th path is L_k.

In order to completely define the failure sequence in that path, an ordering of the reduced resistances must be performed for each of the time steps and the elements must be grouped into components failing at the end of each time step. The probability of occurrence of the k-th failure sequence during [0,T] can be written as:

$$P(F_k) = \int_{\underline{Q}=\underline{q}} P_k(T|\underline{Q}=\underline{q})\; P\Big(\underbrace{\bigcap_{l=1}^{L_k}}_{\substack{\text{for each}\\ \text{time step}}} \; \underbrace{\bigcap_{i=1}^{I_k^l}}_{\substack{\text{for all}\\ \text{present}\\ \text{elements}}} r^l_{n_{kl}(i)} \le r^l_{n_{kl}(i+1)}\Big) dF_{\underline{Q}}(\underline{q}) \qquad (3.8)$$

The second probability corresponds to the specific ordering. In numerical calculations the variables r are represented by their Rosenblatt-transformations. The first probability is:

$$P_k(T|\underline{Q}=\underline{q}) = P^k\, P\Big(\sum_{l=1}^{L_k} T_l^k(r^l_{n_{kl}(1)}) \le t\Big) \qquad (3.9)$$

where the second term corresponds to eq. (3.6) with new notation. The first probability is the probability to be on path k. In these equations, the following notations are used:

* $\underline{I}_k^l$ denotes the numbers of the control points in the k-th path surviving at the beginning and along the l-th time step.
* $n_{kl}(i)$ is an integer function which assigns in ascending order the numbers of the reduced thresholds during the l-th time step.

* $P^k = \prod_{l=1}^{L_k} P_k^l$ is the weighting probability of being on the k-th failure path where

$$P_k^l = \frac{\nu^l_{n_{kl}(i_{kl})} - \nu^l_{n_{kl}(i_{kl}+1)}}{\nu^l_{n_{kl}(1)}}$$

is the weighting probability for the l-th time step which ends at the upcrossing of i_{kl} thresholds corresponding to the elements $n_{kl}(1)$ to $n_{kl}(i_{kl})$ without upcrossing the $n_{kl}(i_{kl}+1)$-th level. Again, the corresponding events are expressed in the standard space by auxiliary standard normal variables making use of $P_k^l = \phi(U_k^l)$ (see Guers et al., 1987, for details)
* $T_l^k(r^l_{n_{kl}(1)})$ is the time to the first upcrossing of the lowest relevant level for the l-th time step, which is also the length of this time step.

The total failure probability can finally be obtained by integration over the uncertain resistance vector $\underline{Q}$, i.e. from

$$P_f(T) = \int_{\underline{q}} P\Big(\bigcup_{k=1}^{K} F_{k|\underline{Q}=\underline{q}}\Big)\, dF_{\underline{Q}}(\underline{q})$$

where $F_{k|\underline{Q}=\underline{q}}$ is the failure event in the k-th sequence conditional on $\underline{Q} = \underline{q}$.

4. MODEL FOR FATIGUE DETERIORATION OF STRUCTURAL ELEMENTS

Fatigue phenomena in metallic structural elements can suitably be modelled by assuming a certain crack initiation period which is followed by the crack propagation period. During the first period there is no substantial reduction of strength against extreme value loading. In the second period a gradual reduction of residual strength takes place. For simplicity of presentation it is here assumed that the crack initiation period is negligibly short (see Guers/Rackwitz, 1986 for a more rigorous, general formulation). A number of crack propagation models have been proposed. It must, however, be recognized that due to the lack of data relatively simple relationships such as the one proposed by Paris-Erdogan usually is accurate enough. For this model and many other alternatives such as the so-called Feddersen scheme it is possible to assess a decreasing threshold function in the load effect space (stress space). This function usually has the form

$$R(\tau) = R(o)[1-K\, E[\sum_{[o,\tau]} (\Delta S_i)^c]]^d \qquad (4.1)$$

where R(o) is the initial resistance and K, c and d are possibly uncertain material properties. The (ΔS_i) denote the damage relevant stress cycle amplitudes. The damage accumulation term $\Sigma(\Delta S_i)^c$ in that equation depends on the counting method of damage relevant stress cycles. If the loading process is fairly narrow banded it is simply ΔS_i = max S_i - min S_i. Then, the expectation term in eq. (4.1) can be written out explicitely. A slight generalisation is achieved if one adopts certain empirical adjustments proposed by Yang (1979). Eq. (4.1) can then be written as

$$R(\tau) = R(o) \left[1 - \frac{X\ K}{R(o)^c}\,\tau\right]^{1/c} \qquad (4.2)$$

with $X\ K = A^c\ K\ (2\sqrt{2})^B\ \Gamma(1+B/2)$ and A and B some additional, usually uncertain parameters also obtainable from fatigue tests.
An important characteristic of that model is the assumption of a slowly deterministically decreasing resistance, which can be verified theoretically and experimentally for high cycle fatigue (Guers/Rackwitz, 1986). The value of the threshold depends primarily on the past load history and only negligibly on the instantaneous stress cycle. The independence of threshold and load process as another consequence of the high cycle assumption together with the limitation to high-reliable systems is equally important for the outcrossing approach because it allows to assume Poisson-distributed conditional upcrossing events.

