Electron density modelling based on Inequality constraints to study the impact of space weather events

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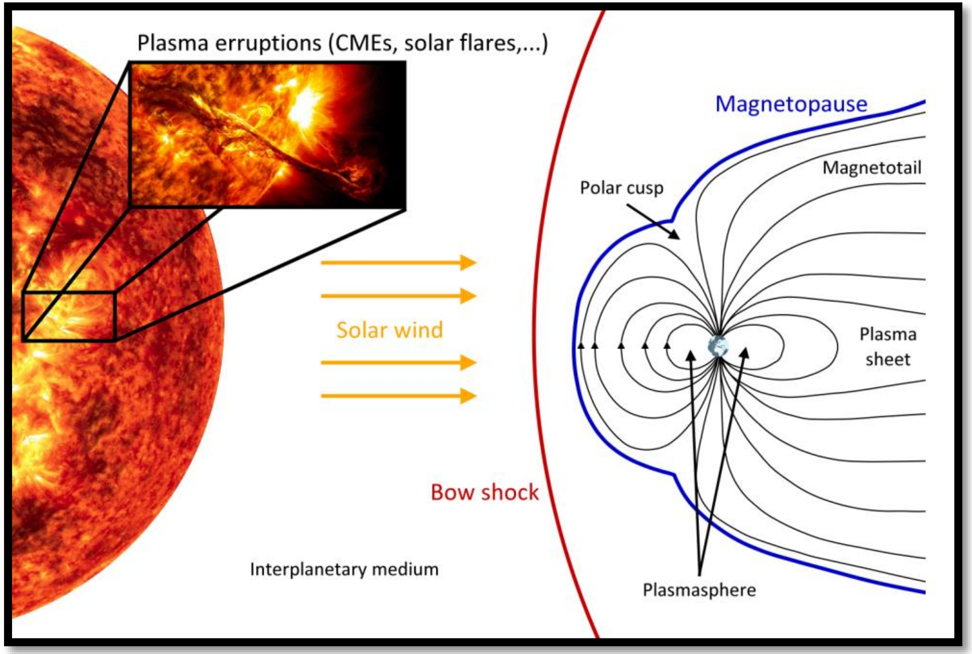
Introduction

- **Space weather events** such as solar storms, CMEs, lead to higher solar irradiance and solar wind.

- Both of them are detrimental to the **dynamics of ionosphere**. The overall ionization and recombination is affected and it results in a differential between the neutral particles and charged particles in the thermosphere – ionosphere system.

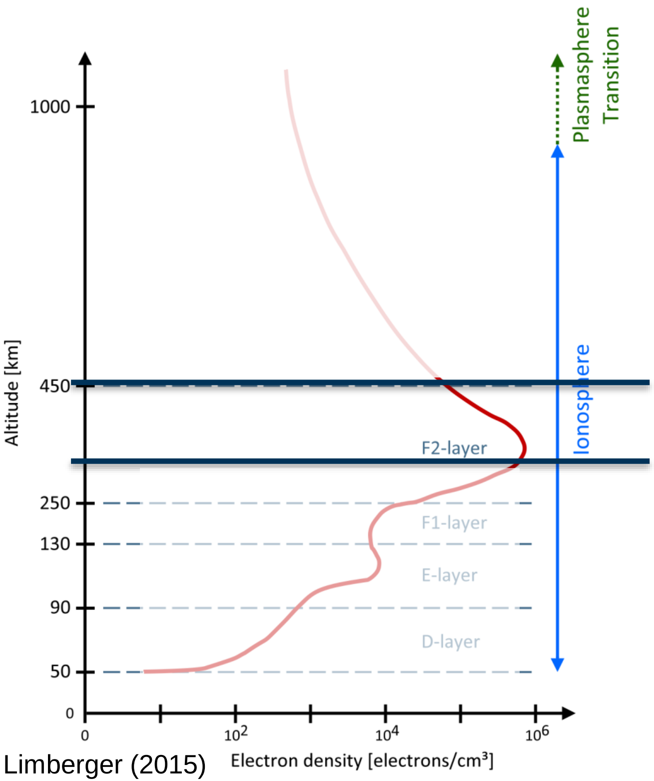
- Essential parameters of the ionosphere undergo changes during these events.



