Delay-aware Wireless Resource Allocation and User Association in LiFi-WiFi Heterogeneous Networks

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***Abstract*—The ever-growing wireless networks demand high capacity, have strict latency requirements, and must support diverse communication services. A LiFi-WiFi heterogeneous network has proven to be a useful tool to satisfy the growing capacity demand. However, to leverage these co-existing, non-interfering technologies, intelligent resource management schemes have to be developed. To support diverse applications with varying delay and data rate requirements, the resource management scheme should consider the Quality of Service (QoS) while allocating wireless resources. In this work, the downlink wireless resources are allocated to users such that the average network packet delay is minimized. Users that are both capable and not capable of multi-homing are considered and a separate optimization problem is formulated for each case. These problems are then solved using a global Branch and Bound-based solver and a genetic algorithm-based Metaheuristic is also proposed. The algorithms are then evaluated with simulations and the results show that the average network packet delay is significantly lowered and each user's strict QoS requirements are satisfied even in a network with heavy traffic flow.**

***Index Terms*—LiFi, delay-aware, HetNet, Multi-Homing, genetic algorithm**

I. Introduction

The number of networked devices is expected to reach a total of 3.6 per person by 2023 [1]. This results in a growing capacity demand. Multiple wireless access technologies integrated into a Heterogeneous Network (HetNet) have emerged as a promising solution to satisfy this demand. An indoor Radio Frequency (RF) HetNet could consist of a cellular femtocell and a Wireless-Fidelity (WiFi) cell. Recently, Light-Fidelity (LiFi) [2] has also been proposed as an access technology operating on the visible light and infra-red spectrum. LiFi, with its large spectrum size [3], does not interfere with RF systems, and a LiFi-RF HetNet can result in a throughput much higher than the individual technologies. In this work, we consider an indoor LiFi-WiFi HetNet.

To leverage the heterogeneity of the different access technologies optimally, intelligent methods for user association and resource allocation have to be developed. There is a large body of work [4]–[6] that focuses on the resource management problem in a LiFi-RF HetNet. However, these papers only focus on optimizing the resource allocation for the sum throughput or data rate of the network while assuring proportional fairness among users. Most works do not consider the delay requirements of the network. The wireless networks of the future are expected to support diverse communication applications with varying QoS requirements. Applications like live video streaming and Voice over Internet Protocol (VoIP) are delay-sensitive and also require a minimum guaranteed bandwidth. Therefore, for QoS flows, it becomes important to optimize the resource allocation considering network packet delay as a performance metric.

Apart from focusing on data rate optimization, most existing works only consider a HetNet in which the user can be served by only one technology at a time. This is due to the existing conventional user equipment that does not support Multi-Homing. But with the increasing support for Multipath transport protocols like Multipath Transmission Control Protocol (MPTCP), it is important to also consider Multi-Homing user devices where the users can be served by more than one wireless access technology simultaneously. This allows the aggregation of wireless resources and better utilization of the heterogeneity of the technologies.

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A. Related Work

In [5], the authors include the data rate QoS metric in the resource allocation optimization objective function in a LiFi-RF HetNet and propose an evolutionary game theory-based algorithm to solve the problem. However, the authors did not consider any delay-related metric. Effective capacity has been introduced in [7] as a method to model the channel in terms of QoS metrics. The authors in [8] allocate resources in a homogeneous RF network by maximizing the effective capacity. The effective capacity has also been used as the maximization objective in a homogeneous LiFi network in [9]. This work was then extended to a LiFi-RF HetNet in [10]. Although there exist multiple works tackling the user association problem in HetNets, not many consider the delay constraint. In this work, we emphasize minimizing the network packet delay while also guaranteeing a certain minimum delay requirement per user to support delay critical applications. The authors in [11] perform a similar optimization where they minimize the network packet delay without any constraints on the delay or bandwidth requirement in an RF-only HetNet. The authors propose a distributed algorithm to solve this problem. However, they do not consider user devices with Multi-Homing capabilities.

