THE RETURN OF KAON FLAVOUR PHYSICS*

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(Received May 28, 2018)

Kaon flavour physics has played in the 1960s and 1970s a very important role in the construction of the Standard Model (SM) and in the 1980s and 1990s in SM tests with the help of CP violation in $K_{\rm L} \to \pi\pi$ decays represented by ε_K and the ratio ε'/ε. In this millennium, this role has been taken over by $B_{s,d}$ and D mesons. However, there is no doubt that in the coming years, we will witness the return of kaon flavour physics with the highlights being the measurements of the theoretically clean branching ratios for the rare decays $K^+ \to \pi^+\nu\bar\nu$ and $K_{\rm L} \to \pi^0\nu\bar\nu$, and the improved SM predictions for the ratio ε'/ε, for ε_K and the K^0–$\bar K^0$ mixing mass difference ΔM_K. Theoretical progress on the decays $K_{\rm L,S} \to \mu^+\mu^-$ and $K_{\rm L} \to \pi^0\ell^+\ell^-$ is also expected. They are all very sensitive to new physics (NP) contributions and the correlations between them should help us to identify new dynamics at very short distance scales. These studies will be enriched when theory on the $K \to \pi\pi$ isospin amplitudes ${\rm Re}A_0$ and ${\rm Re}A_2$ improves. This paper summarizes several aspects of this exciting field. In particular, we emphasize the role of the Dual QCD approach in getting the insight in the numerical Lattice QCD results on K^0–$\bar K^0$ mixing and $K \to \pi\pi$ decays.

DOI:10.5506/APhysPolB.49.1043

1. Introduction

A strategy for identifying new physics (NP) through flavour violating processes in twelve steps has been proposed in [1]. Presently, several of these steps cannot be realized, but this will certainly change in the coming years.

* Presented at the Cracow Epiphany Conference on Advances in Heavy Flavour Physics, Kraków, Poland, January 9–12, 2018.

In this paper, I will concentrate on kaon flavour physics and, in particular, on its main players:

— K^0–$\bar{K}^0$ mixing with the parameter ε_K representing mixing induced CP violation in $K_{\rm L} \to \pi\pi$ decays and the $K_{\rm L} - K_{\rm S}$ mass difference ΔM_K;

— The ratio ε'/ε representing the direct CP violation in $K_{\rm L} \to \pi\pi$ decays relative to the mixing induced one;

— The rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$, CP-conserving and CP-violating ones, respectively;

— Rare decays $K_{\rm L,S} \to \mu^+\mu^-$ and $K_{\rm L} \to \pi^0\ell^+\ell^-$;

— The $K \to \pi\pi$ isospin amplitudes ${\rm Re}A_0$ and ${\rm Re}A_2$ and, in particular, their ratio known under the name of the $\Delta I = 1/2$ rule.

During the last five years, most of the flavour theorists concentrated their efforts on the explanation of the so-called B-physics anomalies in $B \to K(K^*)\ell^+\ell^-$ and $B \to D(D^*)\tau\nu_\tau$ decays. Several hundreds of papers were published on them and numerous workshops have been organized to discuss possible NP behind them. Also this conference was dominated by these anomalies. While I took part in some of these discussions in the context of the so-called P_5' anomaly, I did not write a single paper on the anomalies in the ratios R_K, R_{K^*}, R_D and R_{D^*}. There were two reasons for it. First, I am still not convinced that these four anomalies, related to the violation of lepton flavour universality, will survive the future more precise measurements, to be performed not only by the LHCb experiment but in particular by Belle II. But I do hope very much that they will not disappear as they imply very interesting NP and moreover in view of the absence of direct signals for NP from ATLAS and CMS, we need as many anomalies in flavour physics as possible.

The second reason is that having retired in 2012 I got slower and could simply not compete with much younger researchers in writing so many papers and definitely I did not want to run behind a crowd which could be compared to the crowds on the south-route to the summit of the Mount Everest. For emeriti more pleasant is hiking in Norwegian mountains, simply because one is basically almost alone meeting during the day only few tourists. If one looks at the number of papers written on kaon flavour physics in the last five years, it is evident that working in this field is like hiking in Norway. Therefore, since 2014 I have changed my strategy and concentrated [2], with few exceptions, on kaon flavour physics. A series of reviews on our work appeared in [3–7].

It is not the purpose of my paper to repeat all the material in these reviews but rather concentrate on the 2017 news including also the very recent ones which could not be presented at the Epiphany 2018 as the corresponding papers appeared just recently. However, my paper will, in the first part, concentrate on the main players, not necessarily in the order of their appearance in the list above. This will be the material of Section 2. In Section 3, I will discuss some aspects of the work done in 2017 and 2018. Finally, in Section 4, I will present my shopping list for the coming years.

2. Main players

2.1. The $\Delta I = 1/2$ rule

One of the puzzles of the 1950s was a large disparity between the measured values of the real parts of the isospin amplitudes A_0 and A_2 in $K \to \pi\pi$ decays, which on the basis of usual isospin considerations were expected to be of the same order. In 2018, we know the experimental values of the real parts of these amplitudes very precisely [8]

$$\mathrm{Re}A_0 = 27.04(1) \times 10^{-8}\ \ \mathrm{GeV}, \quad \mathrm{Re}A_2 = 1.210(2) \times 10^{-8}\ \ \mathrm{GeV} \tag{1}$$

and express the so-called $\Delta I = 1/2$ rule [9, 10]

$$R = \frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 22.35\,. \tag{2}$$

In the 1950s, QCD and Operator Product Expansion did not exist and clearly one did not know that $W^\pm$ bosons existed in nature but using the ideas of Fermi [11], Feynman and Gell-Mann [12], and Marshak and Sudarshan [13], one could still roughly estimate the amplitudes $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ to conclude that such a high value of R is a real puzzle.

In modern times, one can recover this puzzle by considering QCD in the large-N limit [2], where N is the number of colours. In this limit, there are no QCD corrections to the Wilson coefficient of the current–current operator $Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}$ representing a simple tree-level $W^\pm$ exchange, and the relevant hadronic matrix elements of this operator can be calculated exactly in terms of pion decay constant F_π and the masses m_K and m_π by just factorizing the operator matrix element into the product of matrix elements of quark currents. One finds then [2]

$$\mathrm{Re}A_0 = 3.59 \times 10^{-8}\,\mathrm{GeV}\,, \qquad \mathrm{Re}A_2 = 2.54 \times 10^{-8}\,\mathrm{GeV}\,, \qquad R = \sqrt{2} \tag{3}$$

in plain disagreement with the data in (1) and (2). It should be emphasized that the explanation of the missing enhancement factor of 15.8 in R through some dynamics must simultaneously give the correct values for $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$. This means that this dynamics should suppress $\mathrm{Re}A_2$ by a factor of 2.1, not more, and enhance $\mathrm{Re}A_0$ by a factor of 7.5. This tells us that while the suppression of $\mathrm{Re}A_2$ is an important ingredient in the $\Delta I = 1/2$ rule, it is not the main origin of this rule. It is the enhancement of $\mathrm{Re}A_0$.

It should also be emphasized that the result in (3) has little to do with the so-called vacuum insertion approximation (VIA) but follows from the Dual QCD approach (DQCD) [14–17] in which the factorization of matrix elements in question can be proven to be the property of QCD in the large-N limit because in this limit QCD at very low momenta becomes a free theory of mesons [18–21]. With non-interacting mesons, the factorization of matrix elements of four-quark operators into matrix elements of quark currents or quark densities is automatic.

The first step towards the explanation of the $\Delta I = 1/2$ rule has been made through the pioneering 1974 calculations in [22, 23] where QCD renormalization group effects between M_W and scales $\mathcal{O}(1\ \mathrm{GeV})$, to be termed *quark–gluon evolution* in what follows, were done at leading order in the renormalization group improved perturbation theory and now can be done at the NLO level. But if one continues to use hadronic matrix elements obtained by factorizing them, the result is both scale- and renormalization scheme-dependent. Moreover, as shown in [2], the ratio R is in the ballpark of 3–4 certainly an improvement, but no explanation of its experimental value. In 1975, an attempt has been made to explain this rule by QCD penguins [24] but in 1986, it was pointed out in the framework of DQCD that the current–current operators and not QCD penguins are responsible dominantly for this rule [16]. This is obtained by performing a *meson evolution* from low scales at which factorization of matrix elements is valid in QCD to scales $\mathcal{O}(1\ \mathrm{GeV})$ at which the resulting matrix elements are combined with their Wilson coefficients evaluated by the known renormalization group methods. As shown in [2], the pattern of meson evolution below 1 GeV that includes also QCD penguins is similar to the one of quark–gluon evolution at short-distance scales so that the matching between these two evolutions although not precise is acceptable. A good summary of the basic structure of DQCD can be found in Sections 2 and 3 of [2].

The DQCD approach to weak decays developed in the 1980s has been improved in [2] through the inclusion of vector meson contributions in addition to pseudoscalars and improved through a better matching to short-distance contributions. Including QCD penguin contribution that at scales $\mathcal{O}(1\ \mathrm{GeV})$ amounts to a 10% effect in $\mathrm{Re}A_0$, one finds [2]

$$\begin{aligned} \mathrm{Re}A_0 &\approx (17.0 \pm 1.5) \times 10^{-8}\ \mathrm{GeV}\,, \qquad \mathrm{Re}A_2 \approx (1.1 \pm 0.1) \times 10^{-8}\ \mathrm{GeV}\,, \\ R &\approx 16.0 \pm 1.5\,. \end{aligned} \tag{4}$$

Even if the result for $\mathrm{Re}A_0$ is not satisfactory, it should be noted that the QCD dynamics identified by us was able to enhance the ratio R by an order of magnitude. We, therefore, conclude that QCD dynamics is dominantly responsible for the $\Delta I = 1/2$ rule. The remaining piece in $\mathrm{Re}A_0$ could come from final-state interactions (FSI) between pions as advocated in [25–31] bringing the values of R in (4) closer to its experimental value. Some support for this claim comes from the recent reconsideration of the role of FSI in the $\Delta I = 1/2$ rule in [32]. As investigated in [33], also NP could enter at some level.