Schmidt (2018)

- The **aim of this presentation** is to show a systematic approach to **modelling the parameters of ionosphere electron density** across multiple ionospheric layers **using constraint optimization approach.**

Electron density modelling – multi layer approach



Limberger (2015)

- Vertical structure of the ionosphere can be divided into different layers.
- The electron density contribution of each of these layers can be mathematically described using profile functions (such as **Chapman function** [Chapman (1931)]).
- For e.g. the F2 layer electron density profile can be represented using three parameters, namely – F2 **peak density**, **peak height** and **scale height** [Limberger (2015)], [Liang (2017)].

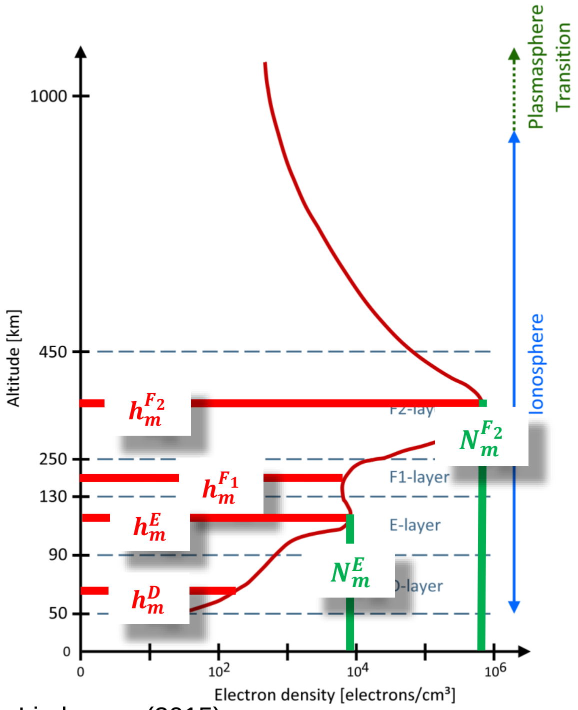
$$N_e(h) = N_m^{F_2} \exp\left(\frac{1}{2}\left(1 - \frac{h - h_m^{F_2}}{H^{F_2}} - \exp\left(\frac{h - h_m^{F_2}}{H^{F_2}}\right)\right)\right)$$

F2 **peak density**

F2 **peak height**

F2 **scale height**

Electron density modelling – ionosphere key parameters



Limberger (2015)

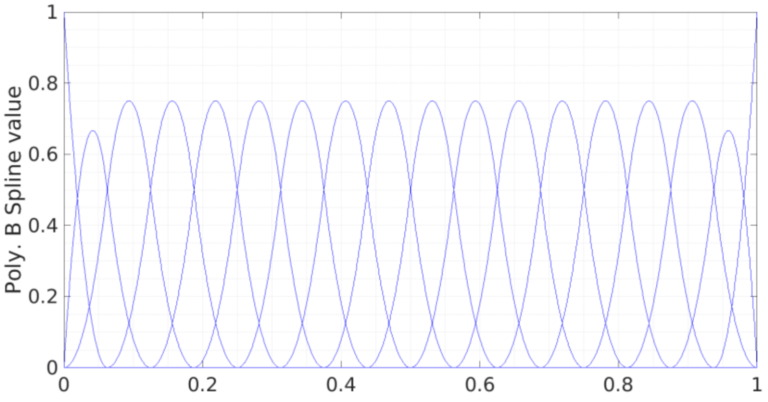
- The different layers are characterized by their respective "key parameters" (KP).
- Under moderate to extreme space weather events, the ionosphere key parameters undergo significant changes (due to the ionosphere expanding and compressing)
- Overall electron density profile is then parameterized by a set of different key parameters.
- Our approach is to represent the key parameters using uniform **B-spline** basis functions (series expansion).
- We **estimate the coefficients** of B-spline series of the different key parameters.

