OUTCROSSING FORMULATION FOR DETERIORATING STRUCTURAL SYSTEMS

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1. Introduction

In many cases the resistance properties of structural systems as well as the loads acting on the structure depend on time. If the loads can be modelled by stationary processes and the resistances are time-invariant the determination of the time-dependent reliability essentially is a problem of load combination for which a number of solution procedures exists applicable not only to structural components but also to entire structural systems. (See references [1,2,3] as representative for the different approaches and [4] as an example for the treatment of structural systems). The only method capable to handle instationary loading with some rigour appears to be the outcrossing approach [5] and the same is true for cases where resistances deteriorate with time e.g. due to load-induced fatigue, corrosion or aging. This has been demonstrated in [6] and elsewhere for structural components. If, however, deterioration occurs in a (redundant) structural system a reliability analysis meets serious complications because structural failure most likely is the result of a sequence of componental failures at different random times each of which changing the stress regime in the structure. Failure phenomena of this type will especially be found in certain types of railway bridges, aircraft structures and in many maritime structures such as ships or offshore platforms. The quantification of the time-dependent reliability is not only the basis for a proper design of such structures but, probably more important, allows the selection of suitable inspection strategies and rational decisions about necessity and time of repairs.

The only studies known to the authors which have addressed this problem so far are due to Martindale/Wirsching [7] and to Rackwitz [8]. The first mentioned reference assumes the distribution of times between componental failures as known but depending on the system states. Reliabilities are determined by the Monte-Carlo-methods.

In this study the widely analytical outcrossing approach proposed in [9] for the determination of time-variant structural reliability under stationary loading and non-deteriorating structural resistances is generalised to fatigue-induced deterioration of structural components.Furthermore, formulations are sought such that reliability calculations can be performed with modern first- and second-order reliability methods. This might enable the analysis of larger systems with many uncertain variables. The formulations are demonstrated at a simple example.

2. Reliability of structural components

Assume a statically reacting, linear-elastic and redundant structural system subject to loads modelled as piecewise stationary (and ergodic) Gaussian vector processes $\underline{L}(\tau)$. Further, suppose that failure can occur in a finite number of preselected control points (hot spots) which will be denoted by elements or components of the structure. For these components it is always possible to derive the load effect process which, here, is assumed to be a scalar process, i.e. $S_j(\tau) = \sum_i a_{ji} L_i(\tau)$, and whose mean and covariance function are easily determined from the properties of $\underline{L}(\tau)$. Component failure occurs whenever $S_j(\tau)$ exceeds some resistance (residual strength) $R_j(\tau)$ for the first time. Componental failure or state changes are understood as a discontinuous change (decrease) in stiffness at that time causing a more or less abrupt redistribution of internal forces in the system in some manner. For simplification, only perfectly brittle elemental failures will be considered. In practical applications one might wish to model the rupture phenomenon more realistically, e.g. by retaining some fraction of the original stiffness. The considerations to come also hold in this case with some minor modifications.

The resistances $R_j(\tau)$ depend on a time-invariant vector of uncertain parameters such as initial strength $R_j(o)$ and parameters determining the strength degradation. These are collected in the uncertain vector $\underline{Q}$ with given distribution function $F_{\underline{Q}}(\underline{q})$. For the moment, $\underline{Q}$ will be kept fixed and, therefore, the considerations in this section are conditional on $\underline{Q} = \underline{q}$. In order to determine the failure probability one needs to know the distribution of the time to failure which, unfortunately, can only be given exactly under very special conditions for the process $S_j(\tau)$ and functions $R_j(\tau)$. However, a rather general asymptotic formula for the failure probability or the distribution of time to failure under the conditions mentioned is:

$$P_{f,j} = P\ (T_j \leq t) = F_{T_j}(t) \sim 1 - \exp\ [- \int_o^t \nu_j(\tau)\ d\tau] \tag{1}$$

Herein, $\nu_j(\tau)$ is the upcrossing rate defined by:

$$\nu_j(\tau) = \lim_{\Delta \to o} \frac{P(\{S_j(\tau) \leq R_j(\tau)\} \cap \{S_j(\tau + \Delta) > R_j(\tau + \Delta)\})}{\Delta} \tag{2}$$

Eq. (1) is valid provided that $\nu_j(\tau)$ exists. For example, it exists for Gaussian processes with continuous, differentiable sample paths. In principle, the residual strength is also a non-stationary process depending on the load effect process. For high cycle fatigue, however, one can assume that asymptotically (large τ) the processes $R_j(\tau)$ not only become uncorrelated with the processes $S_j(\tau)$ but also have vanishing coefficient of variation [6]. Under the same conditions it can be assumed that $R_j(\tau)$ are sufficiently smooth functions with existing derivative. Then, with $R_j(\tau)$ being approximately a deterministic function the following formula for $\nu_j(\tau)$ can be derived [10] (the reference to τ now being ommitted)

$$\nu_j = \omega_o \varphi(r)\ \Psi(\dot{r}/\omega_o) \tag{3}$$

where $r = (R - m_s)/\sigma_s$, $\dot{r}$ is the time derivative of r, ω_o^2 the variance of the derivative of the normalized load effect process $s = (S - m_S)/\sigma_S$ and $\Psi(x) = \varphi(x) - x\Phi(-x)$.

The residual strength function typically has the form

$$R(\tau) = K_1 \left(1 - K_2 \nu_o E[\Delta S^A]\tau\right)^B \tag{4}$$

where the constants can have concrete physical meaning. Assume, for example, the crack propagation law due to Paris/Erdogan [11]

$$\frac{da}{dn} = C(\Delta S\sqrt{\pi a})^m \tag{5}$$

with a the crack length, ΔS the effective stress-range and C and m two material parameters. The crack becomes unstable for $K = \sqrt{\pi a}S > K_c$ with K_c the fracture toughness.

Then, it can be shown that for $m > 2$, $K_1 = K_c\,(\pi a_o)^{-1/2}$, $K_2 = C\,\pi^{m/2} a_o\,(m - 2)^2/4$, $A = m$, $B = (m - 2)^{-1}$. ν_o is the rate of positive zero crossings of $s(\tau)$. For a sufficiently narrow-band process $s(\tau)$ it is finally $E\,[\Delta S^A] = (2\sqrt{2})^A \sigma_S^{\,A}\Gamma(1 + A/2)$ [10,12].

The integral in eq.(1) of the outcrossing rate eq. (3) with threshold function eq. (4) can be approximated fairly well for sufficiently large thresholds r(t) by using the method of Laplace. Following [13] where the application of this method to fatigue reliability problems is investigated in some detail, one obtains

$$I(t) = \int_o^t \nu(\tau)\, d\tau \approx \frac{h(t)}{f'(t)} \exp\,[f(t)][1 - \exp\,[-\,tf'(t)]] \tag{6}$$

with

$$h(t) = \frac{\omega_o}{\sqrt{2\pi}} \varphi \left(\frac{\dot{r}(t)}{\omega_o}\right) \tag{7}$$

$$f(t) = -\frac{1}{2} r^2(t) \tag{8}$$

$$f'(t) = -r(t)\,\dot{r}(t) \tag{9}$$

Eq. (6) can be improved further (See [13]).

