Efficient discretization procedure for integrating structural reliability into Bayesian networks

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ABSTRACT: Structural reliability methods have limitations in near-real time applications, in which probability estimates should be updated with new information. The combination of structural reliability with discrete Bayesian networks can overcome such problems. This requires the discretization of the continuous basic random variables. We develop an efficient discretization scheme, which is based on finding an optimal discretization for the linear FORM approximation of the limit state function. Since the objective is a good approximation of the probability estimate under all possible future information scenarios, the discretization scheme is optimized with respected to the expected posterior error.

1 INTRODUCTION

In structural reliability problems, the performance of a component or system is described through a limit state function (LSF) $g(\mathbf{X})$, where $\mathbf{X}$ is the vector of basic random variables influencing the component/system. System failure is defined as $F = \{g(\mathbf{X}) \leq 0\}$, $f_{\mathbf{X}}(\mathbf{x})$ denotes the joint probability density function (PDF) of $\mathbf{X}$ and the probability of failure is:

$$\Pr(F) = \Pr(g(\mathbf{X}) \leq 0) = \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{1}$$

In the general case, the integral in Equation 1 cannot be solved analytically, and structural reliability methods (SRMs) have been developed to approximate it. These methods include first and second order reliability methods (FORM and SORM), Monte Carlo simulation (MCS) and advanced sampling methods, including importance sampling techniques (IS) and subset simulation (SuS); For an overview see (Rackwitz, 2001, Au and Beck, 2001, Der Kiureghian, 2005, Ditlevsen and Madsen, 2007).
In near-real-time applications, the probability of F must be computed under potentially evolving information. Ideally, this is achieved through Bayesian updating of $\Pr(F)$ with the new information Z to the posterior probability $\Pr(F|Z)$. While such updating is possible with SRM (Straub, 2011), it is often difficult to perform the required computations in near-real-time, due to a lack of efficiency or robustness. A modeling and computation framework that does facilitate efficient Bayesian updating is the discrete Bayesian network (BN). Hence it was proposed to combine SRMs with discrete Bayesian networks for near-real-time computations (Friis-Hansen, 2000, Straub and Der Kiureghian, 2010b, Straub and Der Kiureghian, 2010a).

The size of discrete BNs, and the necessary computational effort, increases approximately exponentially with the number of discrete states of its nodes, which motivates the development of efficient discretization algorithms. While efficient discretization in the context of machine learning and Bayesian networks in general has been investigated by multiple researchers (Dougherty et al., 1995, Kotsiantis and Kanellopoulos, 2006), research on efficient discretization in the context of engineering risk analysis or structural reliability has been limited. In general, it is to be distinguished between static and dynamic discretization. While the former is limited to discretizing once at the onset of solving the problem, the latter is based on an iterative scheme to update the discretization scheme. Static discretization schemes are independent of the evidence, while dynamic discretization schemes change as evidence changes. Dynamic discretization for risk analysis applications has been developed mainly by (Neil et al., 2008). Some considerations on static discretization schemes for reliability problems have been presented in (Friis-Hansen, 2000, Straub, 2009, Straub and Der Kiureghian, 2010a).

1.1 *Bayesian networks*

We provide only a brief introduction to the most important aspects of discrete BNs. For a more in-depth treatment of BNs, the reader is referred to textbooks (Jensen and Nielsen, 2007, Kjaerulff and Madsen, 2013).

BNs are based on directed acyclic graphs (DAGs), to efficiently define a joint probability distribution $p(\boldsymbol{Z})$ over a random vector $\boldsymbol{Z}$. The DAG of a BN, which is often referred to as the qualitative part of a BN, consists of a node for each variable in $\boldsymbol{Z}$ and a set of directed links representing relationships between the nodes. Family terms are used to describe relationships in BNs. As an example, in the BN of Figure 1, Z_3 is a child of Z_2 and Z_4, while Z_2 is a parent to Z_1 and Z_3.

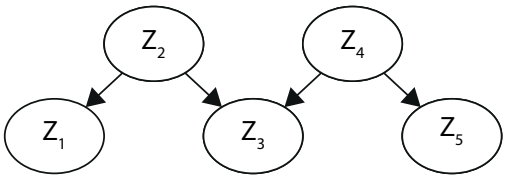

Figure 1. A simple BN.

