Composite Structures 226 (2019) 111159

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Stochastic dynamic analysis of composite plate with random temperature increment

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ARTICLE INFO

Keywords:
Stochastic finite element method (SFEM)
Graphite-epoxy composite
Random mean temperature
Generalized polynomial chaos (gPC)
Non-intrusive method.

ABSTRACT

During the service life, laminated composites may be subject to some random thermal environment. Quantification of the uncertainty in static and dynamic response of the composites under such condition is still a challenging issue. This work presents a stochastic dynamic response analysis of a graphite-epoxy composite plate using generalized polynomial chaos (gPC) expansion due to random mean temperature increment. A stochastic finite element method (SFEM) based on the first-order shear deformation theory (FSDT) is used to describe the free and forced vibration response of the graphite-epoxy composite plate under a uniform distribution of the temperature throughout the plate. Newmark's time integration scheme is used to predict the time-dependent displacement response under dynamic loading. The collocation-based non-intrusive gPC expansion method is used for stochastic dynamic analysis of the graphite-epoxy composite plate. The increment in the temperature is considered as an uncertain parameter and presented by the truncated gPC expansion. The stochastic system response of the plate is projected to the deterministic solver by using the stochastic Galerkin method. The statistical response of eigen frequencies and dynamic displacements of the composite plate at incremental random mean temperature are investigated, and are compared with the results of the Monte Carlo simulation. The numerical studies show a reduction in amplitude of the dynamic mean displacements with the increment in the time and it increases with the increment in the random mean temperature. The characteristics of loading have also significantly influenced the uncertainty in the time-dependent displacement response.

1. Introduction

Applicability of laminated composites in manufacturing important and critical components of the aircraft, rocket, space station, high-speed train, and racing car has widely increased nowadays for exploiting various inherent advantages from their material properties such as high strength-to-weight and high stiffness-to-weight ratios, long fatigue life, and dimensional stability during temperature change. The specific parts of the structures such as the nose and wings of the aircraft experience a wide range of temperature variation during the service life due to the movement at supersonic speeds. Similarly, due to the high speeds, the body of the racing car does also experience elevated temperatures. The increase in temperature is very random in nature depending upon various unpredictable influences. Due to high dimensional stability, low coefficient of thermal expansion (CTE), high strength, and high glass transition temperature graphite-epoxy composite is used to manufacture some of the critical components in the structures. Therefore, the variations in the temperature increment exhibit a significant range of uncertainties in the response of the graphite-epoxy composite structure. Moreover, adequate information on variability of the structural response is essential to design a thermally sensitive part of the structure using graphite-epoxy composite. On the other hand, structural strength of the composite plate is also random in nature. Stochastic studies of the graphite-epoxy plate under dynamic loading is essential to estimate the probability of failure in uncertain thermal environment. Thus, for the safe and reliable design of the structural components subjected to the uncertain temperature and different types of loading conditions instigate to study the stochastic dynamic response of the graphite-epoxy composite plate in the random thermal environment.

The analysis of fiber reinforced composite (FRC) plate in the thermal environment was initiated by Halpin [1], and was followed by Whitney and Ashton [2]. They had presented deterministic elastic response of the symmetric and anti-symmetric composite plates using generalized Duhamel-Neumann form of Hooke's law including the effect of moisture and thermal strain. Ram and Sinha [3] had presented a finite element (FE) method using first-order shear deformation theory

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https://doi.org/10.1016/j.compstruct.2019.111159
Received 11 December 2018; Received in revised form 7 June 2019; Accepted 18 June 2019
Available online 22 June 2019

(FSDT) to determine eigen frequency of the laminated composite plate with increasing uniform temperature and moisture concentration. They had shown that the eigen frequency of the composite plate decreases with increasing temperature and moisture concentration for symmetric and anti-symmetric angle-ply laminates with simply-supported and fixed boundary conditions. Rao and Sinha [4] had developed a three-dimensional FE model to represent the eigen frequency and transient response of the multidimensional composite plate using 20-node isoparametric quadratic elements at elevated temperature and moisture concentration. Mallikarjuna and Kant [5] and Kant and Mallikarjuna [6] had presented large deflection response of the composite plate using higher-order shear deformation theory (HSDT) with the application of C^0 isoparametric element. Rather extensive deterministic studies of the laminated plates and shells had appeared in the literature on the nonlinear transient response albeit without considering thermal and moisture effects [7–10]. Huang and Tauchert [11] had investigated temperature-induced large deflection behavior of the laminated composite plates and spherical panels. Patel et al. [12] had studied hygrothermal effects on thick laminated composite plate using higher-order theory. They had shown that eigen frequencies obtained from the application of the higher-order theory for thin laminated composite plate are comparable to those obtained from the FSDT in hygrothermal environment. Ganapathi et al. [13] had studied nonlinear dynamic response of the thick composite and sandwich plates subjected to thermal and mechanical loading using higher-order theory. Huang et al. [14] had studied the effects of deterministic nonlinear vibration and dynamic response of the FRC plate using the HSDT in the hygrothermal environment. They had obtained nonlinear frequencies and dynamic response of the composite plate by an improved perturbation technique. The nonlinear free vibration analysis and evaluation of transient response of a doubly-curved shell structure, by incorporating Green-Lagrange type nonlinear strain into the FSDT, using FE formulation was presented by Naidu and Sinha [15,16]. They had used Newmark's average acceleration method for the transient analysis conducted from the nonlinear governing equations of motion. Ribeiro and Jansen [17] had presented nonlinear transient response of the composite laminated shallow shells subjected to the simultaneous application of the thermal field and mechanical excitation. The FE model was based on the FSDT with hierarchical basis function. Mahapatra et al. [18,19] had investigated nonlinear frequency response of the singly- or doubly-curved laminated composite shell panels considering the HSDT and Green-Lagrange type nonlinearity. Nanda and Pradyumna [20] had presented nonlinear free vibration analysis and evaluation of transient response of the imperfect laminated composite shell in hygrothermal environment. The formulation was based on the FSDT and von Kármán-type nonlinear kinematics. Biswal et al. [21] had reported a numerical study of free vibration of woven fiber glass-epoxy laminated composite shallow shell under hygrothermal environment using the FSDT; wherein, the numerically simulated results were well supported by the experimental measurements. In all these studies, the elastic parameters had been considered as deterministic, and deterministic dynamic response of the structures was presented at various deterministic temperatures. However, in practical situations the temperature increment is not always deterministic necessarily, rather it is quite random in nature. Therefore, the probabilistic study of the dynamic response of the graphite-epoxy composite plate at elevated random temperature is deemed essential.

Uncertainty quantification in the system response of the FRC plate using the FE method has been investigated in the recent decade considering the aleatory uncertainties due to the randomness in the material properties of the composite. Stochastic static and dynamic analyses of the FRC plate using perturbation method were presented in details by Engelstad and Reddy [22], Park et al. [23], Salim et al. [24], Chen et al. [25], Singh et al. [26], Onkar and Yadav [27], and Lal et al. [28]. In perturbation method, the uncertain parameters are expanded by Taylor's series expansion about the mean value. However, the limitation of this method is, the deviation of the randomness cannot be too large with respect to the mean value of the parameters. The brute-force, Monte Carlo simulation (MCS) method is relatively simple and extensively applied to quantify the uncertainty in the static and dynamic response evaluation of the composite plates. Nevertheless, a large numbers of Monte Carlo (MC) realizations are required for achieving good accuracy in the simulation, which is time consuming and computationally inefficient. Application of the MC-based simulation for studying reliability of the laminated composite plate was shown by Zhang et al. [29]. To address the issue of computational efficiency, the spectrum-based generalized polynomial chaos (gPC) expansion method [30–35] has received a significant attention due to its computational efficiency with reasonable accuracy in the simulation over the sampling-based MCS. Sepahvand et al. [36–38] presented the application of the gPC expansion method to represent the uncertainty in the eigen frequencies and eigen modes of the FRC plates due to the uncertainty in the elastic moduli and fiber orientations. More details on the applications of the method can be found in [39,40].

In the recent past, some studies have been reported which address the uncertainty in the eigen frequency response of the composite plate in the thermal environment. Lal and Singh [41] had investigated the uncertainty in the first eigen frequency arising due to a small level of uncertainty in the individual system parameters of the composite plate at different temperatures. The system parameters included elastic moduli, Poisson's ratio, and thickness of the composite plate. They had used first-order perturbation technique (FOPT) in conjunction with the HSDT in the FE method for the composite plate. Singh and Verma [42] had studied the uncertainty in predicting the buckling load due to the uncertainty in the geometric and material properties of the composite plate at different moisture and temperature conditions using the HSDT and FOPT. They [41,42] had adopted simply-supported and fixed boundary conditions for the plates in the analysis. Kumar et al. [43] had studied stochastic free vibration response of the laminated composite plate resting on elastic foundation under hygrothermal environment using the HSDT. Dey et al. [44] had applied surrogate modeling approach to investigate the stochasticity in the first three natural frequencies of the laminated composite plate due to the uncertainty in the temperature, elastic moduli, and fiber angle/ orientations. They had considered cantilever laminated composite plate for the analysis. Kumar [45,46] had studied the mean and coefficient of variation (COV) of the first mode of linear and nonlinear eigen frequencies, respectively with the increment in the temperature and moisture content considering randomness in the elastic moduli, coefficient of thermal expansion, and moisture content using the FOPT. Nevertheless, the stochastic static and free vibration response of the composite plate due to the randomness in the material properties at various temperatures using the FOPT has been investigated in [45,46], considering limitation in the applicability of the COV equal to 0.1.

The perusal of the earlier works reveals that research is conducted on probabilistic study of the static response and eigen frequency response at higher temperatures due to the uncertainty in the system parameters using the FOPT. However, probabilistic dynamic analysis of the composite plate due to the random temperature increment is completely missing, though it is such an important consideration. Therefore, the present study intends to report the effect of random mean temperature increment on the eigen frequency and dynamic response of the composite plate with various stacking sequences using the gPC expansion method. Moreover, the application of the gPC expansion technique may be able to address the issue regarding consideration of the large uncertainty over the perturbation technique. The collocation-based non-intrusive gPC expansion method is applied to model the stochastic response of the eigen frequencies and the time-dependent displacement field. The stochastic response at each time step has been determined to describe the effect of the temperature uncertainty on the dynamic response of the composite plate. A deterministic FE model has been developed to realize the dynamic response considering the temperature-dependent elastic properties of the graphite-epoxy composite plate. This FE model is developed to analyze the laminated composite plate in the thermal environment to evaluate the structural response from the non-intrusive stochastic model. The major contribution from this paper is, to study the effect of thermal stochasticity by using stochastic finite element method (SFEM) with the application of the gPC to evaluate the uncertainty in the eigen frequency and dynamic central displacement. Numerical dynamic analysis has been carried out with suddenly applied pulse and impulse loading to investigate the variation of uncertainty in the time domain for the cross-ply and angle-ply laminates.