5. APPLICATION FOR A 4-ELEMENT DANIELS SYSTEM

The ideal-brittle Daniels system (Daniels, 1944) is known for its simple mechanics. On the other hand, the degree and efficiency in redundancy of this system is exceptional due to its symmetry. As a consequence it requires more involved reliability calculations than most of the more common structures. For the purpose of illustration of the foregoing concepts a Daniels system with 4 elements is assumed (fig. 1). The tension strengths R_i of the different tendons are assumed to be normally distributed and equicorrelated. The following simple resistance model then holds:

$$R_i = E[R_i] + D[R_i](\sqrt{\rho}\ U_o + \sqrt{1-\rho}\ U_i)$$

where U_o and the U_i's are independent standard normal variables.
For the numerical calculations with this example the following values have been chosen

$$R_i \sim N(E[R_i] = 0.9; D[R_i] = 0.2)$$
$$L(\tau) \sim N(0.5; 0.1)$$
$$\rho = 0.3$$
$$\nu^+ T = 10^6$$

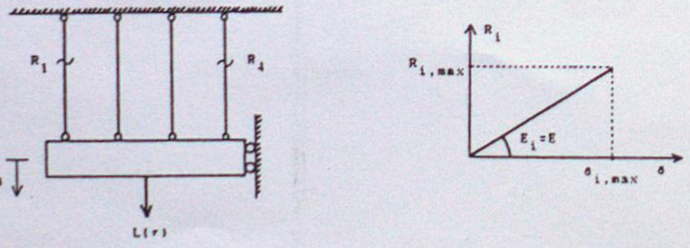

Figure 1: 4 elements Daniels system

The two parameters in eq.(4.2) are assumed to be lognormally distributed with means $E[XK] = 10^{-3}$ resp. $E[c] = 4$ and a coefficient of variation COV = 0.2. B is deterministic, i.e. B = 3. The method of the order statistics is used to calculate the distributions of the ordered resistances $\hat{R}_1 < \hat{R}_2 < \hat{R}_3 < \hat{R}_4$ (Hohenbichler/Rackwitz, 1982). The failure sequences are represented by the ruptures of elements $\hat{1}, \hat{2}, \hat{3}$ and $\hat{4}$, repectively, and differ only by the composition of the components failing at the end of each time step.

The failure tree of the system without deterioration, i.e. for $E[XK] = 0$ is given in fig. 2. The second-order safety indices are shown in parenthesis at every step in the failure tree after the last failed component.The values of the variables at the expansion point for FORM/SORM can be interpreted as the most likely parameter values for the considered failure sequence. In particular, the most likely times between failures can be computed providing important information about the remaining redundancy in the system and, thus, also for further inspection and repair actions. The numbers just above the same component are the time step lenghts in the most likely failure state given as fractiles of the total lifetime. An ordering of the failure sequences is possible. As expected due to the assumption of delayed load redistribution, the sequences with several time steps and thus load redistributions are the most likely to occur (see fig. 2).

In figure 3 the failure tree with fatigue deterioration of the resistances is shown. It can be observed that the safety indices now are generally smaller. The safety index for the sequence $\hat{1}, \hat{2} \cap \hat{3}$, for example drops down from 4.72 to 4.10. Also, the system most likely spends comparatively more time in the first time steps when the resistances are less deteriorated. In the previously mentioned sequence, the time spent in the intact state increases from 26.4% to 58.2%. Of course, the total lifetime is reduced (compare the β-values).

6. DISCUSSION

As mentioned in section 4, the independence of the upcrossing events is a basic condition for the calculation of the first crossing time. This assumption will be more and more violated for the lower thresholds, i.e. at the end of the lifetime of the structure. However, it has been found that because the later times are short as compared to the preceeding ones, the error made remains negligible.

The deterioration model proposed before is adequate for preliminary investigations. It can be easily improved for special cases, for example, by using Vanmarcke's (1975) improvement for the upcrossing rate of narrow-band processes or by using more sophisticated models for componential residual strength.

The application of this methodology to more complicated truss or frame structures does not imply essential changes. Only ordering conditions as in eq. (3.9) of the thresholds then have to be included (Guers et al.,1987). The case of unequal fatigue constant,i.e. where the thresholds can cross others during he lifetime, can also be solved by introducing truncated distributions for the first crossing times.