The total failure probability with the paramters $\underline{Q} = \underline{q}$ now being random is:

$$P_{f,j}(t) \sim 1 - \int_{\underline{Q}} \exp\left[-I_j(t|\underline{q})\right] dF_{\underline{Q}} \tag{10}$$

A serious obstacle in applications can be the multidimensional integration required in eq. (10). It is, however, possible to reformulate the reliability problem such that modern first- and second - order reliability methods become applicable [14]. Introducing the auxiliary standard normal variable as [8]

$$P(T_j(\underline{q}) \leq t) = 1 - \exp\left[-I_j(t|\underline{q})\right] = P(U_{T_j} \leq u) \tag{11}$$

we find that by solving for $T_j(\underline{q})$

$$T_j(\underline{q}) = I_j^{-1}\left[-\ln \phi\,(-U_{T_j})\,|\underline{q}\right] \tag{12}$$

the Rosenblatt-transformation [15] in the required formulation:

$$P_{f,j}(t|\underline{q}) = P(T_j(\underline{q}) - t \leq o) \tag{13}$$

The total failure probability can be given as

$$P_{f,j}(t) = P(T_j(\underline{T}(\underline{U}_{\underline{Q}}) - t \leq o) \tag{14}$$

with $\underline{Q} = \underline{T}(\underline{U}_{\underline{Q}})$ the Rosenblatt-transformation of $\underline{Q}$. Then, for small failure probabilities an accurate probability estimate is [14]

$$P_{f,j}(t) \sim \phi(-\beta) \prod_{i=1}^{n_j} (1 - \kappa_i \delta)^{-1/2} \tag{15}$$

where

$$\beta = \|\underline{u}^*\| = \min \{\|\underline{u}^*\|\} \text{ for } \{\underline{u}: g(\underline{u}) \leq o\} \tag{16}$$

the κ_i's the main curvatures in $\underline{u}^*$, $\delta = \varphi/(-\beta) \; / \; \phi/(-\beta)$ and $g/(\underline{u}) \leq o$ the event in the left-hand side of eq. (14). The inversion of the integral $I_j(t)$ in eq. (12) is best made by Newton's algorithm:

$$T_j(\underline{q})^{(k+1)} = T_j(\underline{q})^{(k)} - \frac{I_j(T_j(\underline{q})^{(k)}) + \ln \phi(-U_{Tj})}{\nu_j(T_j(\underline{q})_j^{(k)})} \tag{17}$$

Formula (3) has been found to be rather conservative for not too large values of r(t) especially for narrow band processes such as wave loading processes and the consideration of crossings of the envelope process $E(\tau)$ of $S(\tau)$ may yield better results. In this case, eq. (3) has to be modified into [12]:

$$\nu_2 = \omega_E \, f_{Ray}(r) \, \Psi \, (\dot{r}/\omega_E) \tag{18}$$

with $\omega_E^2 = \omega_o^2 \, (1- (\lambda_1(\lambda_o\lambda_2)^{-1/2})^2)$ and $f_{Ray}(r) = r \exp [-r^2/2]$ the Rayleigh-density. An even better result is obtained by using the interpolation between eq. (3) end eq. (18) proposed in [16]

$$\nu_3 = \nu_1 \, [1 - \exp [- \nu_2/\nu_1]] \tag{19}$$

The conditions for the validity of the approximation eq. (5) are still fulfilled but eq. (7) is replaced by:

$$h(t) = \frac{\omega_o}{\sqrt{2\pi}} \Psi(\dot{r}(t)/\omega_o) \, [1 - \exp [- \nu_2/\nu_1]] \tag{20}$$

$$\nu_2/\nu_1 = \frac{\omega_E \Psi(\dot{r}(t)/\omega_E)}{\omega_o \Psi(\dot{r}(t)/\omega_o)} \, r(t) \tag{21}$$

Formula (18) should be used in practical applications, not only because it gives more accurate results for a larger range of thresholds and bandwidth parameters $\delta = \lambda_1(\lambda_o\lambda_2)^{-1/2}$ with λ_i the i-th spectral moment of $S(\tau)$ but is also more consistent with the basic Poissonian assumptions underlying eq. (4).

3. System reliability

Consider now system failure which in redundant structures requires several components to fail simultaneously or in a sequence. In general, many different sets of componental failures exist which imply system failure. Consider, for the moment, a certain set or failure path ν consisting of $N = N(\nu)$ components. Further analysis now must distinguish between different cases. Here, only several limiting cases will be investigated in more detail. Assume that during a "local" extreme of the loading process there is a brittle componental failure as discussed in the foregoing section. Internal forces will then be redistributed. The load effect in another component will perform a damped oscillation around the statically redistributed load effect. Two extreme cases can be visualized. If there is small damping and a relatively large eigenfrequency of the oscillations the additional, dynamic load-effect after brittle failure of some component can reach at most twice the difference between the static load-effect before and after the failure (case A). The other extreme is where redistribution corresponds to the case of critical (over) damping. The redistribution follows a negative exponential function (case B). No dynamic overshooting occurs. Unfortunately, the dynamic effects in rupture phenomena have found very little attention in the literature and it is hard to say which of the two limiting cases is closer to reality in a specific application. The authors are inclined to presume that case B is more representative for real failure phenomena in many cases.

Furthermore, we need to consider two other extreme cases defined by two limits of the ratio T_R of (almost perfect) load-effect redistribution time and the predominant period T_o of the load process . The case where redistribution of forces takes a time much shorter than T_o is denoted by case I. For case II, on the other hand, the time required to redistribute the forces is much larger than T_o (See figure 1). During a "local" extreme of the load multiple failures can occur. It is even possible that all components in a failure path fail simultaneously thus causing immediate system col-lapse. An example of the failure tree is given in figure 2. In case I there is 'immediate' load redistribution during one local extreme of the loading process while there is virtually none in case II in this period.

If now partial failure occurs along the failure path at the different failure times it is clear that the time to system collapse is the sum of the times between those partial failures. For non-deteriorating structures, we simply have [9]:

$$P_{f,\nu}(T) = P\left(\sum_{i=1}^{N(\nu)} T_i(\underline{q}) - T \leq 0\right) \tag{22}$$

Herein, T is some prespecified service time of the structure. For the moment, it is assumed for simplicity of notation that only one component fails at the time instants of abrupt changes in the system.