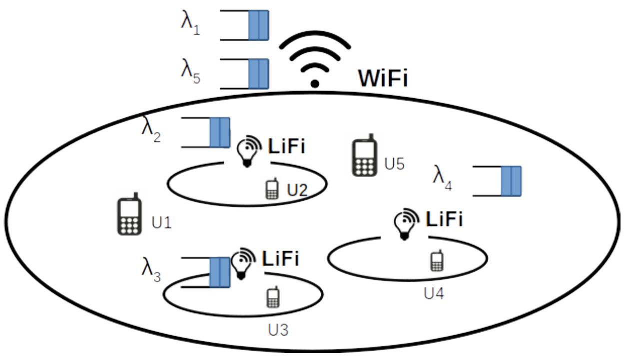


Fig. 1. Architecture of a LiFi-RF HetNet with QoS traffic

B. Contribution

In this paper, we consider a network with communication applications that have diverse delay and data rate QoS requirements. We focus on delay-critical applications in a Hybrid network and formulate a resource allocation Mixed Integer Nonlinear Programming (MINLP) problem with the objective of minimizing the average network packet delay for the QoS flows. We also guarantee the maximum delay and minimum data rate per user based on their requirement. We extend this formulation to a network with Multi-homing users to simultaneously allocate LiFi and WiFi resources. We then provide the solution to these problems using a Branch and Bound-based solver. We also propose a genetic algorithm to solve these problems and then evaluate all solution methods using simulations for varying system parameters.

C. Organization

The rest of the paper is organized as follows. Section II introduces the system model considered. The optimization problems for Hybrid and Multi-homing networks are described in Section III. Section IV details the solution methods to these optimization problems and the methods are evaluated in Section V. Finally, Section VI concludes the paper.

II. System Model

A. Network Architecture

In this paper, we consider an indoor heterogeneous LiFi-WiFi network with a total of N_α Access Points (APs) both LiFi and WiFi as represented in Fig. 1. All LiFi APs function at the same frequency resulting in high interference in the overlapping areas of the LiFi cells. We consider a single WiFi AP and multiple LiFi APs. There are a total of N_μ users that are equipped with LiFi and WiFi receivers. These users can be served by only one technology at a time in a Hybrid network and can be simultaneously served by both technology APs in a Multi-Homing network. Each user has its own QoS traffic that arrives at the network and has to be served in the downlink. The inter-arrival times of the data packets arriving at the downlink are independent for each user and follow an exponential distribution with a mean of $\frac{1}{\lambda_\mu}$ seconds. The length of the packets also follows an exponential distribution with a mean of L_μ bits. Apart from these QoS flows, we also assume that there is some Best Effort (BE) traffic that has to be delivered to the users or delay tolerant traffic that only has bandwidth constraints. We also assume that the APs are always on in order to serve the continuous BE traffic. To serve this traffic, each AP α allocates a resource proportion y_μ to each user μ. The resulting data rate is the resource proportion multiplied by the link data rate $R_{\mu,\alpha}$. Therefore, the service time for the QoS packets to each user is exponentially distributed with a mean of $1/(y_{\mu,\alpha} \times \frac{R_{\mu,\alpha}}{L_\mu})$. Hence the traffic to each user can be modeled as an M/M/1 queue.

In order to decide the resource proportion to be allocated to each user, all LiFi and WiFi APs are connected to a central controller. The controller has an overview of the wireless channel state information of all users in the network. Since the controller has global information, it can perform a centralized resource allocation. This allocation has to be repeated at regular intervals to accommodate the changing channel information. This process not only decides the resource proportion but also decides the AP for the user to associate to, in a Hybrid network.

B. LiFi Channel Model

The channel model for LiFi is as explained in [5]. For the sake of brevity, we only define the data rate. The specifications for the data rate are taken from the documentation for LiFi hardware available in-house [12]. An Adaptive Modulation and Coding (AMC) scheme is used as in [5], and the link data rate between a user μ and an AP α is calculated as,

$$R_{\mu,\alpha} = \frac{2B_L}{Q} \sum_{i=1}^{\frac{Q}{2}-1} q_L(i), \tag{1}$$

where B_L is the LiFi modulation bandwidth and q_L is the spectral efficiency on i-th sub-carrier with a total of Q subcarriers. The maximum capacity offered by a LiFi AP is 54 Mbps.