After heroic efforts over many years, also lattice QCD by means of very sophisticated and tedious numerical calculations made impressive progress towards the explanation of the $\Delta I = 1/2$ rule within the SM. The most recent result from the RBC-UKQCD Collaboration reads [34]

$$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)_{\text{lattice QCD}} = 31.0 \pm 11.1 \tag{5}$$

and, in agreement with the 1986 result from DQCD [16], also this result is governed by current–current operators. But the uncertainty is still very large and it will be interesting to see whether lattice will be able to come closer to the data than it is possible using DQCD. One should also stress that the lattice value for $\mathrm{Re}A_2$ has a much smaller error than $\mathrm{Re}A_0$ and agrees well with the data.

To summarize, from my point of view, the dominant dynamics behind the $\Delta I = 1/2$ rule has been identified within the DQCD approach already in 1986 [16] and has been confirmed through improved calculations in 2014 [2]. This dynamics is very simple. It is just short-distance (quark–gluon) evolution of current–current operators down to scales $\mathcal{O}(1\ \mathrm{GeV})$ followed by meson evolution down to scales $\mathcal{O}(m_\pi)$ at which the hadronic matrix elements factorize and can easily be calculated. I doubt that the remaining piece can be fully explained by NP as this would lead to a very large fine-tuning in ΔM_K as demonstrated in [33]. It is likely that FSI and additional subleading corrections not included in the result in (4) could be responsible for the missing piece. However, I do not think that the present analytic methods like DQCD or the methods advocated by Pich and collaborators, as reviewed recently in [35], are sufficiently powerful to answer the question at which level NP enters the amplitudes $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$. Here, lattice QCD should provide valuable answers and I am looking forward to improved results on these two amplitudes from the RBC-UKQCD Collaboration and other lattice groups. This would provide two additional important constraints on NP models.

2.2. ε_K and ΔM_K

The parameter ε_K and the $K_{\rm L}-K_{\rm S}$ mass difference have already for decades played an important role in the constraints on NP. There is some tendency that ε_K in the SM is below the data [36–39], but certainly one cannot talk presently about an anomaly in ε_K. Indeed, this depends on whether inclusive or exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$ are used and with the inclusive ones SM value of ε_K agrees well with the data. But then, as emphasized in [40], ΔM_s and ΔM_d are significantly above the data. Moreover, this is true for the whole class of CMFV models. Related discussions can be found in [33, 41–44]. This tension increased recently due to the improved lattice calculations [45] and could signal new complex phases beyond the CKM phase as only such phases could decrease ΔM_s and ΔM_d through destructive intereference between SM and NP contributions.

Such new phases could have an impact not only on ε_K but, as emphasized in [46], also on ΔM_K. The point is that ΔM_K is proportional to the real part of a square of a complex coefficient C_K and a new phase modifying its imaginary part will quite generally decrease the value of ΔM_K relative to the SM estimate simply because

$$(\Delta M_K)^{\rm NP} = c\left[({\rm Re}\, C_K)^2-({\rm Im}\, C_K)^2\right] \tag{6}$$

with c being positive. The uncertainty in the SM estimate of ΔM_K is unfortunately still very large [47] so that we cannot presently decide whether a positive or negative NP contribution to ΔM_K if any is required. Future lattice QCD calculations of long-distance contributions to ΔM_K could help in this respect [48, 49]. In DQCD, they are found to amount to $20\pm10\%$ of the measured ΔM_K [2, 50]. In the case of ε_K, such long-distance contributions to ε_K are below 10% and have been reliably calculated in [37, 51].

Now if NP contributes significantly to ε_K and ΔM_K, one has to consider new local operators in addition to the SM operator so that the full operator basis is given as follows [52, 53]:

$$\begin{aligned}
\mathcal{O}_1 &= \bar{s}^\alpha\gamma_\mu P_{\rm L}d^\alpha\ \bar{s}^\beta\gamma_\mu P_{\rm L}d^\beta\,,\\
\mathcal{O}_2 &= \bar{s}^\alpha P_{\rm L}d^\alpha\ \bar{s}^\beta P_{\rm L}d^\beta\,,\\
\mathcal{O}_3 &= \bar{s}^\alpha P_{\rm L}d^\beta\ \bar{s}^\beta P_{\rm L}d^\alpha\,,\\
\mathcal{O}_4 &= \bar{s}^\alpha P_{\rm L}d^\alpha\ \bar{s}^\beta P_{\rm R}d^\beta\,,\\
\mathcal{O}_5 &= \bar{s}^\alpha P_{\rm L}d^\beta\ \bar{s}^\beta P_{\rm R}d^\alpha\,,
\end{aligned} \tag{7}$$

with α,β being colour indices and $P_{\rm R,L}=(1\pm\gamma_5)/2$. Only $\mathcal{O}_1$ is present in the SM. Moreover, also operators with $P_{\rm L}$ and $P_{\rm R}$ interchanged contribute.

The Wilson coefficients of these operators have already been known at the NLO level [54, 55] for almost two decades. Recently, also significant progress in the evaluation of K^0–$\bar{K}^0$ matrix elements by ETM, SWME and RBC-UKQCD lattice collaborations [56–60] has been made.

It is customary to represent the results for the K^0–$\bar{K}^0$ matrix elements of the operators in question in terms of B_i parameters. In the vacuum insertion approximation (VIA), they are simply given by

$$B_1 = B_2 = B_3 = B_4 = B_5 = 1 \qquad \text{(VIA)} \tag{8}$$

and, moreover, do not depend on the renormalization scale μ as predicted by QCD. Already this property of VIA, which is based on the factorization of matrix elements of four-quark operators into products of quark currents or quark densities, is problematic as generally these parameters depend on μ.

Now the RBC-UKQCD Collaboration working at $\mu = 3$ GeV finds [58–60]

$$B_1 = 0.523(9)(7)\,, \qquad B_2 = 0.488(7)(17)\,, \qquad B_3 = 0.743(14)(65) \tag{9}$$

and

$$B_4 = 0.920(12)(16)\,, \qquad B_5 = 0.707(8)(44)\,, \tag{10}$$

with the first error being statistical and the second systematic. Similar results are obtained by EMT and SWME collaborations although the values for B_4 and B_5 from the ETM Collaboration are visibly below the ones from given above: $B_4 = 0.78(4)(3)$ and $B_5 = 0.49(4)(1)$. Except for B_4, all values differ significantly from unity prohibiting the use of VIA.

To our knowledge, no lattice group made an attempt to understand this pattern of values, probably because within lattice QCD which works at scales $\mathcal{O}(2\text{–}3)$ GeV, this pattern cannot be understood. On the other hand, it has been recently demonstrated in [61] that this pattern can be understood within DQCD approach because in this approach, an insight in the QCD dynamics at very low scales up to 1 GeV can be obtained through meson evolution followed by the usual RG QCD evolution as already discussed above in the context of the $\Delta I = 1/2$ rule.

The case of B_1 is well-known. In the large-N limit, one finds $B_1 = 3/4$ [62]. The meson evolution followed by quark–gluon evolution brings it in the ballpark of the lattice result in (9). In this particular case, one usually multiplies the result by the corresponding SD renormalization group factor to find the scale and renormalization scheme-independent $\hat{B}_K = 0.73 \pm 0.02$ [2] in a very good agreement with the world average of lattice QCD calculations $\hat{B}_K = 0.766 \pm 0.010$ [63].

In the case of the BSM operators $\mathcal{O}_i$ with $i = 2$–5, the construction of scale-independent $\hat{B}_i$ parameters, although possible, is not particular useful because $\mathcal{O}_2$ mixes under renormalization with $\mathcal{O}_3$ and $\mathcal{O}_4$ with $\mathcal{O}_5$. This mixing is known at the NLO level [54, 55] and useful NLO expressions for μ dependence of hadronic matrix elements and their Wilson coefficients can be found in [64].

In the large-N limit, one finds [61]

$$B_2 = 1.20\,, \qquad B_3 = 3.0\,, \qquad B_4 = 1.0\,, \qquad B_5 = 0.2 \qquad (\text{large-}N \text{ limit})\,. \tag{11}$$

These results differ significantly from lattice results but apply to $\mu = \mathcal{O}(m_\pi)$, while the lattice results where obtained at $\mu = 3$ GeV. It is, therefore, remarkable that the pattern

$$B_2 < B_5 \leq B_3 < B_4 \qquad (\mu = 3~\text{GeV}) \quad (\text{Lattice QCD}) \tag{12}$$

can indeed be understood within DQCD although there one finds first

$$B_5 < B_4 < B_2 < B_3 \qquad (\mu = \mathcal{O}(m_\pi)) \quad (\text{DQCD})\,. \tag{13}$$

As meson evolution with the inclusion of pseudoscalar mesons can be done only up to $\mu = 0.65 \pm 0.05$ GeV, let us use the standard RG equations to find first lattice values for B_i at $\mu = 1$ GeV, where perturbation theory is still reliable. From central values in (9) and (10), one finds at $\mu = 1$ GeV [61]

$$B_2 = 0.608\,, \quad B_3 = 1.06\,, \quad B_4 = 0.920\,, \quad B_5 = 0.519 \quad (\text{Lattice QCD})\,. \tag{14}$$

Using ETM values for B_4 and B_5, one would find $B_4 = 0.78$ and $B_5 = 0.24$.