For deteriorating structures the problem is somewhat more complex. The second failure time now must depend in the first failure time since the latter determines the two damage accumulation regimes to be considered when computing the second failure time. And the third, forth, ... failure times analogously depend on all previous failure times, respectively. Hence, a possible formulation is:

$$P_{f,\nu}(T) = P\left(\sum_{i=1}^{N(\nu)} T_i(\underline{q})\,|\,T_1, \ldots, T_{i-1}) - T \leq 0\right) \tag{23}$$

A crucial assumption when proceeding further is that the various failure times are conditionally independent. In addition they are exponentially distributed according to eq. (1). This requires that even the failure events at the end of a failure path are rare events and, hence, the Poissonian character of the crossing events can be maintained. From a physical point of view this assumption also implies that after each componental failure the load effect process has a "restart" from the intersection of the safe domains of all still unfailed components. Then, in generalising the approach for component failure as in eqs. (10) to (16) the following transformation appears natural:

$$1 - \exp\left[-\int_0^{T_1} \nu_1(\tau|\underline{q})d\tau\right] = \phi\,(U_{T_1})$$

$$1 - \exp\left[-\int_0^{T_i} \nu_1(\tau|\underline{q}, U_{T_1}, \ldots, U_{T_{i-1}})d\tau\right] = \phi\,(U_{T_i})$$

$$1 - \exp\left[-\int_0^{T_{N(\nu)}} \nu_{N(\nu)}\,(\tau|\underline{q}, U_{T_1}, \ldots, U_{T_{N(\nu)-1}})d\tau\right] = \phi\,(U_{T_{N(\nu)}})$$

(24)

Thus, estimates of path failure probabilities can be obtained by eqs (15) and (16), too.

Finally, if the parameter $\underline{Q} = \underline{q}$ is uncertain and there are M possible paths to system failure the overall failure probability is the probability of the union of the path failure events

$$P_f(T) = P(\bigcup_{\nu=1}^{M} F_\nu) = P(\bigcup_{\nu=1}^{M} \int_{\underline{Q}} (\sum_{i\in N(\nu)} T_i(\underline{q}|T_1,T_2,\ldots) - T \le 0)\, dF_{\underline{Q}}) \quad (25)$$

$$= P(\bigcup_{\nu=1}^{M} (\sum_{i\in N(\nu)} T_i(\underline{T}_{\underline{Q}}(\underline{U}_{\underline{Q}})|T_1,T_2,\ldots) - T \le 0))$$

This probability can be bounded according to [17]:

$$P_f(T) \begin{cases} \le P(F_1) + \sum_{\nu=2}^{m} (P(F_\nu) - \max_{\mu<\nu} (P(F_\nu \cap F_\mu))) \\ \ge P(F_1) + \sum_{\nu=2}^{m} (P(F_\nu) - \sum_{\mu<\nu} P(F_\nu \cap F_\mu)) \end{cases} \quad (26)$$

The consideration of multiple failures which drastically can reduce the redundancy in a system requires a clear distinction between case I and II. In the latter case the formulations given before carry over with the only modification that now the crossings into intersections of componental failure domains have to be considered. For case I with or without dynamic overshooting the calculation of the failure times is more complicated. Due to the normally different load redistribution regimes once the resistance level of a certain component is exceeded, one needs to consider all possible permutations in the set of remaining components for multiple failure. In order to illustrate the basic aspects assume that the set consists of only two elements. Once the level of the first component expe-riences a crossing two cases can occur. Redistribution of internal forces either causes the level of the second component being crossed by the load effect process during redistribution or leads to a new (reduced) resistance level for the second component. In the first case the crossing rate is the crossing rate of component 1 before load redistribution. In the second case the crossing rate corresponds to the crossings of component 2 after load redistribu-tion immediately after a crossing of level 1. Moreover, the cros-sing rate for component 1 alone needs to be computed under the condition that the process enters the failure domain of component 1 but not of component 2. Neither this last crossing rate nor the first one can be determined exactly. However, certain asymptotic results are available which are summarized in [21]. In particular, it is shown that for high levels r_1 the sample path of the process after an upcrossing is a parabola whose peak is exponentially dis-tributed. This leads to the approximate conditional rates:

$$\nu_{R_1 \cap \bar{R}_2} = \nu_{R_1} P(S_m \leq R_1) = \nu_{R_1} [1 - \exp[- (r_1 r_2 - r_1^2]] \quad (27)$$

$$\nu_{R_1 \cap R_2} = \nu_{R_1} P(S_m > R_2) = \nu_{R_1} \exp[- (r_1 r_2 - r_1^2)] \quad (28)$$

Unfortunately, these approximations appear to be good only for rather high levels r_1 and r_2.

If, on the other hand, the first case is present eq. (22) or (23), i.e. where the failure time to collaps is the sum of the failure times of the components, are no more valid. An exact analysis of this situation appears very cumbersome but possible. Here, we pro-pose an approximation. The sums in eqs. (22) or (23) are truncated as soon as the crossing rate or even the probability of first passage of the next component would become smaller than for the previous one all evaluated at the β-point. In this case one does need not to investigate further multiple failures from the remaining set of components in the failure path.

In general, there exist a very large number of paths to system failure. In practical computation it will, therefore, be necessary to limit the analysis to only a few failure paths which prefereably are the dominant (most likely) ones. They can be found by appropriate search algorithms. Suitable algorithms have been proposed by several authors [18,19,20] for time-invariant structural system reliability analyses. The one proposed in [20] which has been adapted to time-variant reliability in [9] might also be used here although it still is considered to be suboptimal.The algorithm can be described as follows. Let there be a set of M = ⟨1, 2, ...,⟩ failure events a finite number of subsets of which leads to system failure. For the intact structure all componental failure probabilities are computed. Each component is the starting point of a time-variant failure tree. The component with largest failure probability, then, is transferred into a failed state which implies an updating of the stiffness matrix of the system. It is now necessary in case II to check whether after load redistribution there is no component whose failure is implied by the failure of the first component. If there are any those have also to be transferred into a failure state. If this is true for all remaining components in the failure path the search is terminated because a path to failure is found. Otherwise, in order to find the next most likely state change in the system the probabilities of joint occurence of the first state change event and the possible consecutive events are computed. If one of these joint probabilities is larger than the previously calculated probabilities the corresponding component is transferred into a failure state. A second updating of the stiffness matrix is performed. At this stage we again need to check the likelihood of multiple failure by use of formula (28) and its generalisations followed by a check of presence of redundancy in the system. The algorithm, then, continues with either two simultaneous and one consecutive or three consecutive failure events involved. If, however, smaller probabilities have been calculated previously, the procedure continues at those components after having restored the system properties back to the degradation state of interest. Eventually, a sequence failure event will be found. Its occurence probability is the joint probability of all events in the sequence which is also the largest probability computed up to now. This terminates the algorithm. A lower bound for the system failure probability, then, is the probability of the union of all complete failure sequences. The lower bound in eq. (26) applies. An upper bound is the probability of the union of all complete and incomplete failure sequences for which the upper bound in eq. (26) should be used. In order to improve these bounds, one might include the next most likely failure paths which can be obtained in an analogous manner. This technique to produce strict probability bounds by a combination of an optimal search for dominant complete and of further incomplete failure paths and of eq. (26) facilitates the analysis of larger systems very much. For real structures such as offshore platforms the choice of the particular search algorithm appears to be only a secondary problem because the various proposed algorithms essentially lead to the same results.