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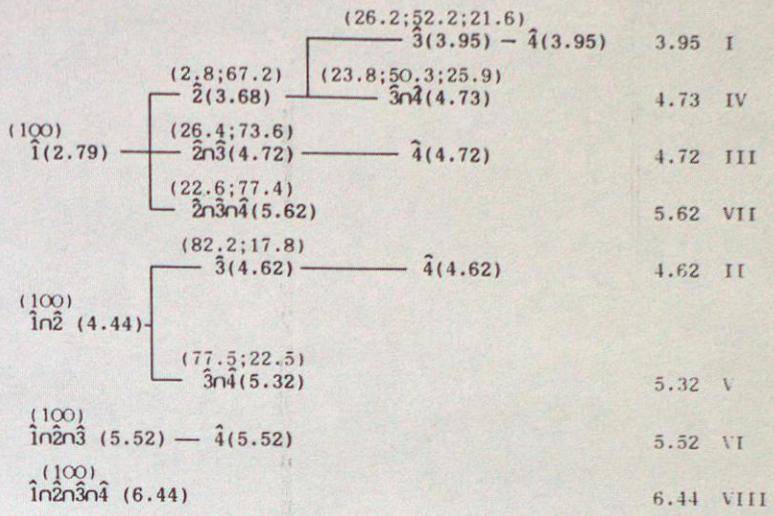


Figure 2: Failure tree for Daniels system with time-invariant resistances

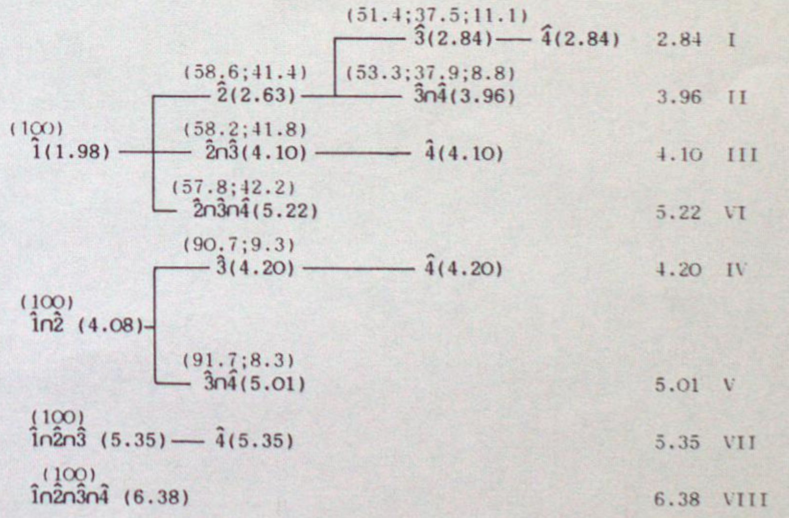


Figure 3: Failure tree for Daniels system with deteriorating resistances

Captions of figures

Figure 1: Load redistribution after brittle failure

Figure 2: Multiple crossings

Figure 3: Daniels system and failure tree

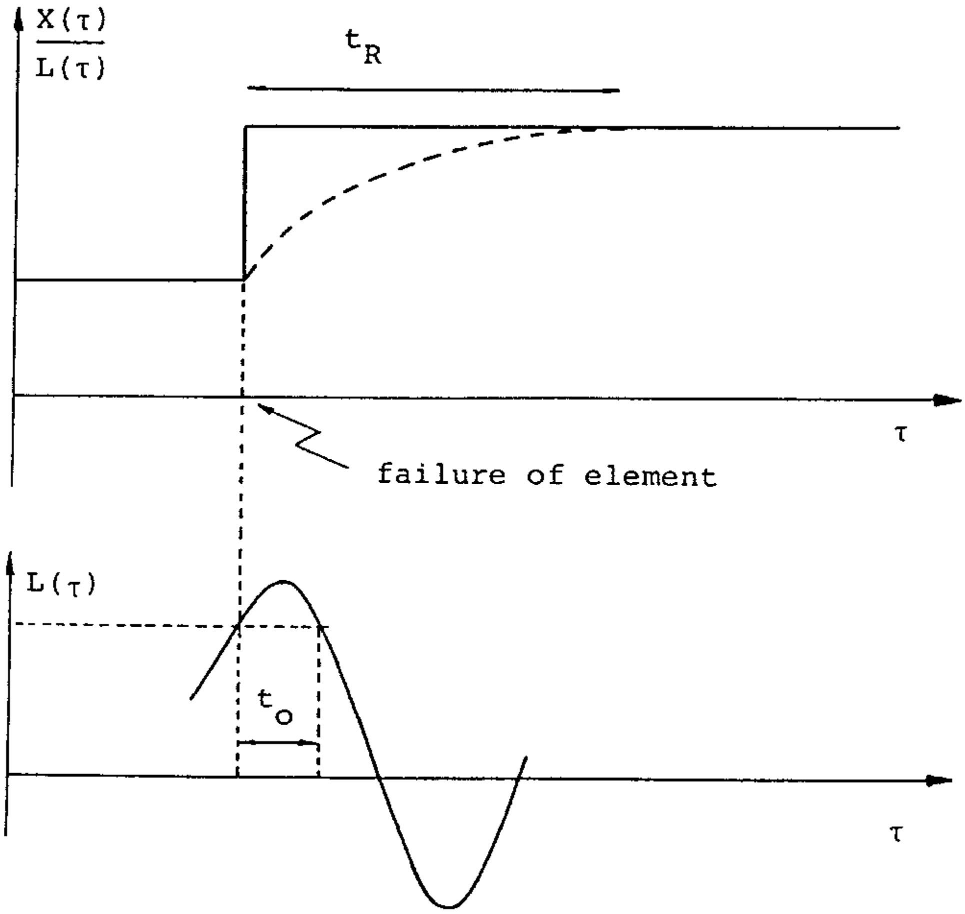
Figure 4: Failure tree of 4-element Daniels-system - First-and second order safety indices