The paper is organized as follows: development of the stochastic formulation of the graphite-epoxy composite laminated plate for the uncertainty in the temperature increment is presented in the next section. A step-by-step procedure for the numerical study is demonstrated in Section 3. Validation of the stochastic model and numerical results are given in Section 4, followed by conclusions in the last section of this paper.

2. Stochastic formulation for the random temperature increment

In the present study, a laminated composite plate of length L, width W, and uniform thickness h is considered consisting of n numbers of unidirectional lamina. It is assumed that each lamina of the composite plate is orthotropic, and bonded together with infinitely thin bonds to act as an integral part of the composite plate. The thickness of the composite plate is considered to be very small as compared to the in-plane dimensions, and shear deformation of the composite plate is constant throughout the thickness. Consequently, the FSDT is employed in the present study considering the desirable accuracy with improved computational efficiency [47]. A shear correction factor is applied here to account for the non-uniform distribution of the transverse shear strain along the thickness of the laminate.

2.1. Constitutive relationship of the composite plate

Mid-plane of the composite plate is considered as a reference plane to evaluate the displacement fields. According to the FSDT, normal to the mid-plane remains straight before and after deformation. The positive sign convention for the in-plane translations u and v, out-of-plane translation w, the rotations of the transverse normal θ_x and θ_y of the composite plate about y and x axes, respectively, and fiber orientations of the lamina are shown in Fig. 1. The generalized displacement vector $\{d\} = \{u \quad v \quad w \quad \theta_x \quad \theta_y\}^{\mathrm{T}}$ of the composite plate at a distance z from the mid-plane is expressed as

$$u = u_0 + z\theta_y,\ \ v = v_0 - z\theta_x,\ \ w = w_0. \tag{1}$$

Here, u_0, v_0, and w_0 are the mid-plane displacements along x, y, and z directions, respectively. Shear rotations φ_x and φ_y in $x-z$ and $y-z$

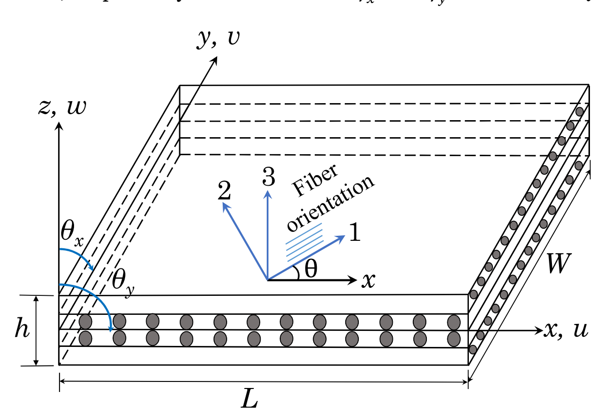

Fig. 1. Laminate geometry, and fiber angle orientations with respect to the global axes.

planes, respectively, are expressed as

$$\varphi_x = w_{,x} + \theta_y,\ \ \varphi_y = w_{,y} - \theta_x. \tag{2}$$

Components of linear strain vector $\{\varepsilon\}$ of the laminate at distance z from the mid-plane are derived from Eq. (1) as

$$\begin{aligned} \varepsilon_x &= u_{0,x} + z\theta_{y,x},\\ \varepsilon_y &= v_{0,y} - z\theta_{x,y},\\ \varepsilon_{xy} &= u_{0,y} + v_{0,x} + z(\theta_{y,y} - \theta_{x,x}),\\ \varepsilon_{xz} &= \varphi_x,\\ \varepsilon_{yz} &= \varphi_y. \end{aligned} \tag{3}$$

Linear strain terms are redefined as $\varepsilon_{0x} = u_{0,x}$, $\varepsilon_{0y} = v_{0,y}$, $\varepsilon_{0xy} = (u_{0,y} + v_{0,x})$, $\kappa_x = \theta_{y,x}$, $\kappa_y = -\theta_{x,y}$, and $\kappa_{xy} = (\theta_{y,y} - \theta_{x,x})$. When composite plate is subjected to the uniform distribution of the temperature, the stress-strain relationship for the k^{th} lamina with reference to the laminate axes $(x,\ y,\ z)$ is written as

$$\{\sigma\}_k = [Q]_k\,[\{\varepsilon\}_k - \{\alpha\}_k\Delta T], \tag{4}$$

in which $\{\sigma\} = \{\sigma_x \quad \sigma_y \quad \sigma_{xy} \quad \sigma_{xz} \quad \sigma_{yz}\}^{\mathrm{T}}$ is stress vector, thermal expansion coefficient vector is written as $\{\alpha\} = \{\alpha_x \quad \alpha_y \quad \alpha_{xy} \quad 0 \quad 0\}^{\mathrm{T}}$ and ΔT is the increment in temperature over reference temperature. Here, $[Q]_k$ is the stress-strain relationship matrix for the k^{th} lamina with reference to the laminate axes, cf. [47,48] for further details. The force and moment resultant of the laminate are obtained by integrating Eq. (4) over the thickness and written as

$$\{F_r\} = [D][\{\varepsilon^*\} - \{e^*\}]. \tag{5}$$

In this equation, the resultant force and moment vector is

$$\{F_r\} = \{N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy} \quad Q_x \quad Q_y\}^{\mathrm{T}}, \tag{6}$$

generalized mid-plain strain vector is

$$\{\varepsilon^*\} = \{\varepsilon_{0x} \quad \varepsilon_{0y} \quad \varepsilon_{0xy} \quad \kappa_x \quad \kappa_y \quad \kappa_{xy} \quad \varepsilon_{xz} \quad \varepsilon_{yz}\}^{\mathrm{T}}, \tag{7}$$

generalized thermal strain vector is

$$\{e^*\} = \{e_x \quad e_y \quad e_{xy} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^{\mathrm{T}}, \tag{8}$$

and $[D]$ is the load-strain relationship matrix of the laminated composite plate. Accordingly, Eq. (5) can be rewritten in a compact form as

$$\{F_r\} = [D]\{\varepsilon^*\} - \{F_N\}, \tag{9}$$

in which the thermal resultant force and moment vector are given by

$$\{F_N\} = \{N_x^N \quad N_y^N \quad N_{xy}^N \quad M_x^N \quad M_y^N \quad M_{xy}^N \quad 0 \quad 0\}^{\mathrm{T}}. \tag{10}$$

Initial strain $\{\varepsilon_{nt}\}$ due to thermal load is represented by the nonlinear portion [3,49] of the overall strain as

$$\begin{aligned} \varepsilon_{xnt} &= \frac{1}{2}(u_{,x}^2 + v_{,x}^2 + w_{,x}^2),\\ \varepsilon_{ynt} &= \frac{1}{2}(u_{,y}^2 + v_{,y}^2 + w_{,y}^2),\\ \varepsilon_{xynt} &= (u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y}),\\ \varepsilon_{xznt} &= (u_{,x}u_{,z} + v_{,x}v_{,z}),\\ \varepsilon_{yznt} &= (u_{,y}u_{,z} + v_{,y}v_{,z}). \end{aligned} \tag{11}$$

The Eq. (11) can be rewritten in a compact form with reference to the Eq. (1) as

$$\{\varepsilon_{nt}\} = \frac{1}{2}[R]\{d^*\}, \tag{12}$$

where $\{d^*\} = \{u_{0,x} \quad u_{0,y} \quad v_{0,x} \quad v_{0,y} \quad w_{,x} \quad w_{,y} \quad \theta_{x,x} \quad \theta_{x,y} \quad \theta_{y,x} \quad \theta_{y,y} \quad \theta_x \quad \theta_y\}^{\mathrm{T}}$, and $[R]$ is the strain-displacement relationship matrix of the nonlinear strain.

2.2. Equations of motion

The equations of motion of the laminated composite plate are presented here using the Hamilton variational principle [48], stated as

$$\int_{t_1}^{t_2} \delta\left(\mathbb{E} - \mathbb{K}\right)\mathrm{d}t = 0, \tag{13}$$

where $\mathbb{E}$ and $\mathbb{K}$ are total potential and kinetic energies, respectively, during the time interval (t_1, t_2). The total potential energy $\mathbb{E}$ can be written as $\mathbb{E} = \mathbb{U} - \mathbb{W}$, where $\mathbb{U}$ represents the strain energy of the plate, and $\mathbb{W}$ is the work done by the externally applied forces. The total potential energy $\mathbb{E}$ of the composite plate in the thermal environment is expressed as

$$\mathbb{E}=\left(\frac{1}{2}\int_{A}\{\varepsilon^*\}^{\mathrm{T}}[D]\{\varepsilon^*\}\mathrm{d}A + \frac{1}{2}\int_{A}\{d^*\}^{\mathrm{T}}[S_r]\{d^*\}\mathrm{d}A\right) - \left(\int_{A}\{d\}^{\mathrm{T}}\{F\}\mathrm{d}A + \int_{A}\{\varepsilon^*\}^{\mathrm{T}}\{F_N\}\mathrm{d}A\right), \tag{14}$$

in which $[S_r]$ is residual stress resultant matrix, see Appendix A.1; and $\{F\}$ is externally applied transverse load vector per unit area in the direction of the generalized displacement vector $\{d\}$. The kinetic energy $\mathbb{K}$ of the composite plate is presented as

$$\mathbb{K} = \frac{1}{2}\int_{A}\{\dot{d}\}^{\mathrm{T}}[\overline{M}]\{\dot{d}\}\mathrm{d}A. \tag{15}$$

Here, $\{\dot{d}\}$ is a generalized velocity vector corresponding to $\{d\}$, and $[\overline{M}]$ is the distributed inertia matrix of the laminated composite plate. Accordingly, mathematical expression for the dynamic motions is formed by combining Eqs. (14) and (15) into the Hamilton variational principle, see Eq. (13), as

$$\delta\int_{t_1}^{t_2}\left[\frac{1}{2}\int_{A}\{\varepsilon^*\}^{\mathrm{T}}[D]\{\varepsilon^*\}\mathrm{d}A + \frac{1}{2}\int_{A}\{d^*\}^{\mathrm{T}}[S_r]\{d^*\}\mathrm{d}A - \int_{A}\{d\}^{\mathrm{T}}\{F\}\mathrm{d}A - \int_{A}\{\varepsilon^*\}^{\mathrm{T}}\{F_N\}\mathrm{d}A - \frac{1}{2}\int_{A}\{\dot{d}\}^{\mathrm{T}}[\overline{M}]\{\dot{d}\}\mathrm{d}A\right]\mathrm{d}t = 0. \tag{16}$$