4. Numerical example

As in [7] and [8] we shall investigate in more detail one of the mechanically simplest redundant systems shown in figure 3. This so-called Daniels-System (after Daniels who first studied its reliability in [22]) has n physical components whose stochastic properties all have the same distribution function. If a component fails its load is distributed equally among the remaining components. These assumptions enable not only a number of simplifications in the formulation but circumvent the problem of considering a large number of failure paths. We shall especially use certain results presented in [23] for the time-invariant case. Yet, we can illustrate all relevant aspects outlined before.

We adapt the following fatigue deterioration model which is somewhat simpler than the one associated with eq. (5) (See, for example, [24]). It is assumed that the decrement of residual strength is proportional to some function of the stress range ΔS and inversely proportional to some power of the actual strength, i.e. the governing differential equation is [25]:

$$\frac{dR_j(\tau)}{d\tau} = - h(\Delta S_j)/(m\ R_j(\tau))^{m-1} \qquad (29)$$

Using the usual S-N-curve information in the form $KNS^b = 1$ for a narrow-band Gaussian load effect process leads to (see also eq. (4)) [23]:

$$R_j(\tau) = R_j(o) \left(1 - K \frac{(2\sqrt{2})^b}{R_j^m(o)} \sigma_{S_j}^b \Gamma(1 + b/2) \nu_o \tau \right)^{1/m} \qquad (30)$$

This is a monotonically decreasing function for any positive m. Assume further that the only uncertain variable is the initial strength R(o) which is normally distributed with mean E[R(o)] and standard deviation D[R(o)]. Alternatively, the parameters K, b, and m can be introduced as random variables, but must not depend on j in order to render the special formulation possible. There are good reasons for a non-negligible inter-element correlation. Therefore, for initial strength values which are positively and equally correlated one has the following representation (Rosenblatt-transformation) for the various $R_j(o)$'s:

$$R_j(o) = E[R(o)] + D[R(o)] \left(U_o \sqrt{\rho} + U_j \sqrt{1-\rho}\right), \; j = 1, \ldots, n \qquad (31)$$

In order to establish the sequence of element failures, we need the order statistics of $(R_1(\tau), R_2(\tau), \ldots, R_n(\tau))$. Since the parameters in the second factor in eq. (28) are assumed constant, the order statistics $\hat{R}_1(\tau) \le \hat{R}_2(\tau) \le \ldots \le \hat{R}_n(\tau)$ can be derived from the order statistics of the $R_j(o)$'s. In [24] it is shown that the order statistics of a vector of independent standard normal variables have Rosenblatt-transformation:

$$\hat{U}_1 = \phi^{-1} [1 - \phi(-U_1)^{1/n}]$$

$$\hat{U}_j = \phi^{-1} [1- \prod_{k=1}^{j} \phi(-U_k)^{1/(n-k+1)}] \tag{32}$$

Therefore,

$$\hat{R}_j(o) = E[R(o)] + D[R(o)](U_o \sqrt{\rho} + \hat{U}_j \sqrt{1-\rho}) \tag{33}$$

with $\hat{U}_j$ as given in eq. (32). Eq. (33) inserted in the corresponding eq. (30) yields the required order statistics $\hat{R}_j(\tau)$.

The numerical calculations to be discussed are performed with the following set of data:

$S(\tau) \sim N\ (0.5,\ 0.1)$,

$R_j(o) \sim N\ (0.8,\ 0.2)$,

$n = 4$,

$\rho = 0.3$,

$\nu_o = 1$,

$T = 10^6$

$m = 10^{-2}$

Furthermore, formula (3) is used throughout instead of the presumeably better formula (18).

At first, the case of time-invariant elemental resistance is investigated by setting $m \to \infty$. Case IB, that is (immediate) load redistribution without dynamic overshooting is calculated as follows.

The extreme value distribution of the load is:

$$F_{\max_{[O,T]} S}(x) = \exp[-\nu(x)\ T] \tag{34}$$

which easily is transformed by:

$$S_{max} = \nu^{-1}\left[-\frac{1}{T}\ln \phi(U_S)\right] \tag{35}$$

The system failure probability must be determined from [24]:

$$P_f(T) = P\left(\bigcap_{j=1}^{n} ((n-j+1)\hat{R}_j(o) - S_{max} \leq O)\right) = P\left(\bigcap_{j=1}^{n} \hat{F}_j\right) \tag{36}$$

The numerical calculations yield the following results in terms of the equivalent or generalized safety index $\beta = -\ \phi^{-1}\ [P(.)]$. The individual event and intersection safety indices are:

$\beta\ (\hat{F}_1) = 2.22$; $\beta\ (\hat{F}_1) = 2.22$;

$\beta\ (\hat{F}_2) = 2.71$; $\beta\ (\hat{F}_1 \cap \hat{F}_2) = 2.81$;

$\beta\ (\hat{F}_3) = 2.25$; $\beta\ (\hat{F}_1 \cap \hat{F}_2 \cap \hat{F}_3) = 2.97$;

$\beta\ (\hat{F}_4) = -0.35$; $\beta\ (\hat{F}_1 \cap \hat{F}_2 \cap \hat{F}_3 \cap \hat{F}_4) = 2.97$;

Table 1: Safety indices (Case IB)

It is obvious from the individual safety indices and from the increments in the system safety indices that the second "component" dominates the system. If this fails there is only a slight increase in reliability. And given that the third component fails, there is a probability of larger than 0.5 that the forth component will also fail due to the significant effect of load redistribution. Note that this formulation does not contain any information about the times of failure. In particular, there is no information whether the component $\hat{1}$ failed some time before $\hat{2}$ or both components failed simultaneously in a single "load wave".

For this case, we also study dynamic overshooting (case IA). Let j-1 components already be broken. An upper bound to the additional dynamic load for component j to break is $\hat{R}_{j-1}$. Therefore, after some rearrangements, eq.(34) is modified to:

$$P_f(T) = P\left(\bigcap_{j=1}^{n} \left((n-j+1)\,\hat{R}_j(o) - \hat{R}_{j-1}(o) - S_{max} \leq 0\right)\right) = P\left(\bigcap_{j=1}^{n} \hat{F}_j\right) \tag{37}$$

The numerical results for case IA are:

$\beta\,(\hat{F}_1) = 2.22$; $\beta\,(\hat{F}_1) = 2.22$;

$\beta\,(\hat{F}_2) = 1.79$; $\beta\,(\hat{F}_1 \cap \hat{F}_2) = 2.65$;

$\beta\,(\hat{F}_3) = -0.51$; $\beta\,(\hat{F}_1 \cap \hat{F}_2 \cap \hat{F}_3) = 2.65$;

$\beta\,(\hat{F}_4) = -3.22$; $\beta\,(\hat{F}_1 \cap \hat{F}_2 \cap \hat{F}_3 \cap \hat{F}_4) = 2.65$;

Table 2: Safety indices (Case IA)

As expected, the order statistics safety indices now decrease substantially. The effect of redundancy is moderate after failure of the weakest component. Therefore, significant dynamic effects, if present, require special consideration in applications. They are neglected in the following.