C. WiFi Channel Model

The IEEE 802.11n standard is used to model the WiFi network. The channel bandwidth is 20 MHz , according to which a WiFi AP's total capacity is 65 Mbps [13]. The channel model is as defined in [5] and the link data rate between a user μ and an AP α is calculated as,

$$R_{\mu,\alpha} = \frac{B_R}{Q} \sum_{i=1}^{Q-1} q_R(i), \tag{2}$$

where B_R is the WiFi bandwidth and q_R is the spectral efficiency on i-th sub-carrier with a total of Q subcarriers.

D. Blockage Model

The Line Of Sight (LOS) signal from LiFi APs can easily be blocked due to the properties of the visible light signal. Therefore, it is important to consider the effect of blockages on the network. The occurrence of blockages is modeled using

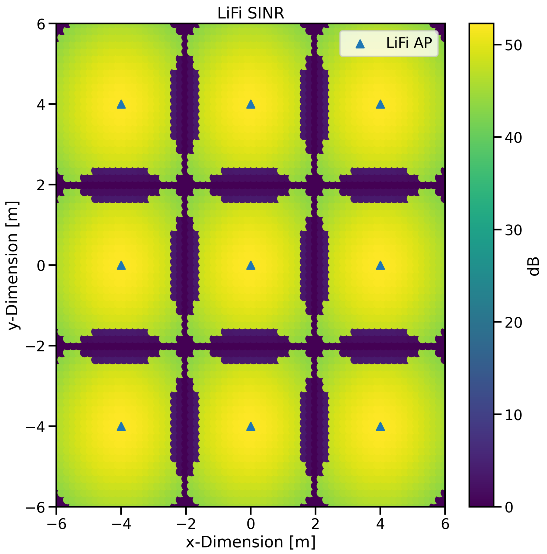


Fig. 2. LiFi SINR in the conference room topology

a Bernoulli distribution. The probability, Pr_{μ}, that a user μ is blocked in a state is given by,

$$Pr_{\mu} = \begin{cases} p & \mu \text{ blocked} \\ 1-p & \mu \text{ not blocked}, \end{cases} \tag{3}$$

where p is user's blocking probability. This model is apt for modeling transient blockages.

E. Network Topology

We consider an indoor conference room topology as this is a common application of a LiFi-WiFi HetNet. The indoor conference room topology has one WiFi AP at the center of the room and LiFi APs positioned in a lattice format with randomly positioned mobile users. The area of the room is 6x6 m^2. The distribution of the LiFi Signal to Interference and Noise Ratio (SINR) in such a room is depicted in Figure 2.

III. Problem Formulation

In this section, the problem formulation for the resource allocation minimizing network delay for a LiFi-WiFi HetNet is described for both Hybrid and Multi-Homing networks. For this purpose we use the average network packet delay as the metric to minimize.

A. Hybrid Networks

In a hybrid network, the user can only be served by one AP at a time. So the resource allocation process has to decide the association of the user to the AP $x_{\mu,\alpha}$ and the resource proportion $y_{\mu,\alpha}$ allocated by this AP. $x_{\mu,\alpha}$ is a binary variable that is 1 when the user μ is connected to the AP α. In such a network, the network packet delay, $\tau_{\mu,\alpha}$, is described as,

$$\tau_{\mu,\alpha} = \frac{1}{y_{\mu,\alpha}\frac{R_{\mu,\alpha}}{L_{\mu}} - x_{\mu,\alpha}\lambda_{\mu}} \tag{4}$$

where $y_{\mu,\alpha}R_{\mu,\alpha}/L_{\mu}$ is the service rate of the data packets to the user μ.