2.3. Stochastic modeling of uncertain parameters

Stochastic response of the dynamical system due to independent and identically distributed (iid) random parameters can be represented by generalized polynomial chaos (gPC) theory. The concept of application of the chaos theory to represent the stochastic response was first coined by Wiener [50]. A set of orthogonal polynomials is used to project the random variables onto the stochastic space. Consider a probability space represented by $(\Omega, \mathcal{F}, P)$, in which Ω is the random sample space, $\mathcal{F}$ is a σ-algebra on Ω, and P is a probability measure on the sample space. Any uncertain parameter $\mathcal{X}: \Omega \to \mathbb{R}$ with finite variance can be expressed as [32]

$$\mathcal{X} = \sum_{i_1=0}^{\infty} a_{i_1}\Psi_{i_1}(\xi). \tag{17}$$

This is the gPC expansion of the uncertain parameter $\mathcal{X}$ in a compact form. The random orthogonal polynomial Ψ_{i_1} is a multidimensional function of random variables, $\xi = \{\xi_i\}, \quad i = 1, 2, ..., n$ in the particular sample space. Selection of the orthogonal polynomial is dependent on the type of sample space of the random variables. It is convenient to use the truncated series for the expansion considering the accuracy and the sample space of the random variables. The unknown coefficients $\{a_{i_1}\}$ are determined by the Galerkin projection technique.

In the present study, uniform temperature increment of the composite plate is considered as a random variable. It is reported earlier that the elastic moduli of the graphite-epoxy composite plate varies with the variation in the temperature [51–53]. The elastic moduli of the graphite-epoxy composite is varied according to the random temperature increment, and consequently the dynamic response of the composite becomes stochastic in nature. Hence, uncertainty in the temperature increment can be represented by the truncated gPC expansion as

$$T(\xi) = \sum_{i_1=0}^{N_1} a_{i_1}\Psi_{i_1}(\xi) = \boldsymbol{a}^{\mathrm{T}}\boldsymbol{\Psi}, \tag{18}$$

in which $\boldsymbol{a} = \{a_{i_1}\}$ is the vector of deterministic unknown coefficients and N_1 is the finite number of terms of the gPC expansion of the random temperature. The orthogonality relation of the multidimensional polynomial functions, $\boldsymbol{\Psi} = \{\Psi_{i_1}(\xi)\}$ is written as

$$\mathbb{E}[\Psi_{i_1}, \Psi_{j_1}] = \mathbb{E}[\Psi_{i_1}^2]\delta_{i_1 j_1} = p_{i_1}^2\delta_{i_1 j_1}, \qquad i_1, j_1 = 0, 1, 2, ..., N_1, \tag{19}$$

in which $\delta_{i_1 j_1}$ and p_{i_1} represent Kronecker delta and the norm of the polynomials, respectively. The unknown coefficients $\{a_{i_1}\}$ are determined using Galerkin projection technique as

$$\{a_{i_1}\} = \frac{1}{\langle\Psi_{i_1}^2\rangle}\int_{\Omega}\left\{T(\xi)\right\}\left\{\Psi_{i_1}(\xi)\right\}\mathfrak{f}(\xi)\mathrm{d}\xi, \tag{20}$$

where $\langle\Psi_{i_1}^2\rangle$ denotes the inner products in the Hilbert space, and $\mathfrak{f}$ is the probability density function (PDF) of random variable ξ. Once $\{a_{i_1}\}$ are known, any statistical property of the random parameter can be calculated. For instance, the expected value μ_T and the variance σ_T^2 take the following forms

$$\mu_T = a_0, \quad \sigma_T^2 = \sum_{i_1=1}^{N_1} a_{i_1}^2 p_{i_1}^2. \tag{21}$$

Due to the increment in the random mean temperature, elastic properties of the composite are varied. The structural response of the composite also becomes uncertain. Accordingly, the random eigen frequency $f(\xi)$ and time-dependent random displacement $d(t, \xi)$ are approximated by truncated finite number of terms N_2 using the gPC expansions as

$$f(\xi) = \sum_{i_2=0}^{N_2} b_{i_2}\Psi_{i_2}(\xi) = \boldsymbol{b}^{\mathrm{T}}\boldsymbol{\Psi}, \tag{22}$$

$$d\left(t, \xi\right) = \sum_{i_2=0}^{N_2} c_{i_2}(t)\Psi_{i_2}(\xi) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{\Psi}. \tag{23}$$

Here, $\boldsymbol{b} = \{b_{i_2}\}$ and $\boldsymbol{c} = \{c_{i_2}(t)\}$ are the deterministic unknown coefficients for the random eigen frequency and random dynamic displacement at each time step, respectively, and $\boldsymbol{\Psi} = \{\Psi_{i_2}(\xi)\}$ is the orthogonal polynomial function.

2.4. Stochastic finite element modeling

Orthotropic composite plate is mathematically modeled by following finite element method (FEM), and the entire plate domain is discretized by eight-node C^0 isoparametric element with five degrees of freedom (DOF) per node. The stochastic element displacement vector $\{d(t, \xi)\}$ is expressed in terms of the stochastic nodal displacement vector $\{d_e(t, \xi)\}$ using elemental interpolation functions $[N]$, and is given by

$$\{d(t, \xi)\} = [N]\{d_e(t, \xi)\}. \tag{24}$$

Accordingly, the random mid-plain strain vector $\{\varepsilon^*(t, \xi)\}$ can be calculated from the stochastic nodal displacement vector $\{d_e(t, \xi)\}$ employing strain-nodal displacement matrix $[B]$, with reference to Eqs. (7) and (24) as

$$\{\varepsilon^*(t, \xi)\} = [B]\{d_e(t, \xi)\}. \tag{25}$$

The stochastic elemental strain energy for an element e is derived with reference to Eq. (16)

$$\int_{t_1}^{t_2}\left[\int_{A_e}\delta\{d_e(t,\xi)\}^{T}[B]^{T}[D(T(\xi))][B]\{d_e(t,\xi)\}\mathrm{d}A_e+\int_{A_e}\delta\{d_e(t,\xi)\}^{T}[G]^{T}[S_r(T(\xi))][G]\{d_e(t,\xi)\}\mathrm{d}A_e-\int_{A_e}\delta\{d_e(t,\xi)\}^{T}[N]^{T}\{F(t)\}\mathrm{d}A_e-\int_{A_e}\delta\{d_e(t,\xi)\}^{T}[B]^{T}[F_N(T(\xi))]\mathrm{d}A_e+\int_{A_e}\delta\{d_e(t,\xi)\}^{T}[N]^{T}[\overline{M}][N]\{\ddot{d}(t,\xi)\}\right]\mathrm{d}t=0, \quad (26)$$

where $[G]$ is the matrix of shape functions given in Appendix A.2. Since, virtual displacement $\delta\{d_e(t,\xi)\}$ is arbitrary in nature, stochastic finite element model of the element e of the laminated composite plate can be presented as

$$\{[K_e(T(\xi))]+[K_{Ge}(T(\xi))]\}\{d_e(t,\xi)\}+[M_e]\{\ddot{d}_e(t,\xi)\}=\{P_e(t)\}+\{P_{Ne}(T(\xi))\}. \quad (27)$$

The stochastic elemental stiffness matrix $[K_e(T(\xi))]$, stochastic elemental geometric stiffness matrix $[K_{Ge}(T(\xi))]$, and stochastic elemental thermal load vector $\{P_{Ne}(T(\xi))\}$ are obtained as

$$\left[K_e\left(T(\xi)\right)\right]=\int_{A_e}[B]^{T}\left[D\left(T(\xi)\right)\right][B]\mathrm{d}A_e, \quad (28)$$

$$\left[K_{Ge}\left(T(\xi)\right)\right]=\int_{A_e}[G]^{T}\left[S_r\left(T(\xi)\right)\right][G]\mathrm{d}A_e, \quad (29)$$

$$\left\{P_{Ne}\left(T(\xi)\right)\right\}=\int_{A_e}[B]^{T}\left\{F_N\left(T(\xi)\right)\right\}\mathrm{d}A_e. \quad (30)$$

The elemental mass matrix $[M_e]$ and elemental dynamic force vector $\{P_e(t)\}$ are given respectively by

$$[M_e]=\int_{A_e}[N]^{T}[\overline{M}][N]\mathrm{d}A_e, \quad (31)$$

$$\left\{P_e(t)\right\}=\int_{A_e}[N]^{T}\left\{F(t)\right\}\mathrm{d}A_e. \quad (32)$$

Elemental static load vector can be developed with reference to the Eq. (32) as

$$\{P_{Se}\}=\int_{A_e}[N]^{T}\{F\}\mathrm{d}A_e, \quad (33)$$

and corresponding global static load vector $\{P_S\}$ is developed after proper assembling. The global stochastic FE model for the forced vibration can be obtained after assembling the element matrices in the following form

$$\{[K(T(\xi))]+[K_G(T(\xi))]\}\{d(t,\xi)\}+[M]\{\ddot{d}(t,\xi)\}=\{P(t)\}+\{P_N(T(\xi))\}. \quad (34)$$

It is stated earlier that elastic properties of the constituent materials of the composite are varied with the variation in the temperature [51–53,3]. Therefore, stiffness matrix, geometric stiffness matrix, and thermal force vector of the composite are expressed as functions of random matrices due to randomness in the temperature. The solution of the stochastic forced vibration problem is sought by Newmark's integration technique, and the randomness in the displacement at each time step is described. The homogeneous solution of Eq. (34) yields the stochasticity in the eigen frequency $f(\xi)$ of the composite plate for the specified boundary conditions. The stochastic representations, as discretized in Eqs. (18) and (23), using truncated gPC expansion method are substituted in Eq. (34), i.e.

$$\{[K(\boldsymbol{a}^{T}\boldsymbol{\Psi})]+[K_G(\boldsymbol{a}^{T}\boldsymbol{\Psi})]\}(\boldsymbol{c}^{T}\boldsymbol{\Psi})+[M](\ddot{\boldsymbol{c}}^{T}\boldsymbol{\Psi})=\{P(t)\}+\{P_N(\boldsymbol{a}^{T}\boldsymbol{\Psi})\}. \quad (35)$$