Figure 5: Failure tree of 4-element Daniels-system - Failure times and branch probabilities

Figure 6: Example truss structure (a=1)

Figure 7: Failure tree of example truss - delayed load redistribution

Figure 8: Failure tree of example truss - immediate load redistribution



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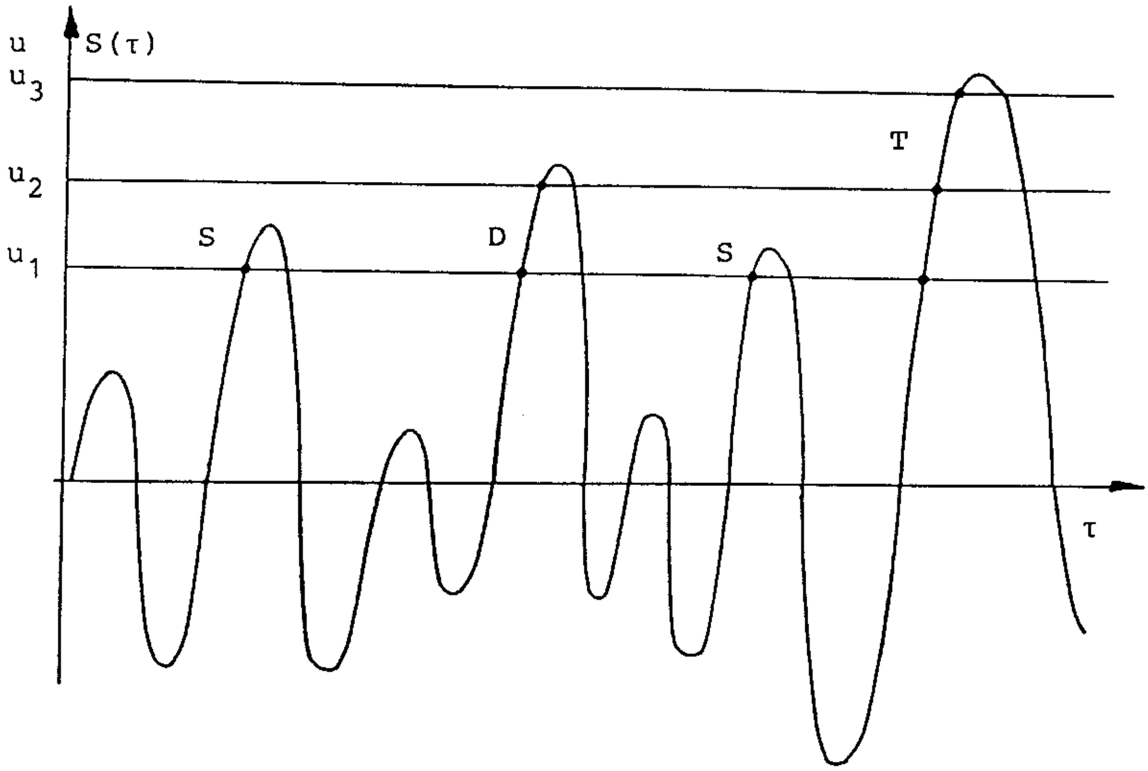
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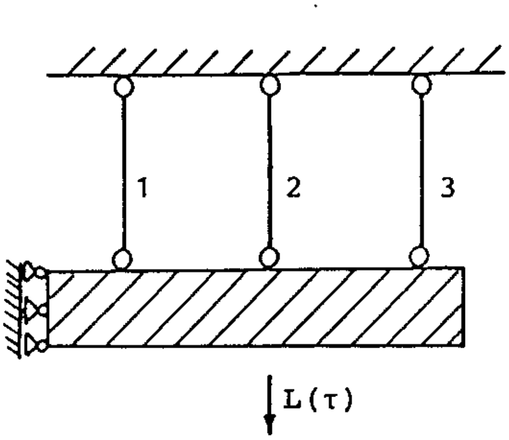
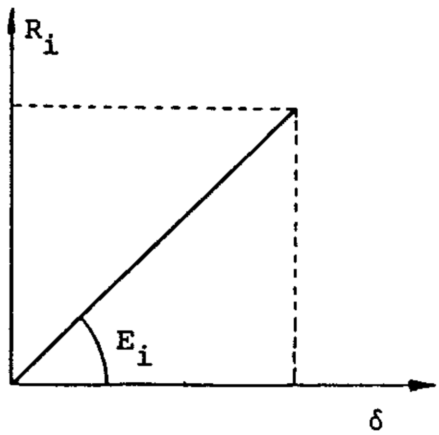
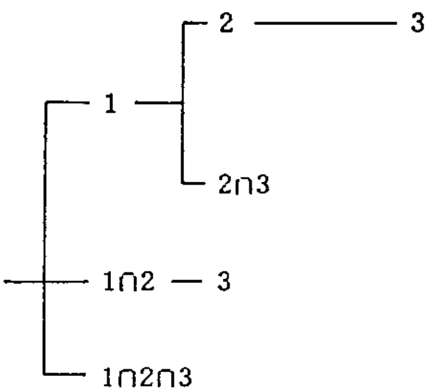