The goal of the resource allocation problem is to minimize the average network packet delay while guaranteeing a maximum delay and minimum data rate for the data flow to the users. Therefore the optimization problem is formulated as,

$$\min_{x_{\mu,\alpha},y_{\mu,\alpha}} \frac{1}{\sum_{\mu=1}^{N_{\mu}}\lambda_{\mu}} \sum_{\alpha=1}^{N_{\alpha}}\sum_{\mu=1}^{N_{\mu}} x_{\mu,\alpha}\lambda_{\mu}\tau_{\mu,\alpha} \tag{5}$$

$$\sum_{\alpha} x_{\mu,\alpha} = 1 \qquad \forall\mu = 1, ...N_{\mu} \tag{6}$$

$$\sum_{\mu} x_{\mu,\alpha}y_{\mu,\alpha} \leq 1 \qquad \forall\alpha = 1, ...N_{\alpha} \tag{7}$$

$$\sum_{\alpha} x_{\mu,\alpha}\tau_{\mu,\alpha} \leq \tau_{thresh,\mu} \qquad \forall\mu = 1, ...N_{\mu} \tag{8}$$

$$\sum_{\alpha} x_{\mu,\alpha}y_{\mu,\alpha}R_{\mu,\alpha} \geq R_{thresh,\mu} \qquad \forall\mu = 1, ...N_{\mu} \tag{9}$$

$$x_{\mu,\alpha} \in \{0,1\}; y_{\mu,\alpha} \in [0,1] \tag{10}$$

The equality constraint in (6) says one user can only associate to one AP. The constraint in (7) describes the limit on the maximum capacity of the AP. The constraints in (8) and (9) define the delay and data rate requirement. Here, we assume, that apart from the QoS packets, there is also other traffic to the user that contributes to the data rate requirement of each user.

B. Multi-Homing Networks

In a Multi-Homing network, the user can be served simultaneously by two technologies and one AP per technology at a time. So the resource allocation process only has to decide the resource proportion $y_{\mu,\alpha}$ to be allocated by both technology APs. The best AP to serve the user is selected based on the maximum SINR offered by the AP. So, the LiFi AP that offers the best SINR among all LiFi APs and the single WiFi AP forms A_{μ} which is the set of best APs for a user μ. In such a network, the network packet delay, τ_{μ}, is described as,

$$\tau_{\mu} = \frac{1}{\sum_{\alpha\in A_{\mu}} y_{\mu,\alpha}\frac{R_{\mu,\alpha}}{L_{\mu}} - \lambda_{\mu}} \tag{11}$$

where $\sum_{\alpha\in A_{\mu}} y_{\mu,\alpha}\frac{R_{\mu,\alpha}}{L_{\mu}}$ is the aggregate service rate offered by both technologies to the user μ.

With the same goal as before, the optimization problem is formulated as,

$$\min_{y_{\mu,\alpha}} \frac{1}{\sum_{\mu=1}^{N_{\mu}}\lambda_{\mu}} \sum_{\mu=1}^{N_{\mu}} \lambda_{\mu,\alpha}\tau_{\mu} \tag{12}$$

$$\sum_{\mu} y_{\mu,\alpha} \leq 1 \qquad \forall\alpha = 1, ...N_{\alpha} \tag{13}$$

$$\tau_{\mu} \leq \tau_{thresh,\mu} \qquad \forall\mu = 1, ...N_{\mu} \tag{14}$$

$$\sum_{\alpha\in A_{\mu}} y_{\mu,\alpha}R_{\mu,\alpha} \geq R_{thresh,\mu} \qquad \forall\mu = 1, ...N_{\mu} \tag{15}$$

$$y_{\mu,\alpha} \in [0, 1] \quad (16)$$

The constraint in (13) describes the limit on the maximum capacity of each AP. The constraints in (14) and (15) define the delay and data rate requirement.

IV. Resource Allocation schemes

The hybrid network optimization problem described in Sec. III is an MINLP problem since it has both integer $x_{\mu,\alpha}$ and real valued $y_{\mu,\alpha}$ variables. The constraints define the feasible region. In general, MINLP problems are mathematically intractable. Branch and bound [14] algorithms are the most commonly used algorithms to tackle mixed integer problems. In a branch and bound algorithm, the optimization problem is recursively split into smaller sub-problems until the sub-problems are easy to solve. To avoid going through all possible enumerations of the variables, the problem is pruned efficiently. In this work, we use the open source Couenne [15] solver that solves MINLP problems with global optimality using a spatial branch and bound algorithm. We also use this solver on the Multi-Homing problem for comparison. The entire optimization framework is modeled in Pyomo [16] which is an open source optimization modeling language implemented in Python. It provides interfaces to various solvers like the Couenne solver.