We observe that B_2, B_3 and B_5, all moved towards their large-N values in (11), while B_4 did not change in LO approximation. These results are already very encouraging. The rest of the job is done by meson evolution. Starting with the values in (11) and performing meson evolution in the chiral limit, one finds at the order of $1/N$ [61]

$$B_2(\Lambda) = 1.2\left[1 - \frac{8}{3}\frac{\Lambda^2}{(4\pi F_K)^2}\right], \qquad B_3(\Lambda) = 3.0\left[1 - \frac{16}{3}\frac{\Lambda^2}{(4\pi F_K)^2}\right], \tag{15}$$

$$B_4(\Lambda) = 1.0\left[1 - \frac{4}{3}\frac{\Lambda^2}{(4\pi F_K)^2}\right], \qquad B_5(\Lambda) = 0.23\left[1 + 4\frac{\Lambda^2}{(4\pi F_K)^2}\right], \tag{16}$$

where Λ is the cut-off of DQCD which allows us to separate the non-factorizable meson evolution from the quark–gluon one. The general trend already observed in the quark–gluon evolution is nicely outlined in the meson evolution with a strong suppression of B_2, an even stronger suppression of B_3, a smooth evolution of B_4 and a strong enhancement of B_5.

Consequently, for $\Lambda = 0.7$ GeV, one finds

$$B_2 = 0.79\,, \quad B_3 = 0.96\,, \quad B_4 = 0.83\,, \quad B_5 = 0.30 \quad (\text{DQCD})\,. \tag{17}$$

We note also that the values for B_4 and B_5 are in between those from RBC-UKQCD and ETM collaborations, and we are looking forward to new improved lattice results for all four parameters in order to see how well DQCD reproduces LQCD numbers in question.

In any case, as the meson evolution has been performed in the chiral limit without the inclusion of vector meson contributions, this result should be considered as not only satisfactory but remarkable as our calculations involved only one parameter, the cut-off scale Λ which in any case should be around 0.7 GeV if only pseudoscalar meson contributions are taken into account. It demonstrates the importance of the QCD dynamics at scales below 1 GeV and gives additional support to our claim that meson evolution is the dominant QCD dynamics responsible for the $\Delta I = 1/2$ rule.

We are not aware of any analytical approach that could provide such insight in lattice QCD results in question. We challenge the chiral perturbation theory experts to provide an insight into the values of B_i from LQCD in their framework, in particular without using lower energy constants obtained from LQCD.

2.3. $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

These two very rare decays are exceptional in the flavour physics as their branching ratios are known for fixed CKM parameters within an uncertainty of 2% which, to my knowledge, cannot be matched by any other meson decay. Indeed, they are theoretically very clean and their branching ratios have been calculated within the SM including NLO QCD corrections to the top-quark contributions [65–67], NNLO QCD corrections to the charm contribution in $K^+ \to \pi^+\nu\bar{\nu}$ [68–70] and NLO electroweak corrections [71–73]. Moreover, extensive calculations of isospin breaking effects and non-perturbative effects have been done [74, 75]. Therefore, once the CKM parameters $|V_{cb}|$, $|V_{ub}|$ and γ will be precisely determined in tree-level decays, these two decays will offer excellent tests of the SM and constitute very powerful probes of NP. Reviews of these two decays can be found in [1, 76–79]. In particular in [80], bounds on $K \to \pi\nu\bar{\nu}$ decays in correlation with the unitarity triangle and $\sin 2\beta$ within models with minimal flavour violation have been derived. See also interesting recent papers of the impact of lepton flavour non-universality on these decays [81–83] and right-handed neutrinos [84].

It is really exciting that after twenty five years of waiting [65, 85], the prospects of measuring the branching ratios for these two *golden* modes with good precision within the next five years are very good. Indeed, the NA62 experiment at CERN has recently found one event of $K^+ \to \pi^+\nu\bar{\nu}$ decay and twenty SM-like events are expected until the end of 2019. Eventually, NA62 expects to measure the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio with the precision of $\pm 10\%$ [86, 87]. Also the KOTO experiment at J-PARC should make a significant progress in measuring the branching ratio for $K_{\rm L} \to \pi^0\nu\bar{\nu}$ [77, 88].

Here, it will suffice to quote parametric expressions for branching ratios $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ in the SM in terms of the CKM inputs [89]

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = (8.39 \pm 0.30) \times 10^{-11} \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^{2.8} \left[\frac{\gamma}{73.2^\circ}\right]^{0.74}, \quad (18)$$

$$\mathcal{B}\left(K_{\rm L} \to \pi^0\nu\bar{\nu}\right) = (3.36 \pm 0.05) \times 10^{-11} \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}}\right]^{2} \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^{2} \times \left[\frac{\sin(\gamma)}{\sin(73.2^\circ)}\right]^{2}. \quad (19)$$

The parametric relation for $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ is exact, while for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, it gives an excellent approximation: for the large ranges $37 \leq |V_{cb}| \times 10^3 \leq 45$ and $60^\circ \leq \gamma \leq 80^\circ$, it is accurate to 1% and 0.5%, respectively. The exposed errors are non-parametric ones. They originate in the left-over uncertainties in QCD and electroweak corrections and other small uncertainties. For $K^+ \to \pi^+\nu\bar{\nu}$, the error is larger due to the relevant charm contribution that can be neglected for $K_{\rm L} \to \pi^0\nu\bar{\nu}$. In the case of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, we have absorbed $|V_{ub}|$ into the non-parametric error due to the weak dependence on it.

The virtue of these formulae is that they allow easily to monitor the changes in the values of branching ratios in question, which clearly will still take place before the values on $|V_{cb}|$, $|V_{ub}|$ and γ from tree-level decays will be precisely known. The error budgets can be found in Fig. 1 of [89]. They tell us, as already inferred from (18) and (19), that for $K^+ \to \pi^+\nu\bar{\nu}$ the crucial CKM element is $|V_{cb}|$ and for $K_{\rm L} \to \pi^0\nu\bar{\nu}$ all three: $|V_{cb}|$, $|V_{ub}|$ and γ.

Using (18) and (19) together with an average provided in [89]

$$|V_{cb}|_{\rm avg} = (40.7 \pm 1.4) \times 10^{-3}, \qquad |V_{ub}|_{\rm avg} = (3.88 \pm 0.29) \times 10^{-3}, \quad (20)$$

one finds with $\gamma = (73.2^{+6.3}_{-7.0})^\circ$

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = (8.4 \pm 1.0) \times 10^{-11}, \quad (21)$$

$$\mathcal{B}\left(K_{\rm L} \to \pi^0\nu\bar{\nu}\right) = (3.4 \pm 0.6) \times 10^{-11}. \quad (22)$$

While the values in (20) will change in time, we expect that both branching ratios will not be modified by more than 15% and the errors will be reduced significantly due to better determination of $|V_{cb}|$, $|V_{ub}|$ and γ.

Experimentally, we have [90]

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right)_{\rm exp} = \left(17.3^{+11.5}_{-10.5}\right) \times 10^{-11}\,, \tag{23}$$

and very recently the NA62 Collaboration observing one event quotes

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right)_{\rm exp} = \left(28^{+44}_{-23}\right) \times 10^{-11} \qquad ({\rm NA62})\,. \tag{24}$$

This result should be improved in 2019. The 90% C.L. upper bound on $K_{\rm L} \to \pi^0\nu\bar{\nu}$ reads [91]

$$\mathcal{B}\left(K_{\rm L} \to \pi^0\nu\bar{\nu}\right)_{\rm exp} \leq 2.6 \times 10^{-8}\,. \tag{25}$$

It should also be improved by KOTO in the coming years.

2.4. ε'/ε *striking back*

One of the stars of flavour physics in the 1990s was the ratio ε'/ε that measures the size of the direct CP violation in $K_{\rm L} \to \pi\pi$ relative to the indirect CP violation described by ε_K. On the experimental side, the world average from NA48 [92] and KTeV [93, 94] collaborations reads

$$\left(\varepsilon'/\varepsilon\right)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}\,. \tag{26}$$

On the theory side, a long-standing challenge in making predictions for ε'/ε within the SM has been the significant cancellation of QCD penguin contributions by electroweak penguin contributions to this ratio. In the SM, QCD penguins give a positive contribution and electroweak penguins a negative one. In the 1980s, when the mass of the top quark was not known and m_t in the ballpark of 50–100 GeV has been used in the analyses of ε'/ε, electroweak penguin contributions governed by Z^0-penguins could be neglected and only QCD penguins and isospin breaking corrections were taken into account. The SM prediction was then close to the one in (26) [95]. The situation changed in 1989 when it was demonstrated in [96, 97] that in the presence of a very heavy top Z^0-penguins, entering ε'/ε with the opposite sign to QCD penguins cannot be neglected leading to a very strong suppression of ε'/ε.

Therefore, in order to obtain a useful prediction for ε'/ε, the relevant contributions of the QCD penguin and electroweak penguin operators must be known accurately. Reviews on ε'/ε can be found in [98–102]. See also recent review in [35] which discusses ε'/ε mainly within a chiral perturbative framework including also some large-N ideas but having nothing to do with DQCD and reaching very different conclusions than those presented below.

As far as short-distance contributions (Wilson coefficients of QCD and electroweak penguin operators) are concerned, they have been known already for more than twenty five years at the NLO level [103–108]. First steps towards the NNLO predictions for ε'/ε have been made in [68, 109, 110]. Recently, an important progress towards the complete NNLO result has been made in [111]. We refer to this paper and the contribution of Cerdà-Sevilla to these proceedings [112].