Conditional probability tables (CPTs) quantitatively define the type and strength of the relationships between the nodes. The entries of the CPT of a variable Z_i are the probabilities for each state of Z_i conditional on all possible combinations of states of its parents: $p(z_i|pa(Z_i))$. The joint probability distribution over all random variables is then the product of these conditional distributions:

$$p(\mathbf{z}) = \prod_{i=1}^{n} p(z_i|pa(Z_i)) \tag{1}$$

The number of parameters needed for defining the CPT of Z_i increases with the number of states of Z_i's parents. For this reason, the number of parameters that need to be defined for a BN can become large if a node has many parents or if its parents have many states. Hence it is desirable to reduce the number of states of the parents. When discretizing a continuous random variable, one aims at limiting the number of discrete states for computational efficiency, while at the same time minimizing the discretization error. Such an optimal discretization is the aim of this paper.

2 TREATMENT OF A RELIABILITY PROBLEM IN A BN

We combine discrete BNs and structural reliability concepts to facilitate updating of failure probabilities under new observations. The general problem setting is illustrated in the BN of Fig. 2. We limit ourselves to component reliability problems. The node 'Component performance' is described by the LSF $g(\mathbf{X})$. The basic random variables $\mathbf{X}$ are included in the BN as parents of 'Component performance'. The nodes M_i represent measurements of individual random variables X_i, and nodes I_j represent factors influencing the basic random variables. Dependence between the variables in $\mathbf{X}$ is modeled either directly by links among them (here $X_2 \rightarrow X_1$ and $X_4 \rightarrow X_5$) or through common influencing factors (here $I_2 \rightarrow X_3$ and $I_2 \rightarrow X_4$).

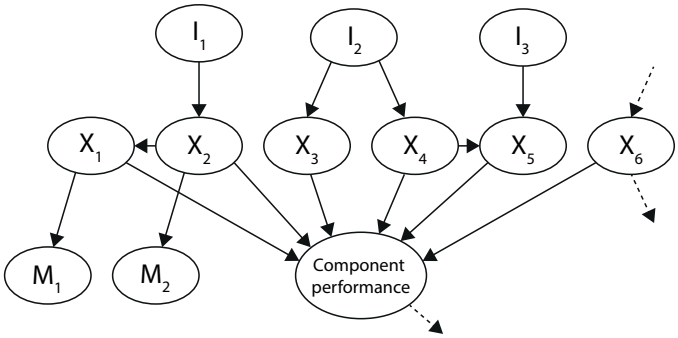

Figure 2. A general BN representing a component reliability problem.

Ultimately, the goal is to predict the component performance, i.e. $\Pr(F)$, conditional on observations of other variables, typically the measurement variables M_i and possibly the influencing variables I_j. Whenever new evidence on these variables is available, the BN should be evaluated in near-real time, utilizing exact inference algorithms.

To this end all continuous random variables are discretized. These include the $\mathbf{X}$, and possibily the M_i and I_j. In the general case, the computational effort for solving the BN is a direct function of the CPT size of 'Component performance'. The size of this CPT is $2\prod_{i=1}^{n} n_i$, with n being the number of random variables in $\mathbf{X}$ and n_i the number of states used for the discretization of X_i. In this paper, we do not describe the discretization of random variables M_i and I_j, since these are typically straightforward and not significant to computational performance.

2.1 *Discretization of basic random variables*

In the scope of this paper we discuss discretization of the basic random variables $\mathbf{X}$. To facilitate illustration, we consider the special case shown in Figure 3, where the performance of the component depends on n statistically independent random variables and is described by a LSF $g(\boldsymbol{X}) = g(X_1, \dots, X_n)$. For all basic random variables X_i, corresponding measurements M_i can be performed. To obtain an equivalent discrete BN, the continuous X_i are replaced by the discrete random variables Y_i, and the LSF is replaced by the CPT of component performance conditional on $\mathbf{Y} = [Y_1; \dots; Y_n]$. For each discrete random variable Y_i with n_i states $1,2, \dots, n_i$ we define a discretization scheme $D_{Y_i} = \left[x_{i,1}, \dots, x_{i,n_i-1}\right]$ consisting of $n_i - 1$ interval boundaries.