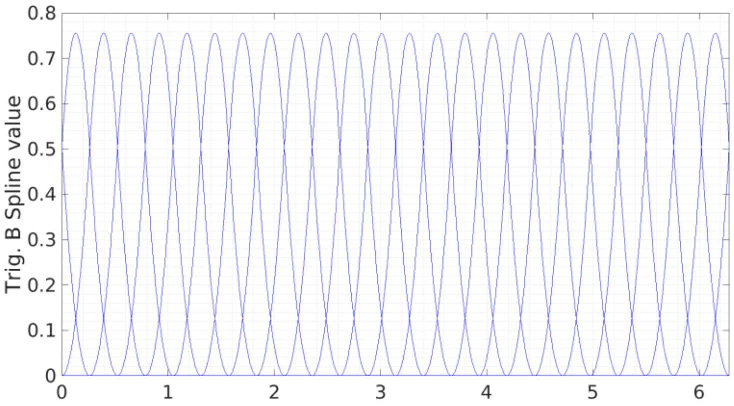
Electron density modelling – B-spline basis functions (Optional)

- B-splines are one of the basis function used in the field of ionosphere modelling [Schmidt (2007)].
- We use **polynomial B-splines** along latitude and **trigonometric B-splines** along longitude.

- B-splines are characterized by their degree and level. We use polynomial B-splines with degree 2 and level 4. We use trigonometric B-splines with degree 2 and level 3. This pairing of B-spline levels is referred to as using **levels (4, 3)**.

- Level (4,3) is comparable to spherical harmonics with degree, order (15, 15) [Schmidt et. al. (2015)].





Electron density modelling – multi layer approach with B-splines

Space weather events

Space weather events affect each layer in possibly different ways due to the **energy absorption** and the **composition of the layer.**

D-Layer	E-Layer	F_1-Layer	F_2-Layer	Plasmasphere

$$N_e(h) = N_e^D(h) + N_e^E(h) + N_e^{F_1}(h) + N_e^{F_2}(h) + N_e^P(h)$$

$$\mathcal{K} = \{N_m^D, h_m^D, H^D, N_m^E, h_m^E, H^E, N_m^{F_1}, h_m^{F_1}, H^{F_1}, N_m^{F_2}, h_m^{F_2}, H^{F_2}, N_0^P, H^P\}$$

- Therefore it is important to model the ionosphere layers in a combined manner.
- For modelling, each key parameter could be expressed as a B-spline series and its corresponding coefficients are to be estimated.

Total num. of splines

$$\kappa(\lambda,\varphi) = \sum_{k_1=0}^{K_{J_1}-1} \sum_{k_2=0}^{K_{J_2}-1} d_{k_1,k_2}^{J_1,J_2}\, N_{J_1,k_1}^2(\varphi)\, T_{J_2,k_2}^2(\lambda)$$

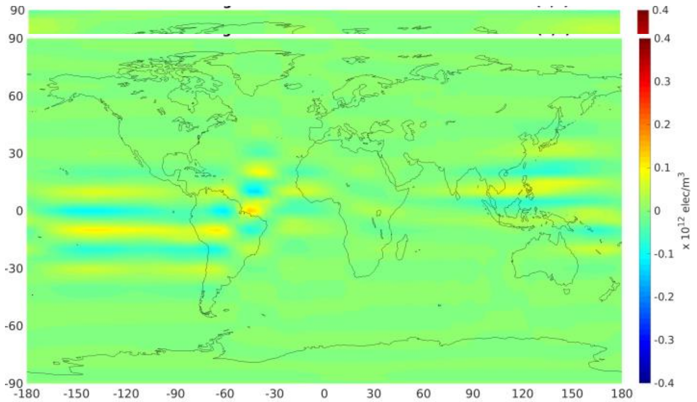
B-spline coefficients

Polynomial B-spline

Trigonometric B-spline

Electron density modelling – multi layer approach with B-splines (optional)

- For B-spline representation, same or different B-spline levels could be used for each key parameter.
- Higher B-spline levels allow representation of finer spatial features of the key parameter at the cost of larger number of coefficients.



Truncation error for F2 peak density using level (3,2)
Truncation error for F2 peak density using level (4,3)

An example is shown below for two distinct scenarios and the number of coefficients to be estimated in both cases.