The unknown deterministic coefficients for the eigen frequencies $\boldsymbol{b}^{T}$ are estimated by minimization of the stochastic error $\{\epsilon_1(t,\xi)\}$ from the homogeneous solution of Eq. (35) as

$$\{\epsilon_1(t,\xi)\}=\{[K(\boldsymbol{a}^{T}\boldsymbol{\Psi})]+[K_G(\boldsymbol{a}^{T}\boldsymbol{\Psi})]\}(\boldsymbol{c}^{T}\boldsymbol{\Psi})+[M](\ddot{\boldsymbol{c}}^{T}\boldsymbol{\Psi}). \quad (36)$$

Similarly, the solution of the unknown deterministic coefficients for time-dependent displacement field $\boldsymbol{c}^{T}$ are derived by minimization of the stochastic error $\{\epsilon_2(t,\xi)\}$ from

$$\{\epsilon_2(t,\xi)\}=\{[K(\boldsymbol{a}^{T}\boldsymbol{\Psi})]+[K_G(\boldsymbol{a}^{T}\boldsymbol{\Psi})]\}(\boldsymbol{c}^{T}\boldsymbol{\Psi})+[M](\ddot{\boldsymbol{c}}^{T}\boldsymbol{\Psi})-\{P(t)\}-\{P_N(\boldsymbol{a}^{T}\boldsymbol{\Psi})\}. \quad (37)$$

The minimization of the stochastic error is carried out by calculating the deterministic response of the system at some specific collocation points, i.e. at the roots of the higher-order orthogonal polynomials, and minimizing the error between these response. The response is calculated by the gPC expansion using least-squares method [32]. The collocation-based non-intrusive method is implemented here to derive the unknown coefficient vectors. In this method, deterministic governing equations of motion are employed as a deterministic solver, and solutions are obtained at the specific collocation points. Selection of the collocation points depends on the choice of the order of the gPC expansion representing the randomness in the dynamical system, cf. [32] for more details.

3. Solution procedure

The solution of the non-intrusive gPC-based stochastic FE model is evaluated in two parts, i.e. the solution of the deterministic finite element model, and the solution of the stochastic model by determining the unknown coefficients while setting the random errors equal to zero at some predefined collocation points. The FE model developed for the laminated composite plate in the thermal environment is used as a deterministic solver, and runs of the deterministic FE model are repeated at the specified realizations of the selected random vector points. The detailed procedure of the numerical simulation, considering temperature uncertainty is summarized here in Algorithm 1 and Algorithm 2.

Algorithm 1: Deterministic analysis using FE model of the composite plate in thermal environment

1 Develop global matrices $[K]$ and $[M]$ as well as force vectors $\{P_S\}$, $\{P(t)\}$, and $\{P_N\}$ of the composite plate at the predefined temperature;
2 Calculate initial displacement $\{\delta_i\}$ from bending equation, $[K]\{\delta_i\}=\{P_S\}+\{P_N\}$;
3 Substitute initial displacement $\{\delta_i\}$ in Eqs. (25) and (9) to yield the residual stress resultants $\{F_r\}$;
4 Develop geometric stiffness matrix $[K_G]$;
5 Determine eigen frequencies and mode shapes of the laminated composite plate in the thermal environment from the homogeneous solution of Eq. (34);
6 Solve Eq. (34) for time-dependent forcing function $\{P(t)\}$ using Newmark's direct time integration method at each incremental time step to obtain the dynamic response.

Gauss quadrature rule is adopted here for integration over the elemental area for calculation of the element matrices. The 3-point Gauss quadrature rule is adopted to compute the bending stiffness matrix; whereas, 2-point Gauss quadrature rule is adopted to calculate the shear stiffness, mass matrix, and force vectors to avoid the shear locking phenomenon in the thin plate. The constant-average acceleration scheme is adopted to solve Newmark's direct time integration method for obtaining stable solution of the linear problem [54]. A deterministic MATLab® code has been developed to evaluate the eigen frequencies and transient response of the laminated composite plate in thermal environment, as stated in Algorithm 1.

Table 1
Elastic moduli of graphite-epoxy lamina at different temperatures, cf. [3]. $G_{13} = G_{12},\ \ G_{23} = 0.5G_{12}$.

	Temperature, T (K)					
Elastic moduli (GPa)	300	325	350	375	400	425
E_{11}	130	130	130	130	130	130
E_{22}	9.5	8.5	8.0	7.5	7.0	6.75
G_{12}	6.0	6.0	5.5	5.0	4.75	4.5

Table 2
Mean μ_T and sd σ_T of the input random parameter.

Random parameter	Type of distribution	μ_T (K)	σ_T (K)
Temp. increment	Normal	25, 50, 75, 100	5, 10, 15, 20

Table 3
Geometric dimension, elastic parameters, density, coefficient of thermal expansion, lamina sequences of the graphite-epoxy laminated composite plate.

	Plate 1	Plate 2
Dimensions (mm)	$L = W = 100,\ h = 1$	$L = W = 100,\ h = 2$
Elastic moduli (GPa)	$E_{11} = 130,\ E_{22} = 9.5,$ $G_{12} = 6.0,\ G_{13} = G_{12},$ $G_{23} = 0.5G_{12}$	See Table 1
Poisson's ratio	$\nu_{12} = 0.3,$ $\nu_{21} = \nu_{12}\frac{E_{22}}{E_{11}}$	$\nu_{12} = 0.3,$ $\nu_{21} = \nu_{12}\frac{E_{22}}{E_{11}}$
Density (kg/mm^3)	$\rho = 1.6 \times 10^{-6}$	$\rho = 1.6 \times 10^{-6}$
Coefficient of thermal expansion (\K)	$\beta_1 = -0.3 \times 10^{-6},$ $\beta_2 = 28.1 \times 10^{-6}$	$\beta_1 = -0.3 \times 10^{-6},$ $\beta_2 = 28.1 \times 10^{-6}$
Lamina sequence	For cross-ply laminate (0°/90°/90°/0°)	For cross-ply laminate (0°/90°/90°/0°), For angle-ply laminate (45°/−45°/−45°/45°)

Algorithm 2: Collocation-based SFEM for analysis of the composite plate due to thermal uncertainty

1 Define deterministic geometry of the model, elastic parameters, lamina sequence of the composite plate, as well as mean, standard deviation (sd), and probability space of the random temperature;
2 Represent the uncertainty in the temperature increment using the gPC expansion as $T(\xi)$, see Eq. (18);
3 Estimate the unknown coefficients a_{i_1} for the random temperature increment using Galerkin projection technique, see Eq. (20);
4 Select the type of orthogonal polynomial $\Psi_{i_2}(\xi)$ based on the random space of the input random variable, and the order of the polynomial function as N_2;
5 Construct the uncertainty in the structural response $f(\xi)$ and $d(t, \xi)$ using the truncated gPC expansion, see Eqs. (22) and (23);
6 Generate the collocation points from the roots of the higher-order polynomial function $\Psi_{i_2}(\xi)$. Number of collocation points should be at least equal to the number of unknown deterministic coefficients b_{i_2} and $c_{i_2}(t)$;
7 Generate the random temperature increment at the predefined collocation points;
8 Realize the structural response using the deterministic FE solver at the pre-generated random incremental temperature. Develop a set of equations from Eqs. (36) and (37) at the predefined collocation points;
9 Calculate the unknown coefficients b_{i_2} and $c_{i_2}(t)$ for the eigen frequency, and the time-dependent displacement, respectively from the above set of equations employing least-squares minimization technique;
10 Estimate the statistical parameters of the structural response, e.g. mean, sd, and corresponding PDF.

The selection of the orthogonal polynomial basis function depends on the type of variability in the random input parameters. For instance, Hermite polynomial is used for normally distributed input parameter, whereas Jacobi polynomial is used if the input random parameter is in Gamma distribution. Following Algorithm 2, a collocation-based SFEM code is developed in MATLab® environment to evaluate the unknown coefficients of the structural response in the gPC expansion method. Predefined deterministic FE solver is used to generate structural response at the collocation points.

Table 4
Suddenly applied transverse load.

	Pulse loading	Impulse loading
Loading (N/mm^2)	$q_0 = 0.001$	$q_0 = 0.001$
Time of excitation (s)	$t_p = 0.25$	$t_{ip} = 0.001$

4. Numerical study

Numerical study is conducted to evaluate uncertainty in the eigen frequency and dynamic response of the graphite-epoxy laminated composite plate due to random temperature increment using the gPC expansion method. The stochastic studies are conducted at the mean temperatures of 325 K, 350 K, 375 K, and 400 K. However, 300 K is considered as a reference temperature. The temperature-dependent elastic properties of the graphite-epoxy composite lamina are illustrated in Table 1. The mean μ_T and sd σ_T of the random temperature increment as input parameters are shown in Table 2. In the present study, elastic moduli, coefficient of thermal expansion, and Poisson's ratio of the composite plate are considered as deterministic. The geometric dimension, elastic properties, density, and stacking sequences of the composite plate considered here are shown in Table 3. The composite plates are subjected to the uniformly distributed transverse loads as given in Table 4.

The polynomial basis function is represented by Hermite polynomial for random input variable, which is normally distributed. The number of unknown coefficients are increased rapidly if the order of the polynomial is increased. Herein, one-dimensional 3rd order Hermite polynomial is used to approximate the stochastic response. Therefore, Hermite polynomial can be presented in term of the random variable ξ as $\Psi_0 = 1,\ \Psi_1 = \xi,\ \Psi_2 = (\xi^2 - 1)$, and $\Psi_3 = (\xi^3 - 3\xi)$. The eigen frequency f and the transverse central $(\frac{L}{2}, \frac{W}{2})$ displacement $d(t)$ of the composite plate are considered here to investigate the uncertainty in the dynamic response due to random mean temperature increment.

4.1. Validation of the FE model

The FE model of the laminated composite plate in thermal environment has been developed, and the frequencies extracted are compared with that reported in the literature. An ANSYS® parametric design language (APDL) code is employed to calculate the eigen frequencies of the composite plate considering the effect of thermal prestress during modal analysis. The eigen frequencies of the simply-supported graphite-epoxy laminated composite plate at the temperature of 300 K and 325 K are evaluated using the present formulation, and are compared with the frequencies reported by Ram and Sinha [3] and ANSYS® simulation to establish validity of the present deterministic formulation, cf. Table 5.