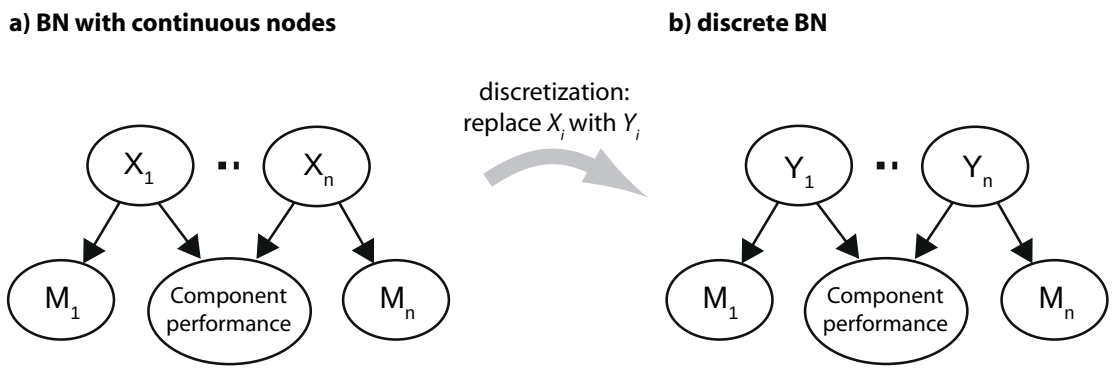

Figure 3. Representation of a basic reliability problem with n independent basic random variables in a BN. Right: original problem with continuous basic random variables X_i, left: discrete BN, in which X_i are substituted with discrete nodes Y_i

Since here the X_i, and thus the Y_i, have no parents, the PMF of Y_i is defined as:

$$p_{Y_i}(j) = \begin{cases} F_{X_i}(x_{i,1}) & for\ y_i = 1 \\ F_{X_i}(x_{i,j}) - F_{X_i}(x_{i,j-1}) & for\ 1 < y_i < n_i \\ 1 - F_{X_i}(x_{i,n_i-1}) & for\ y_i = n_i \end{cases} \quad (2)$$

where $F_{X_i}(x_i)$ denotes the cumulative distribution function (CDF) of random variable X_i. The probability of failure corresponding to the discrete BN in Figure 3 can now be calculated as:

$$Pr(F) = \sum_{y_1=1}^{n_1} \dots \sum_{y_n=1}^{n_n} p_{Y_1}(y_1) \cdot \dots \cdot p_{Y_n}(y_n) \cdot \Pr(F|Y_1 = y_1 \cap \dots \cap Y_n = y_n) \quad (3)$$

This expression holds with n basic random variables that are unconditionally independent, i.e. when there are no direct links between the $Y_i s$ and no common parents to the Y_i (as in Figure 3).

Once measurements from the nodes $\mathbf{M} = [M_1; \dots; M_n]$ are available, the conditional failure probability can be calculated as:

$$Pr(F|\boldsymbol{M}) = \frac{1}{p_{\mathbf{M}}(\mathbf{m})} \sum_{y_1=1}^{n_1} \dots \sum_{y_n=1}^{n_n} p_{Y_1}(\mathrm{y_1}) \cdot p_{M_1|Y_1}(m_1|\mathrm{y_1}) \cdot \dots \cdot p_{Y_n}(\mathrm{y_n}) \cdot p_{M_n|Y_n}(m_n|\mathrm{y_n}) \cdot \Pr(F|Y_1 = \mathrm{y_1} \cap \dots \cap Y_n = \mathrm{y_n}) \quad (4)$$

where $\Pr(F|Y_1 = y_1, \dots, Y_n = y_n)$ is the conditional failure probability of component failure given $Y_1 = y_1$, ..., and $Y_n = y_n$. If only some of the measurements are available, the equation is modified accordingly.

While the computation of the unconditional failure probability following Eq. 3 is exact, the computation of the conditional failure probability through Eq. 4 is only an approximation. The reason for this dependence? The dependence between the measurement variable M_i and the 'Component performance' variable is not fully captured in the discrete BN in Figure 3. This has already been discussed in (Straub and Der Kiureghian, 2010b). In Figure 4 this is illustrated for a reliability problem with only one basic random variable X_i. Both the continuous distribution (Figure 4a) and the corresponding