Next, we study the case that during a "wave" there is essentially no load redistribution and no dynamic overshooting (case IIB). The resistances are still time-invariant. The corresponding numerical calculations are shown in figure 4. In this case, all possible failure paths are investigated. Two safety indices are given for each mode in the failure tree of figure 4. The upper value corresponds to the first-order reliability method, i.e. to $\phi(-\beta)$. The lower value corresponds to eq. (15), i.e. to the more accurate (asymptotic) second-order result. The equivalent safety index is defined as before. It is seen that the second-order corrections are significant in this example. The Rosenblatt-transformation for the order statistics of the $R_j(o)$'s has been given recursively in "ascending" order. An equivalent transformation can also be given in descending order. In [26] it is demonstrated that the numerical results of FORM or SORM can also depend on the special type of Rosenblatt-transformation. The first event, therefore, is computed with both the ascending (arrow indicating upwards) and the descending (arrow indicating downwards) transformation. Comparison of the two sets of results shows that the first-order results can differ by a certain amount whereas the second-order results are sufficiently stable. That this is so can also be taken as a verification of the high numerical accuracy of SORM. In all subsequent figures only the SORM-results are given. The system safety indices along a failure path must increase. But from figure 4 it is evident that after seperate or joint failure of the two weakest elements in the structure little extra reliability is gained when the last two elements are also included in the analysis. This corresponds to the earlier findings. An interesting piece of information about the system behavior is also the (most likely) fraction of time spent in each system state indicated in the last column in figure 4. For example, for the first failure sequence we see that it takes 27 % of the sequence life to failure of the first component, another 59 % to the second component failure and 14 % to the third component failure which is immediately followed by failure of the last component. In the second critical fifth failure sequence 99 % of the sequence life is spent without any failure but sequence failure occurs shortly after joint failure of the first two components. Comparison of the system β's corresponding to case I and case II verifies that case II has higher reliability as it should be, although the difference is not very large in this example.

A similar figure 5 has been produced for components subject to fatigue, i.e. by applying eq.(23) and eq.(28) with the given parameters. Only case IIB is treated. The beta values now are substantially smaller. In addition, the ordering of the failure sequences according to their system safety index is different from that in figure 4. As expected, the sequences now spend more time of their total life time in the earlier degradation states.

Finally, we return to the frequently more realistic case of immediate load redistribution but without dynamic overshooting in order to investigate structural degradation in time (case IB). The algorithm described at the end of the foregoing section is applied. The failure sequences computed by the algorithm are shown in figure 6. At first, component $\hat{1}$ is transferred into a failure state after some time T_1. Next, component $\hat{2}$ is transferred into a failure state at time $T_1 + T_2$. At this stage the redistribution of forces reveals that failure of $\hat{2}$ is most likely followed by failure of $\hat{3}$ and $\hat{4}$. Hence, the sequence is truncated at this point. Investigation of multiple failure further shows that sequence "$\hat{F}_1 \cap \hat{F}_2$" is more likely than sequence "$\hat{F}_1$ then $\hat{F}_2$". The sequence is not investigated because the level of $\hat{3}$ almost coincides with the level of $\hat{1}$. "$\hat{F}_1 \cap \hat{F}_2 \cap \hat{F}_3$". One concludes that for this system the dominant sequence is "$\hat{F}_1 \cap \hat{F}_2$". There will hardly be a chance for repair to save the system once $\hat{F}_1$ or even $\hat{F}_1$ and $\hat{F}_2$ have failed. The overall equivalent safety index should be identical to the safety index already computed for case IB. That this is true only approximately must be attributed to the somewhat inconsistent computation of concept for the event "$\hat{F}_1 \cap \hat{F}_2$" when using eq. (28).

5. Summary and Conclusions

This study is directed towards the quantification of failure probabilities of structural components and systems under time-variant loading with and without fatigue induced strength deterioration. In particular, the degradation behavior of redundant structural systems is investigated in order to be able to design proper inspection and repair strategies. The so-called outcrossing approach is chosen and embedded in modern methods of numerical reliability analysis. Appropriate formulations are given which are numerically feasible although sometimes laborous. The computation schemes are especially suited to highly reliable systems.

The study of time-variant system reliability at first requires a realistic mechanical modelling in order to take account of dynamic effects and/or proper redistribution regimes for the internal forces upon componental failure. As in time-invariant structural reliability with brittle components the effect of structural redundancy usually is moderate. It appears that except in highly redundant structures there is little chance to repair one or more failed components in time. Usually, the failure of those components also imply structural collapse. Therefore, observations of the degradation state of components and the entire structure are very important. Several aspects of the theory and of modelling need to be improved. The most important are:

i. Methods for multiple failure probabilities
ii. Search algorithms for dominant componental failure sequences
iii. Incorporation of inspection observations
iv. Consideration of more realistic load models, e.g. by introduction of stationary sequences of sea states in applications to offshore structures.