The parameters of Plate 1 (Table 3) are used for the validation analysis. The finite element mesh considered here is discretized as 4×4, based on mesh convergence procedure suggested in [55]. First four natural frequencies of the composite plate at temperatures of 300 K and 325 K represent a good agreement with the results reported by Ram and Sinha [3] and ANSYS® simulation (Table 5), which confirms validity of the in-house MATLab® code developed and used for further analysis. Furthermore, the dynamic response of the composite plate at a temperature of 300 K is compared with that reported by Kant et al. [48] and Niyogi et al. [56], and a good agreement is observed in the prediction of the results.

The convergence of the dynamic response, i.e. central displacement at different time steps of the (0°/90°/90°/0°) graphite-epoxy laminated

Table 5
Results of the free vibration analysis of the graphite-epoxy composite Plate 1 at T = 300 K and 325 K.

Mode	Temperature	Present		ANSYS®	Ram and Sinha [3]
Nos.	T (K)	Eigen freq. f (Hz)	NDF[1] λ	Eigen freq. f (Hz)	NDF[1] λ
1	300	14.818	12.083	14.807	–
	325	9.929	8.097	9.917	8.088
2	300	29.434	24.001	29.330	–
	325	23.665	19.196	23.551	19.297
3	300	51.493	41.988	51.343	–
	325	48.428	39.324	48.276	39.324
4	300	62.050	50.600	61.780	–
	325	56.615	46.165	56.336	45.431

[1] Non-dimensional frequency, $\lambda = 2\pi f L^2 (\rho/E_{22}h^2)^{1/2}$

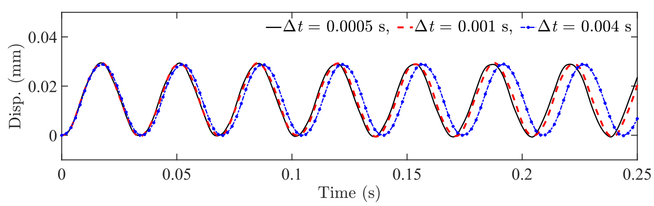

Fig. 2. Convergence studies of Newmark's integration method for a simply-supported (0°/90°/90°/0°) graphite-epoxy Plate 2 subjected to suddenly applied pulse loading, q_0 at T = 325 K.

composite plate, at a temperature of 325 K is shown in Fig. 2 for Plate 2. Newmark's time integration scheme is used for conducting transient analysis of the composite Plate 2. The converged value for time step $\triangle t$ = 0.001 s is adopted in the present analysis. This FE model is subsequently used as a deterministic FE solver to evaluate the uncertainty in the eigen frequencies and the dynamic displacement of the graphite-epoxy laminated composite plate due to the random mean temperature increment.

4.2. Validation of the stochastic model

The gPC expansion method is a robust technique, which precisely predict randomness in the system response due to randomness in the input parameters. Effectiveness of the gPC expansion method is investigated here by comparing with the realizations generated from 10, 000 Monte Carlo simulations (MCS). Table 6 shows the mean μ_f and sd σ_f of the first three eigen frequencies derived using 3rd and 4th order gPC expansion method at a random mean temperature of 325 K, and are compared with the MC simulations of 10, 000 sample realizations.

A comparison of the PDFs for the first three eigen frequencies at the mean random temperature of 325 K for the composite Plate 2 is illustrated in Fig. 3. It is evident that, 3rd order gPC expansion is enough to represent the uncertain response of the composite plate due to random mean temperature increment.

The deterministic FE model of the laminated composite plate under uniform temperature is used to determine the unknown coefficients of 3rd order gPC expansion in Eqs. (22) and (23), at 25 sets of random temperatures using least-squares method. Deterministic dynamic response for each predefined temperature is calculated at every incremental time step using Newmark's step-by-step integration technique. The total time of study is kept as 0.25 s, and the time step considered is 0.001 s. Uncertainty in the time-dependent central displacement using 3rd order gPC expansion method, incorporating the Hermite polynomials, in Eq. (23) as represented in [57]

$$d(t,\xi) = c_0(t) + c_1(t)(\xi) + c_2(t)(\xi^2-1) + c_3(t)(\xi^3-3\xi). \quad (38)$$

The unknown coefficients $c_i(t)$ are derived by solving the stochastic Eq. (38) for a set of 25 collocation points generated from the roots of the 4th order Hermite polynomial at each incremental time step. Time history plots of the unknown coefficients for a simply-supported (0°/90°/90°/0°) graphite-epoxy laminate due to suddenly applied pulse and impulse loading are shown in Figs. 4 and 5, respectively at the mean temperatures of 325 K, 350 K, 375 K, and 400 K. The first coefficient c_0 indicates the mean response of the central displacement, and has dominating influence on both the types of loading conditions. On the other hand, the amplitude of the second coefficient c_1, which influences the sd of the response, is in increasing order with the increase in the random mean temperature. Moreover, the amplitude of c_1 is comparable with the mean response for the impulse loading at the random mean temperature of 400 K. It can be stated that, at the same level of uncertainty in temperature increment, the sensitivity of the dynamic response increases with the random mean temperature increment. Note that, the amplitude of c_0 is decreasing with the increment in time and is increasing with the increment of the random mean

Table 6
Comparison of the statistical results of first three eigen frequencies of the (0°/90°/90°/0°) laminate graphite-epoxy Plate 2 at a mean temperature, T = 325 K.

Method	1st eigen freq. (Hz)		2nd eigen freq. (Hz)		3rd eigen freq. (Hz)	
	μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
MCS (10,000)	27.430	0.392	54.365	0.608	99.289	0.308
3rd order gPC	27.440	0.397	54.378	0.615	99.298	0.311
4th order gPC	27.435	0.397	54.375	0.616	99.291	0.313

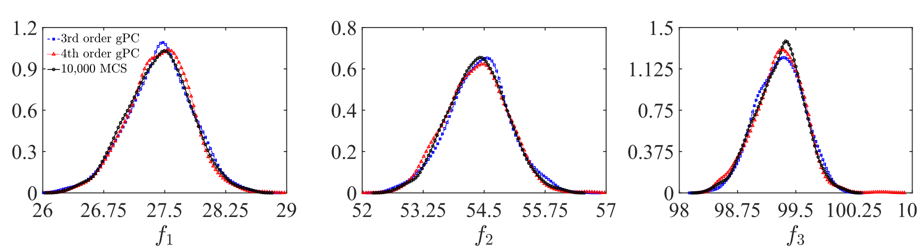

Fig. 3. PDFs of first three eigen frequencies (Hz) obtained using 3rd and 4th order gPC expansions compared with the MCS at mean random temperature of 325 K for Plate 2.

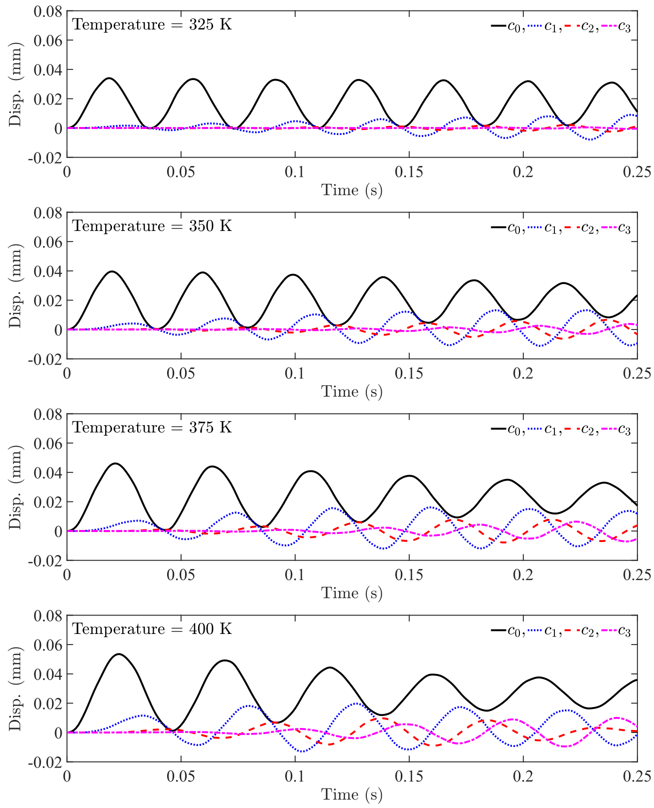

Fig. 4. ime history of the gPC expansion coefficients of central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to the pulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

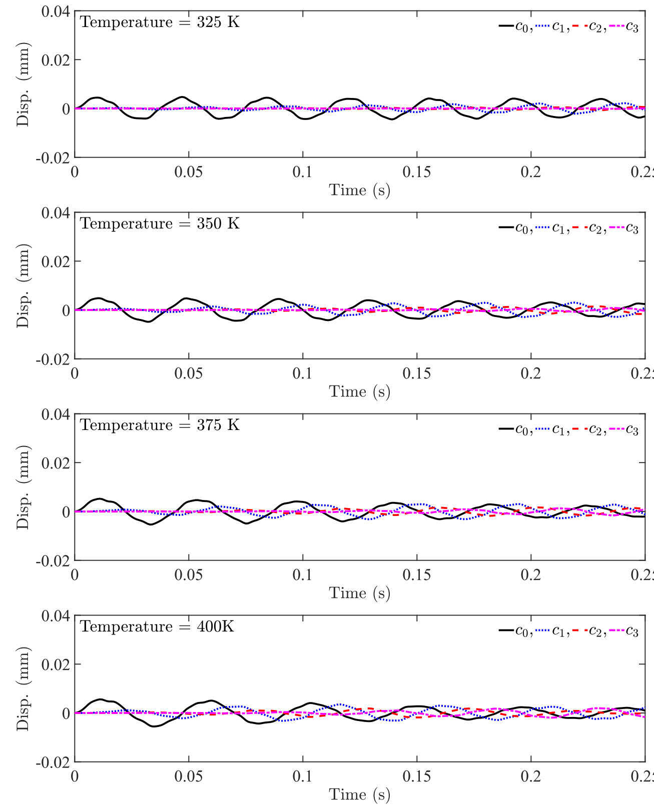

Fig. 5. Time history of the gPC expansion coefficients of central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to the impulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

temperature. The effectiveness of the gPC expansion method is established by the convergence of coefficients, c_1, c_2, and c_3.