discretized distribution (Figure 4b) are updated correctly after observing M_1. However, for Eq. 4 to be correct, also the conditional failure probabilities $\Pr(F|Y_1 = y_1)$ need to be updated. This can be observed in Figure 4a): in interval $Y_1 = 3$, which is the one cut by the limit state surface, the ratio of the probability mass in the failure domain to that in the safe domain changes from the prior to the posterior case. The fact that $\Pr(F|Y_1 = 3) \neq \Pr(F|Y_1 = 3, M_1 = m_1)$ shows that the independence assumption underlying Eq. 4, namely $\Pr(F|Y_i = y_i) \neq \Pr(F|Y_i = 3, M_i = m_i)$ is only an approximation. This error occurs only in the intervals that are cut by the limit state surface. In the simple one-dimensional case of Figure 4 an optimal discretization approach would be to discretize the whole outcome space in two intervals, one capturing the survival and one the failure domain. This discretization would have zero approximation error. However, already in a two-dimensional case, such a solution is not possible. This is illustrated in Figure 5, where the cells cut by the limit state surface are indicated in grey. The failure probability conditional on measurements calculated according to Equation 4 will necessarily be an approximation. The approximation error will be small, if the contribution of the cells cut by the limit state surface (the grey cells in Figure 5) to the total failure probability is small. An efficient discretization will thus limit this contribution with as few intervals as possible.

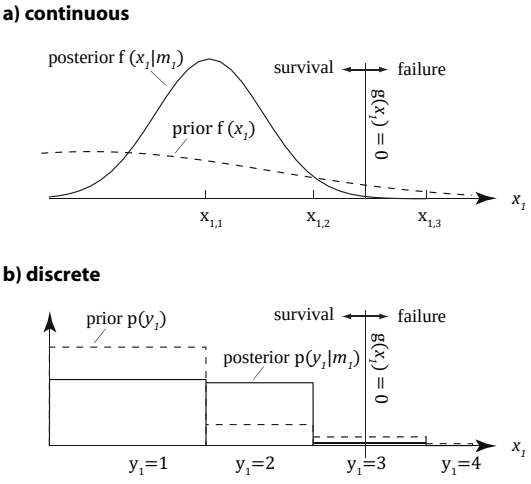

Figure 4. Discretization error in 1D.

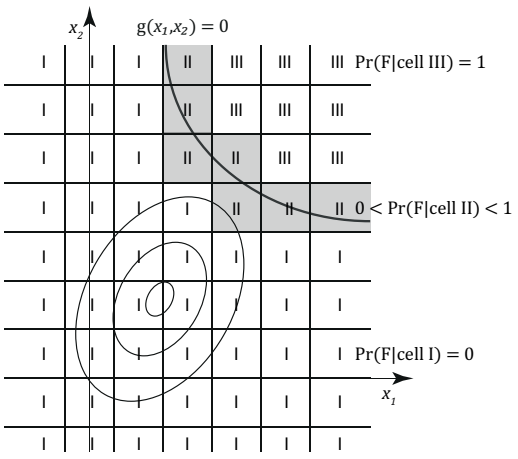

Figure 5. Discretization error in 2D.

3 OPTIMAL DISCRETIZATION OF LINEAR PROBLEMS IN STANDARD NORMAL SPACE

As shown in the previous section, discretization induces an approximation error in the estimate of the posterior failure probability. In this section, we find an optimal discretization that minimizes the expected posterior discretization error for a special case. We consider a reliability problem, transformed to standard normal space, and solve the optimization problem for the FORM approximation of the LSF. The most important concepts of FORM are therefore shortly revisited.

3.1 *First order reliability method (FORM)*

To get an approximation of the probability of failure through FORM, the LSF $g(\mathbf{X})$ is transformed to U-space, where all random variables $U_\mathrm{i} \in \mathbf{U}$ are uncorrelated and have

the standard normal distribution. A suitable transformation for this purpose is the Rosenblatt transformation (Hohenbichler and Rackwitz, 1981). When all basic random variables are independent, this transformation reduces to the marginal transformations: $U_i = \Phi^{-1}\left[F_{X_i}(X_i)\right]$, with Φ^{-1} being the inverse standard normal CDF.
The approximate FORM solution of the probability of failure is obtained by substituting the LSF in U-space $G(\mathbf{U})$ by a linear function $G_L(\mathbf{U})$. This linearization is done at the design point $\mathbf{u}^*$**,** also known as the most likely failure point, which is the point that minimizes $\|\mathbf{u}^*\|$ subject to $G_L(\mathbf{U}) \leq 0$. Since all marginal distributions of the standard multi-normal distribution are standard normal, it can be shown that the FORM probability of failure $\Pr(G_L(\mathbf{U}) \leq 0)$ is:

$$\Pr(G_L(\mathbf{U}) \leq 0) = \Phi(-\beta_{FORM}) \tag{5}$$

where Φ is the standard normal CDF and β_{FORM} is the distance from the origin to the design point, i.e. $\beta_{FORM} = \|\mathbf{u}^*\|$. The problem thus reduces to finding the design point $\mathbf{u}^*$. In the case where $g(\mathbf{U})$ is linear, the FORM solution of the probability of failure is exact, otherwise it is only an approximation, which however is typically good in most practical applications with a limited number of random variables (Rackwitz, 2001).