The situation with hadronic matrix elements is another story and even if significant progress on their evaluation has been made over the last 25 years, the present status is far from being satisfactory. In order to describe the problem in explicit terms, let me write down the NLO formula for ε'/ε presented in [113]

$$\begin{aligned}\frac{\varepsilon'}{\varepsilon} = {} & 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\times 10^{-4}}\right]\left[a\left(1-\hat{\Omega}_{\rm eff}\right)\left(-4.1(8)+24.7\,B_6^{(1/2)}\right)\right.\\ & \left. +1.2(1)-10.4\,B_8^{(3/2)}\right].\end{aligned}\tag{27}$$

This formula has been obtained by assuming that the real parts of the $K\to\pi\pi$ isospin amplitudes A_0 and A_2, which exhibit the $\Delta I=1/2$ rule, are fully described by SM dynamics. Their experimental values are used to determine to a very good approximation hadronic matrix elements of all $(V-A)\otimes(V-A)$ operators [107]. The first and the third term in (27) summarize these contributions. In this manner, the main uncertainties in ε'/ε reside in the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ which represent the hadronic matrix elements of the $(V-A)\otimes(V+A)$ QCD penguin and electroweak penguin operators, Q_6 and Q_8, respectively.

The parameters a and $\hat{\Omega}_{\rm eff}$ summarize isospin breaking corrections and include strong isospin violation $(m_u\neq m_d)$, the correction to the isospin limit coming from $\Delta I=5/2$ transitions and electromagnetic corrections. They can be extracted from [114–116] and are given as follows [113]:

$$a=1.017\,,\qquad \hat{\Omega}_{\rm eff}=(14.8\pm 8.0)\times 10^{-2}\,.\tag{28}$$

The latter value differs from the one quoted in [101] but is equivalent to it as discussed in detail in [113] after equation (16) in that paper.

Expression (27) tells us that a precise determination of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in QCD is crucial. First steps in this direction have been made 30 years ago in [14, 62, 117] by calculating them analytically in DQCD in the large-N limit [14, 62, 117]

$$B_6^{(1/2)}=B_8^{(3/2)}=1\qquad(\text{large-}N\text{ limit})\,.\tag{29}$$

For many years, various authors have estimated $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in a number of other large-N approaches [118–120] finding $B_6^{(1/2)}$ in the ballpark of 3 and $B_8^{(3/2)} > 1$. Similar comment applies to $B_8^{(3/2)}$ in the dispersive approach [121, 122]. With such values, the SM is fully consistent with the data in (26).

The 2015 results from the RBC-UKQCD Collaboration and DQCD approach contradict this picture. Indeed, in 2015, significant progress on the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ was made by the RBC-UKQCD Collaboration, who presented their results on the relevant hadronic matrix elements of the operators Q_6 [34] and Q_8 [123]. These results imply the following values for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ at $\mu = 1.53$ GeV [89, 113]

$$B_6^{(1/2)} = 0.57 \pm 0.19\,, \qquad B_8^{(3/2)} = 0.76 \pm 0.05 \qquad (\text{RBC-UKQCD})\,. \tag{30}$$

While the low value of $B_6^{(1/2)}$ in (30) is at first sight very surprising, a new analysis in DQCD beyond the large-N limit in (29) [124] gives strong support to the values in (30). In fact, Gérard and myself demonstrated explicitly the suppression of both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ below their large-N limit which is caused by meson evolution from scales $\mathcal{O}(m_\pi)$, where (29) is valid to scales $\mathcal{O}(1\ \text{GeV})$ at which one can compare with lattice results. The sign of this evolution is such that both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ evaluated at $\mu = \mathcal{O}(1\ \text{GeV})$ are decreased below unity and the suppression of $B_6^{(1/2)}$ is stronger than the one of $B_8^{(3/2)}$. This pattern is consistent with the perturbative evolution of these parameters above $\mu = \mathcal{O}(1\ \text{GeV})$ [107] and implies a smooth matching between meson and quark–gluon evolutions. Consequently, at scales $\mu = \mathcal{O}(1\ \text{GeV})$, the inequalities

$$B_6^{(1/2)} < B_8^{(3/2)} < 1 \qquad (\text{DQCD}) \tag{31}$$

can be obtained. More specifically, we find

$$B_6^{(1/2)}(m_c) \le 0.60\,, \qquad B_8^{(3/2)}(m_c) = 0.80 \pm 0.10 \tag{32}$$

in agreement with (30). The result for $B_6^{(1/2)}$ is less precise and we cannot exclude values as low as $B_6^{(1/2)} = 0.50$ and as large as 0.70 but there is a strong indication that $B_6^{(1/2)} < B_8^{(3/2)}$. For further details, see [124]. In fact, as we demonstrated in the case of K^0–$\bar{K}^0$ matrix elements and summarized briefly above, DQCD even if not precise provided correct pattern of B_i values obtained by lattice QCD with much higher precision than it was possible so far for $B_6^{(1/2)}$ and $B_8^{(3/2)}$. We are, therefore, confident that future more precise lattice calculations will also confirm the pattern in (31).

In this context, it should be emphasized that in the past, values $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$ were combined in phenomenological applications with the Wilson coefficients evaluated at scales $\mu = \mathcal{O}(1\ \mathrm{GeV})$. The results above show that this is incorrect and the factorization scale is at very low momenta. However to find it out, one has to include non-factorizable contributions as done in [124] and determine the scale at which they vanish.

Inserting the lattice results in (30) into (27), a detailed numerical NLO analysis in [113] gave[1]

$$\varepsilon'/\varepsilon = (1.9 \pm 4.5) \times 10^{-4}\,, \tag{33}$$

roughly 3σ away from the experimental value in (26). A subsequent NLO analysis in [125] using also hadronic matrix elements from lattice QCD confirmed these findings

$$\left(\varepsilon'/\varepsilon\right)_{\mathrm{SM}} = (1.1 \pm 5.1) \times 10^{-4} \qquad (\mathrm{KNT})\,. \tag{34}$$

The difference from (33) is related to a different input but clearly these results are consistent with each other.

While these results, based on the hadronic matrix elements from the RBC-UKQCD lattice Collaboration, suggest some evidence for the presence of NP in hadronic K decays, the large uncertainties in the hadronic matrix elements in question do not yet preclude that eventually the SM will agree with data. In this context, the upper bounds from DQCD in (31) are important as they give presently the strongest support to the anomaly in question, certainly stronger than present lattice results. Indeed, employing the rather precise lattice value for $B_8^{(3/2)}$ in (30) and setting $B_6^{(1/2)} \le B_8^{(3/2)} = 0.76$, one finds, varying all other input parameters, the upper bound

$$\left(\varepsilon'/\varepsilon\right)_{\mathrm{SM}} \le (6.0 \pm 2.4) \times 10^{-4}\,, \tag{35}$$

still $3\,\sigma$ below the experimental data.

As the bound in (31) plays a significant role in the conclusion that NP could be at work in ε'/ε, let us remind sceptical readers about other successes of DQCD that we discussed above. Therefore, I strongly believe that future more precise lattice calculations of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ will confirm the bound in (31) implying that indeed NP contributes significantly to ε'/ε unless the error in the experimental value in (26) has been underestimated. In fact, taking additional information provided below into account, my expectation for the SM value of ε'/ε in the SM is

$$(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (5 \pm 2) \times 10^{-4} \qquad (\text{my expectation for SM})\,. \tag{36}$$

[1] Some authors refer to this result as based on DQCD. Even if DQCD would get similar values, the numbers in (33) are based on $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from LQCD.

Therefore, I strongly disagree with the SM estimate in [35], where the authors using chiral perturbation framework find $\varepsilon'/\varepsilon = (15 \pm 7) \times 10^{-4}$. From my point of view, this paper demonstrates that ε'/ε cannot be predicted reliably within this framework. Indeed within 2σ, one finds, on the one hand, $\varepsilon'/\varepsilon = 3 \times 10^{-3}$ and, on the other hand, 1×10^{-4}. This framework does not include the meson evolution and it is not surprising that the resulting central value of ε'/ε obtained by these authors is so large.

Additional support for the small value of ε'/ε in the SM comes from the recent reconsideration of the role of FSI in ε'/ε [32] and from first NNLO QCD calculations [111] of QCD penguin contributions. It should also be recalled that NNLO corrections to electroweak penguin contributions calculated already in [109] and not included until now in the numerical results presented above increase the role of electroweak penguins by roughly 16% decreasing further ε'/ε. In this case, an effective central value of $B_8^{(3/2)}$ from the RBC-UKQCD Collaboration is increased to $(B_6^{(1/2)})_{\rm eff} = 0.88 \pm 0.06$. However, such effects should be included together with all NNLO corrections.

As far as FSI are concerned, the chiral perturbation theory practitioners, already long time ago, put forward the idea that both the amplitude $\mathrm{Re}A_0$, governed by the current–current operator Q_2–Q_1 and the Q_6 contribution to the ratio ε'/ε could be enhanced significantly through FSI in a correlated manner [27–31] bringing the SM prediction for ε'/ε in the ballpark of experimental data [35]. However, as shown in [32] beyond the strict large-N limit, FSI are likely to be relevant for the $\Delta I = 1/2$ rule, in agreement with [27–31, 126], but much less relevant for ε'/ε. In particular, as demonstrated in [32], the correlation between the $\Delta I = 1/2$ rule and ε'/ε claimed in these papers is broken at the $1/N$ level. Let us hope that new result from the RBC-UKQCD Collaboration will shed light on these different views on ε'/ε.