Level	Mode	Num. coefficients (parameters)
4,3	$N_m^{F_2}$, $h_m^{F_2}$	936
4,3 3,2	$N_m^{F_2}$, $h_m^{F_2}$, $N_m^{F_1}$ H^{F_2}, $h_m^{F_1}$	1296

← 2 parameters : each same level

← 5 parameters : each different levels
Standard level (4,3) for three KP
Reduced level for two KP

Constraint optimization – relevance to electron density modelling

- Using conventional least squares based approach for the estimation of several B-spline coefficients, it is found that the coefficients are **highly correlated** and they are prone to exceeding absolute bounds that are **physically incomprehensible** (such as negative values for F1 layer peak density or peak height).

- Other impractical outcomes could be that the estimated F2 layer peak height is lesser than that of the F1 layer (This cannot be because F2 layer is above the F1 layer !).

- Such problems can be avoided by imposing **constraints** to the electron density modelling.

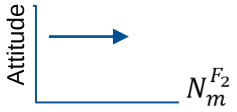
- There not many solvers available that accommodate the inequality constraints in a clear and transparent manner in the modelling for the users to understand the treatment of constraints.

- This is the main motivation for this work and the unknown parameters are estimated subject to **inequality constraints** using optimization approach (motivated from the approaches in [Roese-Koerner (2015)]).

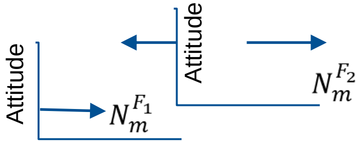
Optimization – constraint types (Optional slide)

Different types of inequality constraints could be applied to the electron density modelling.

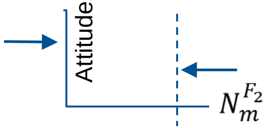
Absolute constraint : Absolute lower bound or upper bound for key parameter
E.g. peak density of F1 layer ($N_m^{F_1}$) $\geq$ 0



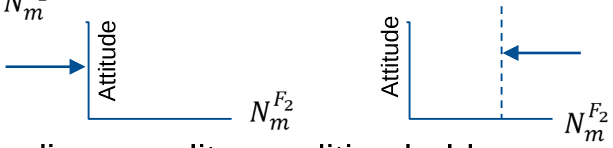
Relative constraint : peak density of F2 larger than that of F1 ($N_m^{F_2} > N_m^{F_1}$)



Bounded constraint : 0.01 $\leq N_m^{F_2} \leq$ 2.5 (x 10^{12} *elec/m*^{3})



Unbounded constraint : 0.01 $\leq N_m^{F_2}$ or $N_m^{F_2}$ < 2.5 (x 10^{12} *elec/m*^{3})



Active constraint : An inequality constraint is active when the corresponding equality condition holds.
(The active set for 0.01 $\leq N_m^{F_2}$ would be $N_m^{F_2}$ = 0.01 (x 10^{12} *elec/m*^{3}))

The constraints on key parameters could be transformed to the corresponding constraints on the B-spline coefficients.