The linearized LSF $G_L(\mathbf{U})$ can be written as:

$$G_L(\mathbf{U}) = \beta_{FORM} - \alpha^{\mathrm{T}}\mathrm{U} \tag{6}$$

where $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_n]$ is the vector of FORM importance measures. These importance measures are defined as:

$$\alpha_i = \frac{u_i^*}{\beta_{FORM}} \tag{7}$$

where u_i^* is the i-th component of the design point coordinates. The $\alpha_i s$ take values between 0 and 1, and it is $\|\mathbf{u}^*\|$, α_i is 0, if the uncertainty on U_i has no influence on $\Pr(G_L(\mathbf{U}) \leq 0)$, and it is 1, if U_i is the only random variable affecting $\Pr(g_L(\mathbf{U}) \leq 0)$.

3.2 *Formulation of an optimization problem*

Evaluating $G_L(\mathbf{U})$ is computationally inexpensive once the design point $\mathbf{u}^*$ is available. Therefore, it is feasible to find the optimal discretization for $G_L(\mathbf{U})$ through optimization. Furthermore, in cases, where $G(\mathbf{U})$ is not strongly non-linear, the optimal discretization found for $G_L(\mathbf{U})$ will be an efficient discretization for $G(\mathbf{U})$ and, once transformed back to the original space also for g(X).
Since the approximation error of the discretization is associated with the measurements of the Xi (or more generally with the information used to update the distribution of X), we introduce equivalent measurements on the random variables U. We assume that independent measurements errors ε_i are associated with each measurement m_i on U_i as:

$$M_i = U_i + \varepsilon_i \tag{8}$$

ε_i is modeled as a normal distribution with zero mean and standard deviation σ_ε.
For the FORM reliability problem, we define the optimal discretization as the one, which minimizes the expected posterior error $\mathrm{E}_M\left[err_{post}(\mathbf{d}, \mathbf{M})\right]$. Here, $\mathbf{d}$ are the discretization parameters and E_M denotes the expectation with respect to the measurements M. err_{post} is a measure for the posterior error, which is here defined as a weighted posterior error:

$$err_{post}(\mathbf{d},\mathbf{M}) = ER'_{rel}(\mathbf{d},\mathbf{M}) = \left| \frac{\log_{10}\left(\widehat{\mathrm{P}}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{d},\mathbf{M})\right) - \log_{10}\left(\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{M})\right)}{\log_{10}\left(\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{M})\right)} \right| \tag{9}$$

$\widehat{\mathrm{P}}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{d},\mathbf{M})$ is the conditional probability of failure calculated with the discretization and $\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{M})$ is the exact conditional probability. Due to the linearity of the problem and all random variables having posterior normal distribution, $\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}(\mathbf{M})$ can be calculated exactly. The error measure of Eq. 9 implies that under/overestimating a conditional probability of failure of 10^{-y} by an order of magnitude is considered a times worse than under/overestimating a conditional probability of failure of $10^{-a\cdot y}$.
The optimal discretization is defined as:

$$\mathbf{d}^{opt} = \arg\min_{\mathbf{d}} \mathrm{E}_M\left[err_{post}(\mathbf{d},\mathbf{M})\right] = \arg\min_{\mathbf{d}} \int_{\mathbf{M}} err_{post}(\mathbf{d},\mathbf{M}) f(\mathbf{m})\, d\mathbf{m} \tag{10}$$