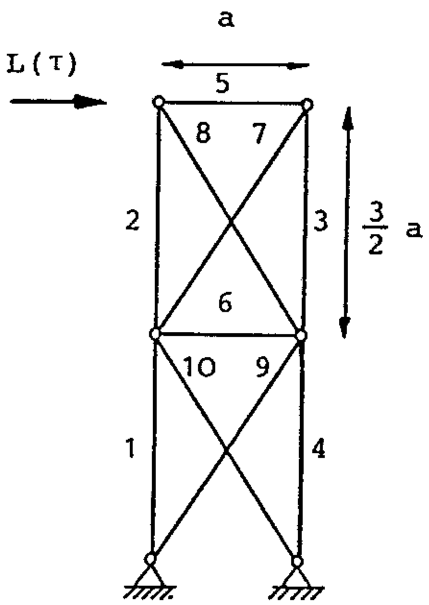


$$\frac{\nu_1-\nu_2}{\nu_1}\ \frac{\nu_2-\nu_3}{\nu_2}\ \frac{\nu_3-0}{\nu_3}\,P(T_1+T_2+T_3\le 0)$$

$$\frac{\nu_1-\nu_2}{\nu_1}\ \frac{\nu_3}{\nu_2}\ P(T_1+T_2\le 0)$$