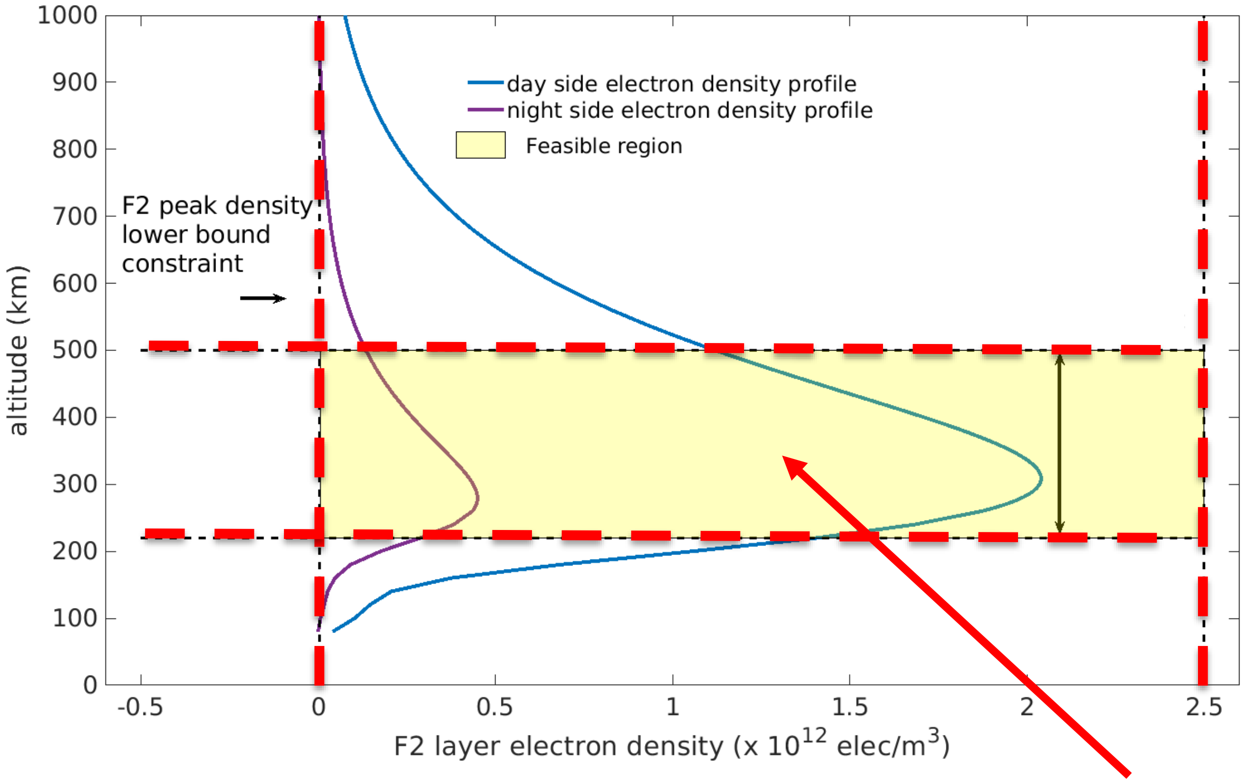
Optimization – imposition of constraints

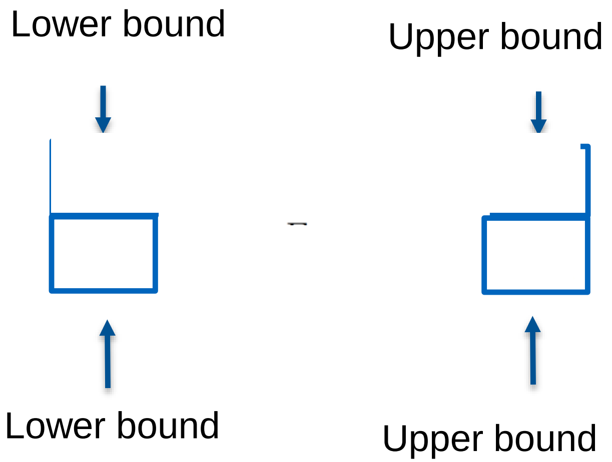
Key parameter	Lower bound	Upper bound
F2 peak density $N_m^{F_2}$ (x 10^{12} *elec/m*^{3})	0.01	2.5
F2 peak height $h_m^{F_2}$ (km)	220	500

We impose bound constraints on two key parameters : F2 peak density and peak height.

- Bounded constraint results in a finite (yet a large) set of candidate points to evaluate the cost function and determine the next set of feasible directions and descent directions.
- The region of space occupied by constraints is called the **feasible region.**
- The residual sum of squares of the electron density becomes the optimization **cost function.**
- The upper and lower bounds for the two key parameters are graphically shown in the next slide.

Optimization – imposition of constraints





Feasible region between the constraints

Inequality constraint optimization - solution

- The main idea in inequality constraint optimization is to seek a certain direction to **traverse** in order to reach an optimal state.

- This involves determining a set of direction vectors, called **feasible directions,** within the **feasible region**. In addition, direction of the gradient of the cost function is also determined. This is used to determine the **descent direction**.

- The algorithm looks for the descent direction and feasible direction until it is no longer possible to traverse along the descent direction within the feasible region and yet **reduce the cost function**

- At this point, the algorithm has found the **local minimum** and solved the optimization problem