This optimization requires the computation of an expected value with respect to the possible measurements before having taken any measurements. This is analogous to a preposterior analysis (Raiffa and Schlaifer, 1961, Straub, 2014). Unlike in traditional preposterior analysis, however, the objective is not to identify an optimal action under future available information, but to find the optimal discretization parameters $\mathbf{d}^{opt}$. The integral in Equation 10 is evaluated through a simple Monte Carlo approach.
The parameters in $\mathbf{d}$ are: n_i: the number of intervals used to discretize each random variable U_i, w_i: the width of the discretization frame in the dimension of U_i and v_i: the position of the midpoint of the discretization frame relative to the design point. These variables are illustrated in Figure 6. For a problem with n basic random variables, the full set of optimization parameters is $\mathbf{d} = [w_1, \dots, w_n, n_1, \dots, n_{n-1}, v_1, \dots, v_n]$. As the computational efficiency of the final BN is a direct function of the size of the largest CPT, i.e. the size of the CPT associated to the node component performance, we constrain its size. To this end, we define c_{up} as the maximum number of free parameters of the CPT of the node component state. This puts a constraint on the number of intervals n_i per random variable:

$$c_{up} \leq \prod_{i=1}^{n} n_i \tag{11}$$

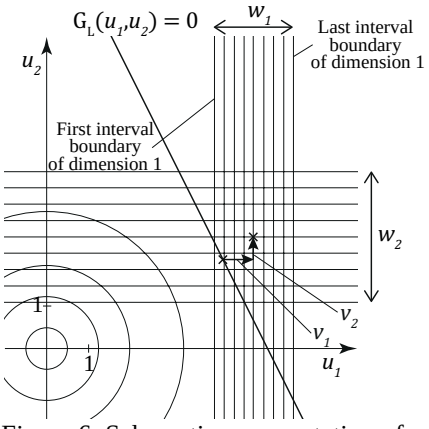

Figure 6. Schematic representation of a discretization of a linear 2D reliability problem. w_i is the distance between the first and the last interval boundary in dimension i. v_i is the position in dimension i of the midpoint of the discretization frame relative to the design point.

The optimization is performed using a two-level approach. The optimization of the continuous parameters width w_i and position of the discretization frame v_i is carried out using unconstrained nonlinear optimization conditional on the n_is. The optimization of the discrete n_is is performed through a local search.

4 NUMERICAL INVESTIGATION

We optimize the discretization for the FORM reliability problem as described by Equation 6 for two and three dimensions. We fix the reliability index at $\beta_{FORM} = 4.26$, corresponding to a probability of failure of 10^{-5}. The standard deviation of the additive measurement error is set to either $\sigma_\varepsilon = 0.5$ or $\sigma_\varepsilon = 1.0$. Different combinations of α_i values are selected, to investigate the effect of the parameter sensitivity on the optimal discretization of the $U_i s$. In all investigated cases, we find that the position of the midpoint of the optimal discretization frame coincides with the design point, i.e. $v_i^{opt} = 0$. Furthermore, the optimal number of intervals n_i^{opt} is essentially the same for all random variables in all investigated cases. We therefore conclude that these two optimization parameters may be fixed at $= 0$ and $n_i = c_{up}^{1/n}$.

The optimal discretization widths w_i^{opt}, however, vary significantly with the importance measures α_i. At first sight, the dependence of w_i^{opt} on α_i is not obvious, but a clear trend can be observed by plotting the probability mass enclosed by w_i^{opt} against α_i, as shown in Figure 7. The width w_i describes the domain in which a fine discretization mesh is applied, see Figure 6. The results of Figure 7 indicate that the probability mass contained within this interval should be a direct function of the random variable's importance, as expressed through α_i. The more important the variable, the finer the discretization should become. The observed relationship between this probability mass and α_i is almost deterministic, and a function can be fitted, see Figure 7. Neither the dimensionality of the problems nor the standard deviation of the measurement error appear to have any influence on this relation. However, in additional investigations it was found that the relation does depend on the prior failure probability of the problem (i.e. on β_{FORM}) and on the number of intervals n_i used to discretize the domain. The discussion of these effects is outside the scope of this paper.

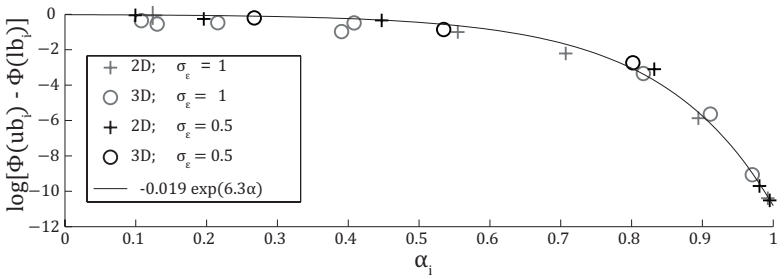

Figure 7. Logarithm of the probability mass enclosed by the discretization frame plotted against α_i. Φ denotes the standard normal CDF and ub_i respectively lb_i the last (upper) and the first (lower) interval bound in dimension i.