Apart from the branch and bound-based solver, we also propose a genetic metaheuristic algorithm to solve the problem with lower complexity without sacrificing the optimality of the solution. Genetic Algorithms (GAs) are metaheuristic algorithms based on the theory of natural selection where the fittest individuals of each generation are selected for reproduction to produce the population of the next generation. A typical genetic algorithm consists of the following stages.

1) Initial Population: The initial population consists of possible solutions to the optimization problem. We start with a random initial population of integer and real values for the integer and real valued variables.
2) Fitness: The fitness function or the objective function decides the best individual/solution by assigning a fitness score. This score decides if those individuals will be chosen for reproduction. The fitness functions we consider are the objective functions as described in (5) and (12).
3) Selection: In this stage, the fittest individuals according to the fitness score are selected.
4) Crossover: In this stage, the parent individuals are combined to produce one or more offspring. These offspring form a part of the next generation. The offspring are generated based on the variable bounds as described in (10) and (16). In this work, we use the Simulated Binary Crossover (SBX) [17] operator to perform the crossover with integer and real variables.
5) Mutation: Some of the new offspring can be mutated with a certain probability to maintain diversity in the new generation and to avoid fast convergence of the algorithm which would result in a local optimum. In this work, the decision to mutate is sampled from the same probability distribution as the crossover.
6) Termination: The process terminates when the algorithm converges and the variable solutions are feasible solutions.

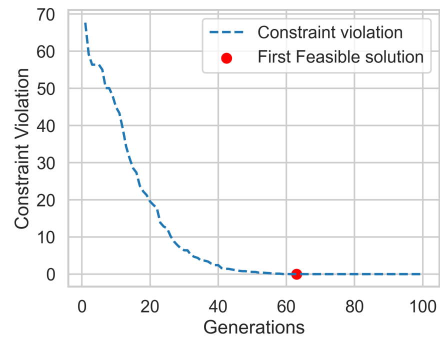


Fig. 3. Convergence of the genetic algorithm

To define the feasibility of the solution, the constraints have to be modeled. Typically, metaheuristics work with unconstrained problems and to add constraints, manipulation of the objective or fitness function is required. Constraints can be added to the fitness function in terms of a penalty function. The violation of a constraint penalizes the value of the objective function, thus making that individual undesirable. The genetic algorithm ranks all individuals according to their feasibility and prefers the feasible solutions over the others. The algorithm converges when the constraints are satisfied and the objective function is minimized. Fig. 3 shows the average convergence time for the genetic algorithm used. The constraint violation reaches 0 and the algorithm converges within a few tens of generations or iterations.

V. Evaluations and Discussions

The Hybrid and Multi-Homing optimization problems have been solved using the solution methods described in the Sec. IV and have been extensively evaluated in simulations. The simulations are performed for the conference room scenario for different blocking probabilities, receiver orientations, data rate requirements, and traffic patterns. The results of this evaluation are presented in this Section. The simulation parameters used are described in Table I. Fig. 4 shows the Cumulative Distribution Function (CDF) of the average network packet delay for a packet of mean length of 1000 bits arriving at an average arrival rate of 1000 packets per second. All users have the same traffic pattern and have a LiFi blocking probability of 0.1. The results for the Couenne solver-based solution (Hybrid, Multi-homing) and the genetic algorithm (Hybrid GA and Multi-homing GA) are compared with a simple maximum Signal to Noise Ratio (SNR) algorithm (Max-SNR) since this is the strategy implemented in devices in the market currently. Since the Max-SNR does not consider the QoS requirements, the delay has a wide range. But it can be seen that the minimum delay is comparable with the optimized