4.3. PDF of the eigen frequencies

For safe design of the composite plate, and to estimate the factor of safety at elevated temperature the probabilistic analysis is necessary over the deterministic analysis. The mean and sd of the first three eigen frequencies of the symmetric cross-ply laminate at different random mean temperatures is presented in Table 7. Fig. 6 represents the PDF of the first three eigen frequencies at various random mean temperatures in increasing order and corresponding deterministic eigen frequencies at the mean temperature. It is observed from Table 7 and Fig. 6 that, deterministic values of the eigen frequencies lie near the maximum probability density. The sd, which represents the dispersion of the probability plot, is increased with the increase in the random mean temperature. The sd of the eigen frequencies at a temperature of 325 K is less in comparison with the higher random mean temperature. This indicates the fact that, the random mean temperature increment influences the variation in the elastic properties of the composite, and thereby increment in the level of uncertainty in the frequency response at the higher random temperature.

4.4. Stochastic dynamic response of laminated composite plates

The effect of uncertainty in the temperature increment on the time-dependent transverse central displacement for the symmetric cross-ply and angle-ply laminated composite plates for the suddenly applied pulse and impulse loading are investigated. The time-dependent deterministic and mean values of the central displacement for simply-supported cross-ply laminate under the suddenly applied pulse and impulse loading are plotted in Figs. 7 and 9, respectively. The ratio of the sd and mean values of transverse displacement at each time step is derived to measure the variation in the level of uncertainties in the dynamic response due to the random mean temperature increment for the symmetric cross-ply laminate in time domain, which are reported in Figs. 8 and 10 for the suddenly applied pulse and impulse loading, respectively. It is observed from Figs. 7 and 9 that deterministic and mean central displacements are in increasing order due to corresponding degradation in the material properties due to the increment in the random mean temperature under both types of loading. However, the mean central displacements are decaying with the time as compared to the deterministic values, and the decay is faster with the increment in the random mean temperature under both types of loading. This decay in the mean amplitude in time domain is due to the increasing randomness with the temperature increment [58]. The ratio of the sd and mean for the symmetric cross-ply laminate in Fig. 8 represents sudden peak at troughs of the corresponding time-dependent displacement plot for the pulse loading. The peak value is decreased with an increment in the random mean temperature. However, for the impulse loading in

Table 7
Statistics of the first three eigen frequencies (Hz) for a simply-supported (0°/90°/90°/0°) laminate due to the mean random temperature increment for Plate 2.

Temperature	1st mode		2nd mode		3rd mode	
	μ_f	σ_f	μ_f	σ_f	μ_f	σ_f
325 K	27.444	0.397	54.378	0.615	99.298	0.311
350 K	25.431	0.788	51.332	1.193	97.664	0.641
375 K	23.565	1.070	48.517	1.602	96.186	0.821
400 K	21.865	1.374	46.034	1.958	94.958	0.969

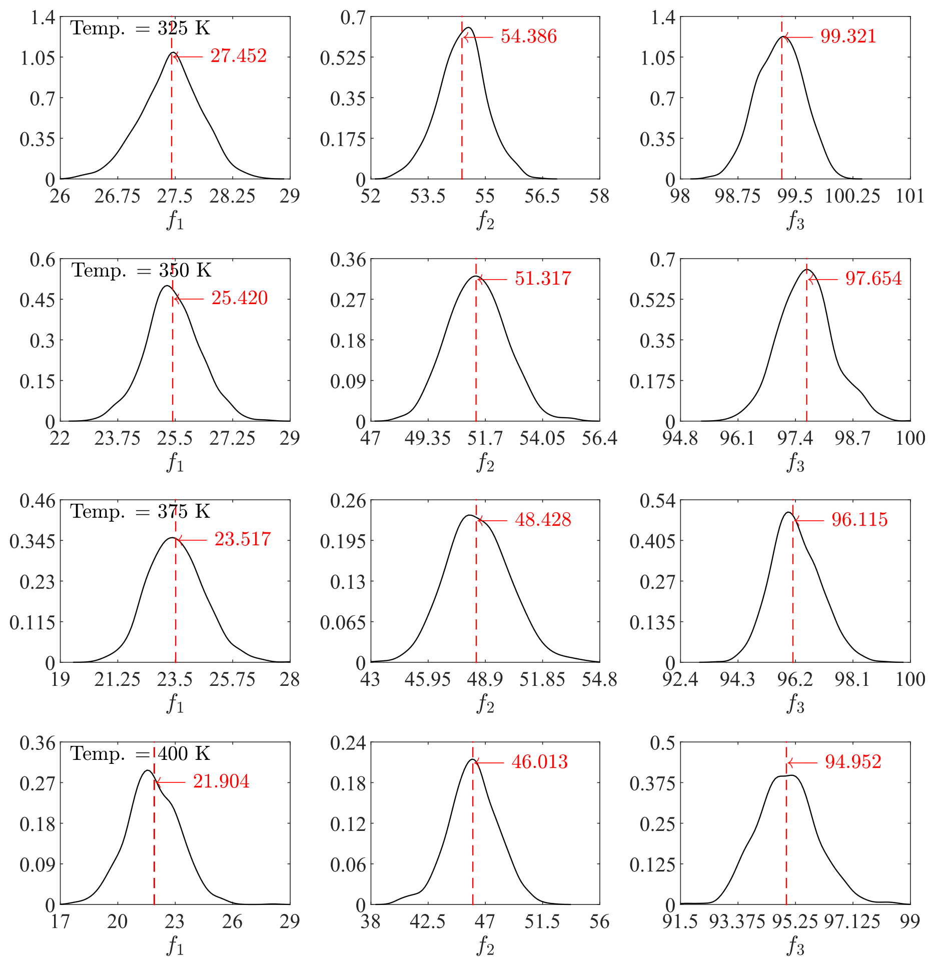

Fig. 6. PDF of first three natural frequencies (Hz) of a simply-supported (0°/90°/90°/0°) laminate due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively, and corresponding deterministic value (red dashed line) for Plate 2.

Fig. 10 sudden peak for sd/mean plot is observed in-between crests and troughs of the time-dependent displacement plot. The value of sd/mean is increased with the random mean temperature increment. It can be stated that characteristics of loading does influence the level of uncertainty in the dynamic displacement for uncertain temperature increment.

Figs. 11 and 13 present a comparison of the deterministic and the mean response of the central displacements for symmetric angle-ply laminate under pulse and impulse loading, respectively, and corresponding plots of sd/mean are shown in Figs. 12 and 14. When uncertainty in the response of the symmetric cross-ply and angle-ply laminates is compared, the rate of decay in the mean displacement with respect to time shows a comparable performance. The value of the sudden peak of sd/mean of corresponding time-dependent displacement for symmetric angle-ply laminate is more as compared to the symmetric cross-ply laminate.

Due to the suddenly applied impulse loading, the level of uncertainty in displacement is increased at delayed time domain response with an increment in the random mean temperature; whereas, the level of uncertainty is decreased at delayed time domain response of the central displacement under suddenly applied pulse loading with an increment in the random mean temperature. Thus, level of uncertainty in the dynamic displacement is significantly varied in time domain with the increment in the random mean temperature. Hence, prior to the engineering application, uncertainty quantification in the dynamic response of the composite plate with various anticipated loading conditions and lamina sequences subject to random temperature field is essential to ensure safety in its design.

It can be concluded from the earlier discussion in this section that, in case of the pulse loading the mean value and sd of the dynamic central displacement is decreasing at the delayed time domain due to the random mean temperature increment. However, in case of the impulse loading the amplitude of the mean central dynamic displacement is diminishing in time domain though the dispersion is increasing in time domain near the mean position of the amplitude along with the random mean temperature increment. Therefore, statistical parameters of the stochastic dynamic response due to the uncertain thermal parameters are also influenced by the characteristics of the applied loading. Application of four-layered symmetric cross-ply and angle-ply laminates does not have significant influence on the stochastic dynamic response characteristics.

4.5. PDF of peak displacement

The effect of temperature uncertainty on the peak dynamic displacement of the composite plate for the symmetric cross-ply and angle-ply laminates is demonstrated in Figs. 15 and 16, respectively with the applied pulse and impulse loading. The distribution of the peak central displacement due to 0.001 N/mm^2 pulse and impulse loading are plotted at various random mean temperatures in incremental order with the same level of uncertainty using the gPC expansion method. It is observed that the dispersion of the PDF increases with the increment in the random mean temperature, specifically under the pulse loading. Under the pulse loading, at a random mean temperature of 325 K the distributions are more symmetric, however at the higher random mean temperature the distribution became non-Gaussian and unsymmetric.

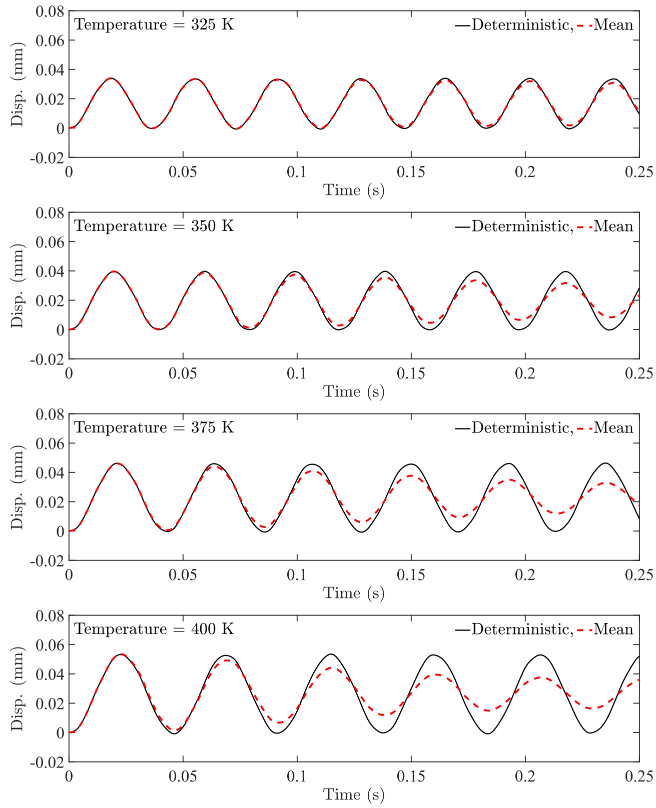

Fig. 7. Comparison of time history of the deterministic central displacement and mean of the central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to pulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