Constraint optimization – computation of feasible direction

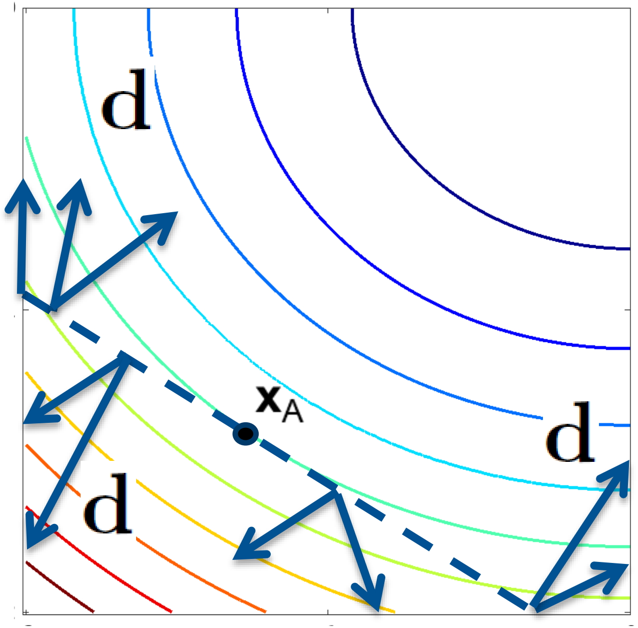
- Consider the iso-contours of the cost function within the feasible region [square boundary]. Cost function is color coded by the contours (Red is high and blue is low)

- Any direction which when traversed by a **small step** (from x_A), retains the point within the **feasible region** is a valid **feasible direction.**

  "**Feasible direction(s)**" denoted as **d** (blue arrows)

- In our electron density modelling problem, any direction that keeps the traversal within the bounds of the constraints imposed by the key parameters is a valid feasible direction

$$0.01 \leq N_m^{F_2}(\lambda, \phi) \leq 2.5 \text{ (x } 10^{12} \; elec/m^3\text{)}$$
$$220 \leq h_m^{F_2}(\lambda, \phi) \leq 500 \; km$$



Constraint optimization – computation of descent direction

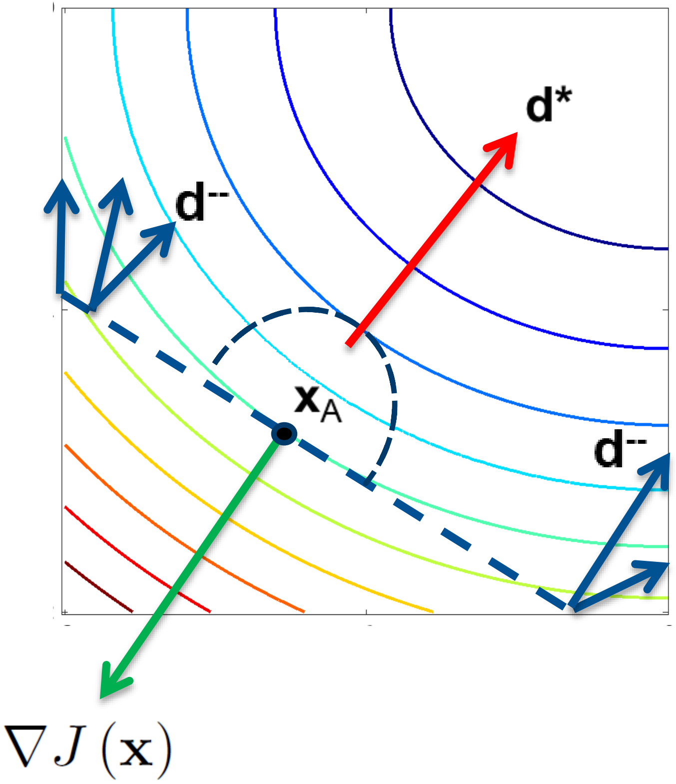
- Among the computed set of "**Feasible directions**" (**d**), there are some directions (**d**⁻⁻) along which the gradients of the cost function reduces.

- There is one particular direction (**d***) in which the gradients of the cost function reduces in the steepest manner.

  Descent directions **d** satisfies $\nabla J(\mathbf{x})\,\mathbf{d} < 0$

- Descent direction and feasible direction sets have overlap. This indicates that it is still possible to proceed along the descent direction.

- Direction of the gradient of cost function at $\mathbf{x}_A$ is normal to the local tangent and along increasing *J* (green arrow).



Electron density modelling – implementation aspects and assumption (Optional)

We have used a standard 5° x 5° uniform grid to create pseudo observations of electron density along latitude and longitude.