TABLE I
SIMULATION PARAMETERS

Parameter	Abbreviation	Value
Power of a LiFi LED	P_L	20 W
Half power beam width	$\theta_{1/2}$	90°
Physical area of received PD	A_p	10^{-4} m^2
FoV of the receiver	FoV	60°
Noise spectral density of LiFi links	N_L	10^{-21} A^2 Hz^{-1}
WiFi AP transmit power	P_R	20 dBm
Noise power of RF	σ	-57 dBm
Number of LiFi APs	N_L	9
Number of WiFi APs	N_R	1
Number of users	N_μ	12

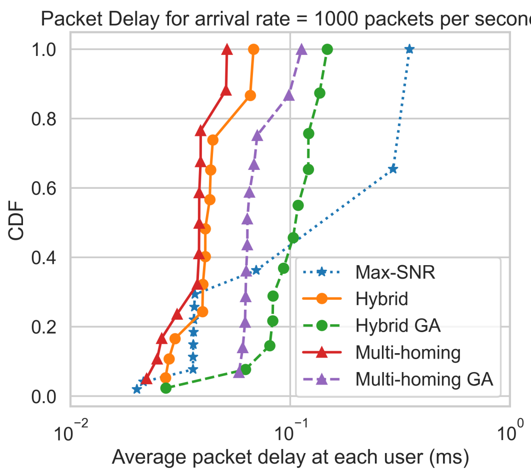
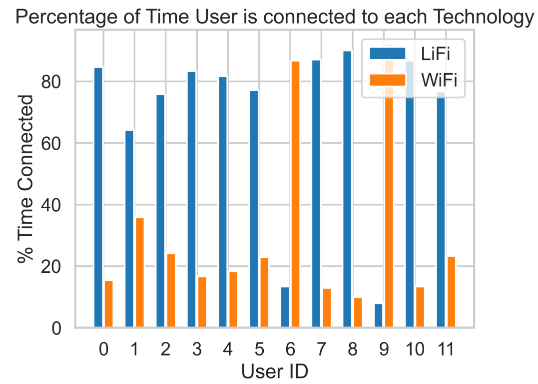
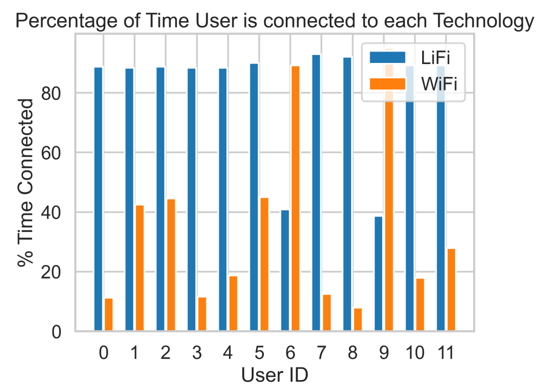


Fig. 4. CDF of the average packet delay for an arrival rate of 1000 packets per second



(a) Connection indicator for a Hybrid Network



(b) Connection indicator for a Multi-Homing Network

Fig. 5. Fraction of time that each user is connected to a certain wireless access technology

solution. This is because, typically, in a small-range LiFi cell the entire bandwidth of an AP is only shared by a few users resulting in a high capacity per user. Since overlapping cells interfere, the user usually has a choice of connecting to one LiFi and/or one WiFi AP. The GA solutions perform worse than the global optimizer as expected, but the difference is small enough that the solutions can be accepted. Overall, we also see that the average network packet delay is quite small in the sub-milliseconds range. This emphasizes the advantage of using small-range LiFi cells with high data rate density. If the user is connected to both technologies the delay is less as can be observed in the difference between the Hybrid and Multi-homing results.

Fig. 5 provides a detailed look into the percentage of time that each user was connected to certain access technology for the same scenario. The user is deemed connected when the data rate received is more than 0. The results show that the user is mostly connected to LiFi except when the light blockage would make it impossible. In the multi-homing scenario, we see that the user is still not always connected to WiFi and this is because of the maximum capacity constraint on the AP.