While after the completion of NNLO corrections to Wilson coefficients the fate of ε'/ε in the SM will be in the hands of lattice gauge theorists, one should not forget all the efforts made by renormalization group experts over almost 30 years that allowed to determine the Wilson coefficients of the relevant operators precisely. Without such calculations, the matching of short-distance contributions to long-distance contributions represented by hadronic matrix elements would not be possible and, consequently, the prediction for ε'/ε would be poorly known even if lattice QCD would reach satisfactory precision. For a historical account of these NLO and NNLO efforts, see [127].

A number of authors investigated what kind of NP could give sufficient upward shift in ε'/ε and what would then be implications for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$. The summary of these studies can be found in the reviews in [5–7] so that I will make only general comments on them. The up-todate list of relevant papers is collected in Table I. In these models, ε'/ε can be enhanced significantly without violating existing constraints. An exception are leptoquark models which we will discuss in the final part of this presentation.

TABLE I

Papers studying implications of ε'/ε anomaly.

NP scenario	References	Correlations with
LHT	[128]	$K_L \to \pi^0\nu\bar{\nu}$
Z-FCNC	[46, 129, 130]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
Z'	[46],	$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ and ΔM_K
Simplified models	[131]	$K_L \to \pi^0\nu\bar{\nu}$
331 models	[132, 133]	$b \to s\ell^+\ell^-$
Vector-like quarks	[134]	$K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ and ΔM_K
Supersymmetry	[135–139]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
2-Higgs doublet model	[140, 141]	$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$
Right-handed currents	[142, 143]	EDMs
Left–right symmetry	[144]	EDMs
Leptoquarks	[145]	All rare kaon decays

We have seen that one of the reasons for a large uncertainty in the SM prediction for ε'/ε was the strong cancellation between QCDP and EWP contributions. As stressed in [46] beyond the SM, quite generally either EWP or QCDP dominate NP contributions and theoretical uncertainties are much smaller because no cancellations take place. We refer to [46] for the discussion of this point.

Finally, in all models listed in Table I, only modifications of the Wilson coefficients of SM operators by NP contributions have been considered. However, generally, other operators with different Dirac structures, like the ones in (7) could be responsible for the observed ε'/ε anomaly. To my knowledge, the relevant hadronic matrix elements of these operators have never been calculated in QCD. We hope to present the first results for them in DQCD soon.

On the other hand, the $K \to \pi\pi$ matrix element of the chromomagnetic penguin operator has been calculated in DQCD [146] and found to be significantly smaller than previously expected in agreement with the earlier lattice QCD calculation by the ETM group of related $K \to \pi$ matrix element of this operator [147].

2.5. $K_{L,S} \to \mu^+\mu^-$ and $K_L \to \pi^0\ell^+\ell^-$

We will be only very brief about these decays. All are subject to LD uncertainties. $K_{\rm L} \to \mu^+\mu^-$ is CP conserving, while $K_{\rm S} \to \mu^+\mu^-$ is CP violating and $K_{\rm L} \to \pi^0\ell^+\ell^-$ are dominated by indirect CP violation. Yet in the presence of NP both $K_{\rm S} \to \mu^+\mu^-$ and $K_{\rm L} \to \pi^0\ell^+\ell^-$ could still be dominated by direct CP violation. In any case, all three decays constitute in certain models an important constraint on model parameters. A recent example are leptoquark models in case one would like to remove the ε'/ε anomaly with the help of leptoquarks. We will discuss this in Section 3.3.

3. Recent news

3.1. SMEFT for Z mediated new physics

3.1.1. Preliminaries

It is interesting to ask next what would be the implications of the ε'/ε anomaly for rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$. This question can only be answered in concrete NP scenarios and we have listed above a number of papers where such implications have been studied. In particular in [46], a number of correlations between ε'/ε and $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ have been presented dependently on NP scenario considered.

Here, we will summarize such implications in a simple scenario with FCNCs appearing already at tree-level and being mediated by Z-boson exchange. While studies of this type have been presented already some time ago [46, 131, 148], a rather recent analysis in [129] in the framework of SMEFT demonstrates that in these papers, important contributions to $\Delta F = 2$ transitions generated by renormalization group effects above the electroweak scale have not been included. A related analysis can be found in [130].

Let us then see how such simple models look from the point of view of the SMEFT framework and how the analyses in [46, 131, 148] are affected by these new contributions. We will follow here [129] and for our presentation, we will recall the $\Delta F = 2$ operators in the basis of [55]

$$O_{\rm VLL} = [\bar{s}\gamma_\mu P_{\rm L} d]\,[\bar{s}\gamma^\mu P_{\rm L} d]\,, \qquad O_{\rm VRR} = [\bar{s}\gamma_\mu P_{\rm R} d]\,[\bar{s}\gamma^\mu P_{\rm R} d]\,, \tag{37}$$

$$O_{\rm LR,1} = [\bar{s}\gamma_\mu P_{\rm L} d]\,[\bar{s}\gamma^\mu P_{\rm R} d]\,, \qquad O_{\rm LR,2} = [\bar{s} P_{\rm L} d]\,[\bar{s} P_{\rm R} d]\,, \tag{38}$$

where the summation over colour indices in every current or quark density has been made. We show only operators that are relevant in the case of Z exchanges. Equivalent discussion can be made with the operator basis $\mathcal{O}_i$ of [54] in (7), which we used previously.

The importance of Z-mediated FCNC processes has increased recently in view of the absence of direct NP signals at the LHC. As the neutral Z is particularly suited to be a messenger of possible NP even at scales far beyond the reach of the LHC, the SMEFT framework is very well suited for the proper description of the basic structure of such models. In this manner, the gauge invariance under the SM group can be kept under control and as we will see renormalization group effects, not only from QCD as done already in [46, 131, 148], but also from electroweak gauge interactions and in particular from top Yukawa couplings can be taken properly into account [129].

3.1.2. Some details

Let us then assume that new particles with a common mass Λ have been integrated out at some scale $\mu_\Lambda \gg \mu_{\rm ew}$, giving rise to the SMEFT framework [149]. The field content of the SMEFT Lagrangian are the SM fields and the interactions are invariant under the SM gauge group. The corresponding Lagrangian can be written as

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm dim-4} + \sum_a \mathcal{C}_a \mathcal{O}_a \,. \tag{39}$$

Here, $\mathcal{L}_{\rm dim-4}$ coincides with the SM Lagrangian and a non-redundant set of operators of dimension six (dim-6), $\mathcal{O}_a$, has been classified in [150]. The anomalous dimensions (ADM) necessary for the RG evolution from μ_Λ to $\mu_{\rm ew}$ of the SM couplings and the Wilson coefficients $\mathcal{C}_a$ are known at one-loop [151–153]. Given some initial coefficients $\mathcal{C}_a(\mu_\Lambda)$, they can be evolved down to $\mu_{\rm ew}$, thereby resumming leading logarithmic (LLA) effects due to the quartic Higgs, gauge and Yukawa couplings into $\mathcal{C}_a(\mu_{\rm ew})$.

It is customary to parametrize FC-quark couplings of the Z as [148]

$$\begin{aligned}\mathcal{L}^{\rm NP}_{\psi\bar\psi Z} &= Z_\mu \sum_{\psi=u,d} \bar\psi_i \gamma^\mu \left(\left[\Delta^\psi_{\rm L}(Z)\right]_{ij} P_{\rm L} + \left[\Delta^\psi_{\rm R}(Z)\right]_{ij} P_{\rm R}\right)\psi_j \,,\\ P_{\rm L,R} &= \tfrac{1}{2}(1 \mp \gamma_5)\end{aligned} \tag{40}$$

with $[\Delta^\psi_{\rm L,R}(Z)]_{ij}$ being complex-valued couplings. We keep the flavour indices to be arbitrary as the discussion applies not only to $(ij = sd)$ but also $(ij = bd)$ and $(ij = bs)$ relevant for $B_{s,d}$ systems.

On the other hand, the operators of SMEFT that induce FC quark couplings to Z are given as follows. The ones with left-handed (LH) quark currents are[2]

$$\mathcal{O}^{(1)}_{Hq} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left[\bar q^i_{\rm L}\gamma^\mu q^j_{\rm L}\right], \qquad \mathcal{O}^{(3)}_{Hq} = \left(H^\dagger i \overleftrightarrow{D}^a_\mu H\right)\left[\bar q^i_{\rm L}\sigma^a\gamma^\mu q^j_{\rm L}\right]. \tag{41}$$

[2] In order to simplify notations, we suppress flavour indices on the operators.

The ones with right-handed (RH) quark currents are

$$\mathcal{O}_{Hd} = \left(H^\dagger i \overleftrightarrow{\mathcal{D}}_\mu H\right)\left[\bar{d}^i_{\rm R}\gamma^\mu d^j_{\rm R}\right], \qquad \mathcal{O}_{Hu} = \left(H^\dagger i \overleftrightarrow{\mathcal{D}}_\mu H\right)\left[\bar{u}^i_{\rm R}\gamma^\mu u^j_{\rm R}\right]. \quad (42)$$

Here, H is the Higgs field, σ^a are Pauli matrices and $\mathcal{D}_\mu$ covariant derivative that includes the $W^\pm$ and Z^0.