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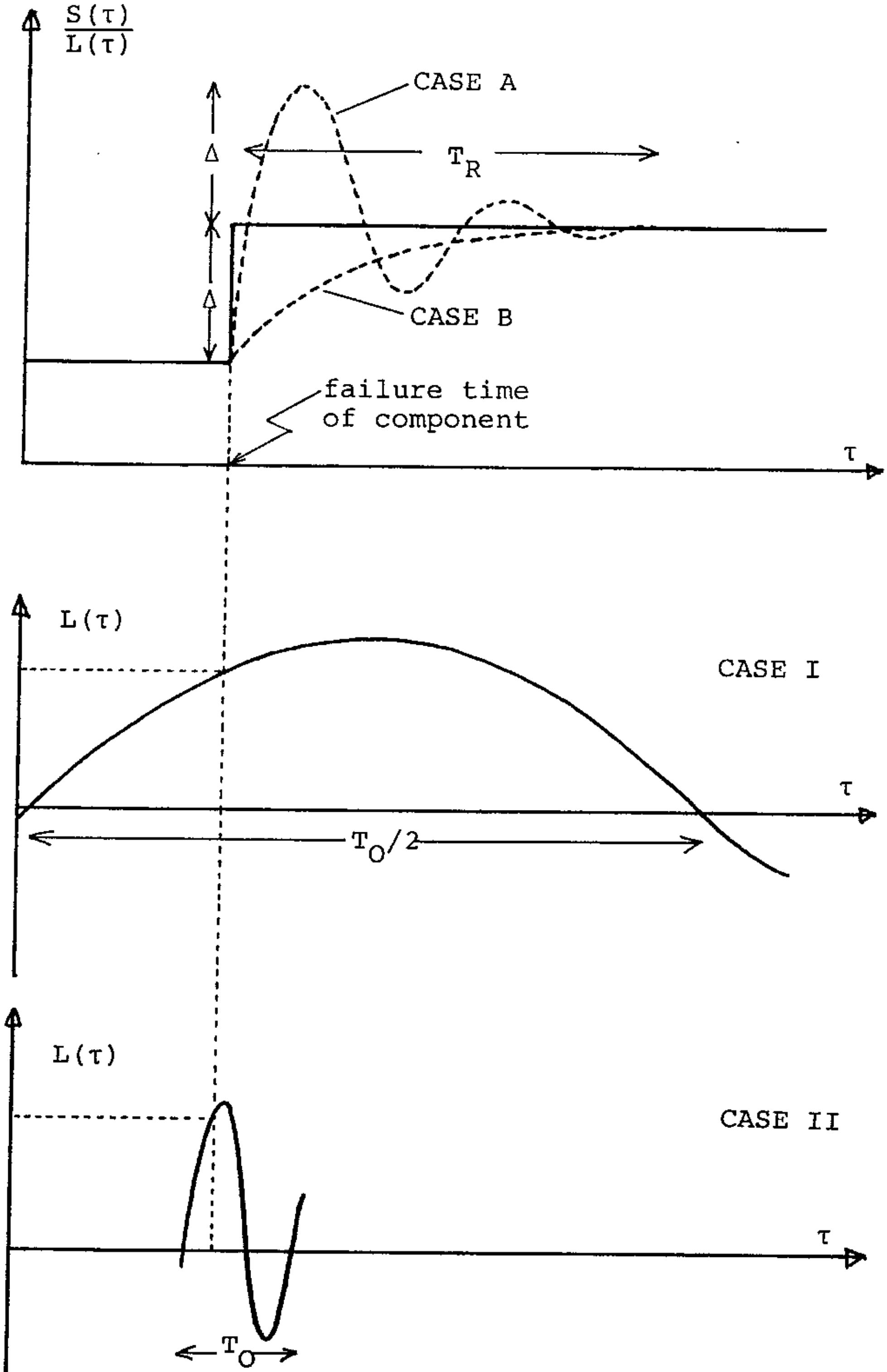