$$\frac{\nu_1-\nu_3}{\nu_1}\ \frac{\nu_2-0}{\nu_2}\ P(T_1+T_3\le 0)$$

$$\frac{\nu_3}{\nu_1}\ P(T_1\le 0)$$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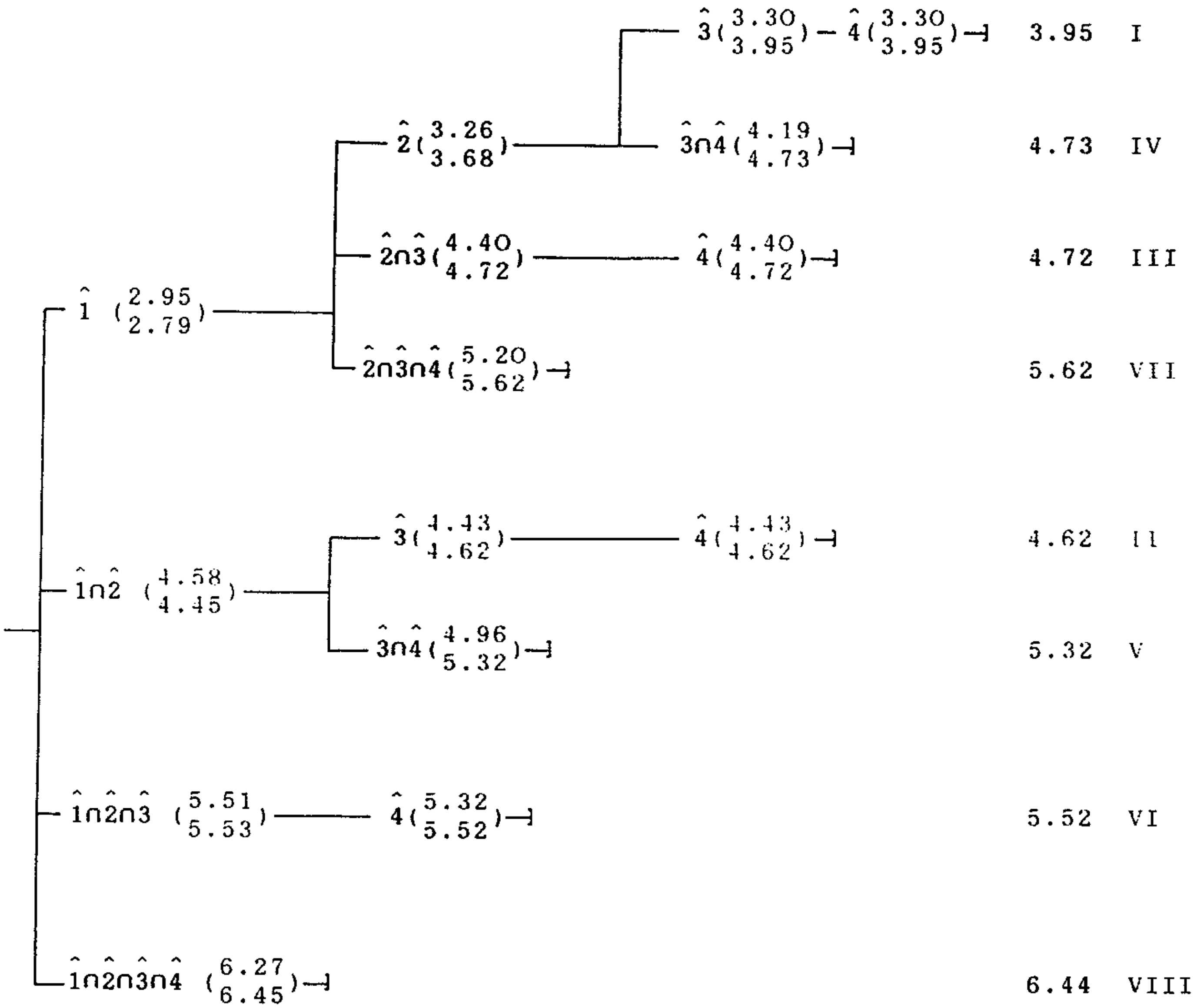
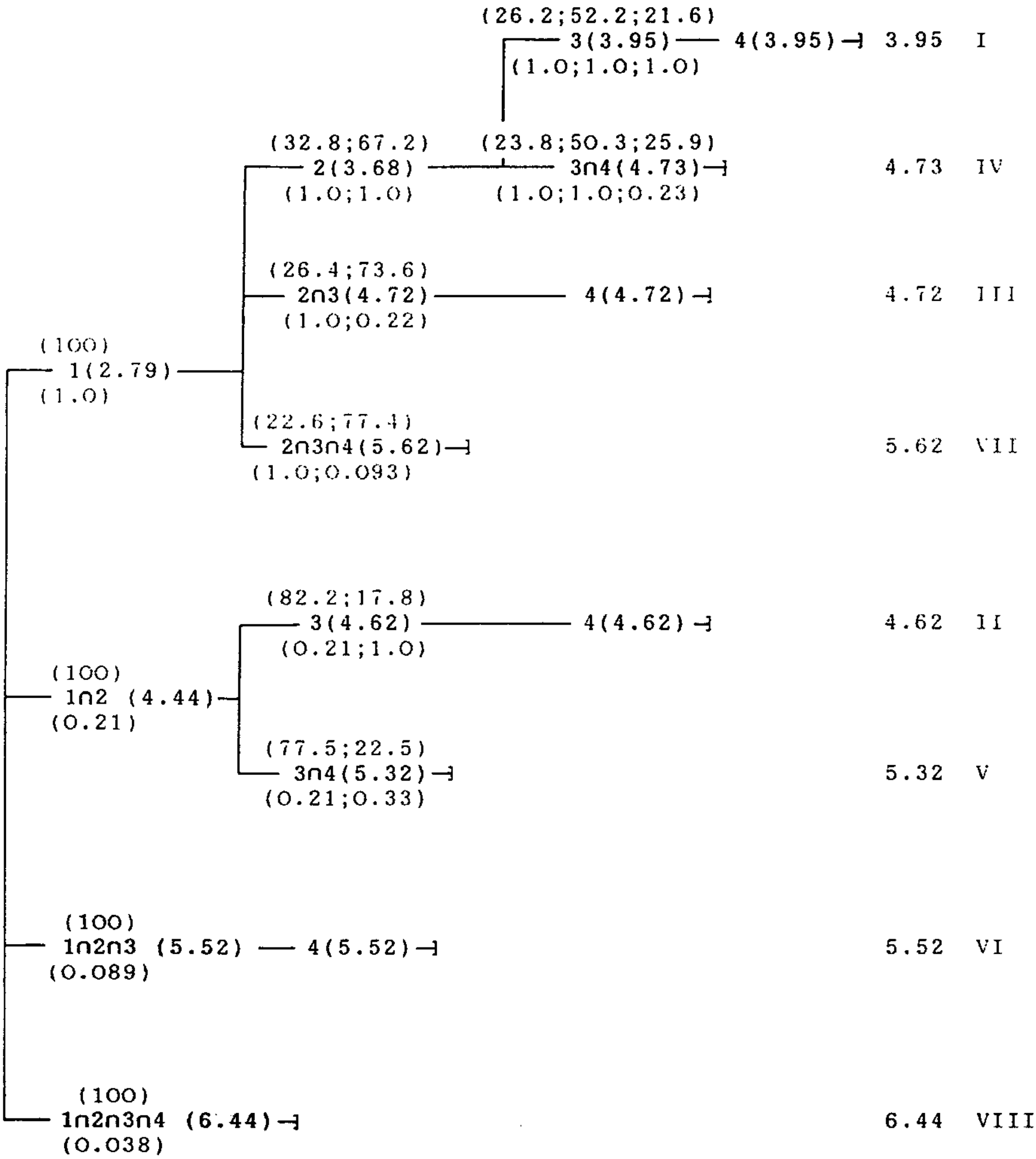