The complex-valued coefficients of these operators are denoted by

$$\left[\mathcal{C}^{(1)}_{Hq}\right]_{ij}, \quad \left[\mathcal{C}^{(3)}_{Hq}\right]_{ij}, \quad [\mathcal{C}_{Hd}]_{ij}, \quad [\mathcal{C}_{Hu}]_{ij}. \quad (43)$$

The Z couplings in (40) can now be expressed in terms of the latter couplings as follows [129]:

$$\begin{aligned} [\Delta^u_{\rm L}(Z)]_{ij} &= -\frac{g_Z}{2}v^2\left[\mathcal{C}^{(1)}_{Hq} - \mathcal{C}^{(3)}_{Hq}\right]_{ij}, \qquad & [\Delta^u_{\rm R}(Z)]_{ij} &= -\frac{g_Z}{2}v^2[\mathcal{C}_{Hu}]_{ij}, \\ \left[\Delta^d_{\rm L}(Z)\right]_{ij} &= -\frac{g_Z}{2}v^2\left[\mathcal{C}^{(1)}_{Hq} + \mathcal{C}^{(3)}_{Hq}\right]_{ij}, \qquad & \left[\Delta^d_{\rm R}(Z)\right]_{ij} &= -\frac{g_Z}{2}v^2[\mathcal{C}_{Hd}]_{ij}, \end{aligned} \quad (44)$$

with $v = 246$ GeV being the Higgs vacuum expectation value.

As $\mathcal{C}_a = \mathcal{O}(1/\Lambda^2)$, the couplings in (44) are $\mathcal{O}(v^2/\Lambda^2)$. If one considers $\Delta F = 1$ transitions, the leading contributions are just tree-level Z exchanges with one of the vertex given by (40) and (44) and the second flavour conserving vertex being the SM one. Evidently, such diagrams are $\mathcal{O}(v^2/\Lambda^2)$ and generate dimension-six contributions in (39). This is, in fact, what has been done in [46, 131, 148]. So far, so good.

However, in the latter papers, the $\Delta F = 2$ processes have been described also by simple tree-level Z exchange, this time having on both ends of the Z propagator the FC vertices in (44). Evidently, such a contribution is $\mathcal{O}(v^4/\Lambda^4)$ and generates one of the dimension-eight contributions in (39). While for Λ being $\mathcal{O}(1~\text{TeV})$ such contributions cannot be neglected, for sufficiently large $\Lambda \geq 5$ TeV, they cannot compete with dimension-six contributions which are $\mathcal{O}(v^2/\Lambda^2)$.

The question then arises what are these dimension-six contributions to $\Delta F = 2$ processes that represent Z-mediated NP. This question has been answered in [129] allowing to identify new effects which have been missed in previous literature. These are:

1. In the presence of right-handed FC Z couplings, *i.e.* $\mathcal{C}_{H_d} \neq 0$ or $[\Delta^d_{\rm R}(Z)]_{ij}$, inspection of the renormalisation group (RG) equations due to Yukawa couplings in [152] yields that at $\mu_{\rm ew}$, the left–right $\Delta F = 2$ operators $O_{\rm LR,1}$ in (38) are generated and are enhanced by the large leading logarithm $\ln \mu_\Lambda/\mu_{\rm ew}$. Such operators are known to provide very important contributions to $\Delta F = 2$ observables because of their

enhanced hadronic matrix elements and an additional enhancement from QCD RG effects below $\mu_{\rm ew}$, in particular in the K-meson system. As a result, *these* operators — and not $O^{ij}_{\rm VRR}$ in (37), as used in [46, 131, 148] — dominate $\Delta F = 2$ processes. The results in [152] allow the calculation of this dominant contribution including only leading logarithms but this is sufficient for our purposes and even for scales μ_Λ as high as 20 TeV a good approximation is to keep only leading logarithms.

2. Because of the usual scale ambiguity present at leading order (LO), the next-to-leading order (NLO) matching corrections of $\mathcal{O}_{Hd}$ to $\Delta F = 2$ processes at $\mu_{\rm ew}$ within SMEFT have to be calculated. One NLO contribution is obtained by replacing the flavour-diagonal lepton vertex in the SM Z-penguin diagram by $[\mathcal{C}_{Hd}]_{ij}$, which again generates the operator $O^{ij}_{\rm LR,1}$ simply because the flavour-changing part of the SM penguin diagram is LH. In fact, this contribution has been first pointed out in [130] and used for phenomenology. Unfortunately, such contributions are by themselves gauge-dependent, simply because the function $C(x_t)$ present in the SM vertex is gauge-dependent. Hence, while the observation made in [130] was important, the analysis of these new contributions presented there was incomplete[3]. In [129], the missing contributions have been calculated using SMEFT, obtaining a gauge-independent contribution. However, the LO contribution is not only more important due to the large logarithm $\ln\mu_\Lambda/\mu_{\rm ew}$, but has also opposite sign to the NLO term, allowing to remove the LO scale dependence. Moreover, being strongly enhanced with respect to the contributions considered in [46, 131, 148], it has very large impact on the phenomenology; in particular, as discussed in detail in [129] and summarized briefly below correlations between $\Delta F = 2$ and $\Delta F = 1$, observables are drastically changed.

3. The situation for LH FC Z couplings is different from the RH case both qualitatively and quantitatively: inspecting again the RG equations in [152] one finds that the two operators $\mathcal{O}^{(1)}_{Hq}$ and $\mathcal{O}^{(3)}_{Hq}$ in SMEFT listed above generate only the $\Delta F = 2$-operator $O_{\rm VLL}$ in (37) that is dominant already in the SM. The operator structure is then the same as in [148]. The resulting NP effects are then much smaller than in the RH case, because no LR operators are present. But now comes an important difference from [148]. The correlations between $\Delta F = 1$ and $\Delta F = 2$ processes are weakened very significantly: while $\Delta F = 1$ transition amplitudes are proportional to the sum $\mathcal{C}^{(1)}_{Hq} + \mathcal{C}^{(3)}_{Hq}$, the leading

[3] Meanwhile, the authors of [130] included additional contributions and confirmed the results in [129].

RG contribution to $\Delta F = 2$ processes is proportional to $\mathcal{C}_{Hq}^{(1)} - \mathcal{C}_{Hq}^{(3)}$, that is proportional to $[\Delta_{\rm L}^{u}(Z)]_{ij}$. The appearance of the u-quark coupling in a process involving d-quarks only is the consequence of $\mathrm{SU}(2)_{\rm L}$ gauge invariance: left-handed up- and down-quark couplings belong to doublets under $\mathrm{SU}(2)_{\rm L}$ symmetry. Consequently, we have more free parameters and correlations between $\Delta F = 1$ and $\Delta F = 2$ processes are hence only present in specific scenarios, *e.g.* when the couplings are given in terms of the fundamental parameters of a given model that can be determined in other processes. This is in stark contrast to the contributions considered in [46, 131, 148], where the same couplings enter both classes of processes and no involvement of specific models was necessary. Of course, correlations remain in each sector separately, since both are governed by two complex couplings, but as previously only one complex coupling was present, one needs more observables to determine them model-independently. Moreover, in models where $\Delta F = 2$ and $\Delta F = 1$ observables are correlated, the constraints become weaker allowing for larger NP effects in rare decays.

4. Also for the operators $\mathcal{O}_{Hq}^{(1,3)}$, the NLO contributions to $\Delta F = 2$ corresponding to the replacement of the flavour-diagonal lepton vertex in the SM Z-penguin diagram by $\mathcal{C}_{Hq}^{(1,3)}$ are gauge-dependent. Including the remaining contributions to remove this gauge dependence, one finds two gauge-independent functions of x_t, analogous to $X(x_t)$, $Y(x_t)$ and $Z(x_t)$ known from the SM. Since the NLO contributions are different for $\mathcal{C}_{Hq}^{(1)}$ and $\mathcal{C}_{Hq}^{(3)}$, it is not just their difference contributing to $O_{\rm VLL}$ anymore, but also their sum.
5. At NLO, also new gauge-independent contributions are generated which are unrelated to tree-level Z exchanges and only proportional to $\mathcal{C}_{Hq}^{(3)}$, analogous to the usual box diagrams with $W^{\pm}$ and quark exchanges. They turn out to be important for gauge-independence and depend not only on the coefficients for the quark transition under consideration, but also on additional couplings to the possible intermediate quarks in the box diagrams. However, when the hierarchies in CKM elements are taken into account, $\mathcal{C}_{Hq}^{(3)}$ for the quark transition under consideration is the only free entry in this part.

It should be stressed in this context that the contributions to $\Delta F = 2$ transitions from FC quark couplings of the Z could be less relevant in NP scenarios with other sources of $\Delta F = 2$ contributions. Most importantly, $\Delta F = 2$ operators could receive a direct contribution at tree-level at the scale μ_{Λ}, but also in models where this does not happen Z contributions could be subdominant. Examples are models in which the only new particles are vector-like quarks (VLQs), where box diagrams with VLQ and Higgs exchanges generate $\Delta F = 2$ operators at one-loop level [134, 154], which were found in these papers to be larger than the Z, contributions at tree-level. However, in [154], the new effects listed above have not been included. As shown in [134] for right-handed FC Z couplings, these box contributions are dwarfed by the LR operator contributions mentioned at the beginning of our list in kaon mixing, whereas in B mixing they are comparable.

We will now summarize the phenomenological impact of these new effects on the analysis in [46]. To this end, we will follow the strategy that has been proposed in that paper as this will show us where this strategy could still be successful and where it has to be modified. The main point of this strategy was the determination of FC Z couplings from ε'/ε and ε_K and to use their values to predict branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$, $K_{\rm L} \to \pi^0\nu\bar{\nu}$ and NP contribution to ΔM_K. As we will see, this strategy is still successful in the case of RH scenario even if numerical results are rather different from those presented in [46] because of the contributing left–right operators. At first sight, this strategy must be significantly modified in the case of the LH scenario because of an additional coupling present in $\Delta F = 2$ transitions. However, it turns out, as far as $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ are concerned, that the strategy in [46] remains successful as $K_{\rm L} \to \mu^+\mu^-$ and ε'/ε and not ε_K are the dominant constraints for these two decays in the LH scenario.

It should be emphasized that our critical comments about the simplified approach in [46, 131, 148] do not apply to Z' models considered in these papers. We will discuss these models subsequently.