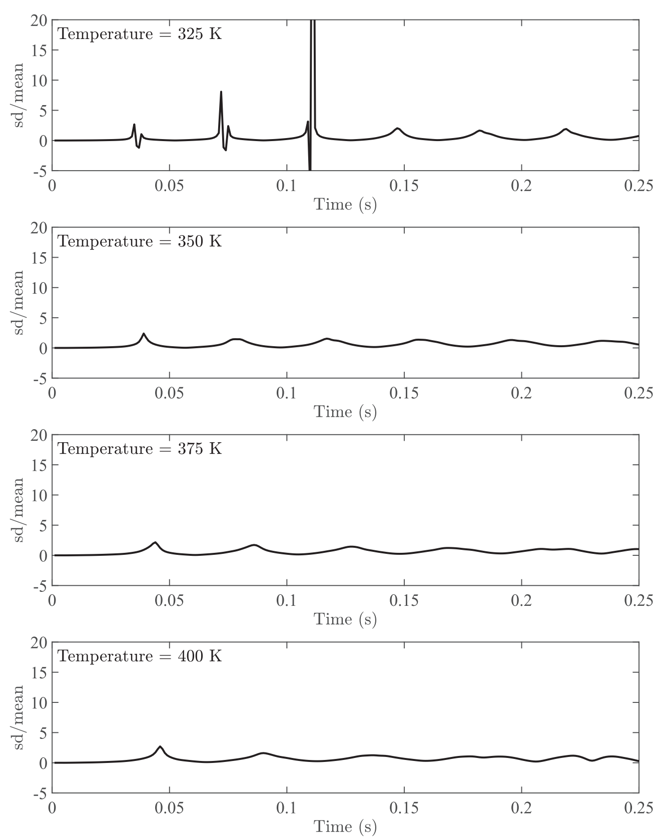

Fig. 8. Time history of sd/mean of the central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to pulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

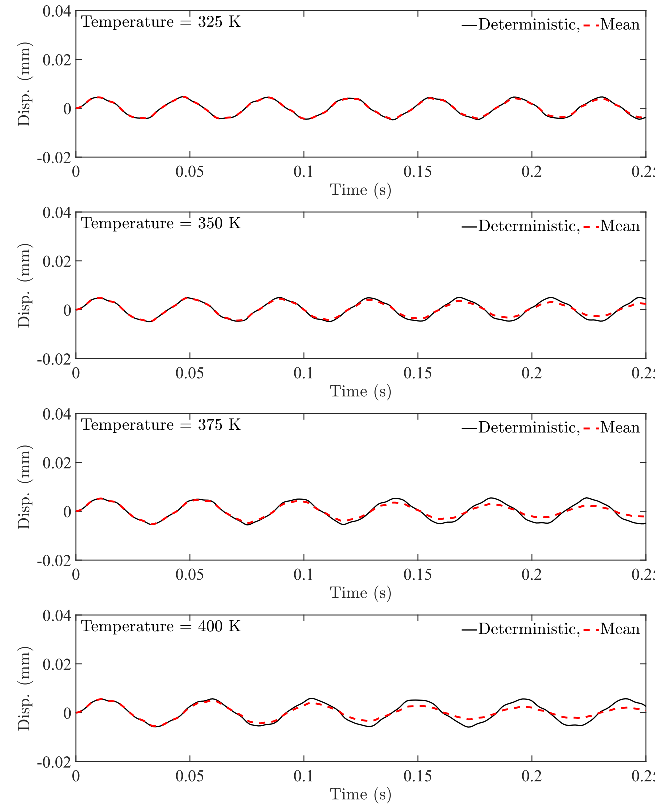

Fig. 9. Comparison of time history of the deterministic central displacement and mean of the central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to impulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

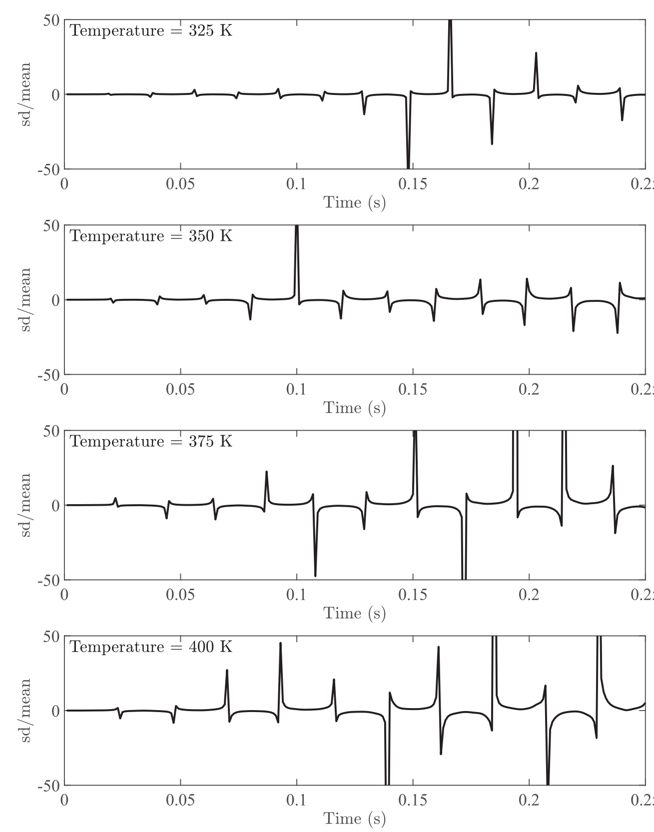

Fig. 10. Time history of sd/mean of the central displacement for a simply-supported (0°/90°/90°/0°) laminate subjected to impulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

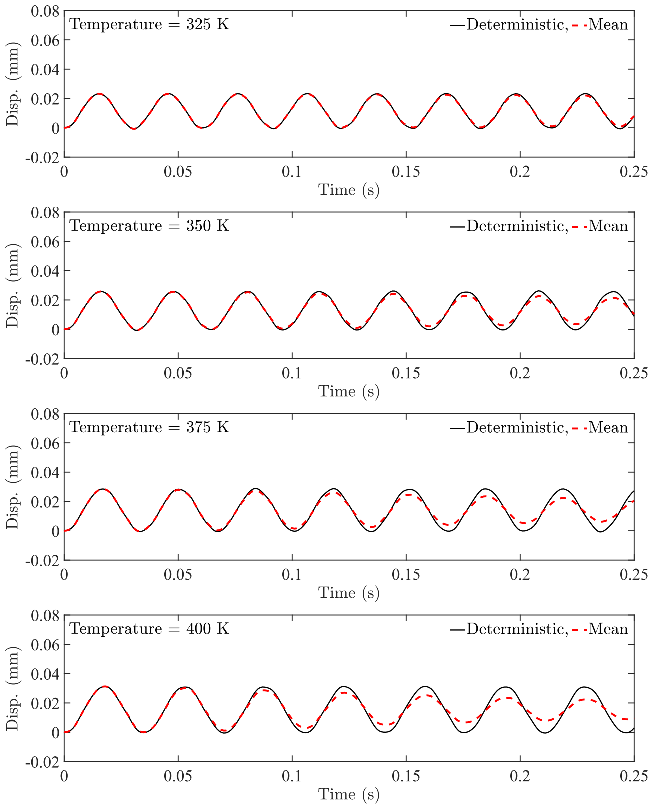

Fig. 11. Comparison of time history of the deterministic central displacement and mean of the central displacement for a simply-supported (45°/−45°/−45°/45°) laminate subjected to pulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

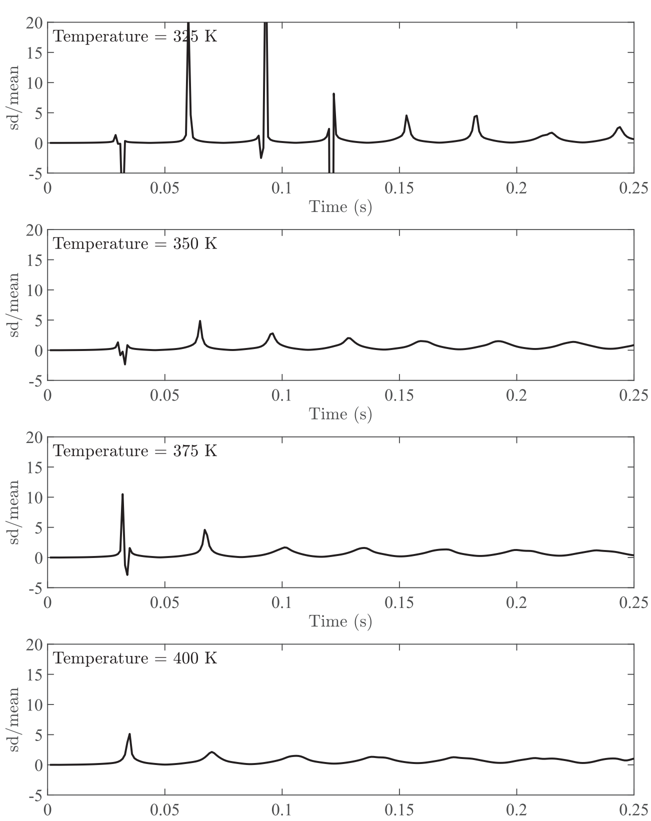

Fig. 12. Time history of sd/mean of the central displacement for a simply-supported (45°/−45°/−45°/45°) laminate subjected to pulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

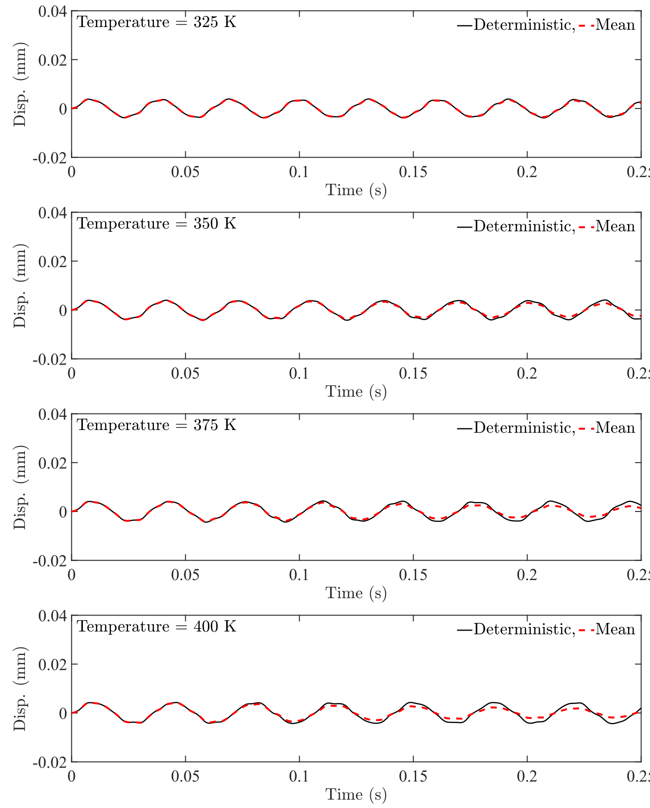

Fig. 13. Comparison of time history of the deterministic central displacement and mean of the central displacement for a simply-supported (45°/−45°/−45°/45°) laminate subjected to impulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