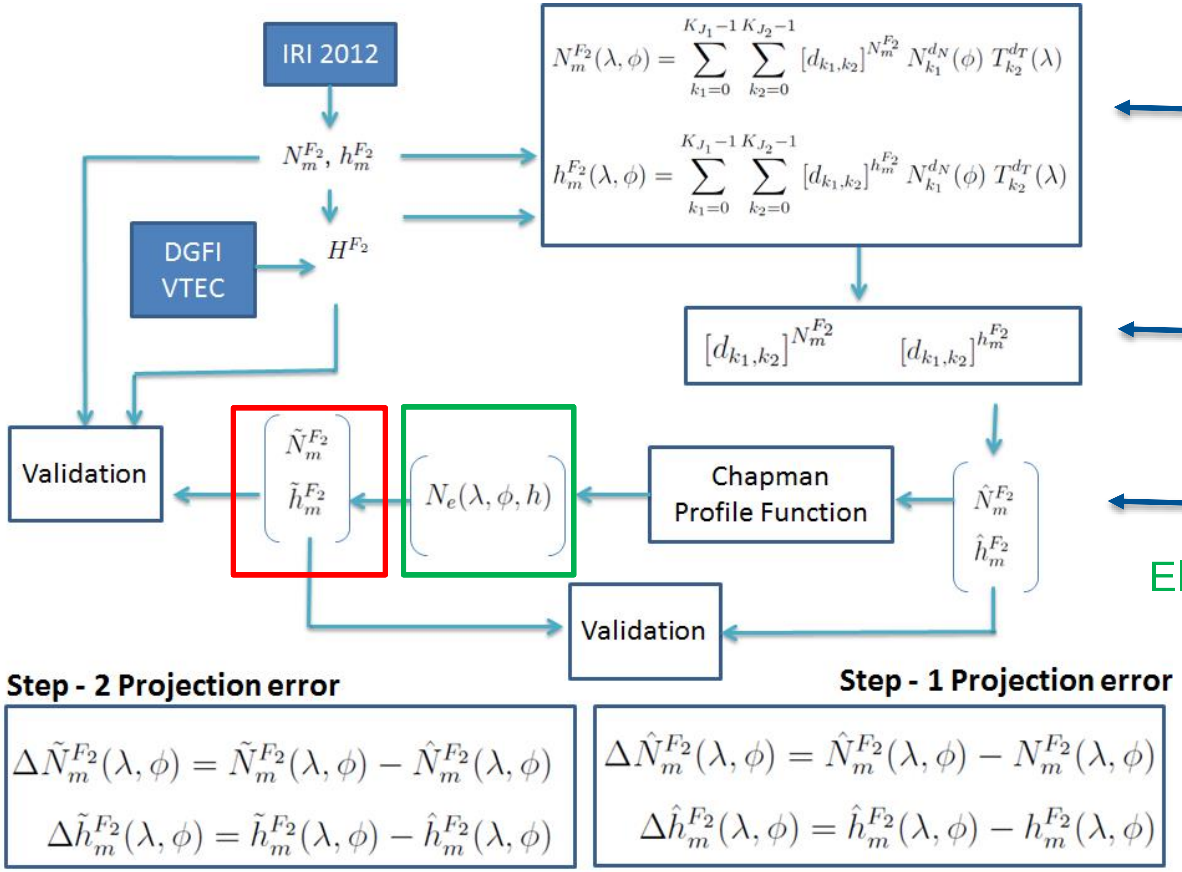
We have also used a variable sampling along the vertical to generate the electron density profile. The nominal altitude step side is 10 km in the D, E and F1 layer. We have used a relatively denser sampling rate (5 km step size) for the F2 layer. Above the F2 layer we have used a step size of 20 km.

We have set the lower limit and upper limit of ionosphere at 90 km and 1000 km respectively.

For the closed loop validation, we have ensured a rank-sufficient system for modelling the global variations of F2 peak density and F2 peak height.