In the strategy in [46], the central role is played by ε'/ε and ε_K for which in the presence of NP contributions, we have

$$\frac{\varepsilon'}{\varepsilon} = \left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm SM} + \left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm NP}, \qquad \varepsilon_K \equiv e^{i\varphi_\epsilon}\left[\varepsilon_K^{\rm SM} + \varepsilon_K^{\rm NP}\right]. \tag{45}$$

As the size of NP contributions is not precisely known, the strategy of [46] is to parametrize this contributions as

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{\rm NP} = \kappa_{\varepsilon'} \times 10^{-3}, \qquad 0.5 \le \kappa_{\varepsilon'} \le 1.5 \tag{46}$$

and

$$(\varepsilon_K)^{\rm NP} = \kappa_\varepsilon \times 10^{-3}, \qquad 0.1 \le \kappa_\varepsilon \le 0.4\,. \tag{47}$$

The ranges for $\kappa_{\varepsilon'}$ and κ_ε only indicate possible size of NP contributions as argued in [46] but can also be treated as free parameters.

3.1.3. Lessons on NP patterns in Z scenarios

The summary of the lessons is rather brief. On the other hand, the presentation in [46] is very detailed with numerous analytic expressions. We stress the differences in numerics due to new contributions identified in [134].

Lesson 1: In the LHS, a given request for the enhancement of ε'/ε determines the coupling $\mathrm{Im}\Delta_{\mathrm{L}}^{sd}(Z)$. Similar in the RHS the coupling $\mathrm{Im}\Delta_{\mathrm{R}}^{sd}(Z)$ is determined.

Lesson 2: In LHS, there is a direct unique implication of an enhanced ε'/ε on $K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$: *suppression* of $\mathcal{B}(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu})$. This property is known from NP scenarios in which NP to $K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$ and ε'/ε enters dominantly through the modification of Z penguins. The known flavour diagonal Z couplings to quarks and leptons and the sign of the matrix element $\langle Q_8\rangle_2$ determines this anticorrelation which has been verified in all models with only LH flavour-violating Z couplings.

Lesson 3: The imposition of the $K_{\mathrm{L}} \to \mu^+\mu^-$ constraint in LHS determines the range for $\mathrm{Re}\Delta_{\mathrm{L}}^{sd}(Z)$ which with the already fixed $\mathrm{Im}\Delta_{\mathrm{L}}^{sd}(Z)$ would allow to calculate the shifts in ε_K and ΔM_K if not for new contributions identified in [129] which were not included in [46]. There it was concluded that these shifts are very small for ε_K and negligible for ΔM_K. However, this conclusion is not valid in the presence of these new contributions. Moreover, in concrete models, new contributions beyond Z exchange are possible. For instance, in VLQ models, box diagrams with VLQs can indeed provide contributions to ε_K and ΔM_K that are larger than coming from tree-level Z exchange provided the masses of VLQs are far above 3 TeV [134, 154]. In any case, $K_{\mathrm{L}} \to \mu^+\mu^-$ determines the allowed range for $\mathrm{Re}\Delta_{\mathrm{L}}^{sd}(Z)$.

Lesson 4: With fixed $\mathrm{Im}\Delta_{\mathrm{L}}^{sd}(Z)$ and the allowed range for $\mathrm{Re}\Delta_{\mathrm{L}}^{sd}(Z)$, the range for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ can be obtained. But in view of uncertainties in the $K_{\mathrm{L}} \to \mu^+\mu^-$ constraint, both an enhancement and a suppression of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ are possible and no specific pattern of correlation between $\mathcal{B}(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is found. In the absence of a relevant ε_K constraint, this is consistent with the general analysis in [155]. $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ can be enhanced by a factor of 2 at most due to bound on NP contribution to $K_{\mathrm{L}} \to \mu^+\mu^-$ that hopefully will be improved in the future.

Lesson 5: As far as the correlation of ε'/ε with $K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$ is concerned, analogous pattern is found in RHS, although the numerics is different: suppression of $\mathcal{B}(K_{\mathrm{L}} \to \pi^0\nu\bar{\nu})$ with increasing $\kappa_{\varepsilon'}$ However, the new contributions from LR operators to ε_K have dramatic impact on the results for $K^+ \to \pi^+\nu\bar{\nu}$ presented in [46]. Now, not $K_{\mathrm{L}} \to \mu^+\mu^-$ but the constraint from ε_K determines the allowed enhancement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$. While in [46] an enhancement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ up to a factor of 5.7 was possible, now only an enhancement up to a factor of 1.5 is possible.

Lesson 6: In a general Z scenario in which the underling theory contains all the operators in (41) and (42) and simultaneously dimension-eight LR operators are present, the pattern of NP effects can change relative to LH and RH scenarios because of many parameters involved independently of whether new contributions considered in [129] are taken into account or not. As demonstrated in [46], the main virtue of the general scenario is the possibility of enhancing simultaneously ε'/ε, ε_K, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ which is not possible in LHS and RHS. Thus, the presence of both LH and RH flavour-violating currents is essential for obtaining simultaneously the enhancements in question when NP is dominated by tree-level Z exchanges. We refer to examples in [46]. Then the main message from this analysis is that in the presence of both LH and RH, new flavour-violating couplings of Z to quarks, large departures from SM predictions for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ are still possible. Similar conclusions have been reached in [130].

3.2. Lessons on NP patterns in Z' scenarios

Z' models exhibit quite different pattern of NP effects in the K-meson system than the LH and RH Z scenarios. In Z scenarios, only electroweak penguin (EWP) Q_8 and Q_8' operators can contribute in an important manner to ε'/ε because of flavour-dependent diagonal Z coupling to quarks. However, in Z' models, the diagonal quark couplings can be flavour universal so that QCD penguin operators (QCDP) (Q_6, Q_6') can dominate NP contributions to ε'/ε. Interestingly, the pattern of NP in rare K decays depends on whether NP in ε'/ε is dominated by QCDP or EWP operators [46]. This is, in fact, a new finding, mainly because nobody studied NP contributions of QCDP to ε'/ε before.

Another striking difference from Z scenarios, known already from previous studies, is the increased importance of the constraints from $\Delta F = 2$ observables as a simple Z' exchange generates six-dimensional operator alone without any interferences with SM contributions that played such an important role in Z cases. This has two virtues in the presence of the ε'/ε constraint:

— The real parts of the couplings are determined for not too a large κ_ε from the ε_K constraint.

— There is a large hierarchy between real and imaginary parts of the flavour violating couplings implied by ε'/ε anomaly in QCDP and EWP scenarios. As shown in [46], in the case of QCDP, imaginary parts dominate over the real ones, while in the case of EWP, this hierarchy is opposite unless the ε_K anomaly is absent. This is related to the fact that strong suppression of QCDP to ε'/ε by the factor of

1/22 coming from $\Delta I = 1/2$ rule requires a large imaginary coupling in order to enhance significantly this ratio. This suppression is absent in the case of EWP and this coupling can be smaller.

Because of this important difference in the manner QCDP and EWP enter ε'/ε, there are striking differences in the implications for the correlation between $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ in these two NP scenarios if significant NP contributions to ε'/ε are required.

We refer to numerous plots in [46] which show clearly the differences between QCDP and EWP scenarios. More details, in particular analytic derivation of all these results, can be found there. We extract from these results the following lessons:

Lesson 7: In the case of QCDP scenario, the correlation between $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ takes place along the branch parallel to the Grossman–Nir (GN) bound.

Lesson 8: In the EWP scenario, the correlation between $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ proceeds away from this branch for diagonal quark couplings $\mathcal{O}(1)$ if NP in ε_K is present and it is very different from the one of the QCDP case as seen in the plots in [46] allowing a clear distinction between QCDP and EWP scenarios.

Lesson 9: For fixed values of the neutrino and diagonal quark couplings in ε'/ε, the predicted enhancements of $\mathcal{B}(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ are much larger when NP in QCDP is required to remove the ε'/ε anomaly than it is the case of EWP. This is simply related to the fact, as mentioned above, that the $\Delta I = 1/2$ rule suppresses QCDP contributions to ε'/ε so that QCDP operators are less efficient in enhancing ε'/ε than EWP operators. Consequently, the imaginary parts of the flavour-violating couplings are required to be larger, implying then larger effects in rare K decays. Only for the diagonal quark couplings $\mathcal{O}(10^{-2})$, the requirement of shifting upwards ε'/ε implies large effects in $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ in EWP scenario. See [46] for a detail discussion of this point.

Lesson 10: In QCDP scenario, ΔM_K is *suppressed* and this effect increases with increasing $M_{Z'}$, whereas in the EWP scenario, ΔM_K is *enhanced* and this effect decreases with increasing $M_{Z'}$ as long as real couplings dominate. Already on the basis of this property, one could differentiate between these two scenarios when the SM prediction for ΔM_K improves.

In summary, assuming that the ε'/ε anomaly will be confirmed by lattice QCD and the results from NA62 and KOPIO, for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ will be available, it will be easy to select between various scenarios presented above.

3.3. Leptoquark models and ε'/ε anomaly

We will next turn our attention to leptoquark models and investigate how these models confront the ε'/ε-anomaly. We have mentioned already that several NP scenarios are able to provide sufficient upward shift in ε'/ε and obtain agreement with experiment, see Table I. These include in particular tree-level Z' exchanges with explicit realisation in 331 models [132, 133] or models with tree-level Z exchanges [129, 130] with explicit realisation in models with mixing of heavy vector-like fermions with ordinary fermions [134] and Littlest Higgs model with T-parity [128]. Also simplified Z' scenarios [46, 131] and the MSSM [135–139] and 2-Higgs doublet models [140, 141] are of help here. But the interest in studying LQ models arose not from ε'/ε anomaly but from their ability in the explanations of B-physics anomalies with selected papers in [156–161]. General information on LQ models can be found in [162, 163]. In Table II, we list various LQ models.