Figure 1: Illustration of extreme cases

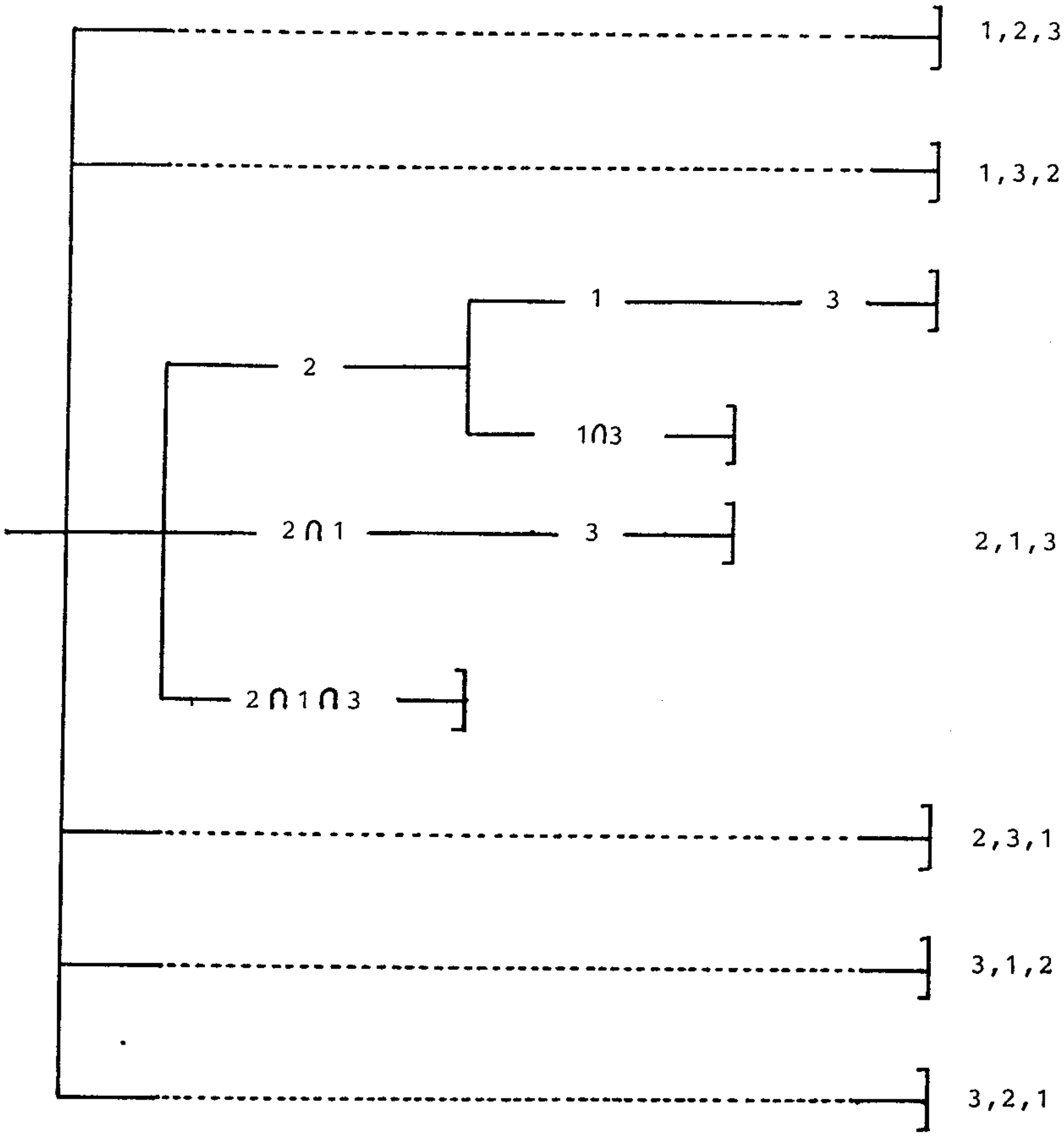

Figure 2: Failure tree of a 3-components system

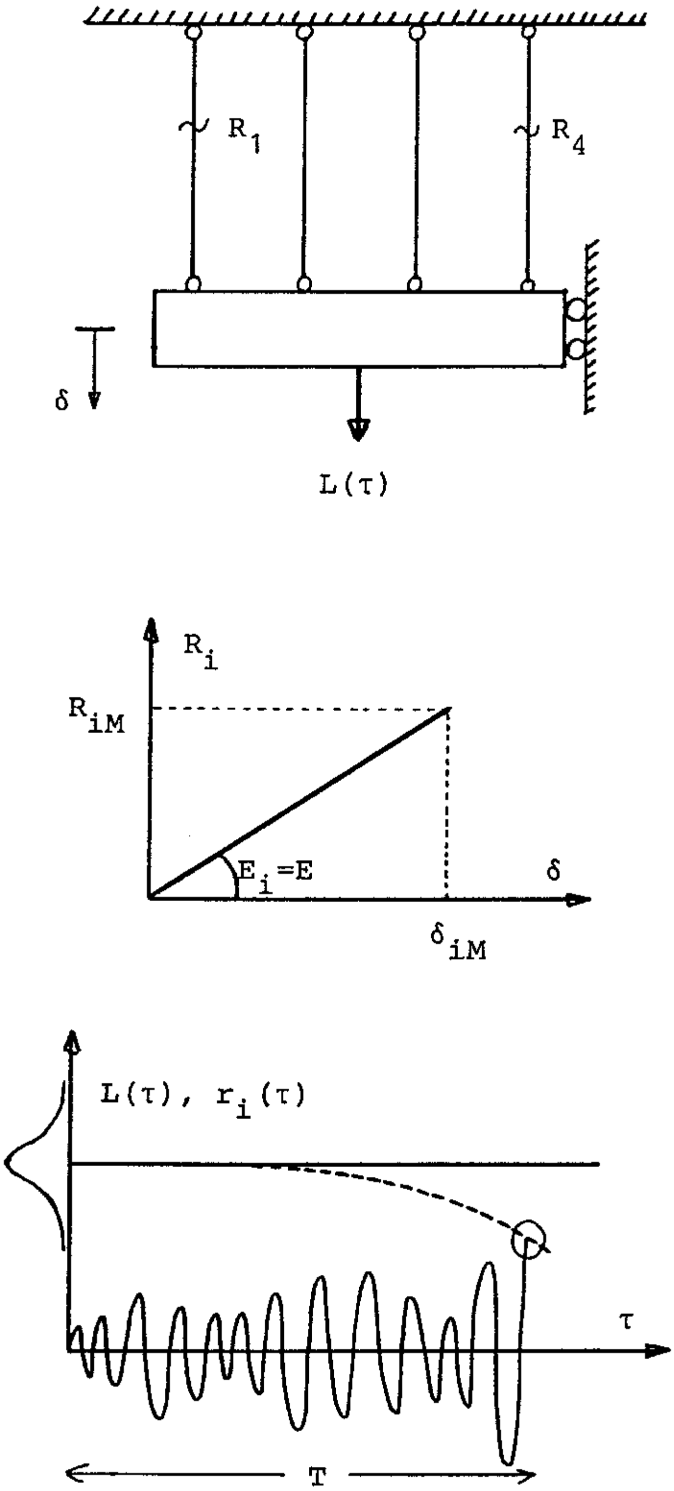

Figure 3 : (Non-) Deteriorating Daniels System

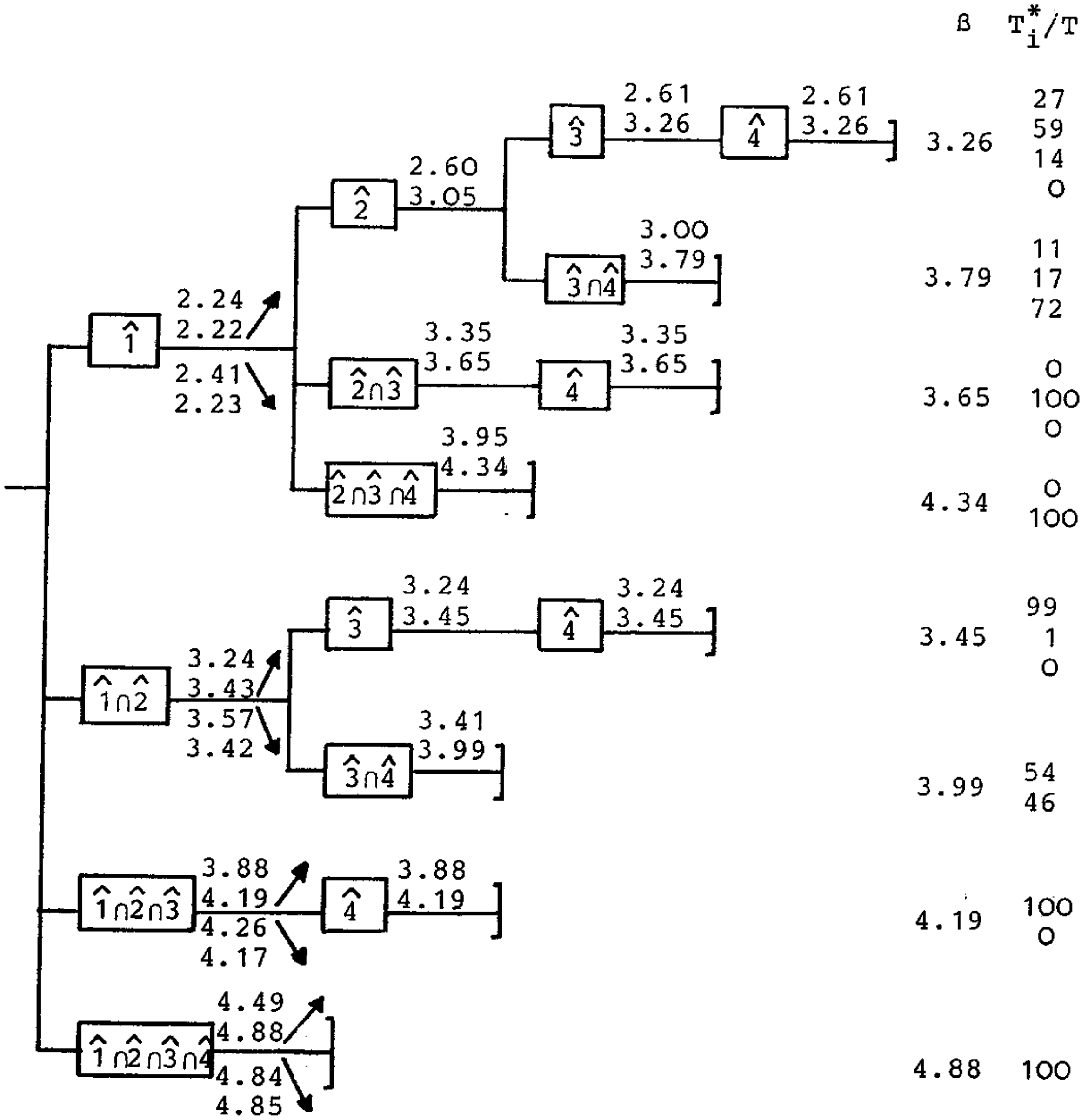