5 APPLICATION

We apply the results presented in Section 4 to define an efficient discretization for a general limit state with non-normal random variables. The approximation errors made by this discretization are investigated for different measurement outcomes.
Failure is defined through the LSF $g(\mathbf{x})$:

$$g(\mathbf{x}) = a - \prod_{i=1}^{n} X_{\mathrm{i}} \tag{12}$$

The basic random variables X_1 to X_n are mutually uncorrelated and distributed as $X_1 \sim LN(0,0.5)$ and $X_2, \dots, X_n \sim LN(1,0.3)$. The values of the parameter a are chosen, such that the prior failure probability is in the range of $1E-5$ to ensure that the results from section 4 are applicable. In the case of $n = 2$ dimensions it is $a = 30$; for $n = 3$ dimensions $a = 100$; and for $n = 4$ dimensions $a = 400$. Measurements $M_{\mathrm{i}} = \mathrm{m}_{\mathrm{i}}$ are available for all basic random variables; they are associated with multiplicative measurement errors $\varepsilon\_\mathrm{i} \sim \mathrm{LN}(0,0.71)$. For this special case, an exact $P_{\mathrm{F|M}}$ is available since all posterior random variables are also lognormal and they all enter the LSF in multiplicative format.

Table 1: Evaluation of the discretization error for different measurements **m**. n is the number of random variables, a is the constant of the LSF, Eq. 12; $\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}$ denotes the analytical conditional failure probability and $\widehat{\mathrm{P}}_{\mathrm{F}|\boldsymbol{M}}$ the conditional failure probability calculated with the discrete BN.

n	a	**m**	$\mathrm{P}_{\mathrm{F}\|\boldsymbol{M}}$	$\widehat{\mathrm{P}}_{\mathrm{F}\|\boldsymbol{M}}$	Absolute error	Relative error [%]
2	30	[6.9,5.1]	$3.8E-4$	$3.8E-4$	$3E-6$	1
2	30	[2.4,2.9]	$1.1E-5$	$1.1E-5$	$7E-7$	6
2	30	[1.3,5.5]	$4.2E-6$	$4.4E-6$	$3E-7$	6
2	30	[0.5,2.4]	$5.4E-8$	$6.7E-8$	$1E-8$	25
2	30	[0.2,0.5]	$4.5E-11$	$9.8E-11$	$5E-11$	117
3	100	[2.2,6.0,9.1]	$1.5E-4$	$1.5E-4$	$1E-7$	0
3	100	[1.7,3.0,2.9]	$9.8E-6$	$1.1E-5$	$1E-6$	10
3	100	[0.4,2.2,0.8]	$9.4E-9$	$1.3E-8$	$4E-9$	41
4	400	[2.0,1.4,5.8,7.8]	$2.0E-5$	$2.1E-5$	$1E-6$	7
4	400	[0.7,1.1,7.3,2.6]	$4.2E-7$	$5.2E-7$	$1E-7$	23
4	400	[0.3,0.7,2.7,1.9]	$2.0E-9$	$3.2E-9$	$1E-9$	57

For the discretization of the outcome space, $n_i = 10$ intervals are used per dimension, the discretization frame was centered at the design point and the discretization widths w_i were determined from the exponential function shown in Figure 7. In Table 1 the estimated conditional probability of failure $\widehat{\mathrm{P}}_{\mathrm{F}|\boldsymbol{M}}$ together with the exact failure probabilities $\mathrm{P}_{\mathrm{F}|\boldsymbol{M}}$ are shown for different measurement cases **m**.

6 SUMMARY AND CONCLUSIONS

Solving and updating structural reliability problems in a Bayesian Network (BN) framework is of interest especially in near-real time applications, where failure probabilities have to be updated frequently after obtaining new information, often by non-experts. In this paper, we proposed an approach to discretize continuous reliability problems, such that they can be solved and updated in the BN framework. By solving an op-

timization problem for linear problems in standard normal space, corresponding to a FORM analysis, we derived a heuristics for discretizing the outcome space of a reliability problem. As shown in a simple application example, the heuristic provides an efficient optimization also for non-normal, non-linear reliability problems.

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