TABLE II

Leptoquark models.

Scalar leptoquark	$\mathrm{SU(2)_L}$	Vector leptoquark
S_1	singlet	U_1
$\tilde{S}_1$	singlet	$\tilde{U}_1$
R_2	doublet	V_2
$\tilde{R}_2$	doublet	$\tilde{V}_2$
S_3	triplet	U_3

Already from the beginning, one can expect that the ε'/ε anomaly will be a challenge for those LQ analyses of B-physics anomalies in which all NP couplings have been chosen to be real and those to the first generation set to zero. It should also be realised that the anomalies $R(D)$ and $R(D^*)$ although being very significant can still be explained in some LQ models through a tree-level LQ exchange. On the other hand, the ε'/ε anomaly, being even larger, if the bound on ε'/ε in [32, 124] is assumed, can only be addressed in these models at one-loop level. This shows that the hinted ε'/ε anomaly is a big challenge for LQ models.

These expectations have been confirmed by a very detailed analysis in [145]. Assuming a mass gap to the electroweak (EW) scale, the main mechanism for LQs to contribute to ε'/ε turns out to be EW gauge-mixing of semi-leptonic into non-leptonic operators. In [145], also one-loop decoupling for scalar LQs has been performed, finding that in all models with both left-handed and right-handed LQ couplings, that is S_1, R_2, and V_2 and U_1, box-diagrams generate numerically strongly enhanced EW-penguin operators Q_8 and Q_8' already at the LQ scale. This behaviour is rather special for LQs as in most models, Q_8 and Q_8' operators cannot be generated at high scale even at NLO, and are generated only in the RG running to low-energy scale from the operators Q_7 and Q_7', respectively. A good example is the SM and all NP models discussed by us until now.

Investigating correlations of ε'/ε with rare kaon processes $K_{\rm L} \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_{\rm L}\to\pi^0\ell\bar{\ell}$, $K_{\rm S} \to \mu\bar{\mu}$, ΔM_K and ϵ_K, one finds then that even imposing only a moderate enhancement of $(\varepsilon'/\varepsilon)_{\rm NP} = 5 \times 10^{-4}$ to explain the current anomaly, hinted by the Dual QCD approach and RBC-UKQCD lattice QCD calculations, leads to conflicts with experimental upper bounds on rare kaon processes. They exclude all LQ models with only a single coupling as an explanation of the ε'/ε anomaly and put serious constraints on parameter spaces of the models S_1, R_2, and V_2 and U_1 where the box diagrams can, in principle, provide a rescue to LQ models provided both left-handed and right-handed couplings are non-vanishing. However, then the presence of left–right operators contributing not only to ε'/ε but also to D^0–$\bar{D}^0$ and K^0–$\bar{K}^0$ mixings requires some fine tuning of parameters in order to satisfy all constraints. In the case of V_2 and U_1, the analysis of box diagrams can only be done in a UV completion.

Future improved results on $K^+ \to \pi^+\nu\bar{\nu}$ from the NA62 Collaboration, $K_{\rm L} \to \pi^0\nu\bar{\nu}$ from the KOTO experiment and $K_{\rm S} \to \mu\bar{\mu}$ from the LHCb will even stronger exhibit the difficulty of LQ models in explaining the measured ε'/ε, in case the ε'/ε anomaly will be confirmed by improved lattice QCD calculations. Hopefully also improved measurements of $K_{\rm L}\to\pi^0\ell\bar{\ell}$ decays will one day help in this context.

The main messages of [145] are then the following ones. If the future improved lattice calculation will confirm the ε'/ε anomaly at the level $(\varepsilon'/\varepsilon)_{\rm NP} \geq 5 \times 10^{-4}$, LQs are likely to be not responsible for it. But if the ε'/ε anomaly will disappear one day, large NP effects in rare K decays that are still consistent with present bounds will be allowed. The analysis in [145] is rather involved and we will not present it here. However, it is an excellent arena to practice the technology of SMEFT and anybody who wants to test their skills in SMEFT should study [145] in detail.

4. Outlook

4.1. Visions

Let us begin the final section with a dream about the discovery of NP in $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ decays as

$$\mathcal{B}\left(K^+ \to \pi^+\nu\bar{\nu}\right) = (18.0 \pm 4.0) \times 10^{-11} \qquad (\text{NA62},\ 2019)\,, \tag{48}$$
$$\mathcal{B}\left(K_{\rm L} \to \pi^0\nu\bar{\nu}\right) = (12.0 \pm 4.0) \times 10^{-11} \qquad (\text{KOTO},\ 2021)\,, \tag{49}$$

and the confirmation of the ε'/ε anomaly as

$$\varepsilon'/\varepsilon = (5 \pm 3) \times 10^{-4} \qquad (\text{RBC-UKQCD, 2018})\,. \tag{50}$$

Looking at various plots in the literature, it is clear that such a combination of anomalies would be truly tantalizing with a big impact on our field. On the other hand, if NA62 will find $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio significantly below 15.0 in these units, the claim for NP will be much weaker and we will have to wait until KOTO measures the branching ratio for $K_{\rm L} \to \pi^0\nu\bar{\nu}$. As I already stated at several places in this paper, I have no doubts that ε'/ε anomaly will stay with us but as of today it is hard to predict at which level.

Assuming then that the lattice values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ will not be modified significantly and the ε'/ε anomaly will stay with us with $\kappa_{\varepsilon'} = 1.0$, the by now old measurement of ε'/ε will allow to exclude certain scenarios and favour other ones. However, this will also depend on the allowed size of NP in ε_K, ΔM_K and rare $B_{s,d}$ decays. In particular, it is crucial that the present anomalies in B decays will be clarified as this will help to identify proper flavour symmetry at short-distance scales and their breakdown. This is also the case of visible tensions between $\Delta M_{s,d}$ and ε_K.

4.2. Open questions

There is no doubt that in the coming years, K-meson physics will strike back, in particular through improved estimates of SM predictions for ε'/ε, ε_K, ΔM_K and $K_{\rm L,S} \to \mu^+\mu^-$ and through crucial measurements of the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$. Correlations with other meson systems, lepton flavour physics, electric dipole moments and other rare processes should allow us to identify NP at very short-distance scales [1] and we should hope that this physics will also be directly seen at the LHC.

Let us then end our short review by listing most pressing questions for the coming years. On the theoretical side we have:

— **What is the value of $\boldsymbol{\kappa_{\varepsilon'}}$** that we defined in (46)? Here, the answer will come not only from lattice QCD but also through improved values of the CKM parameters, completion of NNLO QCD corrections and from an improved understanding of FSI and isospin breaking effects. The recent analysis in the large-N approach in [32] indicates that FSI are likely to be relevant for the $\Delta I = 1/2$ rule in agreement with previous studies [27–31, 126], but much less relevant for ε'/ε. It is important that other lattice QCD groups calculate $B_6^{(1/2)}$ and $B_8^{(3/2)}$, because at the end their values are most important for ε'/ε. However, if this anomaly persists, it will be mandatory to calculate hadronic matrix elements of new operators that are absent in the SM. I am confident that in DQCD we will be able to calculate them soon.

— **What is the value of $\boldsymbol{\kappa_\varepsilon}$?** Here, the reduction of CKM uncertainties and the theoretical ones in η_{cc} are most important. But the analysis in [40] indicates that if no NP is present in ε_K, it is expected to be found in $\Delta M_{s,d}$.

— **What is the value of $\boldsymbol{\Delta M_K}$ in the SM?** Here, lattice QCD should provide useful answers. As pointed out in [46], the sign of possible departure from data could help in distinguishing between different origins of the ε'/ε anomaly. Moreover, as pointed out in the context of VLQ models in [134], the knowledge of the allowed size of NP contributions to ΔM_K will have an impact on NP in $K^+ \to \pi^+\nu\bar{\nu}$ is these models.

— **What are the precise values of Re$\boldsymbol{A_2}$ and Re$\boldsymbol{A_0}$?** Again, lattice QCD will play the crucial role here although the main dynamics behind this rule was identified long time ago in the DQCD approach.

On the experimental side we have:

— **What is $\boldsymbol{B(K^+ \to \pi^+\nu\bar{\nu})}$ from NA62?** We should possibly get some information already in 2019.

— **What is $\boldsymbol{B(K_{\rm L} \to \pi^0\nu\bar{\nu})}$ from KOTO?** We should know it around the year 2021.

— **Do $\boldsymbol{Z'}$ or other new particles like VLQs with masses in the reach of the LHC exist?** We could know it already this year.

Definitely, there are exciting times ahead of us! But in order to distinguish between various NP scenarios and study flavour symmetries and their breakdown, correlations with $B^0_{s,d}$–$\bar{B}^0_{s,d}$ mixing observables and decays like $B_{s,d} \to \mu^+\mu^-$, $B \to K(K^*)\ell^+\ell^-$, $B \to K(K^*)\nu\bar{\nu}$ and $B \to D(D^*)\tau\nu_\tau$ will be crucial.

I would like to thank the organizers of Epiphany 2018 for inviting me to such an interesting workshop and a very pleasant atmosphare. In addition, I appreciate the comments by Marcin Chrząszcz on the present text. I would like to thank all my collaborators for wonderful time we spent together. This year particular thanks go to Jean-Marc Gérard, Christoph Bobeth, Martin Jung, David Straub and my new impressive collaborator Jason Aebischer. The research presented here was done and financed in the context of the ERC Advanced Grant project "FLAVOUR"(267104) and was partially supported by the DFG cluster of excellence "Origin and Structure of the Universe".

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