The behavior of the network is then evaluated for varying LiFi-specific parameters like receiver orientation and blocking probability while fixing the packet length to 9000 bits, the arrival rate to 20 pkts/s, the delay requirement to 10 ms, and the rate requirement to 5 Mbps for all users. Fig. 6 shows the average network packet delay for varying blocking probabilities. With an increasing blocking probability, the chance of connecting to the LiFi AP reduces and more users share the resources of the WiFi AP resulting in an increased delay. Here also we observe that although multi-homing offers a better delay, the difference is not high. Fig. 7 shows the average network packet delay for varying user device orientations. This

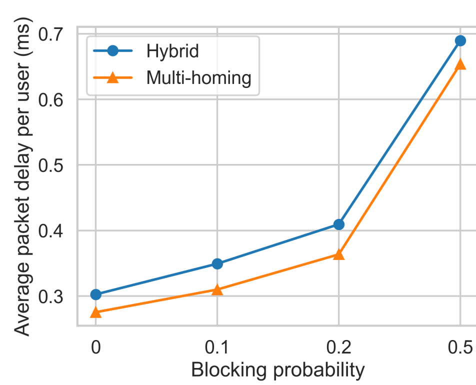


Fig. 6. Average packet delay for varying LiFi LOS blocking probabilities for a Hybrid and Multi-homing Network

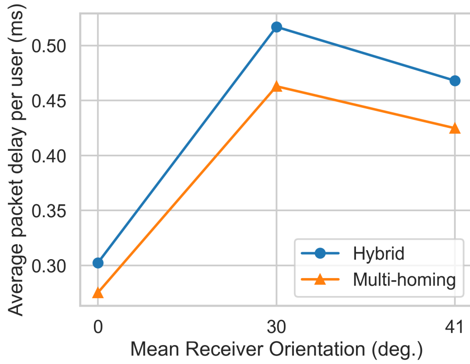


Fig. 7. Average packet delay for varying LiFi receiver elevation angles

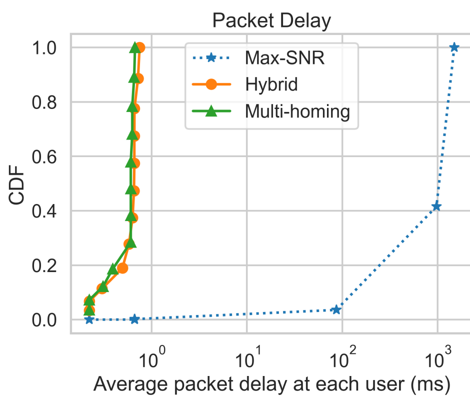


Fig. 8. CDF of average packet delay for a high traffic scenario

is an important parameter in LiFi, since the user orientation determines whether the received signal is within the Field of View (FOV) of the receiver. When the user device is parallel to the ground and facing up the elevation angle is 0° and the user receives the maximum amount of signal from the AP facing the user and hence the delay is minimum. 30° and 41° are the typical elevation angles of a mobile and stationary user respectively [18]. We then consider a high traffic network with an arrival rate of 1500 pkts/s for packets of length 9000 bits. We constrain the delay to 1 ms and the data rate to 15 Mbps. The results are shown in Fig. 8. In this case, the Max-SNR performs much worse than the optimized algorithms since the AP allocations are sub-optimal and cannot handle large traffic. Whereas, our proposed delay-optimized algorithms handle the high traffic and result in a sub-millisecond packet delay on average for all users in the network.

VI. Conclusion

In this paper, the joint wireless resource allocation and user association problem has been analyzed with the goal of minimizing the average network packet delay in order to support delay-critical applications. This analysis has been performed for both Hybrid as well as Multi-homing LiFi-WiFi HetNets. The problem has been formulated as a delay minimization problem with constraints on the maximum delay and achievable data rate of each user. This problem has then been solved using a Branch and Bound algorithm-based solver, Couenne. The problem has also been solved using the proposed Genetic Algorithm. The solution methods have been extensively evaluated using simulations and the results show that the network delay can be significantly reduced by this method compared to the currently existing max-SNR-based methods. Moreover, we see that strict delay and rate requirements are satisfied even in networks with a heavy QoS traffic flow. The results also show that a LiFi-WiFi HetNet is suitable for low latency networks, potentially achieving even a sub-millisecond latency which is only improved by using Multi-homing user devices.

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