Figure 4: Failure tree

Non-deteriorating resistances, case II B

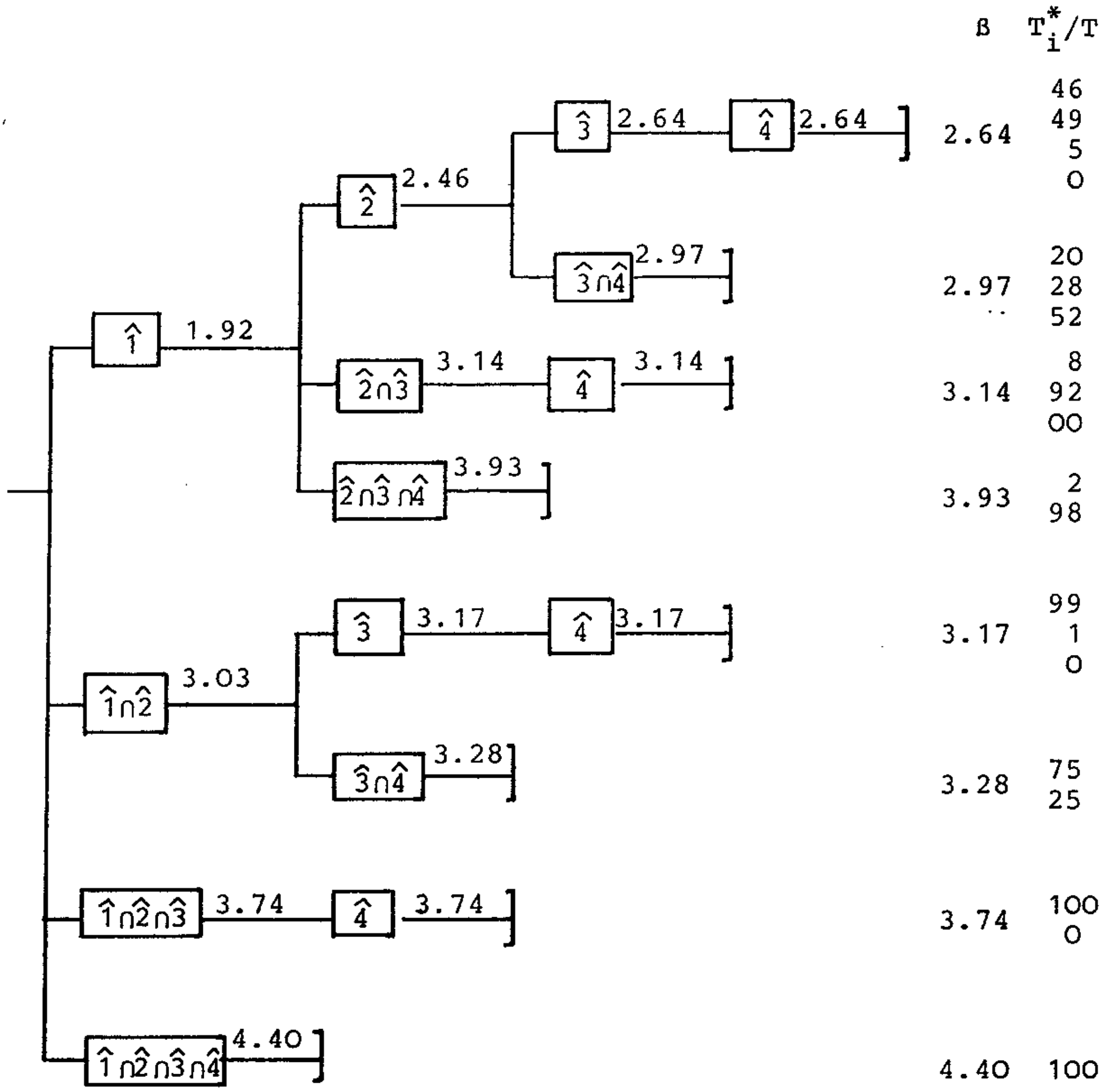

Figure 5: Failure tree

Deteriorating resistances, case II B

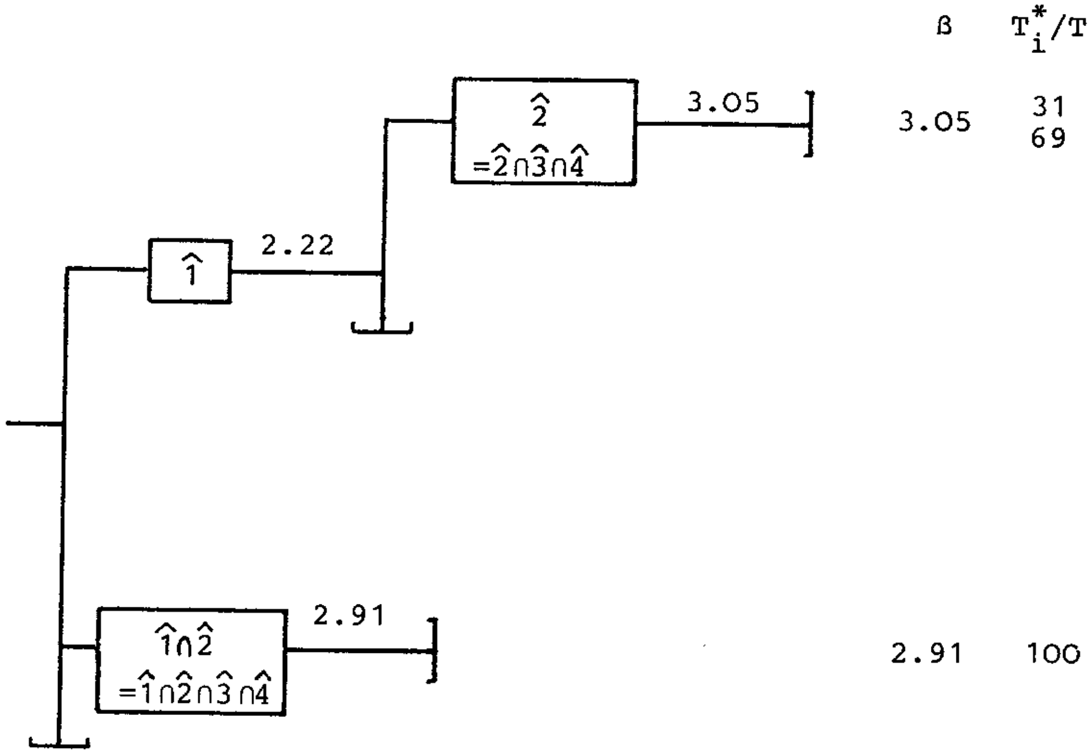

Figure 6: Truncated failure tree

Non-deteriorating resistances, case I B