$\hat{3}\binom{3.30}{3.95}$ — $\hat{4}\binom{3.30}{3.95}$ ⊣ 3.95 I
$\hat{2}\binom{3.26}{3.68}$
$\hat{3}\cap\hat{4}\binom{4.19}{4.73}$ ⊣ 4.73 IV
$\hat{2}\cap\hat{3}\binom{4.40}{4.72}$
$\hat{4}\binom{4.40}{4.72}$ ⊣ 4.72 III
$\hat{1}\ \binom{2.95}{2.79}$
$\hat{2}\cap\hat{3}\cap\hat{4}\binom{5.20}{5.62}$ ⊣ 5.62 VII
$\hat{3}\binom{4.43}{4.62}$
$\hat{4}\binom{4.43}{4.62}$ ⊣ 4.62 II
$\hat{1}\cap\hat{2}\ \binom{4.58}{4.45}$
$\hat{3}\cap\hat{4}\binom{4.96}{5.32}$ ⊣ 5.32 V
$\hat{1}\cap\hat{2}\cap\hat{3}\ \binom{5.51}{5.53}$
$\hat{4}\binom{5.32}{5.52}$ ⊣ 5.52 VI
$\hat{1}\cap\hat{2}\cap\hat{3}\cap\hat{4}\ \binom{6.27}{6.45}$ ⊣ 6.44 VIII