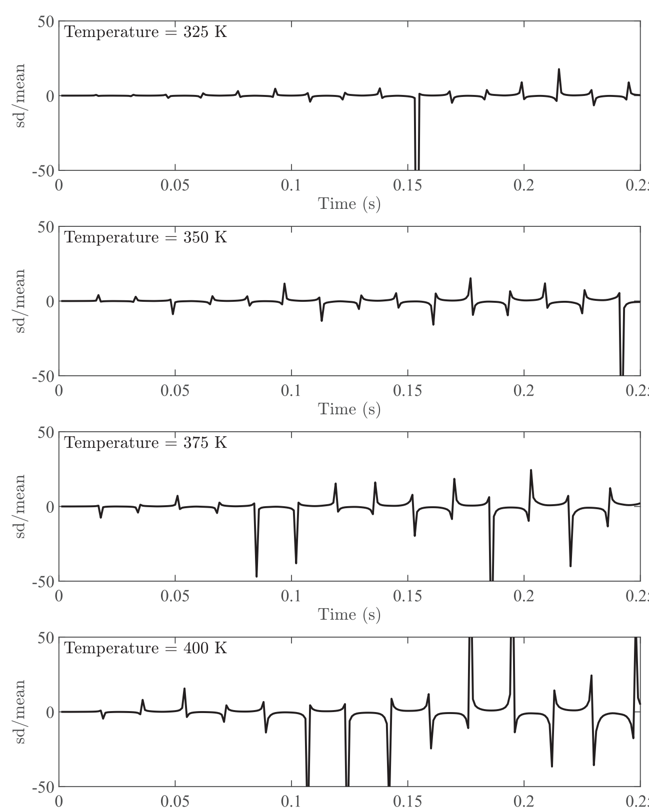

Fig. 14. Time history of sd/mean of the central displacement for a simply-supported (45°/−45°/−45°/45°) laminate subjected to impulse loading due to the randomness in temperature at 325 K, 350 K, 375 K, and 400 K, respectively for Plate 2.

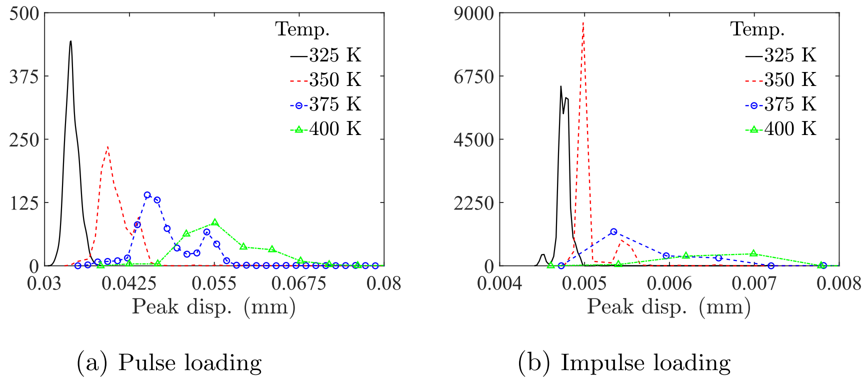

Fig. 15. PDF of peak central displacement for simply-supported (0°/90°/90°/0°) laminate subjected to pulse and impulse loading, respectively due to the randomness in temperature for Plate 2.

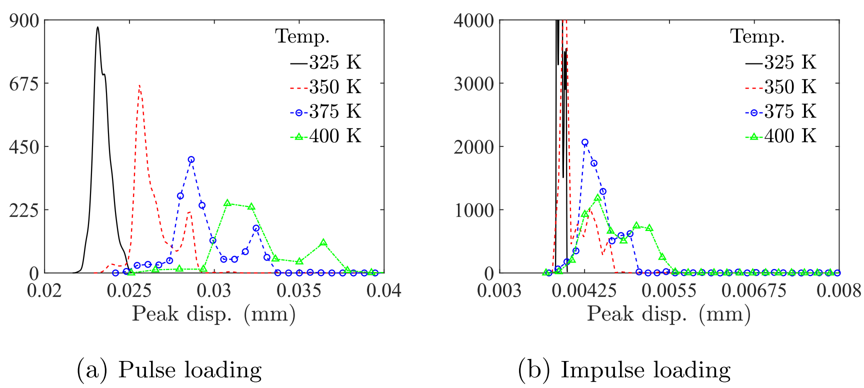

Fig. 16. PDF of peak central displacement for simply-supported (45°/−45°/−45°/45°) laminate subjected to pulse and impulse loading, respectively due to the randomness in temperature for Plate 2.

Under the impulse loading, the distributions are non-Gaussian, and larger part of the distributions overlap with each other at different random mean temperatures. Moreover, due to the impulse loading on the symmetric angle-ply laminate distributions of the peak central displacement are non-Gaussian, and noticeably unsymmetrical. Hence, appropriate evaluation of distribution of the peak displacement response is recommended for various lamina sequences with different loading conditions due to uncertainty in the temperature increment prior to real-field applications. Likewise, the peak failure stress induced in the composite plate is varied due to random mean temperature increment. A reliability analysis due to random thermal increment is necessary prior to application in the thermally sensitive part of the structure. This study has revealed the necessity of conducting further studies on the graphite-epoxy composite plates due to the random thermal environment.

5. Conclusions

The stochastic dynamic response of the graphite-epoxy composite plate under the applied pulse and impulse excitations, considering randomness in the incremental temperature is presented. The non-intrusive generalized polynomial chaos (gPC) expansion method is implemented for the stochastic simulations. The first-order shear deformation theory (FSDT) is adopted to analyze the thin composite plate under uniform temperature increment, and this deterministic finite element (FE) solver is used to generate the response at prescribed collocation points. The major advantage of the applicability of the gPC expansion method is to represent the mean and sd of the time-dependent dynamic response at each time step with reduced computational efforts. The convergence of the polynomial form of the dynamic response indicates the reduction in the error while representing the stochasticity in the temperature by using orthogonal polynomial. The computational accuracy of the gPC expansion method is well compared with the Monte Carlo simulations (MCS). Stochastic dynamic response of the composite plate due to thermal uncertainty is efficiently described here with the application of the gPC expansion method. The key findings from this study are summarized below.

1. The mean eigen frequency of the composite plate decreases with the increment in the random mean temperature, as elastic moduli of the composite plate decrease with the temperature.
2. The standard deviation (sd) of the eigen frequencies of the composite plate increases with the increment in the random mean temperature which imply an increment in the variation of the degraded elastic properties of the composite.
3. The stiffness of the composite plate decreases with the increment in the temperature, and subsequently amplitude of the dynamic displacement is increasing with the increment in the temperature. Moreover, the mean amplitude of the dynamic displacement of the composite plate decays gradually in the time domain with the random mean temperature increment.
4. The level of uncertainty in the dynamic response under impulse loading is higher due to higher rate of decay in corresponding mean transient response in time domain in comparison with the pulse loading.
5. The level of uncertainty in the dynamic displacement response in the delayed time domain is more under the suddenly applied impulse loading. Under the applied pulse loading, the level of

uncertainty in the dynamic displacement is decreasing in the delayed time domain. The PDF of the stochastic dynamic response in the random thermal environment should be studied for the various types of loading at several time steps before practical application.

6. The mean dynamic response is increasing with the increment in the random mean temperature due to the degradation in the material properties of the graphite-epoxy composite plate. Moreover, level of uncertainties are significantly varied in time and temperature domains. Thus, the stochastic studies of graphite-epoxy composite plate due to random thermal increment exhibited necessity over the deterministic analysis.
7. The distribution of the peak displacement at the lower random temperature is symmetrical, more evidently for the pulse loading. For the higher random temperature, the distribution became unsymmetric and non-Gaussian.
8. Statistical properties of the dynamic response are not much influenced by providing four layers of symmetric cross-ply and angle-ply laminates.

The presented methodology for quantifying uncertainty can be efficiently applied to complex structures. A deterministic FE model with complex geometry and advanced engineered materials can be developed in ANSYS® and corresponding modal analysis in thermal environment then can estimate the dynamic response efficiently. Non-intrusive gPC expansion can be efficiently used to estimate the stochastic parameters of the eigen frequencies and time-dependent dynamic response by limited numbers of realization of the deterministic FE model at predefined collocation points.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The first and third authors gratefully acknowledge the financial support extended by the Deutscher Akademischer Austauschdienst (DAAD) under Research Grants- Doctoral Programmes in Germany, and that by the Alexander von Humboldt Foundation under Fellowship for Experienced Researcher.

Appendix A

A.1. Appendix

$$[S_r] = \begin{bmatrix} N_x^r & N_{xy}^r & 0 & 0 & 0 & 0 & 0 & 0 & M_x^r & M_{xy}^r & 0 & Q_x^r \\ N_{xy}^r & N_y^r & 0 & 0 & 0 & 0 & 0 & 0 & M_{xy}^r & M_y^r & 0 & -Q_y^r \\ 0 & 0 & N_x^r & N_{xy}^r & 0 & 0 & -M_x^r & -M_{xy}^r & 0 & 0 & Q_x^r & 0 \\ 0 & 0 & N_{xy}^r & N_y^r & 0 & 0 & -M_{xy}^r & -M_y^r & 0 & 0 & -Q_y^r & 0 \\ 0 & 0 & 0 & 0 & N_x^r & N_{xy}^r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{xy}^r & N_y^r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -M_x^r & -M_{xy}^r & 0 & 0 & \frac{N_x^r h^2}{12} & \frac{N_{xy}^r h^2}{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -M_{xy}^r & -M_y^r & 0 & 0 & \frac{N_{xy}^r h^2}{12} & \frac{N_y^r h^2}{12} & 0 & 0 & 0 & 0 \\ M_x^r & M_{xy}^r & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_x^r h^2}{12} & \frac{N_{xy}^r h^2}{12} & 0 & 0 \\ M_{xy}^r & M_y^r & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_{xy}^r h^2}{12} & \frac{N_y^r h^2}{12} & 0 & 0 \\ 0 & 0 & Q_x^r & -Q_y^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_x^r & -Q_y^r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

A.2. Appendix

$$[G] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ N_{i,y} & 0 & 0 & 0 & 0 \\ 0 & N_{i,x} & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}.$$

Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.compstruct.2019.111159.

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