The inequality constraints are applied consistently at all locations (grid points) globally

Estimated key parameters – F2 layer peak density and peak height



B-spline series for ionosphere key parameters

"True set" of B-spline coefficients (reference)

True set of key parameters

Electron density observations

Final set of estimated key parameters

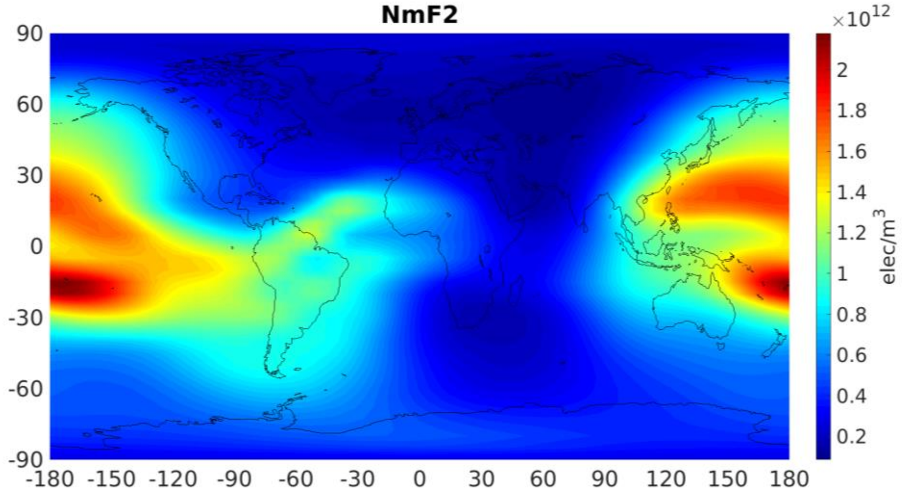
Projection error for validation

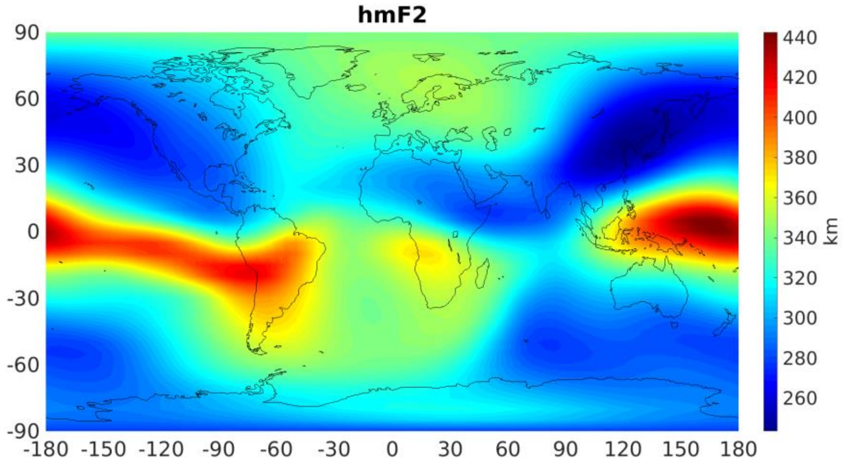
Estimated key parameters – F2 layer peak density and peak height

	Relative projection error Median (%)	Relative projection error Std. dev. (%)
$N_m^{F_2}$	~1e-5	~1e-2
$h_m^{F_2}$	~1e-7	~1e-4

$$\left[\Delta \tilde{N}_m^{F_2}(\lambda,\phi)\right]^{\text{rel.}} = \frac{\tilde{N}_m^{F_2}(\lambda,\phi) - \hat{N}_m^{F_2}(\lambda,\phi)}{\hat{N}_m^{F_2}(\lambda,\phi)}$$

$$\left[\Delta \tilde{h}_m^{F_2}(\lambda,\phi)\right]^{\text{rel.}} = \frac{\tilde{h}_m^{F_2}(\lambda,\phi) - \hat{h}_m^{F_2}(\lambda,\phi)}{\hat{h}_m^{F_2}(\lambda,\phi)}$$





Summary

- Estimation of B-spline coefficients of ionosphere key parameters is performed using **constraint optimization** approach.

- In order to validate the optimization procedures, we use pseudo observations (based on IRI model). This allows an **close loop validation** of the estimated B-spline coefficients with a "**true reference**" set of coefficients.

- Different types of **Inequality constraints** could be imposed on the absolute values of the key parameters.

- The number of constraints could be **scaled up** depending on the total number of parameters to be estimated.

- The approach is well suited for modelling (mutually correlated) ionosphere key parameters for multiple layers simultaneously.

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