(26.2;52.2;21.6)
3(3.95)
(1.0;1.0;1.0)
4(3.95)
3.95 I
(32.8;67.2)
2(3.68)
(1.0;1.0)
(23.8;50.3;25.9)
3∩4(4.73)
(1.0;1.0;0.23)
4.73 IV
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2∩3(4.72)
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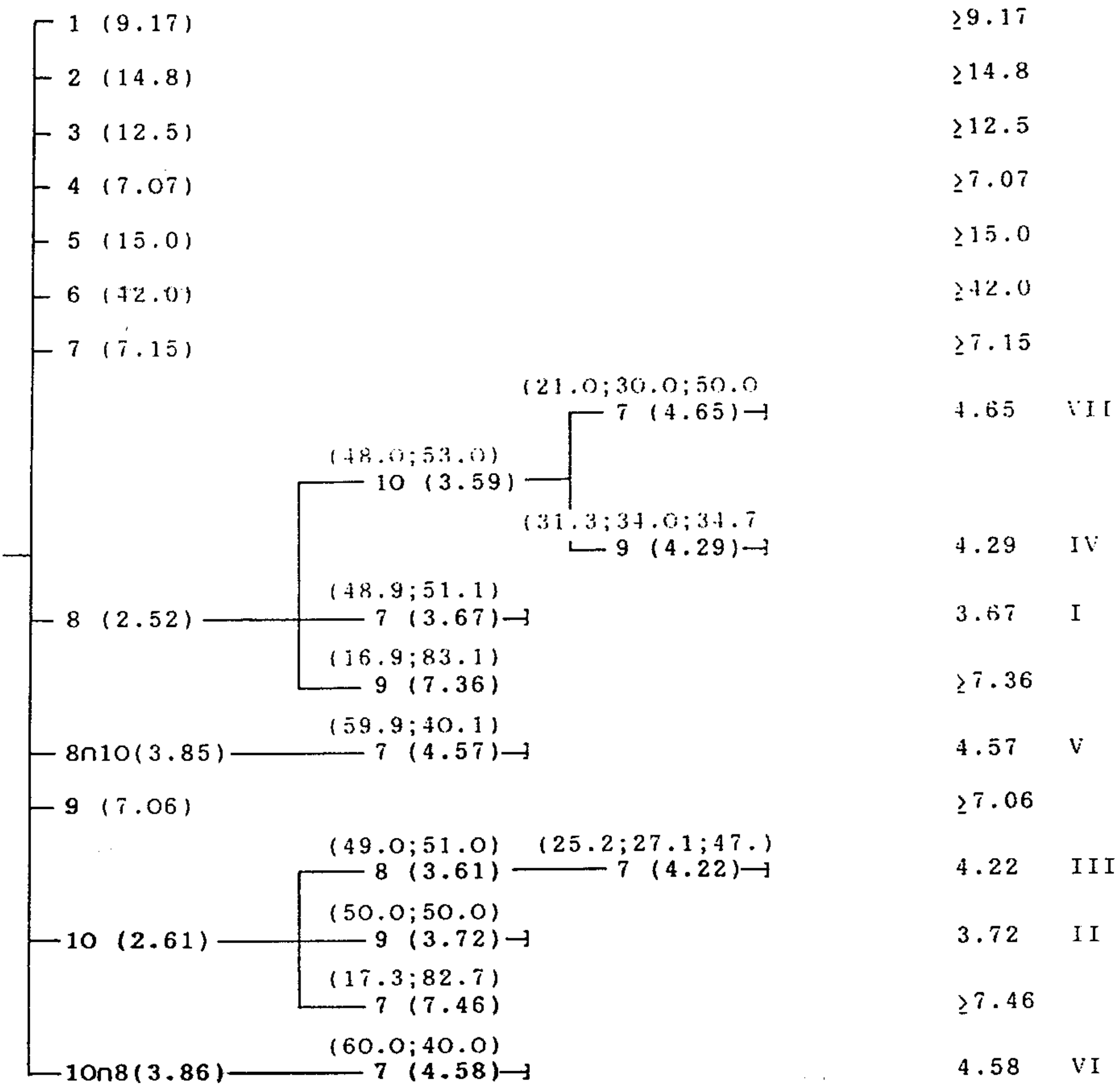

1 (9.17) ≥9.17
2 (14.8) ≥14.8
3 (12.5) ≥12.5
4 (7.07) ≥7.07
5 (15.0) ≥15.0
6 (42.0) ≥42.0
7 (7.15) ≥7.15
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7 (4.65) 4.65 VII
(48.0;53.0)
10 (3.59)
(31.3;34.0;34.7
9 (4.29) 4.29 IV
(48.9;51.1)
8 (2.52)
7 (3.67) 3.67 I
(16.9;83.1)
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7 (4.22) 4.22 III
(50.0;50.0)
10 (2.61)
9 (3.72) 3.72 II
(17.3;82.7)
7 (7.46) ≥7.46
(60.0;40.0)
10∩8(3.86)
7 (4.58) 4.58 VI

1 (9.17)			≥9.17	
2 (14.8)			≥14.8	
3 (12.5)			≥12.5	
4 (7.07)			≥7.07	
5 (15.0)			≥15.0	
6 (42.0)			≥42.0	
7 (7.15)			≥7.15	
8 (2.52)	(48.0;52.0) 10 (3.59)	(21.0;30.0;50.0) 7 (4.65) ⊣	4.72	XI
	(32.4;67.6) $10\cap 7$ (4.60) ⊣		4.60	VII
	(48.9;51.1) 7 (3.67) ⊣		3.67	I
	(16.9;83.1) 9 (7.36)		≥7.36	
$8\cap 10$ (3.81)	(43.2;56.8) 7 (4.54) ⊣		4.54	V
$8\cap 10\cap 7$ (4.66) ⊣			4.66	IX
$8\cap 7$ (3.86) ⊣			3.86	III
9 (7.06)			≥7.06	
10 (2.61)	(49.0;51.0) 8 (3.67)	(25.2;27.1;47.) 7 (4.22) ⊣	4.22	XII
	(33.5;66.5) $8\cap 7$ (4.61) ⊣		4.61	VIII
	(50.0;50.0) 9 (3.72) ⊣		3.72	II
	(17.7;82.3) 7 (7.46)		≥7.46	
$10\cap 8$ (3.82)	(43.2;56.8) 7 (4.55) ⊣		4.55	VI
$10\cap 8\cap 7$ (4.67) ⊣			4.67	X
$10\cap 9$ (3.90) ⊣			3.90	IV